Accounting 9 (2023) 191–202

Contents lists available at GrowingScience

Accounting

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A fair-biased allocation of investment between economic sectors using social accounting matrix multiplier analysis

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C H R O N I C L E

Article history:
Received September 18, 2022
Received in revised format October 17 2022
Accepted February 14 2023
Available online February, 15 2023

Keywords:
Data envelopment analysis
Investment allocation
Social Accounting Matrix
Multiplier analysis
Fair allocation methods

A B S T R A C T

Economists have exploited the social accounting matrix in linear economic models to analyze the effect of some variables such as government spending, investment, and export on other economic indicators including total output, employment, household income, and economic growth. In this paper, the influence of investment injection on some economic indicators is analyzed. In addition, to gather different indicators in a general view, these economic indicators are applied as inputs and outputs in a data envelopment analysis model. Overall, to get to the best possible economic conditions, a fair revenue allocation method is used based on a data envelopment analysis model to determine each economic sector's quota of investment. Next, a kind of fair-biased allocation method improving the economic conditions in comparison to the prior model is proposed. Finally, the whole process for Iran's social accounting matrix and subsequent results are presented.

1. Introduction

Resource allocation methods have been used in different areas of investigation. Since economic development plays a vital role for all countries, one of the most significant issues is how to allocate an amount of investment between different economic sectors to obtain the highest possible economic growth, employment rate, fair distribution of income, and so on. So far, policy makers have mainly determined the most important economic sectors by either selecting one of the existing criteria or putting together all indices and deciding the best units. In other words, they have hardly benefited from new approaches like data envelopment analysis in operations research to determine the most important sectors and allocate investment among them. Understanding a comprehensive method to diagnose the priority in economic sectors to allocate investment can be very helpful in development planning. Development planning has a key role in the process of reaching the objectives such as added value, income, and employment rate, which define a better society especially in developing countries. In the second stage of development planning (Zuvekas, 2015), social accounting matrix (SAM) can present not only the inter-relationships between economic sectors (like input-output models only considering the relationship among production activities) but also the inter-relationships between economic sectors and other accounts. Therefore, SAM-multipliers reveal a more realistic picture of economic flows than input-output models. By defining a saving-investment account as an exogenous variable, the effects of other accounts on each other can be observed. In other words, if one unit of extra investment is injected into an economic sector, its effect on total output, employment, added value, government income, and urban-rural ratio income is observed simultaneously. There are different methods in resource allocation by the help of operations research (Argyris et al., 2022; Diakonikolas et al., 2021; Karthiban & Raj, 2020; C. Li et al., 2022; Yu et al., 2019). One of them are Data Envelopment Analysis (DEA). Data envelopment analysis has been developed as a tool to evaluate and compare different groups, sectors or units with multiple criteria divided into inputs and outputs, which are desired to be in minimum and maximum levels respectively. Having been known by Charnes et al. (Charnes et al., 1978),

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doi: 10.5267/j.ac.2023.2.001

data envelopment analysis has been applied in various branches of science (Abdin et al., 2022; Amini et al., 2019; Wichapa et al., 2020). In addition, there are new methods based on data envelopment analysis, which result in prior achievements such as modern approaches in investment allocation or resource allocation (Jiang et al., 2021; Lozano & Villa, 2004; Ryan et al., 2021). However, DEA is applied in case of having more than one comparable criterion. Also, one of the most significant ways to decide how to allocate an amount of funds among different units is game theory. Moreover, some works have been done to combine data envelopment analysis and game theory like DEA game (Borrero et al., 2016; Chen et al., 2006; Goncalves Machado et al., 2016; Liang et al., 2008; Nakabayashi & Tone, 2006; Namazi & Khodabakhshi, 2022; Wen et al., 2022; Wu et al., 2009; Yaya et al., 2020). Thus, DEA can be one of the practical tools to be applied in this kind of problem.

To sum up, in this paper, at first, we explain SAM-based economic models and obtain some of SAM-multipliers which are substantial in policymaking. Required imports and urban-rural ratio income multipliers, which are desired to be at minimum level, are inputs of DEA model; total output, employment, and added value multipliers as well as government income, which are desired to be at maximum level, are outputs of DEA model. Then, exploiting data envelopment analysis and subsequent resource allocation methods, based on the mentioned measures obtained from SAM, we rank economic sectors and define each sector's quota of investment. Moreover, to allocate investment, we propose a new method, which has a more positive effect on economic conditions than the prior method using DEA.

SAM methodology and SAM-multiplier analysis and their characteristics are discussed in section 2; and, basic data envelopment analysis model and a relative model to rank economic sectors based on SAM-multipliers as evaluative measures are discussed in section 3. Some fair resource allocation methods and their usage to allocate investment between economic sectors and our proposed allocation method are represented in section 4; empirical results for Iran's economic data are analyzed in section 5; and finally conclusion is made in section 6.

2. SAM-multipliers

Social accounting matrix (SAM) is a square matrix with identical rows and columns, including production activities, factors of production (labor and capital), other institutions (rural and urban households), and other accounts (saving-investment and rest of the world). Each row defines receipt (income) of corresponding account, whereas each column defines expenditure of that account. The receipt and expenditure of each account are equal since there is no surplus in accounts and there is an equilibrium in the economy. SAM reveals a static picture of all economic flows (Pyatt, 1988) including transactions between production activities, payments of production activities to factors of production, household income from factors of production, government income (direct tax) and saving accounts from household income, investment in production activities, and final demand from production activities. Since SAM, besides production activities transactions, contains distribution of income between various groups of households (like urban and rural) and income resources of each account, it would show direct and indirect changes in economic conditions when there is a shock in an exogenous variable. For example, SAM inverse studies investigate the influence of economic growth on inequality of income distribution (Pieters, 2010) or the influence of tourism on the economic condition of a country (Akkemik, 2012).

Before looking at SAM achievements, there are some fundamental questions to consider, i) What the SAM-multiplier model is, ii) Which kind of questions it can answer and what sort of problems it can solve, iii) How to choose endogenous and exogenous variables, and iv) What is the drawback of SAM-multiplier analysis. First, these questions are answered.

When one or more accounts (in social accounting matrix) are defined as exogenous accounts, a linear system of equations will be obtained which is SAM-multiplier model. The help of this linear system reveals the influence of a unit change in any exogenous account on other endogenous accounts. Government account, saving-investment account, and rest of world account are usually chosen as exogenous accounts. As a result, the transactions between other endogenous accounts are considered when one unit is injected in one of these three exogenous accounts. Despite comprehensive picture of economic transactions, there are some shortages in these models. SAM is applied in two ways: SAM-multiplier models and computable general equilibrium (CGE) models. In SAM-multiplier models, the relationships between all accounts are assumed linear. To be more exact, a shock in an exogenous variable does not change the behavior of any account and the influence would be linear. On the contrary, in CGE models, which are closer to the reality, the behavior of production activities and institutions could change if there would be a shock in an exogenous account (Defourny & Thorbecke, 1984; Pyatt & Jeffery, 1979).

Table 1 presents a schematic framework of our social accounting matrix. A saving-investment account is defined as the only exogenous variable. It is assumed that there is no limitation on trade-off with the rest of the world account since it is defined as an endogenous variable. By the help of dividing each element of SAM, elements of T_{ij} matrix block, to the summation of that column X'_j, the SAM multiplier model is obtained. That is,

$$S_{ij} = T_{ij}/X'_j \;\; or \;\; S_{ij}X'_j = T_{ij} \qquad (1)$$

$$X_i = X_j'$$
$$X_i = S_{ij}X_j' + T_{i6}$$
$$X = SX + T_{i6}$$
$$X = (I - S)^{-1}T_{i6}$$

Note that X_i and X_j', for each i and j are not of the same number of elements. In other words, X_1' could be a 1×100 vector (X_1 a 100×1 vector), while X_2' could be a 1×3 vector (X_2 a 3×1 vector).
The last equation shows SAM-multiplier model and $(I - S)^{-1}$ is SAM inverse, which represents beneficial information. Influence of one extra investment on each economic sector, here T_{16}, is revealed in SAM inverse.

Table 1
Schematic framework of social accounting matrix

	Production activities	Factors of production	Households (rural-urban)	Enterprises	Government	Capital account (investment)	Rest of world	Total receipts
Production activities	T_{11}		T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	X_1
Factors of production	T_{21}							X_2
Households (rural-urban)		T_{32}	T_{33}					X_3
Enterprises			T_{43}					X_4
Government	T_{51}		T_{53}					X_5
Capital account (saving)			T_{63}		T_{65}		T_{67}	X_6
Rest of world	T_{71}							X_7
Total expenditures	X_1'	X_2'	X_3'	X_4'	X_5'	X_6'	X_7'	

For instance, summation of factors of production elements in the first column shows the increase of added value due to a unit of investment injection in the first sector. In addition to added value, total output, increased employment, government income, required imports, and distribution of urban-rural income can be achieved. Moreover, it is favorable to know which sectors lead to a greater number of other sectors stimulation caused by the same amount of investment, which can be measured by standard deviation. Then, a multiple criteria approach is required to determine best sectors pertaining to all information from SAM inverse.

2.1 An example of social accounting matrix multiplier analysis

An example of a social accounting matrix from Hosoe et al. (2010) is illustrated in this section. It consists of two production activities, bread and milk, two factors of production, capital and labor, indirect tax, tariff, house of hold, government, saving-investment account and rest of world account. Saving-investment account is defined as an exogenous account and is omitted from the calculation of S and $(I - S)^{-1}$. In other words, the influence of investment in each sector on the income of house of hold or income of government and the total output in the economy is evaluated, while the interaction among all accounts except saving-investment account is regarded.

Table 2
An example of social accounting matrix

	BRD	MLK	CAP	LAB	IDT	TRF	HOH	GOV	INV	EXT	TOT
BRD	21	8					20	19	16	8	92
MLK	17	9					30	14	15	4	89
CAP	20	30									50
LAB	15	25									40
IDT	5	4									9
TRF	1	2									3
HOH			50	40							90
GOV					9	3	23				35
INV							17	2		12	31
EXT	13	11									24
TOT	92	89	50	40	9	3	90	35	31	24	463

The S matrix of Eq. (1) is shown. It is a 9×9 matrix, obtained by dividing each element of the social accounting matrix to the total amount of related column, excluding the saving-investment account.

$$S=\begin{bmatrix} 0.2283 & 0.0899 & 0 & 0 & 0 & 0 & 0.2222 & 0.5429 & 0.3333 \\ 0.1848 & 0.1011 & 0 & 0 & 0 & 0 & 0.3333 & 0.4000 & 0.1667 \\ 0.2174 & 0.3371 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1630 & 0.2809 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0543 & 0.0449 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0109 & 0.0225 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 1.0000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 1.0000 & 0.2556 & 0 & 0 \\ 0.1413 & 0.1236 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Applying the equations in (1), the matrix $(I-S)^{-1}$, which is of beneficial information, is achieved. The two first columns of this matrix would present the effect of investment on the production activities of bread and milk. That is, one unit of investment in bread will cause a 5.9549 (3.5292+2.4257) higher level of production in the economy, or one unit of investment in milk will cause a 5.7147 (2.3688+3.3459) higher level of production in the economy. Moreover, one unit of investment in bread leads to 2.8417 (1.5849+1.2568) higher level of income for house of hold, and one unit of investment in milk leads to 2.9689 (1.6428+1.3261) higher level of income for house of hold. One unit of investment in bread increases 1.1199 and one unit of investment in milk increases 1.1388 of government income. Finally, the last row of this matrix gives information about required import in the two production activity sectors due to investment in each sector. The amount of 0.7985 and 0.7483 of imported goods are required owing to investment in the two production activity sectors.

$$(I-S)^{-1}=\begin{bmatrix} 3.5292 & 2.3688 & 2.3056 & 2.3056 & 2.8634 & 2.8634 & 2.3056 & 2.8634 & 1.5712 \\ 2.4257 & 3.3459 & 2.3329 & 2.3329 & 2.6552 & 2.6552 & 2.3329 & 2.6552 & 1.3662 \\ 1.5849 & 1.6428 & 2.2876 & 1.2876 & 1.5175 & 1.5175 & 1.2876 & 1.5175 & 0.8021 \\ 1.2568 & 1.3261 & 1.0312 & 2.0312 & 1.2127 & 1.2127 & 1.0312 & 1.2127 & 0.6399 \\ 0.3008 & 0.2791 & 0.2302 & 0.2302 & 1.2750 & 0.2750 & 0.2302 & 0.2750 & 0.1468 \\ 0.0929 & 0.1009 & 0.0775 & 0.0775 & 0.0908 & 1.0908 & 0.0775 & 0.0908 & 0.0478 \\ 2.8417 & 2.9689 & 3.3188 & 3.3188 & 2.7302 & 2.7302 & 3.3188 & 2.7302 & 1.4420 \\ 1.1199 & 1.1388 & 1.1558 & 1.1558 & 2.0635 & 2.0635 & 1.1558 & 2.0635 & 0.5631 \\ 0.7985 & 0.7483 & 0.6141 & 0.6141 & 0.7328 & 0.7328 & 0.6141 & 0.7328 & 1.3909 \end{bmatrix}$$

Even in this simple example of a social accounting matrix with only two economic sectors, investment in the bread sector would cause a higher level of production in the economy in comparison with the milk sector, while investment in the milk sector would cause a higher level of income of house of hold in comparison with investing in bread sector. The question to be answered in this paper is in which proportion the investment has to be divided among the sectors to obtain the best economic conditions.

3. Data Envelopment Analysis

For many years, evaluating various units of the same organization with multi measures was a controversial issue due to disagreement in weighting the measures. With the advent of data envelopment analysis (DEA) models, a considerable amount of controversy has been solved. DEA measures efficiency of decision making units (DMU) while trying to find units which are better than others with any weight under some constraints. In contrast with parametric approaches to evaluate units, DEA defines a nonparametric frontier using the best units. To this aim, measures would be divided into two groups: inputs (the less the better) and outputs (the more the better). There is an extended range of DEA models to apply in various situations (Cook & Seiford, 2009). Consider a set of n DMUs (j), each of which has m inputs x_{ij} ($i=1,\dots,m$) and s outputs y_{rj} ($r=1,\dots,s$). To measure the efficiency of each unit, DEA uses traditional ratio $\sum u_r y_{rj}/\sum v_i x_{ij}$, while u_r and v_i are variable weights. DEA models are to find variables u_r and v_i while trying to obtain the maximum amount of efficiency $\sum u_r y_{rj}/\sum v_i x_{ij}$. Thus, CCR DEA model is represented as:

$$\begin{aligned} & z=\max \sum_r u_r y_{ro} \Big/ \sum_i v_i x_{io} \\ & s.t.\ \ \sum_r u_r y_{rj}-\sum_i v_i x_{ij} \le 0 \quad for\ all\ j \\ & u_r \ge 0 \ \ r=1,\dots,s\,,\ \ v_i \ge 0 \quad i=1,\dots,m \end{aligned} \tag{2}$$

This CCR model measures efficiency of units based on their distance to the created nonparametric frontier. Moreover, the CCR model separates efficient units from inefficient units with the efficiency score of one. Therefore, it is impossible to distinguish between efficient units all of which are on the defined frontier. To solve this problem, Andersen and Petersen (1993) presented a ranking model to discriminate between efficient units. In Andersen and Petersen (AP) model, the DMU, which is under consideration, is omitted from constraints. If the unit under consideration is an inefficient unit in the CCR model, the frontier will not change. Nevertheless, if that unit is on the frontier, after omitting the related constraint, the frontier will change.

$$\begin{aligned} & z = \max \sum_r u_r y_{ro} \Big/ \sum_i v_i x_{io} \\ & s.t. \quad \sum_r u_r y_{rj} - \sum_i v_i x_{ij} \leq 0 \quad for\ all\ j\ except\ j = o \\ & u_r \geq 0 \quad r = 1, \dots, s\ , \quad v_i \geq 0 \quad i = 1, \dots, m \end{aligned} \tag{3}$$

Unlike CCR model (2) allowing the maximum amount of one, in model (3), efficient units can take an amount more than one owing to new frontier. The AP model makes it possible to rank all DMUs with multi-input and multi-output. We shall apply this model to rank economic sectors with measures mentioned in section 2. AP model is among ranking models (Adler et al., 2002; Alirezaee, 1999; S. Li et al., 2007). Each economic sector is of importance in some respects. Data envelopment analysis can be very helpful in gathering all respects, which are SAM-multipliers as ranking indicators to put all economic sectors in a general view.

4. Investment allocation

There are different kinds of methods to solve resource allocation problems. Among them, some rely on data envelopment analysis. In some cases, they regard resources like new input in DEA models and try to keep the efficiencies unchangeable (Korhonen & Syrjänen, 2004; Yan et al., 2002). In some other cases, the goal is to improve the efficiency of units while they allocate resources like Beasley (2003) and Lozano et al. (2009). Despite mentioned methods, in this section, we do not consider resources as new inputs in DEA models. The goal is to allocate existing investment between economic sectors to achieve the best possible amount of economic indicators. To be more exact, it is favorable to allocate a specific amount of investment to the sectors causing more employment or economic growth as outputs or more investment to the sectors requiring less imports as input in the DEA model. Four different ways are applied to achieve this goal. The first one is a kind of fair cost or revenue allocation proposed by Khodabakhshi and Aryavash (2014). As the second one, we propose a new method, which improves the prior fair revenue allocation method. The third and fourth ones are simply the maximum and minimum proportional allocation related to each economic sectors' optimistic and pessimistic quota, which are obtained in Khodabakhshi and Aryavash (2014).

The first approach is proposed by Khodabakhshi and Aryavash (2014), which uses optimistic and pessimistic allocations. That is, once it solves problem (4) with the objective of maximization for each unit and once problem (4) with the objective of minimization for each unit. Then, it defines the share of parameter, which determines share of optimistic and pessimistic allocation in total revenue allocation.

$$\begin{aligned} & \min and \max \quad f_o = \frac{\sum_r u_r y_{ro}}{\sum_i v_i x_{io}} \\ & \underbrace{\frac{\sum_r u_r y_{r1}}{\sum_i v_i x_{i1}}}_{f_1} + \underbrace{\frac{\sum_r u_r y_{r2}}{\sum_i v_i x_{i2}}}_{f_2} + \dots + \underbrace{\frac{\sum_r u_r y_{ro}}{\sum_i v_i x_{io}}}_{f_o} + \dots + \underbrace{\frac{\sum_r u_r y_{rn}}{\sum_i v_i x_{in}}}_{f_n} = F \end{aligned} \tag{4}$$

Then, in our proposed method, with the main idea of AP model, the share of o^th unit is omitted from the constraint of model (4). In other words, in line with AP model (3) which tries to highlight the strength points of decision making units, here, in the new model, we try to highlight the strength and weak points of units. Units with strength points in some criteria and less weakness in all inputs and outputs will gain higher levels of allocation in comparison to model (4). The model is as followed:

$$\begin{aligned} & \min and \max \quad f_o = \frac{\sum_r u_r y_{ro}}{\sum_i v_i x_{io}} \\ & \underbrace{\frac{\sum_r u_r y_{r1}}{\sum_i v_i x_{i1}}}_{f_1} + \underbrace{\frac{\sum_r u_r y_{r2}}{\sum_i v_i x_{i2}}}_{f_2} + \dots + \underbrace{\frac{\sum_r u_r y_{ro-1}}{\sum_i v_i x_{io-1}}}_{f_{o-1}} + \underbrace{\frac{\sum_r u_r y_{ro+1}}{\sum_i v_i x_{io+1}}}_{f_{o+1}} + \dots + \underbrace{\frac{\sum_r u_r y_{rn}}{\sum_i v_i x_{in}}}_{f_n} = F \end{aligned} \tag{5}$$

To convert the model to a linear form, the mentioned process in Khodabakhshi and Aryavash (2014) has been done. The linear form of the new model is what follows:

$$\min and \max \quad f_o = \sum_r u_r y_{ro} \tag{6}$$

$$s.t. \quad \sum_i v_i x_{io} = 1$$

$$\sum_r u_r y_{rj} - \sum_i \gamma_{ij} x_{ij} = 0 \quad for\ all\ j\ except\ j = o$$

$$\sum_{j \neq o} \gamma_{ij} = F v_i \quad \forall i = 1, \dots, m$$

$$u_r > 0\ ,\ \ v_i > 0\ \ , \gamma_{ij} > 0 \quad \forall r, i, j$$

While F is the total investment which is to be allocated, the maximum amount of last model will give the optimistic allocation and the minimum amount will give the pessimistic allocation. Then, the λ parameter will be obtained by the following equations system:

$$\begin{cases} f_j = f_j^{min} \lambda + f_j^{max}(1 - \lambda)\ \ , j = 1, \dots, n \\ F = \sum_{j=1}^{n} f_j \end{cases} \tag{7}$$

In this new method, the share of each sector is gained immediately after solving the equations system. The only difference between the prior model (Khodabakhshi & Aryavash, 2014) and this new model is omitting the constraint in the second group of constraints in model (6) and omitting the variable in the third group of constraints in model (6). In this new method, as in the AP model, units with better inputs and outputs are allowed to choose more amount of revenue or investment. To interpret the new proposed model, Fig. 1 depicts the two achieved hypercubes by model (4) and model (5) in a three dimensional example. Smaller box defines the obtained hypercube by the prior model and the bigger box defines the hypercube obtained by the new proposed model. Since the decision making units are able to choose higher numerical objectives in the new model, the pertained box would be bigger than the box obtained by the prior model. Furthermore, the hypercube vertex presenting the minimum amount of objective for all units is closer to the plane defined by the amount of in comparison with the prior model. That is, in the new model, the minimum amounts chosen by units are more important than those of the prior model are. To be more exact, having less weaknesses in all inputs and outputs for a decision making unit is of more significant role in comparison to having some strength points in some criteria. On the contrary, in the prior model, there is not any biased choice to the two hypercube vertices defining the minimum and maximum objectives. Thus, in the new model, having relatively good amounts in all criteria is more important to allocate more investment.

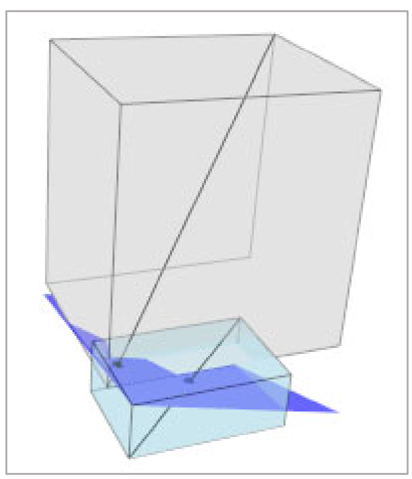

Fig. 1. The two hypercube achieved by the prior model and the new proposed model

Finally, in the third and the fourth approach to allocate investment, the ratio of each sector's optimistic and pessimistic quota obtained by model (4) is allocated to the summation of the quota of all scores. Using this ratio, the quota of each sector of investment is obtained.

5. Empirical results

In this section, we employ the social accounting matrix of Iran to accomplish the whole process of investment allocation. Our social accounting matrix has 71 economic sectors as production activities, 20 household groups with different income levels (10 urban and 10 rural), service compensation, mixed income, and operating surplus as the component of added value, and also enterprises, government, saving-investment and rest of world accounts.

In the first place, to determine the indicators of the DEA model, the saving-investment account of the matrix is chosen as an exogenous account to assess the influence of other endogenous accounts on each other. That is, it is favorable to examine the effect of an injection of investment to each economic sector on other accounts including production activities accounts (economic sectors), rural and urban household accounts, production factors accounts (added value), enterprises, government, and rest of world accounts. Having exploited SAM inverse with saving-investment account as an exogenous account, we define some beneficial information achieved from the inverse matrix as DEA model's measures. We consider the total output (summation of each sector's demand from other sectors), employment (product of employment matrix and part of SAM inverse), added value (summation of factors of production in SAM inverse), and government income (from government account in SAM inverse) as outputs in DEA model and standard deviation of economic sectors' demand from each other, requiring imports, and urban-rural income ratio as inputs in DEA model.

Table 3
SAM multipliers and super efficiency scores for economic sectors

		inputs			outputs				
		Std.	import	Urban/rural	Total output	employment	Added value	Gov. income	scores
1	Private primary education	0.913796	0.310476	3.980753	4.095056	0.029765	2.507084	0.324416	1.675323
2	Manufacture of wearing apparel	1.056374	1.205959	3.914462	4.486151	0.043582	2.190336	0.375275	1.489023
3	Animal husbandry	1.226096	0.424429	3.821623	4.88587	0.01237	2.590572	0.313809	1.098649
4	Manufacture of motor vehicles, trailers and semi-trailers	1.18114	0.737582	3.884301	4.831264	0.008032	2.252267	0.418289	1.050887
5	crude petroleum and natural gas	0.934743	0.181841	3.929735	2.773011	0.004433	2.016644	0.385179	1.04648
6	Electricity	0.951826	0.218989	3.952972	3.310075	0.006074	2.182331	0.365428	1.033634
7	Manufacture of wood and products of wood	1.035442	0.843227	3.873256	4.540723	0.017	2.280019	0.405895	1.031639
8	Nonresidential real estate activities	0.928265	0.333149	3.814207	4.221238	0.007701	2.572716	0.324931	1.028754
9	Activities auxiliary to financial intermediation	0.894346	0.217716	3.929205	3.152547	0.006058	2.167881	0.366688	1.02739
10	Manufacture of fabricated metal products	1.024138	0.573257	3.886532	4.682146	0.011567	2.349273	0.381558	1.027058
11	Buying, selling land activities	0.917107	0.305277	3.828602	3.946515	0.013519	2.58094	0.31996	1.026857
12	Renting real estate activities	0.962216	0.319287	3.801334	4.022883	0.007273	2.618472	0.316961	1.021048
13	Religious and political	0.935639	0.335602	3.943134	4.339565	0.009785	2.430502	0.330447	1.020863
14	Mining	0.892212	0.316803	3.935142	3.569498	0.006951	2.207399	0.366083	1.020138
15	Manufacture of radio, television and communication equipment	1.062703	1.23901	3.888477	4.4425	0.008032	2.147708	0.418599	1.018831
16	Residential real estate activities	0.933626	0.188859	3.932825	2.903735	0.00475	2.00145	0.389057	1.017504
17	Private adult education	0.896115	0.281139	3.918008	3.757994	0.016788	2.332041	0.348783	1.01325
18	Manufacture of food products	1.250599	0.621774	3.855012	4.946984	0.01057	2.409118	0.353322	1.012748
19	Public primary education	0.940191	0.326957	4.042512	4.028847	0.011797	2.691379	0.287689	1.011714
20	Residential construction	0.973222	0.384046	3.916505	4.448628	0.011505	2.363318	0.355011	1.011644
21	Other construction	0.999961	0.378862	3.931684	4.437965	0.010074	2.307313	0.362357	1.010519
22	Manufacture of electrical machinery	1.026035	0.765421	3.890933	4.586629	0.00794	2.25339	0.399726	1.007564
23	Agriculture and gardening	1.138489	0.497137	3.817613	4.390675	0.014546	2.579524	0.312241	1.006128
24	Manufacture of rubber and plastics products	1.037867	0.691682	3.897007	4.584488	0.008289	2.249347	0.399837	1.005985
25	Manufacture of machinery and equipment	1.034609	1.03366	3.889878	4.513534	0.008191	2.194385	0.409137	1.005889
67	Public administration and defense	0.970466	0.388068	4.002563	4.220717	0.011074	2.521017	0.318119	0.956404
68	Manufacture of basic metals	1.172515	0.69241	3.903634	4.526273	0.007382	2.228376	0.385146	0.953743
69	Air transport	1.032209	0.787404	3.934093	4.464666	0.00697	2.226084	0.365779	0.947749
70	Banks	0.979488	0.362841	3.998596	4.017573	0.008306	2.449418	0.330956	0.94138
71	Manufacture of chemicals and chemical products	1.25567	0.657813	3.916593	4.376262	0.006807	2.200395	0.377344	0.936223

Table 3 presents the SAM multipliers (these 4 outputs and 3 inputs) obtained from SAM inverse for each economic sector. For instance, a 1000 unit of investment injection in the manufacture of clothing leads approximately to 43% increased employment, or crude petroleum and natural gas are of the least required imports, 0.1818 monetary units for a unit of injection. Public primary education sector induces 2.69 monetary units as the highest amount of added value for a unit of investment. Manufacture of food products requires the highest amount of inputs from other sectors, 4.94 monetary units for a unit of investment, whereas its need is provided by stimulating a smaller number of other sectors, with standard deviation 1.25. Hence, a question is raised which is how to analyze economic sectors while there are various indicators to study economic relationships.

In the second stage, the obtained measures are applied in the AP model to rank all economic sectors. The last column in Table 3 shows each economic sector's super efficiency score. Their super efficiency scores have sorted economic sectors. Private primary education is of the highest score and manufacture of chemicals and chemical products is of the least. Although some sectors like agriculture and gardening, with one of the least amounts of urban-rural income ratio, have one of the best amounts of indicators, they are not on top of the list, e.g. 23th rank. On the other hand, the electricity sector without having any of the three best indicators is in the sixth rank. We are looking for a way to allocate a fixed amount of investment between economic sectors to achieve the best amount of economic indicators in the whole society.

Finally, the four investment allocation methods, which are prior fair allocation, new fair-biased allocation, maximum proportional allocation, and minimum proportional allocation methods are used to achieve the amount of investment, which is to be allocated to each sector. The results are presented in Table 4 for each method.

It is noticed that if the whole investment is allocated to some of the best sectors obtained by AP model, the most possible amount of economic indicators is achieved. Nevertheless, it must be considered that the demand from each sector is limited and the affluent outputs can remain unused. Hence, diversification in the allocation methods is of a substantial role and it is the rationale behind choosing model (4) and model (5) to be applied in this case.

Table 4
Investment allocation for economic sectors

		Prior fair allocation	Fair-biased allocation	Max proportional	Min proportional
1	Private primary education	3072121	3124607	3738748	1937240
2	Manufacture of wearing apparel	2739804	2777806	3945098	687882.3
3	Animal husbandry	1423967	1423131	1374053	1508940
4	Manufacture of motor vehicles, trailers and semi-trailers	1076105	1066381	1152705	945697.5
5	crude petroleum and natural gas	1742117	1733269	2200534	961695.9
6	Electricity	1584640	1579068	1751043	1301352
7	Manufacture of wood and products of wood	1393501	1382485	1618164	1011028
8	Nonresidential real estate activities	1440073	1441784	1333317	1621817
9	Activities auxiliary to financial intermediation	1608970	1603884	1776566	1323650
10	Manufacture of fabricated metal products	1287730	1287243	1187517	1458336
11	Buying, selling land activities	1817020	1825215	1729755	1965582
12	Renting real estate activities	1439722	1439521	1379760	1541804
13	Religious and political	1535104	1543677	1302218	1931575
14	Mining	1380030	1378656	1328040	1468537
15	Manufacture of radio, television and communication equipment	913531.3	898495.6	1101790	593036
16	Residential real estate activities	1731124	1722877	2142877	1030145
17	Private adult education	2184225	2197422	2287168	2008974
18	Manufacture of food products	1260156	1256829	1220226	1328135
19	Public primary education	1517082	1519384	1425041	1673775
20	Residential construction	1544836	1553559	1311952	1941303
21	Other construction	1501834	1510441	1257205	1918295
22	Manufacture of electrical machinery	1040751	1031027	1111005	921146.9
23	Agriculture and gardening	1499126	1495709	1550060	1412414
24	Manufacture of rubber and plastics products	1099026	1091357	1128746	1048431
25	Manufacture of machinery and equipment	955045.4	942162.8	1092412	721189.4
67	Public administration and defense	1419419	1423716	1245606	1715324
68	Manufacture of basic metals	1021164	1011677	1082451	916828.1
69	Air transport	955879.1	944522.6	1054620	787780.6
70	Banks	1357989	1361134	1192867	1639096
71	Manufacture of chemicals and chemical products	986420.8	976393.4	1056110	867779.3

In Table 4, the amount of allocated investment to each sector in prior and new fair-biased allocation is presented. These amounts are very near but those of maximum proportional allocation and minimum proportional allocation are completely different. It is noticeable that the distance between maximum and minimum amount allocated to sectors in the four methods are logically different. In other words, in prior fair allocation, the range of allocation is between 3072121and 851250.7, while for new fair-biased allocation this range is longer, between 3124607 and 836130.7, since a constraint and a variable are omitted from the linear form of model (4) and the feasible region is different. This range for maximum proportional allocation and minimum proportional allocation is higher and lower respectively in both sides of the interval, with maximum proportional allocation between 3945098 and 988116.4, and minimum proportional allocation between 2025326 and 538466.4. It should be noted that some sectors, with a minimum amount of investment in prior and new fair-biased allocation, are not included in Table 4 due to lack of space.

In Table 4, the sectors allocated more investment in fair-biased methods in comparison to prior fair methods are highlighted among which are the private primary education sector and the manufacture of clothing apparel sector. As these two sectors are highly good in employment, they would be given more investment in the new model. In addition, the private primary education sector has fewer weak points than the manufacturing or clothing sector since this sector is weak in required import indicators. In animal husbandry sector, although good in total output indicators, it has two weaknesses in standard deviation and government income indicators. As a subsequent result, it would be given a slightly less allocation in the new fair-biased model. It is important to note that the amount of is about 0.5 in prior fair allocation and about 0.6 in the new model. That is, the new model is just a little biased to the weaknesses. Fig. 2 and Fig. 3 show the results of investment allocation to economic sectors on the economic indicators. As an example, in Fig. 2, the required import in maximum proportional allocation is much more than three other allocations and this amount in minimum proportional allocation, is much less than the three others. In maximum proportional allocation, 49035087.08 monetary unit imports are required while in minimum proportional allocation, this amount is 43877570. In addition, in prior fair allocation, 47126609 monetary unit imports are required but in our new method less imports in comparison with the prior fair allocation are needed, 47039986 monetary units. Results in urban-rural income ratio indicators are a little different. It means that they are almost the same in the four methods, 3926576, 3926666, 392568105.2, and 392810102 amounts respectively for prior fair allocation, new fair-biased allocation, maximum proportional allocation, and minimum proportional allocation methods.

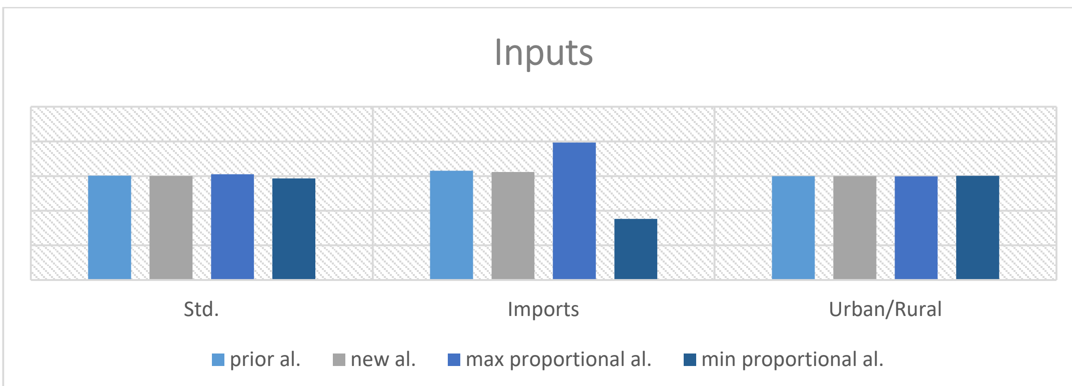

Fig. 2. The normalized amount of input indicators for the whole society

Like input indicators, results can be interpreted for output indicators in Fig. 3. The most impressive output indicator is employment, which leads to 1129952 people employed by our new method, 1127033 people employed by prior fair allocation, 1175940 people employed in maximum proportional allocation, and 1043772 people employed in minimum proportional allocation method. Other output indicators are almost the same for the two fair allocation and fair-biased allocation methods. For instance, although the amount of added value in the new method is slightly more than prior one, extra economic growth, owing to investment injection, is 3.71 percent for both approaches. According to Fig. 2 and Fig. 3, prior fair allocation and our new method are more reliable than the two maximum and minimum proportional allocations, which cause dramatic increase or decrease in some indicators such as imports and employment respectively.

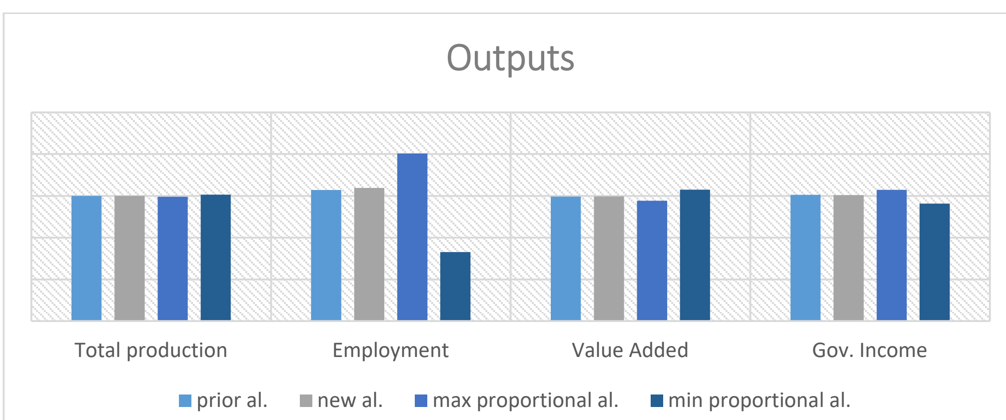

Fig. 3. The normalized amount of output indicators for the whole society

Overall, the new method for investment allocation leads to better results in comparison to prior fair revenue allocation, as it is observable in imports and employment indicators. This happens because of the extended feasible region in the new method to allocate investment.

6. Conclusion

SAM inverse matrix presents some valuable information about economic sectors. For example, it gives which economic sectors would lead to a more employment rate, more economic growth, more total output, less required imports, and less urban-rural income ratio owing to injection of the same amount of investment. We applied a DEA model to evaluate economic sectors using SAM multipliers as inputs and outputs. In addition, we proposed an improved method to determine the amount of investment allocated to each sector. Having exploited the improved method, we achieved about 1130000 people employed and extra 3.7 percent economic growth.

In future works, it is also possible to apply SAM inverse matrix to achieve more economic indicators like resource intensity. Researchers can exploit various DEA models and resource allocation methods using different kinds of economic indicators achieved from SAM to get the best economic and environmental conditions.

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