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An EOQ model with stepwise ordering cost and the finite planning horizon under carbon cap-and-trade regulations

Tzu-Chia Chen[a,b*] and Shu-Yan Yu[a,b]

[a]*BA Program, International College, Krirk University, Bangkok , Thailand*
[b]*Soi Ramindra 1, Khwaeng Anusawari, Khet Bang Khen, Bangkok 10220, Thailand*

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A B S T R A C T

In this study, under the carbon cap-and-trade mechanism, the ordering cost presents a stepwise function for ordering quantity, and the optimal economic ordering quantity model aims to explore the manufacturer's total cost minimization in the finite planning horizon, in combination with the actual situation that the product will produce carbon emissions during transportation and storage. The economic order quantity (EOQ) model with stepwise ordering cost is applicable to the decision environment in which goods are utilized by sea, by rail or by air (e.g., the order cost is charged in addition to the basic fixed cost, the importer of raw materials will pay an additional freight related to delivery, such as the rent for the use of container numbers.). A heuristic algorithm is also proposed to analyze the relevant properties of the optimal solution of the model and to solve the optimal order times and quantities of the manufacturer under the constraint of carbon policy.We further compared the optimal order times with the case without carbon constraint and the order times corresponding to the manufacturer's realization of the minimum carbon emission, and obtained the conditions for the manufacturer to achieve low cost and low emission under the carbon policy.Finally, the theoretical results of the model are verified by numerical examples,and the influence of relevant parameters on the inventory strategy of manufacturers is discussed. The results show that under the carbon cap-and-trade policy, there is an optimal ordering strategy that minimizes the total cost of the manufacturer in the finite horizon. When the demand of the manufacturer is under finite horizon and the carbon policy is equal to the specific multiplier of orders, the manufacturer can achieve a win-win result of low cost and low emissions.

1. Introduction

Climate change is one of the biggest challenges facing countries, governments, businesses and people in the coming decades. In the face of the accompanying threat, not only the international standards for calculating the carbon footprint of products have begun to be established, but also the major international procurement companies have begun to require the degree of warming impact caused by the labeled life cycle of products. The results of enterprise carbon footprint calculation are the best opportunity to assist enterprises in examining process reduction and cost reduction in order to promote supply chain greenhouse gas management. In the international big companies through the industry supply chain system global demand manufacturer questioned, product Carbon Footprint for greenhouse gas (Carbon Footprint of Products, CFP) scrutiny, or under the pressure of public disclosure of Carbon product information, in terms of Taiwan's manufacturing, products' Carbon Footprint and Carbon disclosure has become one of the important topics have to face after ISO 14064-1 (organization level for the quantification and reporting of greenhouse gas emissions and removals) implementation.

* Corresponding author.
E-mail address: tzuchiachen1688@gmail.com (T.C. Chen)

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Product carbon footprint is defined as an activity or the whole product life cycle process of the direct and indirect CO_2 emissions, emissions of carbon footprint still must contain emissions produced during raw material production and manufacturing, the manufacturing and assembly of the product itself, the product is used, until the product discarded or recycled. Therefore, mastering the carbon footprint of products can effectively reduce manufacturing costs and create industrial low-carbon competitive advantages, thus shaping product differentiation, green enterprises and new industrial values. Export-oriented industry in Taiwan, the semiconductor supply chain system has a key position, in the face of international purchasing company with the main sales channels for exposing carbon product information of pressure, companies need to establish product carbon footprint management mechanism, through the integration of the middle and lower reaches of the carbon footprint of supply chain management, import carbon footprint measures validate guidance, strict environmental requirements and conform to the international sourcing companies, and gradually implement the industrial carbon reduction and low carbon products, can effectively meet the customer requirement. However, these measures bring huge cost burden to import manufacturers and weaken their enthusiasm for energy conservation and emission reduction. Enterprise decision-makers are more eager to balance the relationship between economic interests and environmental impacts by changing the operation mode and operation strategy, so as to realize the coordination and unification of the two. Therefore, as an effective market regulation mechanism, the Carbon Emission Trading Scheme (CETS) also provides a platform for both profit and energy saving and emission reduction for import manufacturers. Based on the Cap-and-Trade Principle (C&T), carbon emission rights are traded as commodities in the market, thus encouraging relevant participants to reduce carbon emissions voluntarily. As a consequence, how to effectively optimize the inventory of semiconductor product supply chain industry in Taiwan under the constraint of carbon policy has become a research hotspot in the field of operation management.

In Taiwan semiconductor industry, many manufacturers have to import chemical raw materials from abroad. In addition to the basic fixed cost, the importer of raw materials will pay an additional freight related to delivery, such as freight for the use of container numbers. Therefore, ordering cost has a stepwise function of ordering quantity. In addition, the importer and the customer signed a long-term supply contracts for a finite horizon in which the chemical raw material importer supplies the demand of the customer. At the end of these contracts, the importer still has an inventory and the customer pays only a fee related to the quantities used. For example, Company A is the largest supplier of electronic gases in Taiwan. Its customer base includes international wafer manufacturing companies such as TSMC and UMC. Over the next year, the company contracts with the customer to shake hands in a number of bulk gases, specialty gases, bulk specialty gases and electronic chemicals to each wafer manufacturer. After the end of the contract period, the remaining products will still belong to Company A. Since the products belong to industrial gas, an additional processing fee will even be added to process the remaining products. Therefore, Company A hopes that there will be no remaining products after the end of the limited contract period. Under the constraints of the above ordering scenario and carbon policy, this study will construct its inventory model to determine the optimal order quantities.

2. Literature Review

This study will discuss related literature in three parts. The first part discusses the literature on inventory in finite planning horizon. The second part discusses the literature on inventory of stepwise transportation cost, and finally discusses the enterprise inventory decision under carbon policy.

Under the finite planning horizon, many scholars consider that some parameters in the economic ordering model will change with the increase of time. Some scholars have studied how manufacturers modify the order quantity when the purchase cost changes over time (Lev & Soyster, 1979; Goyal, 1975). Lev and Weiss (1990) proposed an inventory model that can divide the planning horizon into two categories, and discuss how to determine the inventory strategy of manufacturers with finite or infinite planning period. Sivazlian and Stanfel (1975) considered how to correct the order quantity in the later period when the demand would change due to the increase of time. In addition, aiming at other relevant studies on demand parameters, Goyal et al. (1975) considered to put forward an algorithm to find the best inventory decision of the manufacturer under the condition of random demand with allowable shortage for goods. Parker and Kapuscinski (2004) considered that in the supply chain of the two echelons, when the demand is random and the capacity of upstream suppliers is limited, they used dynamic programming to figure out the order quantity of each echelon from upstream suppliers. Dai and Qi (2007) study how to adjust the quantity o orders from each supplier when the manufacturer wants to order from multiple suppliers, and use dynamic planning to obtain the inventory policy of the manufacturer. In this kind of literature, the optimal ordering quantity can be obtained by adjusting the ordering policy of the manufacturer or seeking a new algorithm.

Stepwise transportation costs can analyzed as with fixed costs and the variable costs associated order quantities, that is, when the order quantity exceeds the capacity limit of one container, the additional rental cost of one container is required. This is the variable cost that the manufacturer needs to consider. Nahmias and Cheng (2009); Bramel and Simchi-Levi (1997) believes that it can be seen as a question of getting a discount based on order quantity. Lee (1986) pointed out that in consideration of the stepwise transportation cost, in order to make full use of the container capacity to minimize the total cost, the optimal economic quantities is multiple of the container capacity. In view of this stepwise transportation cost, Alp et al. (2003) take additional factors such as weather and road status into consideration under the stepwise transportation cost, so the time needed for product transportation is random. Under this condition, the planning period is divided into many nodes and dynamic planning is used to find the optimal order quantity. Russell and Krajewski (1991) study the less-than-truckload (LTL) problem and believe that the cost function in LTL problem has the same characteristics. They point out that if the minimum cost obtained by the economic ordering quantities model can only be used as the lower bound of LTL problem. This study also proposes an algorithm to modify the optimal order quantity. Carter et al. (1995) modified the algorithm proposed by Russell and Krajewski (1991) to find a more effective solution. Rieksts and Ventura (2008) discussed how to determine the optimal order quantity under the influence of the stepwise cost function and the condition of finite and infinite planning horizon, respectively. This study also proves that the firm can find optimal ordering time interval under the finite and infinite planning horizon.

At present, some scholars have studied enterprise operation decisions under carbon policy. For example, Hua et al. (2011) extended the classical EOQ model to the case that carbon constraint was considered. They derive inventory strategy of the enterprise under the carbon cap and trade policy and analyze the enterprise to implement low cost and low emission conditions. Chen et al. (2013), based on the EOQ model considering the enterprise quota, a carbon tax on carbon emissions inventory strategy under the policy, they analyze and compare the influence of different carbon policy to the enterprise inventory strategy. Toptal et al. (2014) respectively considered the enterprises in the carbon cap and trade, carbon and carbon tax policy under the joint ordering and carbon emissions decisions. They analyzed three kinds of carbon policy joint inventory and reduction of carbon emissions strategy for the enterprise, the influence of this study on carbon cap and trade policy under the enterprise investment reduction technology can achieve a win-win result of low cost and low emissions. Knour and Schaefer (2014) studied the carbon cap and trade emissions, carbon tax policy under the enterprise joint inventory and transportation strategies, including the mode of transportation of less-than-truck carload transportation and full truck carload transportation. Their research shows that under the constraint of carbon policy, enterprises are more willing to adopt the transportation mode of less-than-truck load to reduce costs and reduce carbon emissions. He et al. (2015) established inventory optimization models under carbon cap-and-trade policies and carbon tax policies. When the purchase price of carbon emission rights is not lower than the sale price, they derive the inventory strategy of enterprises under the carbon cap-and-trade policy and compare it with the carbon tax situation, so as to obtain the corresponding management enlightenment. The characteristics of the above literatures are to analyze the inventory strategy of enterprises under the carbon policy based on the traditional EOQ model. Song and Leng (2012) analyze the impact of carbon policies on business ordering strategies from other perspectives, such as Newsvendor issues.

From the above analysis, we construct an inventory model considering the impact of carbon policy on the inventory strategy of industrial gas enterprises in the finite planning horizon. Due to the special requirement of high residual cost in the ordering and storage process of industrial gas products, ordering such products for multiple times will consume more resources and release more carbon dioxide. Moreover, under the constraint of carbon policy, the decision-making environment and operational objectives of enterprises ordering industrial gas products have changed. Reality based on the above background, this study analyzed the carbon cap and trade policy of a manufacturer, the influence of the finite planning horizon inventory strategy when industrial gas products purchasing plan for finite horizon, and ordering cost of a stepwise function under the condition of transportation cost, on the target of minimizing the total cost of the establishment of a carbon cap and trade policy under the proposed inventory model. First, the uniqueness and existences of the optimal ordering strategy is proved and the optimal ordering times are solved. Then, the conditions for the manufacturer to achieve low cost and low emission are obtained by comparing with the carbon model with no carbon constraint and with the carbon emission minimum as the target. Finally, the theoretical results are verified by an example. The innovation of this paper lies in (1) Considering the impact of carbon policy on inventory strategy under the condition that the order cost is a stepwise function in the finite planning horizon. (2) Compared with the inventory model under the current carbon policy, study the ordering strategies of various industrial gas products under the constraint of carbon emission, and analyze the conditions for such products to achieve low cost and low emission. The remaining chapters of this study are as follows. The third chapter is the model construction and solution. The fourth chapter is case and data analysis. The fifth chapter is the conclusion.

3. The Proposed Model

3.1. Problem Descriptions

Consider a supply chain consisting of a industrial gas supplier and a wafer manufacturer whose customer demands are known. A wafer manufacturer needs to order from overseas suppliers' gas products to meet customer demand in order to meet the needs, large amount of the industrial air product is delivered through a shipping container. In addition to the basic fixed cost, the importer of raw materials will pay an additional freight related to delivery, such as the rent for the use of container numbers, while the ordering cost presents a stepwise function for ordering quantity. Assuming that the capacity of a single container is P and the unit shipping cost is R, the ordering cost is shown in Fig. 1. Manufacturer's total cost includes: order cost, carrying cost and carbon transaction cost. When the carbon emission is lower than the carbon quota, the manufacturer can sell the excess carbon emission right through the carbon trading market. The trading volume is positive and the carbon trading cost is negative (the carbon trading income is positive). On the contrary, insufficient carbon emission rights should be purchased, with negative trading volume and positive carbon trading cost (negative carbon trading income).

Carbon emission is composed of two parts: carbon emission from transportation and carbon emission from storage space. Transport carbon emissions depend on the selected transport vehicle and the amount of cargo carried, assuming that the transport vehicle remains the same. Storage carbon emissions consist of fixed emissions from storage facilities and variable emissions associated with the volume of goods stored. The price of carbon is affected by a country, a region or a global quota, not by the amount of quota allocated to an individual manufacturer. Since the manufacturer contracts with the customer for a time horizon T, it is assumed that the supplier will order again each time the ordered product is run out to ensure that the ending inventory is zero and meet the demand of the finite planning horizon. The remaining assumptions are the same as the traditional EOQ inventory model, as described in the next section.

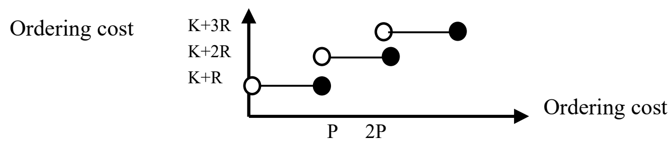

Fig. 1. A stepwise ordering cost

3.2. Problem Assumption

1. Upstream suppliers have unlimited capacity.
2. No shortage of goods is allowed.
3. Ignore transportation time (or assume a fixed constant).
4. The residual value of the inventory is assumed to be zero.
5. If the inventory fails to meet the demand during the current horizon, unmet demand is not backlogged.
6. The supplier has determined the length of planning horizon for signing the contract with the manufacturer.
7. The costs related to container transportation are predetermined.

3.3. Notations

The related notations are given as follows:
Parameters:

T : Length of planning horizon (year).
D : Customer demand rate (demand/year).
h : Manufacturer's inventory carrying cost. ($ / year).
K : The fixed ordering cost ($), the fixed expenditure for each order.
R : The variable ordering cost ($), the cost of using a carrier to transport. (Example: the rent for the use of container numbers)
P : The capacity of transportation vehicle (Example: the container carrying capacity).
A: CO_2 emissions limit within the planning horizon (tons).
C: The market price per tons of CO_2 ($).
e_0: CO_2 emissions of an empty vehicle (tons).
e: CO_2 emissions per unit of transportation (tons, variable transportation carbon emission factor).
g_0: The fixed carbon emissions from storage (tons).
g: Carbon emissions per unit of storage (tons, variable storage carbon emission factor).
B: Carbon rights trading volume limit during the planning period (tons).
$\lceil x \rceil$: One of the Gaussian functions. For any positive real number x, the smallest integer greater than x.

Decision variables:

Q_j : The manufacturer's i order quantity. i =1, 2, 3, ...

Objective functions:

$TC(Q_1,\ldots, Q_i,\ldots,Q_m)$: Q_j's total cost of each order quantity during the planning period.
$CE(Q_1,\ldots, Q_i,\ldots,Q_m)$: Manufacturer's total carbon emissions during the planning period under m orders.

3.4. Problem Formulation and Solving Methods

Under the finite planning period, the total cost of the manufacturer includes the total carrying cost, the total ordering cost and the carbon emission transaction cost during the planning period. The total order cost is the total of each order cost of the manufacturer during the planning period T, but the difference from the transitional EOQ model is that the manufacturer's single order cost needs to add the container transportation cost related to the order quantity in addition to a fixed fee, the transportation cost is determined by the number of containers required for different order quantities. In addition, under a given planning period, the total demand is fixed, so the purchasing cost is fixed and not included in the cost calculation. Considering the carbon emission (rights) trading mechanism (CETS) and the principle of carbon total control and trading (C&T), the total cost model for each order quantity of Q in the planning period is m times as shown below:

$$\min \quad TC(Q_1,\cdots,Q_i,\cdots,Q_m)=\sum_{i=1}^{m}C(Q_i) \tag{1}$$

subject to

$$\sum_{i=1}^{m}Q_i = DT \tag{2}$$

$$\left[e_0 m + e(DT) + g_0 + \sum_{i=1}^{m} g\frac{Q_i}{2}\right] + B = A \tag{3}$$

where

$$\sum_{i=1}^{m}C(Q_i)=\sum_{i=1}^{m}h\cdot\frac{Q_i}{2}\cdot\frac{Q_i}{DT}+\sum_{i=1}^{m}\left(K+R\left\lceil\frac{Q_i}{P}\right\rceil\right)-CB \tag{4}$$

The first item in formula (4) is the inventory carrying cost, because the interval from the i order to $i+1$ ($i=1,\cdots,m$) is Q_i /(DT), and the average inventory is calculated as $Q/2$. The second item is the order cost. In addition to the required fixed cost K, the required number of containers $\lceil Q_i/P\rceil$ needs to be calculated according to the order quantity Q_i of this time to calculate the required cost. The third item is carbon trading costs （$-CB$） . In formula (3), we adopt the approach of Hua et al. (2011), the carbon emission trading volume during the planning period is B from the difference between the carbon allowance A during the planning period and the total carbon emissions per unit time $CE(Q_1,\cdots,Q_i,\cdots,Q_n)$ that includes transportation carbon emissions $(e_0+e\,Q_i)/(Q_i/DT) = e\,DT + e_0\cdot(DT)/Q_i$ and storage carbon emissions $g_0+g\,(Q_i/2)$. In addition, the restriction of formula (2) is to prevent the total order quantity from exceeding the total demand.

We will propose an ordering strategy and prove that this ordering strategy can minimize the total cost of the manufacturer in this study. The basic information related to this ordering strategy is as follows:

1. Except for the last order, the order quantity is the same each time, so that the order quantity for each time before the last time is Q, the planning period T can be divided into $\lceil DT/Q\rceil$ order periods and the quantity ordered for the last time is $DT-(\lceil DT/Q\rceil-1)\cdot Q$.
2. The average inventory level per cycle is $Q/2$, and the last cycle is $\frac{DT-(\lceil DT/Q\rceil-1)\cdot Q}{2}$.
3. The length of each cycle is Q/DT , and the last cycle is $\frac{DT-(\lceil DT/Q\rceil-1)\cdot Q}{DT}$.
4. The manufacturer has to bear the transportation cost of $R\cdot\lceil DT/Q\rceil$ in each cycle, and $R\cdot\left\lceil\frac{DT-(\lceil DT/Q\rceil-1)\cdot Q}{P}\right\rceil$ in the last cycle.

The total cost $TC(Q)$ of the manufacturer in the plan (Eq. (1)) can be expressed as follows:

$$TC(Q) = H(Q)+S_1(Q)+S_2(Q)+TF_1(Q)+TF_2(Q)\ -CB \tag{5}$$

Among them, $H(Q)$ is the total inventory carrying cost during the planning period； $S_1(Q)$ is the total fixed cost of the order, which is related to the total number of orders； $S_2(Q)$ is the total order variable cost, which is related to the number of containers required for the order； $TF_1(Q)$ is the total transportation carbon emission cost, $TF_2(Q)$ is the total storage carbon emission cost, and CB is still the carbon transaction cost. These costs can be expressed as follows:

$$H(Q)=h\cdot\frac{Q}{2}\cdot\frac{Q}{DT}\cdot\left(\left\lceil\frac{DT}{Q}\right\rceil-1\right)+h\cdot\left(\frac{DT-(\lceil DT/Q\rceil-1)\cdot Q}{2}\right)\cdot\left(\frac{DT-(\lceil DT/Q\rceil-1)\cdot Q}{DT}\right) \tag{6}$$

$$S_1(Q)=K\cdot\left\lceil\frac{DT}{Q}\right\rceil \tag{7}$$

$$S_2(Q)=R\cdot\left\lceil\frac{Q}{P}\right\rceil\cdot\left(\left\lceil\frac{DT}{Q}\right\rceil-1\right)+R\cdot\left\lceil\frac{DT-(\lceil DT/Q\rceil-1)\cdot Q}{P}\right\rceil \tag{8}$$

$$TF_1(Q) = Ce\ DT + Ce_0 \cdot \left\lceil \frac{DT}{Q} \right\rceil \tag{9}$$

$$TF_2(Q) = Cg_0 + C \cdot g \cdot \frac{Q}{2} \cdot \frac{Q}{DT} \cdot \left(\left\lceil \frac{DT}{Q} \right\rceil - 1 \right) + C \cdot g \cdot \left(\frac{DT - (\lceil DT/Q \rceil - 1) \cdot Q}{2} \right) \cdot \left(\frac{DT - (\lceil DT/Q \rceil - 1) \cdot Q}{DT} \right) \tag{10}$$

Next, we will discuss the optimal solution properties and solutions of Eq. (5) under the constraints of Eq. (2) and Eq. (3).

Theorem 1: Let $LTC(Q) = H(Q) + S_1(Q) + TF_1(Q) + TF_2(Q)$, when $Q = DT/m_1$, and $m_1 = \left\lceil \frac{-1+\sqrt{1+2(h+C\cdot g)DT^2/(K+C\cdot e_0)}}{2} \right\rceil$, $LTC(Q)$ will have the minimum at $Q = DT/m_1$.

Proof: Using the solution method of Schwartz (1972), we can obtain the following relational expression of m:

$$\begin{aligned} & mK + \frac{hDT^2}{2m} + CeDT + C \cdot e_0 \cdot m + Cg_0 + Cg\left[\frac{DT}{2m}\right]T - CB \\ & \quad = (m+1)K + \frac{hDT^2}{2(m+1)} + CeDT + Ce_0(m+1) + Cg_0 + Cg\left[\frac{DT}{2(m+1)}\right]T - CB \end{aligned} \tag{11}$$

After simplification, the best total number of orders m_1 is the smallest integer that satisfies Eq. (11) as follows:

$$m(m+1) \geq \frac{(h+Cg)DT^2}{2(K+Ce_0)}, \tag{12}$$

$m_1 \geq \frac{-1+\sqrt{1+2(h+C\cdot g)DT^2/(K+C\cdot e_0)}}{2}$ or $m_1 \leq \frac{-1-\sqrt{1+2(h+C\cdot g)DT^2/(K+C\cdot e_0)}}{2}$ (negative disagreement)

Hence $m_1 = \left\lceil \frac{-1+\sqrt{1+2(h+C\cdot g)DT^2/(K+C\cdot e_0)}}{2} \right\rceil$.

Next, we will discuss the optimal solution properties and solution of $S_2(Q)$.

Lemma 1: To ship products with a total demand of DT, at least $\lceil DT/Q \rceil$ containers are required. Therefore, if the order quantity Q_1 each time is an integer multiple of the container capacity P (i.e. $Q = iP$, i is any positive integer), it can be known that the number of containers used is the smallest quantity.

Proof: (Contradiction) When the number of orders is m and the order quantity Q_1 is iP, the required number of containers N_1 is $\mathrm{i}\cdot(m-1) + \left\lceil \frac{DT-(m-1)\mathrm{i}P}{P} \right\rceil = (m-1) + \left\lceil \frac{DT}{P} \right\rceil$. Assuming there exists another order quantity $Q_2 = \mathrm{i}P + \alpha$ ($0 < \alpha < P$, α is also an arbitrary positive integer), and the number of containers used by Q_2 is smaller than Q_1, it can be found that the container quantity N_2 required for order quantity Q_2 is $(\mathrm{i}+1)\cdot(m-1) + \left\lceil \frac{DT-(m-1)(\mathrm{i}P+\alpha)}{P} \right\rceil = (m-1) + \left\lceil \frac{DT-(m-1)\alpha}{P} \right\rceil \geq \left\lceil \frac{DT-(m-1)\mathrm{P}}{P} \right\rceil = \mathrm{N}_1 + 1$ and contradicts the assumption, so it can be proved that $Q=iP$ and i is any positive integer, so The number of containers used is the minimum number.

Given that the total number of orders is m (that is, the range of order quantity is $\frac{DT}{m} \leq Q \leq \frac{DT}{m\text{-}1}$), $S_2(Q)$ can be simplified as follows:

$$S_2(Q) = R\left\lceil \frac{Q}{P} \right\rceil \cdot \left(\left\lceil \frac{DT}{Q} \right\rceil - 1 \right) + R \cdot \left\lceil \frac{DT - (\lceil DT/Q \rceil - 1)\cdot Q}{P} \right\rceil = R\left\lceil \frac{Q}{P} \right\rceil (m-1) + R \cdot \left\lceil \frac{DT-(m-1)\cdot Q}{P} \right\rceil = S_2^{(1)}(Q) + S_2^{(2)}(Q) \tag{13}$$

Among them, $S_2^{(1)}(Q)$ can be defined as the first (m-1) orders, each time the order quantity is Q, the total shipping cost required. $S_2^{(2)}(Q)$ is the required shipping cost of the last order. In order to further explain the nature of $S_2(Q)$, the following will analyze the nature of the two costs of $S_2^{(1)}(Q)$ and $S_2^{(2)}(Q)$ in the interval $\left[\frac{DT}{m}, \frac{DT}{m\text{-}1}\right)$.

Lemma 2: If there are two order quantities Q_3, Q_4, and $\frac{DT}{m} \leq Q_3 \leq Q_4 \leq \frac{DT}{m\text{-}1}$, the following results can be obtained: (1) In the previous *(m-1)* order, the number of containers used each time is $\lceil Q/P \rceil$, and this number will increase or remain unchanged as Q increases. In other words, $0 \le S_2^{(1)}(Q_4) - S_2^{(1)}(Q_3) \le (m-1)R$. (2) For the last order, the number of containers used is $\left\lceil \frac{DT-(m-1)\cdot Q}{P} \right\rceil$, and this number will decrease or remain the same as Q increases. In other words, $0 \le S_2^{(2)}(Q_3) - S_2^{(2)}(Q_4) \le R$.

Proof: Obviously, $R(m-1)\left\lceil \frac{Q}{P} \right\rceil$ is a non-decreasing function of Q and $R \cdot \left\lceil \frac{DT-(m-1)\cdot Q}{P} \right\rceil$ is also a non-increasing function of Q, so it is proved.

According to the foregoing theorem and lemma, the characteristics of the total cost $TC(Q)$ can be summarized as follows:

Theorem 2: If the order quantity $Q=DT/m_1$ obtained by Theorem 1, and Q is an integer multiple of P, then $TC(Q)$ have the minimum at $Q = \frac{DT}{m_1}$.

Proof: According to Theorem 1, $LTC(\frac{DT}{m^*})$ is the minimum value of $LTC(Q)$. According to Lemma 1, when Q is an integer multiple of container capacity P, it will only cause the smallest total order variable cost $S_2(Q)$. Therefore, if $Q = \frac{DT}{m_1}$ is also an integer multiple of P, it can be known that both $LTC(Q)$ and $S_2(Q)$ are the minimum values, so it is the optimal solution of the total cost.

Theorem 3: When there exists $Q' \le Q$ satisfy $\left\lceil \frac{DT}{Q'} \right\rceil = \left\lceil \frac{DT}{Q} \right\rceil = m$ and $\left\lceil \frac{DT-(m-1)\cdot Q'}{P} \right\rceil = \left\lceil \frac{DT-(m-1)\cdot Q}{P} \right\rceil = b$, $TC(Q') \le TC(Q)$ can be obtained.

Proof: If $Q' \leqq Q$, and assuming that the manufacturer's total number of orders is m, we know that $\frac{DT}{m} \le Q' \le Q \le \frac{DT}{m\text{-}1}$, and $LTC(Q)$ can be expressed as follows:

$$
\begin{aligned}
LTC(Q \mid \frac{DT}{m} \le Q \le \frac{DT}{m\text{-}1}) &= (h+Cg)\left[\frac{Q}{2}(m-1)\frac{Q}{D} + \left(\frac{DT-(m-1)Q}{2}\right)\left(\frac{DT-(m-1)Q}{D}\right)\right] \\
&\quad + m(K+Ce_0) + CeDT + Cg_0 + Cg\left\lceil \frac{DT}{2m} \right\rceil T - CB \\
&= \frac{(h+Cg)}{2D} m(m-1)\left(Q - \frac{DT}{m}\right)^2 + \frac{(h+Cg)}{2D}\frac{D^2T^2}{m^2} + m(K+Ce_0) \\
&\quad + CeDT + Cg_0 + Cg\left\lceil \frac{DT}{2m} \right\rceil T - CB
\end{aligned}
\tag{14}
$$

Therefore, it can be seen that $LTC(Q)$ is a strictly increasing function of Q in $\left[\frac{DT}{m}, \frac{DT}{m\text{-}1}\right]$. Since both Q' and Q make the number of containers required for the last order is b, we know that $S_2^{(2)}(Q') = S_2^{(2)}(Q) = bR$. From Eq. (5), we can know that $TC(Q') < \le TC(Q)$.

It can be seen from Theorem 3 that if the manufacturer's order quantity is Q, an equal or better order quantity Q' must be found to make $TC(Q')$ smaller, but Q' must meet the three conditions of Theorem 3. Next, we will use Theorem 3 to find the Q range that can make $TC(Q)$ hace the minimum.

Theorem 4: Continuing theorem 3, if $b \ge \frac{DT}{mp}$, then the feasible solution range of $TC(Q)$ is $\left[\frac{DT}{m}, \frac{DT-P(b-1)}{m-1}\right]$ and $TC(Q)$ has a minimum value at $Q' = \frac{DT}{m}$; if $b < \frac{DT}{mp}$, then the feasible solution range of $TC(Q)$ is $\left[\frac{DT-Pb}{m-1}, \frac{DT-P(b-1)}{m-1}\right]$ and has a minimum value at $Q' = \frac{DT-Pb}{m-1}$.

Proof:

1. If Q makes the total number of orders of the manufacturer m, we know: $\frac{DT}{m} \le Q \le \frac{DT}{m\text{-}1}$, so the last order quantity of the manufacturer must be in the following range: $0 \le DT-(m-1)Q \le \frac{DT}{m}$.

2. When Q also makes the number of containers required for the manufacturer's last order as b, it is known $(b-1)P \le DT-(m-1)Q \le \frac{DT}{m} \le bP$.

3. According to the first point, since the last order quantity cannot be greater than $\frac{DT}{m}$, if $b \ge \frac{DT}{mp}$, $(b-1)P \le DT-(m-1)Q \le \frac{DT}{m} \le bP$, i.e. $\frac{DT}{m} \le Q < \frac{DT-P(b-1)}{m-1}$. If $b < \frac{DT}{mp}$, $(b-1)P \le DT-(m-1)Q \le bP \le \frac{DT}{m}$, i.e. $\frac{DT-Pb}{m-1} \le Q < \frac{DT-P(b-1)}{m-1}$, Therefore, the theorem can be proved according to Theorem 3.

Let's explore the feasible solution range of m and b.

1. Scope of b： Since when the total number of orders is m, the manufacturer's order quantity $Q \ge \frac{DT}{m}$ means that the last order quantity $DT-(m-1)Q$ is at most $\frac{DT}{m}$. So the knowable range is: $\left\lceil \frac{DT}{mP} \right\rceil \ge b \ge 0$. This means that as long as the range of m can be found, the feasible solution range of b can be known.

2. Scope of m：

(1) Let the order quantity $Q_L = \left\lceil \frac{DT/m}{P} \right\rceil \cdot P$ and the order quantity Q^* is the optimal solution to this problem, so $LTC(Q_L)+S_2(Q_L) \ge LTC(Q^*)+S_2(Q^*)$. According to Lemma 1, since Q_L is the smallest order quantity that is greater than DT and is an integer multiple of P, $S_2(Q_L)$ is the smallest total order variable cost.

(2) Therefore, we can get $LTC(Q_L)-LTC(Q^*) \ge S_2(Q^*)-S_2(Q_L) \ge 0$, that is, $LTC(Q_L) \ge LTC(Q^*)$. But from Theorem 2 we know that $LTC(Q^*) \ge LTC(\frac{DT}{m^*})$, so $LTC(Q_L) \ge LTC(Q^*) \ge LTC(\frac{DT}{m^*})$.

(3) $LTC(Q_L) \ge LTC(\frac{DT}{m^*}) = m^*(K+Ce_0) + \frac{(h+C\cdot g)DT^2}{2m^*} + C\cdot e(DT)+Cg_0-CA$, since $LTC(\frac{DT}{m^*})$ is a quadratic function of m* and m* needs to meet the restriction of a positive integer, the upper bound $m_{\max}$ and lower bound $m_{\min}$ of the feasible solution of m can be obtained as follows:

$$m_{\max} = \left\lceil \frac{LTC(Q_L)+C\cdot e(DT)+Cg_0-CB+\sqrt{(LTC(Q_L)+C\cdot e(DT)+Cg_0-CA)^2-2(K+C\cdot \mathrm{e}_0)(h+Cg)DT^2}}{2(K+C\cdot \mathrm{e}_0)} \right\rceil \tag{15}$$

or

$$m_{\min} = \left\lfloor \frac{LTC(Q_L)+C\cdot e(DT)+Cg_0-CB-\sqrt{(LTC(Q_L)+C\cdot e(DT)+Cg_0-CA)^2-2(K+C\cdot \mathrm{e}_0)(h+Cg)DT^2}}{2(K+C\cdot \mathrm{e}_0)} \right\rfloor \tag{16}$$

(Negative disagreement)

From the discussion above, the original problem can be expressed as a nonlinear integer programming problem with two decision variables as follows:

$$\min \mathrm{TC}(Q) = \begin{cases} TC\left(Q=\frac{DT}{m}\right) & \text{if } b \ge \frac{DT}{mp} \\ TC\left(Q=\frac{DT-Pb}{m-1}\right) & \text{if } b < \frac{DT}{mp} \end{cases}$$

subject to (17)

$m_{\min} \le m \le m_{\max}$

$m,b \in \mathbb{N}$ (positive integer).

where

$$TC\left(Q=\frac{DT}{m}\right) = \frac{(h+C\cdot g)DT^2}{2m} + m(K+Ce_0) + R(m-1)\left\lceil \frac{DT}{mP} \right\rceil + R\left\lceil \frac{DT-(m-1)\cdot(DT/m)}{P} \right\rceil + C\cdot e(DT)+Cg_0-CB \tag{18}$$

$$\begin{aligned} TC\left(Q=\frac{DT-Pb}{m-1}\right) = {} & \frac{(h+C\cdot g)(DT-bP)^2}{2D(m-1)} + \frac{(h+C\cdot g)b^2P^2}{2D} + m(K+Ce_0) \\ & + R(m-1)\left\lceil \frac{DT-bP}{(m-1)P} \right\rceil + Rb + C\cdot e(DT)+Cg_0-CB \end{aligned} \tag{19}$$

Since this nonlinear integer programming problem contains Gaussian symbols, it is not easy to find a closed-form solution. Therefore, this study uses the property that the integer solution obtained under the feasible solution range must be a finite number. Find the range of *m* and *b*, and use the global search method to solve.

Next, the process of finding the best solution will be explained. The process of finding the most suitable solution is shown in Figure 2. First, according to Theorem 1, this study can find an order quantity $Q=\frac{DT}{m_1}$ that can minimize *LTC(Q)*. If the order quantity $Q=\frac{DT}{m_1}$ is an integer multiple of *P*, it means that *Q* minimizes the variable cost of ordering. It can be determined that *Q* is the optimal solution for the total cost, and the solution process is ended. But if it is not true, this study proposes a method to find the minimum value of the total cost as shown in Fig. 2.

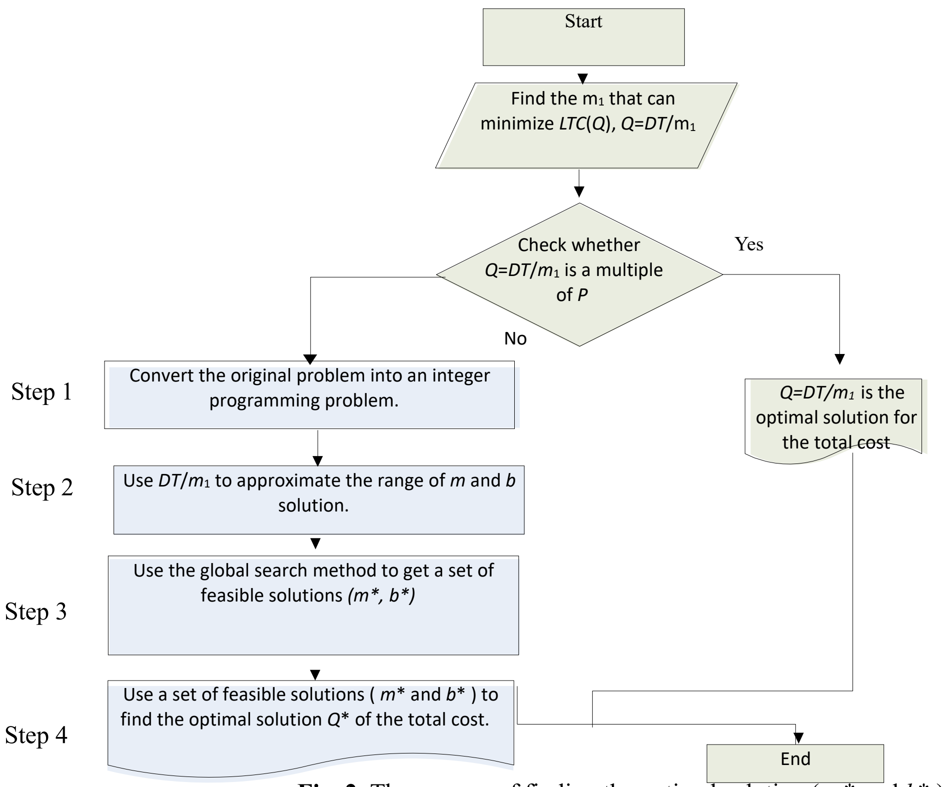

Fig. 2. The process of finding the optimal solution (*m** and *b**)

Finally, we will discuss how manufacturers can achieve a win-win ordering strategy with the lowest carbon emissions and the lowest total cost.

Theorem 5: Continuing Theorem 4, if the minimum value of $TC(Q)$ occurs when $Q=\frac{DT}{m}$ is an integer multiple of *P* and $b\geq\frac{DT}{mp}$, then the manufacturer has the minimum carbon emissions at the same time.

Proof: Carbon emissions of the manufacturer with total number of orders *m* is as follows:

$$CE(Q\mid\frac{DT}{m}\leq Q\leq\frac{DT}{m\text{-}1})=g\left[\frac{Q}{2}(m-1)\frac{Q}{D}+\left(\frac{DT-(m-1)Q}{2}\right)\left(\frac{DT-(m-1)Q}{D}\right)\right]+me_0+eDT+g_0+g\left[\frac{DT}{2m}\right]T$$

$$=\frac{g}{2D}m(m-1)\left(Q-\frac{DT}{m}\right)^2+\frac{g}{2D}\frac{D^2T^2}{m^2}+me_0+eDT+g_0+g\left(\frac{DT}{2m}\right)T \tag{20}$$

when $Q=\frac{DT}{m}$, the manufacturer has a minimum carbon emission of $\frac{g}{2D}\frac{D^2T^2}{m^2}+me_0+eDT+g_0+g\left(\frac{DT}{2m}\right)T$. Therefore, it can be known from Theorem 1 and Theorem 4 that when $Q=\frac{DT}{m}$ is an integer multiple of *P* and $b\geq\frac{DT}{mp}$, the manufacturer has the smallest carbon emissions. At this point, we can use Theorem 1 and Theorem 4 to get the ordering strategy that allows manufacturers to have the smallest carbon emissions.

4. Numerical Analysis

4.1. Numerical examples

This section verifies the above theoretical results through the application of numerical examples, and discusses the influence of relevant parameters on the manufacturer's optimal strategy and carbon emissions in combination with sensitivity analysis, so as to obtain the corresponding management inspiration. Other related parameters in the case of ordering cost respectively in a fixed cost (K) is 20 \$, ordering cost changes in the cost (R) of 10 \$/piece, the manufacturer's inventory carrying costs (h) = 2 \$/piece, the planning phase of the length of time (T) for one year, the rate of customer demand, D is 1000 units of gas per year, bearing capacity limit of the vehicle (P) of 35 parts, used to transport goods vehicles when the light carbon emissions (e_0) around 450 tons of CO_2, goods transport unit variable carbon emission factor (e) is about 0.2 tons of CO_2 / piece, The fixed carbon emission (g_0) of the storage unit is about 500 tons of CO_2, the variable carbon emission factor (g) of the storage unit is about 1 ton of CO_2/ piece, the carbon trading volume (B) allocated by the enterprise is 500 tons of CO_2, the carbon emission allowance (A) is 5,000 tons, and the carbon trading price (C) of CO_2 is about 0.3 \$/ton of CO_2. The steps are as follows: 1. Find the value of m_1 that can minimize *LTC(Q)* as 3, but because Q is not an integer multiple of P, we will proceed to the next step. 2. The feasible solutions for m and b are: $1 \le m \le 5$, $0 \le b \le 29$, m and b are all positive integers. The optimal solution of one of the above problems is (m^*, b^*) = (3, 10). 4 After all possible solutions are substituted by the global search method. Due to $b^* > DT / m^* P$, it can be known that the optimal order quantity $Q^* = DT / m = 1000 / 3 = 333.33$, which means that when the manufacturer orders 333.33 units of gas for three times, the lowest total cost is 1208.333, and the carbon emission at this time is 2666.67 units, which is also the ordering policy that can bring the minimum carbon emission. In the case of no carbon cap-and-trade policy, the manufacturer's (m^*, b^*) is (6, 4). Since $b^* \le DT / m^* P$, the optimal order quantity $Q^* = (DT - Pb^*) / (m^* - 1) = 172$, and the optimal total cost is 577.52, that is, when the manufacturer orders 172 units of gas each time, after five times, the manufacturer orders 140 units of gas the last time. Due to carbon emissions by 2666.67 in this case is less than 5000 carbon quotas, the manufacturer under the carbon cap and trade policy can sell some of the carbon emissions to cost of subsidies, but because manufacturers to increase the total transport carbon emissions and total carbon storage costs of two parts, which makes the manufacturers than the total cost of the value is still no carbon constraints of earned value higher than 630.813, it also shows that carbon cap and trade policy to increase the total cost of the manufacturers. However, it can be known from Theorem 5 that in this example, the manufacturer can still take into account the win-win result of the lowest total cost and the minimum carbon emission to decide the optimal order times.

4.2. Parameter analysis

Using the parameter values in the basic calculation example, we further analyzed the influences of T, K, R, P and C on the total cost and carbon emission of retailers under the carbon cap-and-trade policy. The corresponding trend change curve is shown in Fig. 3 to Fig. 7. As can be seen from Fig. 3, with the increase of the length of the planning period T, the cost value of the manufacturer under the carbon cap-and-trade policy shows a trend of gradual increase, while the cost value without carbon constraint also keeps increasing with the increase of T. Manufacturers in carbon cap and trade policy under the total cost value has been greater than the cost under the restriction of carbon-free value and the difference between them increased with the increase of T, this is because the manufacturers have to bear the total transportation cost of carbon emissions and total carbon storage costs, although can get a certain additional carbon emissions by selling income, but with the increase of T is gradually enable manufacturers in the total cost value is greater than total cost under the restriction of no carbon value. With the increase of T, the number of order cycles corresponding to the carbon cap-and-trade policy and the no-carbon constraint of the manufacturer increases in a step-by-step manner. On the other hand, with the increase of T, the difference between the minimum carbon emission and the carbon emission under minimum of the manufacturer's total cost shows a fluctuating growth, which indicates that the larger the length of the planning period T is, the more detrimental the manufacturer will be to achieve low cost and low emission. As R increases, the total cost of the manufacturer increases and the total carbon emission decreases. However, as R exceeds 20, it can be observed from Figure 4(a) that the total carbon emission of the manufacturer is no longer affected by the change of R. This is because in this example, the optimal order times m and order quantity Q^* have already minimized the transportation times, so it is impossible to reduce carbon emission by changing the order times m and order quantity Q^*. With the increase of P, the optimal order quantity Q^* may increase to reduce the total order number, so the minimum total cost and carbon emission will decrease accordingly (Figure 4(b) and Figure 4(c)). We are also interested in the effect of different combinations of R and P on the total cost and carbon emissions. For example, if the manufacturer can choose two different vehicles to transport, and the transportation cost and capacity limits are (R,P)=(10,35) and (R,P)=(20,70) respectively, which vehicle will have the lower total cost or carbon emission? As can be seen from Fig. 5(a), it can be inferred that when the variable cost R is smaller and the vehicle capacity P is larger, the total cost can be smaller. According to Fig. 5(b), as long as the R/P value is 0.2857(10/35), the difference of total cost of different (R,P) combinations is not significant, which is within 2%. According to Figure 5(c), given two vehicle capacities (P=21, P=35), the relationship between different R/P values and total costs is analyzed. Considering the R/P values in the range of 0.05 to 2, 50 combinations are made for each of the two different vehicle capacities P and R. The analysis results show that as long as the R/P value is low, the total cost can be obtained, and on the basis of the same R/P value (R, P), the combination with a small value also has a relatively small carbon emissions, this is because the combination of smaller (R,P) values results in a slight decrease in total transport carbon emissions (Fig. 5 (d)). Based on the above analysis, it is concluded that when the manufacturer chooses different loads to transport products with fixed other parameters, the decision is only made according to the R/P value. If the R/P value is lower, the total cost and carbon emission can be lower.

As can be seen from Fig. 6(a), with the increase of carbon emissions trading price C, value increases in the total cost of manufacturers under the carbon policy but value of total cost under no carbon cap-and-trade policy remains the same, and the former has been greater than the latter, this is because the manufacturers have to bear the total transportation cost of carbon emissions and total carbon storage costs, although can get a certain additional carbon emissions by selling income, but with the increase of carbon emissions trading price C, carbon emission costs of manufacturers in the total transportation and storage process is gradually greater than the costs of no carbon cap-and-trade policy. At this time, manufacturers will take the strategy of reducing the number of orders and increasing the quantity of each order to reduce carbon emissions. Therefore, with the increase of carbon emission trading price C, the optimal order times of the manufacturer under the carbon policy show a trend of step-by-step decrease (Fig. 6(b)), while the corresponding order times remain unchanged respectively in the absence of carbon constraints. On the one hand, it shows that the change of carbon emission trading price changes the manufacturers' inventory strategy under the carbon policy. On the other hand, when the carbon emission is less than the carbon limits, increasing the carbon emission trading price is conducive to encouraging the manufacturer to reduce carbon emission, and also enables the manufacturer to obtain a lower cost. For example, as shown in Figure 6(c), when the value of C is greater than 0.5, the difference between the manufacturer's carbon emissions and the minimum carbon emissions decreases from 449.745 to 436.775 units. The Fig.7 (a) shows that carbon caps a change on the order number m when other parameters remain the same, and there was no effect within the planning horizon, such as C = 0.3, the limitation of carbon increased from 5000 to 15000, the manufacturer's order number m is 3, the order quantity Q^* is 333(/time) hasn't changed, but the higher the price of carbon trading C, the less the order number m (figure 6 (b)), and the more order quantity Q^* (Fig. 7 (b)). This shows that the transaction price of manufacturers operating effect significantly, under normal circumstances, carbon limitations allocation influenced by national policy, a single enterprise or organization can only accept the current scheme, and operating under the established carbon caps, but the price of carbon trading C is a market product, which directly affects the sale income or purchase cost of manufacturers, thus the effect is more obvious.

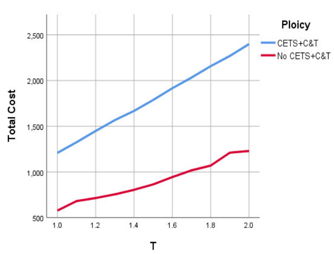

Fig. 3(a) The impact of different T on total cost under two policies

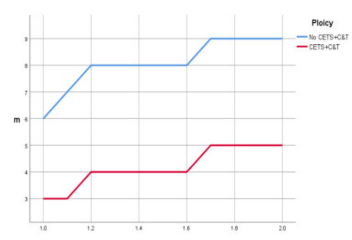

Fig. 3(b) The impact of different T on the number of orders (m^*) under two policies

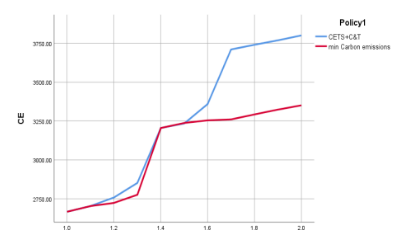

Fig. 3(c) The impact of different T values on carbon emissions under two policies

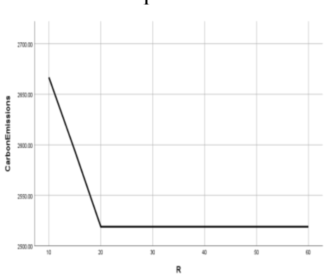

Fig. 4(a) The impact of different R values on carbon emissions

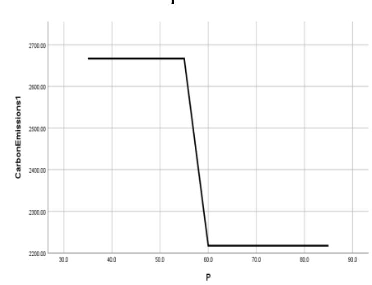

Fig. 4(b) The impact of different P values on carbon emissions

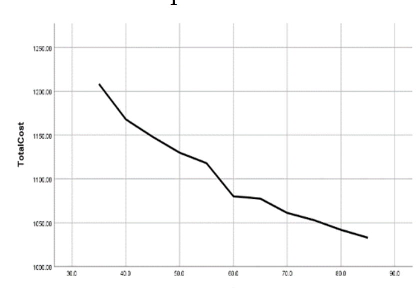

Fig. 4(c) The impact of different P on total cost

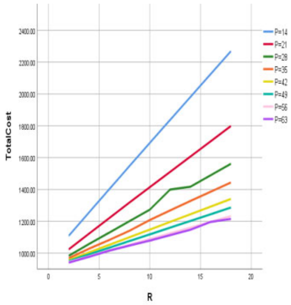

Fig. 5(a) The impact of different *R* and *P* on total cost

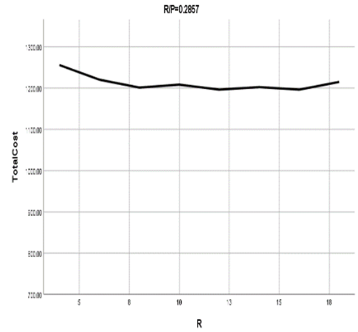

Fig. 5(b) When *R/P* is fixed, the impact of different R on the total cost

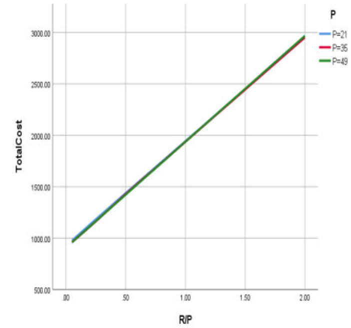

Fig. 5(c) The impact of different *R/P* values on the total cost

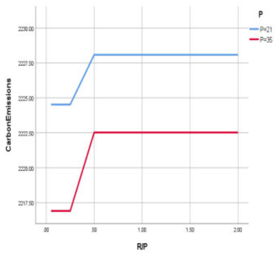

Fig. 5(d) The impact of different *R/P* values on carbon emissions

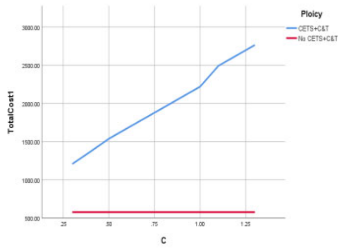

Fig. 6(a) The impact of different *C* values on total cost under two policies

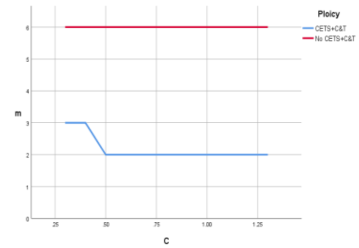

Fig. 6(b) The impact of different *C* values on the number of orders (*m**) under two policies

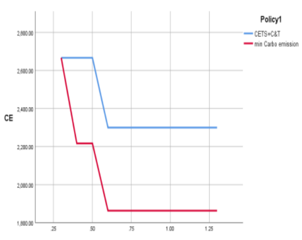

Fig. 6(c) The impact of different *C* values on carbon emissions under two policies

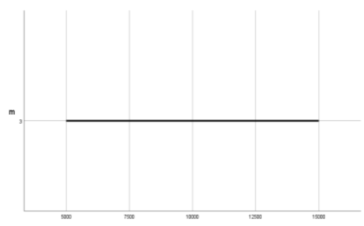

Fig. 7(a) The impact of different *A* values on the number of orders (*m**)

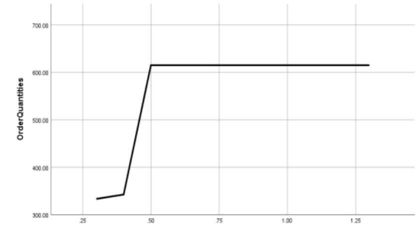

Fig. 7(b) The impact of different *C* values on the order quantities (*Q**)

5. Conclusions

In view of the actual situation that the product will generate carbon emissions during the ordering and storage process and the ordering cost presents a stepwise function to the order quantity, we aim to explore the optimal economic ordering quantity model with the goal of minimizing the total cost of the manufacturer in the finite planning horizon. First, build an inventory optimization model under the carbon cap and trade mechanism, analyze and solve the manufacturer's optimal ordering strategy during the finite planning horizon. Then compare the manufacturer's optimal ordering strategy under the carbon allowance and transaction policy with the corresponding optimal ordering strategy when there is no carbon constraint and the manufacturer releases the minimum carbon emissions, and the conditions are inferred that the manufacturer achieves low cost under the carbon policy and low emission. Finally, numerical experiments are combined to verify the theoretical results and analyze the influence of some parameters on the manufacturer's optimal ordering strategy and total cost. The conclusions of this research are as follows: (1) Under the carbon cap and trade mechanism, there is a unique optimal number of order cycles and optimal order quantities to minimize the total cost of the manufacturer during the finite planning horizon; (2) When the manufacturer's total demand under the carbon allowance and transaction mechanism is a certain multiplier of the number of orders, the manufacturer can achieve a win-win result of low cost and low emissions.

The analysis of calculation experiments and examples show that: Under the constraints of carbon cap and trade mechanism, the number of orders m of manufacturers decreases and the quantities *Q** increases, and the total cost is also higher than in the case of no carbon restrictions. In the parameter analysis, it is found that the changing trend of the total cost is affected by the carbon trading price, the capacity limit P of the vehicle and the variable cost *R* in the ordering cost. When the carbon transaction price increases, the total cost of the manufacturer increases with the increase in the transaction price; If the *R/P* value is lower, it is obvious that lower total costs and relatively small carbon emissions can be obtained. In addition, the larger the length of the finite plan horizon, the less conducive it is for manufacturers to achieve low cost and low emissions. When carbon emissions are less than carbon allowances, increasing the price of carbon emissions trading will help manufacturers achieve low cost and low emissions. Although this study considers the carbon emission model of the transportation mode in the real environment, which has the characteristics of the ordering cost presents a stepwise function for ordering quantity, there are still limitations in many assumptions, and it does not consider the occurrence of defective products, delayed delivery, and backlog of goods. The additional costs and emissions of the company ignore the impact of transportation routes and human factors on carbon emissions. Our research conclusions are suitable for the operational decision-making of a single enterprise. Besides that it is also necessary to analyze the benefits of carbon trading mechanisms for production, transportation and inventory management from the perspective of the joint decision-making and profit distribution of the entire supply chain.

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