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A *DE Novo* multi criteria heterogeneous group decision making approach for green performance assessment of CNC machine tools

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A B S T R A C T

In the contemporaneous sustainable manufacturing scenario and fourth industrial revolutions, requirements of most cutting-edge CNC machine tools are indispensable for finished products with high accuracy, precision and green complaints in particular. Such requirements have impelled the advanced manufacturing industries to evaluate and choose the proper CNC machine tools for best customized performances. In the face of proper and effective green evaluation, this paper incorporates a heterogeneous expert group based decision framework considering multiple significant technical and green criteria by assessing relative importance of diverse conflicting criteria having substantial contribution in performance analysis of CNC machine tools. As a demonstration of the suggested mathematical model, three real life decision making problems related to 3-axes CNC machine tools based on the collected quantitative and linguistic data from catalogues, manufacturer's portals, questionnaires, customer reviews etc. are established. The calculated findings are close to those obtained by previous researchers as well as are verified by well-established techniques. Besides, sensitivity and statistical analysis are performed to examine the robustness and stability of the ranking orders of the alternatives as well as to investigate the efficacy and consistency of the proposed method. Hence thus proposed formulated MCDM approach proves to be a highly effective and reliable decision making tool for choosing the most suitable CNC machine tools.

1. Introduction

Green manufacturing scenario and industry 4.0 encourage more sustainable manufacturing solutions. Switching to CNC machining, modern manufacturing systems can be improved to produce a lower carbon footprint. One of the major advantages is that CNC machining is greener becauseit can be performed electronically. CAD files can be transferred electronically between both customer end and manufacturing end that minimizes to and fro travel resulting in lower carbon emission from transportation. Most automotive and aerospace industries have started adopting Internet of Things (IoT) that integrates CNC systems, central computing systems as well as with other mechanical equipment enabling a major transformation in manufacturing capabilities. The globalizations of green businesses have impelled the automation manufacturers to invest in cutting-edge computer controlled equipment such as CNC machine tools in the advanced manufacturing sector. Modern manufacturing scenario is changing promptly due to growth in using CNC machine tools that scales up efficiency, quality, accuracy, precision as well as reducing material waste and machining time during various stages of manufacturing. Due to global economic reforms and dynamic challenges in the engineering field, developing and underdeveloped countries are currently financing profoundly in the manufacturing sector (Ic, Yurdakul & Erasian,2012). A large number of small scale and medium scale industries (SSI &MSI) have initiated transformation from conventional manufacturing to digital manufacturing particularly to satisfy the needs of the global manufacturing standard. In this research article a considerable numberof diverse and inconsistent machine tool assessment criteria including green data are considered. The technical data related to CNC machine tools specifications are taken as criteria for performance evaluations. Here quantitative criteria are taken as X-Y-Z tool travels, maximum load capacity,spindle speed, motor power, number of

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tool stations, maximum tool diameter,T-slot width, number of T-slots, X-Y-Z rapid traverse. Qualitative criteria are as CNC controller, use of local materials, life expectancy. Green criteria aretaken as CO_2 emission, energy consumption, toxicity and noise level. All those data are directly obtained from such CNC machine tools which are already operational in the Indian manufacturing industries, advanced training centers and tool rooms. They are collected through questionnaires to technicians, CNC specialists, operators and engineers. So the machine tool evaluation problem can be regarded as a multiple criteria or attribute decisionmaking (MCDM) problem with respect to beneficial and non-beneficial criteria (Aghdaie,Zolfani & Zavadskas, 2013; Onut, Kara, Efendigil, 2008).

1.1 *Heterogeneous expert group and green based assessment*:

This research article proposes a heterogeneous expert group based decision support system which aids in reaching a collective and unbiased decision while choosing the best alternative. Heterogeneous nature of decision making approach is highly pertinent to subsequent real life complex decision problems. It integrates diverse technical skills, domains,expertise, qualifications, genders, cultural backgrounds and perspectives ofthe decision making environments. Moreover, a heterogeneous expert group may substantially outdo a homogeneous one when it comes to implementing multifaceted tasks particularly in conducting research and designing processes. But in repetitive tasks such as flying sector, healthcare sector etc. the homogenous decision makers often do better. A homogeneous decision making group includes individuals whose expertise, domains and assessment capability are supposed to be the same which is just contrary to the real life exact assessment of the decision environment. A homogenous expert group is supposed to be convergent thinkers whereas a heterogeneous expert group is required to be divergent thinkers. So, a heterogeneous expert group is highly proficient in making judicious decisions in the real industrial environment (Dey, Bairagi, Sarkar &Sanyal, 2017; Wu, Ahmad & Xu, 2016).This paper specifically emphasizes on green criteriain order to assess CNC machine tools. Authors found inadequate information related to green criteria in previous findings as discussed in the literature survey. Green approach generally considers multiple criteria such as environmental, social, health, security and economic one that have immense impact on the society and the universe as a whole. To ensure cleaner production processes and to benefit the society at large, green criteria must be taken into account while selecting suitable CNC machine tools. Here, in this paper, more or less green criteria considered during selection processes are carbon emission, toxicity, noise effect, use of local materials, power saving, work fluid contaminationand iron dust pollution. Carbon emission: Some basic components of the manufacturingsystem such as electricity, cutting fluid, lubricant, wearing of cutting tools,material consumption, disposal of chips etc. mainly contribute to the carbonemission. The minimal CO_2 emission from CNC based manufacturingsystems results in lesser adverse impact on air quality and subsequently reducesglobal warming. Toxicity: The CNC machine operators and techniciansare highly exposed to toxic or corrosive chemicals used in lubricants andcoolants during machining operations. These toxic chemicals can cause irritationand even pass through the skin of the operators resulting in health threats. CNC machines are required to be equipped so that minimal toxicity level in thesurrounding atmosphere is ensured. Use of local materials: More use of local materials in CNC machinetool components results in reducing transit significantly for maintenance andservice. The reduced transportation in material procurements significantly saves fuels which in turn reduces CO_2 emission. Noise effect: The main actuating system of CNC machine tools mainly depends on the gear transmissions system. It's the main source of noise pollution. Apart from that lubrication system, fit and tolerance inbearing, job load acting in the spindle bearing also contribute to the noisepollution. A good quality machine tool is highly required to comply with a green manufacturing system. Power saving: More energy utilization results in morecarbon emission in the atmosphere. More power consumption has an adverseeffect on the product life cycle. The optimal power utilization is highly requiredin a sustainable manufacturing system. Work fluid contamination: It is mechanicalimpurities arising out of the lubricating system. During machining operation thesemechanical impurities cause smoke concentration and toxicity in the surroundingatmosphere. CNC machine tools are required to be equipped with a mechanism so that the impurities deposition becomes as low as possible. Iron dust pollution: Iron dust mainly forms during machining operation. It is injurious to inhalation. A good CNC machine tool is required to be equipped with an iron dust collector so that minimal mixing of it with the atmosphere is ensured. Thus green criteria play an important role in the sustainable manufacturing system and help in reducing global warming. Moreovergreen criteria encompass a green economy which is the exercise of sustainabledevelopment. The significance of the green economy is that it enhances financial prudence to become more sustainable and low-carbon development. Therefore,judicious evaluation of CNC machine tools under a green environment is animportant and vital decision aspect when upgradation and survival of engineering facilities are concerned. The improper assessment of CNC machines can impact on the overall performance of the manufacturing system such as the productivity, adaptability, reliability, flexibility, and approachability.However, the CNC machine tool evaluation problem is not only a rigorous and inflexible problem but also a difficult task for engineers and other technical persons or managers due to a lack of deep technological implications (Nguyen,Dawal, Nukman & Aoyama, 2014).

1.2 *Literature review*:

The substantial contributions of the previous researches boost the modern research trend. The route of the current research work has been derivedfrom the contributions of the several prominent researchers. In the face of the fourth generation industrial revolution appropriate selection of CNC machinetools can enhance customer oriented product quality and optimization in the manufacturing process. Here significant findings related to decision making models or approaches adopted by those renowned researchers are presented. Sahin and Aydemir (2021) suggested a wide ranging approach for properly evaluating and choosing the CNC machine tool with respect to determinedcriteria and with the help of diverse decision makers. The authors have adopted the Best-Worst method to assess the alternatives. Yang, Guo and Hailong (2021) developed a decision support system for evaluation of subcomponents of high performance Lathe machines in order to optimize the overall performance. The authors have applied the Weakness Ranking Method in conjunction with TOPSIS method to rank weaknesses of the sub components in reaching the decisionprocess. Li, Wang and Chen (2020) proposed an integrated MCDM technique for evaluation of machine tools. The authors have used fuzzy decision-making trial and evaluation laboratory (FDEMATEL) and entropy method for measuring weight.In this paper VIKOR method has been used for ranking of the alternatives. Patiland Kothavale (2020) formulated an AHP based MCDM approach to rank subsystemsof the CNC turning center. Vafadar, Rad and Hayward (2019) proposed an integrated framework by assessing chosen parameters of the flexible drilling machines to aid inreaching the judicious decision by decision makers. Du, Zheng, Wu and Tang (2019) developed a decision making framework to evaluate feasible remanufactured high performance machining systems. The authors have considered the Entropy method to calculate weight of the multiple qualitative and quantitative criteria followed by adopting Analytic hierarchy process (AHP) and extension theory to choose thebest possible alternative. Breaz, Bologa, Racz and Crenganis (2019) suggestedan integrated approach of AHP and fuzzy set theory to formulate a decision model in which a no. of feasible CNC based turning centers were evaluated tochoose the best possible one. Ding, Jiang, Hjhang, Cai and Liy (2018) formulated a hybrid decision making approach to assess CNC machine tool components for remanufacturing purposes. The authors have integrated an analytic hierarchy process (AHP) and modified TOPSIS methods to evaluate guide ways for the machining system to reach an optimal decision. Camci, Temur and Beskese (2018) developed a hesitant fuzzy analytic hierarchy based decision making approach to choose CNC router for small and medium scale woodwork manufacturing industries. Mondal,Kundu, Chatterjee and Chakrabarty (2017) developed a multi attribute baseddecision model to assess the CNC machine tool alternatives. The authors have formulated data envelopment analysis to reach proper decision making inselecting the best one. Dey, Bairagi, Sarkar and Sanyal (2017) proposed a multicriteria decision making technique in conjunction with a heterogeneous expertbased decision making group to assess and select warehouse location in the supplychain system. Wu, Ahmad and Xu (2016) formulated a MCGDM model to select the suitableCNC machine tool by adopting fuzzy VIKOR method. The preferences of thedecision makers have been expressed here by qualitative variables to weighcriteria significance and the performance evaluations. Bologa, Breaz, Racz and Crenganis (2016) proposed a decision making tool to evaluate 3 axes and 5 axesCNC machines. They employed fuzzy based MATLAB software to aid in reaching optimal decision processes. Bologa, breaz, Racz and Crenganis (2016) developed anAHP based decision model to evaluate 5-axes machine tools. Nguyen, Dawal,Nukman and Aoyama (2015) suggested a hybrid approach to aid in reaching aproper decision for choosing the most suitable CNC machine tool. They established fuzzy AHP and fuzzy COPRAS based multi-attribute decision making models tochoose the most suitable CNC machining systems. Nguyen, Dawal, Nukman andAoyama (2014) established a fuzzy Analytic Network Process (ANP) and COPRAS-Gbased MADM model in conjunction with a group of experts to assess and choose themost suitable CNC machine tool. Aghdaie, Zolfani and Zavadakas (2013) establishedan integrated MADM model in which step wise weight assessment ratio analysis(SWARA) and complex proportional assessment of alternatives with grey relations(COPRAS-G) to assess the CNC machine alternatives. Ayag and Ozedemir (2012) presenteda hybrid MCDM approach in which modified TOPSIS and fuzzy ANP were integratedto carry out performance evaluation of CNC machine tools. Ic, Yurdakul andErasian (2012) developed an AHP based decision model to carry out performanceassessment of components of the CNC machining centers. Bairagi. (2022)formulated a novel decision making technique to select the industrial robot employed in the material handling system. The author developed a new MCDM techniquenamed as Technique of Accurate Ranking Order (TARO) to carry out performance assessment of industrial robots used in material handling operations. Dey etal. (2012) developed a fuzzy MOORA based vendor selection framework to choosethe most suitable vendor in a supply chain management system.Taha and Rostam (2011) suggested a decision making system to evaluate most applicable CNC turning centers by employing an integrated approach fuzzy analytic hierarchy process (fuzzy AHP) and preference ranking organizationmethod for enrichment evaluation (PROMETHEE). The authors adopted MATLAB software to evaluate the weights of conflicting criteria to reach an optimal decision. Taha and Rostam (2011) developed a decision support framework inwhich a fuzzy analytic hierarchy process (fuzzy AHP) and artificial neural network have been integrated to choose the most suitable horizontal CNC turningcenter for an automated manufacturing system. Ozgen, Tuzkaya and Ozgen (2011) formulateda hybrid MCDM technique for CNC based press machine tool selection problem. Theauthors have considered integrating the modified DELPHI, Analytical HierarchyProcess and Preference Ranking Organization Method for Enrichment Evaluations(PROMETHEE) approaches as well as with fuzzy sets theory to express the vagueness for correlating the results of the decision-makers. Samdevi et al. (2011)have adopted hybrid MCDM approach to rank the vertical CNC machining centeralternatives. They have combined fuzzyanalytic hierarchy process (AHP) and grey relational analysis methods in orderto take a proper decision. Wang, Shaw and Chen (2010) developed a fuzzy multiple attributedecision-making (FMADM) technique to aid in proper decision making related tochoosing suitable CNC machines for an automated manufacturing system. Athawale and Chaskrabarty (2010) developed a decision support system tosolve CNC machine selection problems. The authors have applied the TOPSIS method toassess alternatives on the basis of system features and costs. Alberti,Ciurana, Rodrguez and Ozel (2009) developed a decision making framework forevaluating high speed milling machine tools based on machine features and Process parameters. Ic, and Yurdkul (2009) developed a MCDM technique by incorporating fuzzy technique for order preference by similarity to idealsolution (FTOPSIS) to assist in the decision making of CNC machining centers. Dağdeviren (2008) proposed a hybrid MCDM approach to choose the most appropriate milling machine tool. They integrated analytic hierarchy process(AHP) and preference ranking organization method for enrichment evaluations(PROMETHEE) to reach the proper decision in choosing the best alternative. Onut,Kara and

Efendigil (2008) applied fuzzy TOPSIS for assessment of CNC milling machines. The author adopted a fuzzy analytical hierarchy process to evaluate the weight of conflicting criteria. Ayag (2007) developed a decision model by formulating an integrated MCDM technique to evaluate the feasible alternatives.The author incorporated fuzzy AHP and simulation techniques to select the best CNC lathe machine. Cimren, Catay and Budak (2006) developed a MCDM technique for evaluating CNC machine tools. The authors have employed the analytic hierarchy process to rank the alternatives considering qualitative decision criteria related to machine features. Ayag and Ozdemir (2005) adopted a fuzzy AHP approach for assessing and choosing the most suitable CNC vertical turning centeramong a set of feasible machine tools. The authors have carried out the Benefitto Cost ratio approach in conjunction with AHP to reach the judicious decision.

Table 1
Detailed literature review and research gap findings

Authors and year	Methods	Area of Application	Unaddressed findings
Sahin et al. (2021)	Best-Worst method	CNC Machine tool	Fuzziness , Inadequacy in green factors, quantitative criteria
Yang et al. (2021)	Weakness ranking method	Subsystems of heavy-duty machine tools	Green factors and Group decision making process
Li et al. (2020)	DEMATEL, entropy weighting & defuzzification VIKOR	CNC machine tool	Green factors
Patil et al. (2020)	AHP	sub-systems of the CNC turning center	Green factors and Group decision making process
Vafadar et al. (2019)	Integrated feasibility analysis	Drilling modular machine tools	Green factors and GDM
DU et. al. (2019)	AHP-entropy weight & extension theory	Remanufactured basic heavy-duty machine tool	Green factors and Group decision making process
Breaz et. al (2019)	AHP and fuzzy logic	CNC turning center	Green factors and GDM
Ding et al. (2018)	AHP & CD-TOPSIS	Guide way for machine tool	Green factors, CNC machine tool
Camci et. al (2018)	Hesitant fuzzy AHP method	CNC router	Green factors and Group decision making approach
Mondal et al. (2017)	Data Envelopment Analysis & MADM	CNC Machine tool	Green factors, Fuzziness and Group decision making
Wu. et al. (2016)	MCGDM & fuzzy VIKOR	CNC machine tool	Green factors
Bologa et al. (2016)	Fuzzy Logic system & MATLAB	Multi axes CNC milling machines	Green factors , Group decision making, MCDM
Bologa et al. (2016)	AHP	5-axes machine tools	Green factors , Group decision making, MCDM
Nguyen et al. (2015)	Fuzzy AHP & Fuzzy COPRAS	CNC machine tool	Green factors
Nguyen et al. (2014)	Fuzzy ANP & COPRAS-G	CNC machine tool	Quantitative attributes
Aghdaie et al. (2013)	SWARA & COPRAS-G	CNC machine tool	Fuzziness, Green factors
Ayag et al. (2012)	Modified TOPSIS & alpha-cut based fuzzy ANP	CNC machine tool	Group decision making process, Green factors
Ic et al. (2012)	AHP	CNC machining center components	Group decision making process, Green factors
Taha et al. (2011)	A hybrid fuzzy AHP-PROMETHEE	CNC turning center	Green factors and Group decision making process
Taha et al. (2011)	fuzzy AHP–ANN	CNC turning center	Green factors
Ozgen et al. (2011)	Modified DELPHI, Fuzzy AHP , PROMETHEE	CNC based Press machine tool	Green factors
Samvedi et al. (2011)	Fuzzy AHP & Grey relational analysis	Vertical CNC machining centers	Group decision making, Green factors
Wang et al. (2010)	FMADM	CNC milling & Lathe machine tools	Green factors and Group decision making process
Athawale et al. (2010)	TOPSIS	CNC machine tools	Green factors, Group decision making process, Fuzziness
Alberti et al. (2009)	Artificial neural network	CNC High speed milling mchine tools	Green factors, Group decision making process
Ic et al. (2009)	MACSEL	CNC machining centers	Green factors and Group decision making process
Dağdeviren. M (2008)	AHP and PROMETHEE	Basic milling machine	Green factors and Group decision making process
Onut et al. (2008)	Fuzzy TOPIS & Fuzzy AHP	CNC Vertical machining center	Green factors and Group decision making process
Ayag.Z (2007)	AHP and Simulation	CNC lathe machine tool	Green factors & Group decision making process
Cimren et al. (2006)	AHP	CNC machine tool	Green factors, Group decision making process
Ayag et al. (2005)	Fuzzy AHP	CNC Vertical turning center	Insufficient Green factors and Group decision making
Tabucanon et al. (1994)	AHP with Dbase III, DBMS & Turbo Pascal compiler	CNC machining center	Green factors & Group decision making process
Myint et al. (1994)	AHP & Goal programming model	CNC machine system	Green factors and GDM

Tabucanon, Batanov and Verma (1994) developed a smart decision making model tosolve the machine selection problem. The authors have adopted Analytic Hierarchy Process in conjunction with Dbase III & DBMS, Expert System shell(EXSYS) and Turbo Pascal compiler to establish the proper decision. Myint and Tabucanon (1994) framed a graphical interaction based decision making toolconsisting of goal programming (GP) and analytic hierarchy process (AHP) to aid decision makers to choose the best CNC machine alternative. Based on the literature reviews the unaddressed findings are presented in Table 1.

Detailed literature reviews show that previous researchers have adopted enormously fuzzy AHP/ANP in solving the decision problem. The AHP/ANP imparts reasonable results in assessing and ranking the feasible alternatives. These techniques are employed successfully in the industrial environment with simultaneous assessment of subjective and objective attributes (Nguyen et.al, 2014). However, fuzzy ANP is an extended version of AHP. But Fuzzy ANP is insufficient to handle the indefinite information, thus fuzzy logic is employed in conjunction with ANP to interpret the experts' decisions. Furthermore, fuzzy ANP has its own inability to calculate complex computation. During the application of the fuzzy ANP, the experts are essentially required to frame answers to a considerable number of pairwise comparisons. To overcome the difficulties arising from Fuzzy ANP/AHP this paper introduces a novel MCDM approach by assessing relative importance of diverse conflicting criteria having significant influence in performance analysis of alternatives.

In view of the unaddressed findings from detailed literature reviews it is found that few past researchers have considered group decision making (GDM) approaches and green criteria for choosing the most suitable machine tool. Here three case studies on three different types of CNC machine tools (CNC vertical machining centers, CNC horizontal machining centers and CNC horizontal turning centers) have been taken to illustrate the proposed decision framework. The originalities of the proposed method are as follows.

- The research work proposes a new MCDM technique capable of incorporating four core decision factors namely performance rating of alternatives, weights of the criteria, impact of the experts' judging capability and degree of reputation of the manufacturers. Neither any open literature nor any algorithm of past researchers has yet considered the fourth factor into the evaluation and decision making process.
- Heterogeneous group decision making process is adopted to achieve more accurate assessment and appropriate decision.
- Key factors influencing green selection of CNC machine tools have been significantly considered in the decision making procedure.

This study is organized as follows: Section 1 describes a detailed literature review and research gap. Section 2 describes proposed methodology for evaluation of alternatives. Section 3 describes the numerical examples and illustrations based on research problems. Section 4 describes the results and discussions of the numerical examples. Section 5 describes conclusion and future research direction.

2. Proposed Methodology

This section presents the proposed approach for finding the performance analysis of the suitable alternatives. Stepwise formulation of the proposed approach is as follows.

Step 1: Formation of decision matrix consists of performance rating of each alternative with respect to each criterion. If there are m number of alternatives and n number of criteria then there will be m$\times$n number of entries that is performance rating of the alternatives. Here A_i denotes alternative, C_j denotes j^{th} criterion and x_{ij} denotes performance rating of alternative A_i with respect to criteria C_j. The decision matrix may be expressed as follows:

$$D = \begin{array}{c} \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & \cdots & C_j & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix} \end{array} \tag{1}$$

Step 2: Construction of criteria weight matrix in terms of linguistic variables assessed by each of experts' experience and perception. If there are p number of experts and n number of criteria, there will be p$\times$n number of assessment in linguistic terms. The experts will award each of the criteria using the described set of nine degrees of linguistic variables. The criteria weight matrix may assume the following form as expressed in Eq. (2). Here E_i represents i^{th} expert and L_{ij}^{c} denotes the weight of the criterion C_j in linguistic terms awarded by i^{th} expert.

$$C_1 \quad \cdots \quad C_j \quad \cdots \quad C_n$$

$$C_W = \begin{array}{c} E_1 \\ \vdots \\ E_i \\ \vdots \\ E_p \end{array} \begin{bmatrix} L_{11}^c & \cdots & L_{1j}^c & \cdots & L_{1n}^c \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{i1}^c & \ldots & L_{ij}^c & \ldots & L_{in}^c \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{p1}^c & \ldots & L_{pj}^c & \ldots & L_{pn}^c \end{bmatrix} \tag{2}$$

Step 3: Building the expertise weight matrix in terms of linguistic variables. If there are p numbers of alternatives then there will be $p \times p$ number of assessment values in the matrix. The nine degrees of linguistic variables accommodated in Table 9 will add as a source of the assessment values. Where L_{ij}^E denotes linguistic variable expressing weight of Expert E_i assessed by expert E_j. The expertise weight matrix may be represented as follows:

$$E = \begin{array}{c} \\ E_1 \\ \vdots \\ E_i \\ \vdots \\ E_p \end{array} \begin{array}{c} \begin{array}{ccccc} E_1 & \cdots & E_j & \cdots & E_p \end{array} \\ \begin{bmatrix} L_{11}^E & \cdots & L_{1j}^E & \cdots & L_{1p}^E \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{i1}^E & \ldots & L_{ij}^E & \ldots & L_{ip}^E \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{p1}^E & \ldots & L_{pj}^E & \ldots & L_{pp}^E \end{bmatrix} \end{array} \tag{3}$$

Step 4: Formation of degree/level of reputation of manufacturer matrix. If p be the number of experts and q be the number of manufacturers then in this matrix there will be $p \times q$ number of elements, each represents the degree of reputation of manufacturer. In the following matrix M_i denotes i^{th} manufacturer, E_j represents j^{th} expert and L_{ij}^M denotes linguistic variable expressing weight of manufacturer M_i assessed by expert E_j. The degree/level of reputation of manufacturer matrix can be represented in the form of Eq. (4).

$$R = \begin{array}{c} \\ M_1 \\ \vdots \\ M_i \\ \vdots \\ M_q \end{array} \begin{array}{c} \begin{array}{ccccc} E_1 & \cdots & E_j & \cdots & E_p \end{array} \\ \begin{bmatrix} L_{11}^M & \cdots & L_{1j}^M & \cdots & L_{1p}^M \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{i1}^M & \ldots & L_{ij}^M & \ldots & L_{ip}^M \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{q1}^M & \ldots & L_{qj}^M & \ldots & L_{qp}^M \end{bmatrix} \end{array} \tag{4}$$

Step 5: The linguistic variables are not suitable for ultimate decision making. Therefore it is required to be transformed into a corresponding crisp number. An appropriate conversion technique is required for the purpose. The following Eq. (5) is recommended for conversion of a fuzzy number into corresponding crisp number to transform it into numerical value to facilitate the computation procedure.

$$x_{ij} = \frac{l_{ij} + 4m_{ij} + u_{ij}}{6} \tag{5}$$

Step 6: Normalization required to be accomplished to convert raw data into specified range in general 0 to1. It also makes the raw data into dimensionless numbers. Sometimes some non-beneficial data is converted into a beneficial one or vise-versa. The current algorithm recommends the following technique for the normalization procedure.

$$y_{ij}^{\theta} = \begin{cases} \dfrac{x_{ij} - (x_j)_{\min}}{(x_j)_{\max} - (x_j)_{\min}}, j \in B \\ \dfrac{(x_j)_{\max} - x_{ij}}{(x_j)_{\max} - (x_j)_{\min}}, j \in C \end{cases} \tag{6}$$

where, $y_{ij}^{\theta} = y_{ij}^{c}$ for criteria weight, $y_{ij}^{\theta} = y_{ij}^{r}$ for performance rating of alternatives, $y_{ij}^{\theta} = y_{ij}^{E}$ for weight of experts, $y_{ij}^{\theta} = y_{ij}^{M}$ for weight of manufacturers, B stands for benefit criteria, C stands for cost criteria or Non- benefit criteria and $(x_j)_{max}$ represents maximum value under j^{th} criterion. $(x_j)_{min}$ denotes the minimum value under j^{th} criterion.

Step 7: In this current investigation it is recommended to transform criteria weight, weight of expertise and degree of reputation within a prescribed range. It is also suggested the transformation or standardization of criteria weight, weight of expertise and degree of reputation by using the following Eq. (7) – Eq. (9) respectively.

$$\omega_i^{CS} = \frac{\omega_i^{C}}{10}, \text{ for criteria weight} \tag{7}$$

$$\omega_i^{ES} = \frac{\omega_i^{ES}}{10}, \text{ for experts weight} \tag{8}$$

$$\omega_i^{RS} = \frac{\omega_i^{R}}{10}, \text{ for reputation weight} \tag{9}$$

Step 8: The weight of an expert is required to be assessed to have proper impact on the decision making procedure. This has been done using two equations. The following formula is recommended for computation of aggregate value of the weight of j^{th} expert.

$$e_j = \left[\prod_{j=1}^{P} \left(y_{ij}^{E} \right) \right]^{\frac{1}{p}} \tag{10}$$

where, p means number of experts.

Step 9: In this step the normalization of aggregate weight of experts is carried out in such a way that it falls into desired range 0 to 1 but the sum of the weights is not necessarily unity. Eq. (11) is recommended for the computation of normalized weight of the j^{th} expert.

$$\overline{e}_j = \frac{e_j}{\max(e_j)} \tag{11}$$

Here, $\overline{e}_j$ is the normalized weight of j^{th} expert.

Step 10: Since criteria weight is assessed by experience and knowledge of heterogeneous group decision makers therefore the corresponding weight of criteria should be modified by the respective decision makers' judgment capability or importance weights. This investigation recommends the following corrected/modified criteria weight matrix for the purpose.

$$M = \begin{array}{c} \\ E_1 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & \cdots & C_j & \cdots & C_p \end{array} \\ \begin{bmatrix} y_{11}^{c}\overline{e}_1 & \cdots & y_{1j}^{c}\overline{e}_j & \cdots & y_{1p}^{c}\overline{e}_p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{i1}^{c}\overline{e}_1 & \dots & y_{ij}^{c}\overline{e}_j & \dots & y_{ip}^{c}\overline{e}_p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{n1}^{c}\overline{e}_1 & \dots & y_{nj}^{c}\overline{e}_j & \dots & y_{np}^{c}\overline{e}_p \end{bmatrix} \end{array} \tag{12}$$

where, $\overline{e}_j$ is the relative weight of j^{th} expertise relative weight.

Step11: The sum of all criteria is taken as unity in general. For carrying out the accomplishment of obtaining the aggregate weight value of the criteria set as 1(unity) an equation is introduced. The Eq.(13) is proposed for the calculation of integrated weight of criteria by a heterogeneous experts group.

$$w_j = \frac{\left[\left[\prod_{j=1}^{p}\left(y_{ij}^{c}\overline{e}_j\right)\right]^{\frac{1}{p}}\right]}{\sum_{j=1}^{n}\left[\left[\prod_{j=1}^{p}\left(y_{ij}^{c}\overline{e}_j\right)\right]^{\frac{1}{p}}\right]} \tag{13}$$

where, n is number of criteria chosen for evaluation of alternatives.

Step 12: The degree of reputation of the manufacturers of CNC machine tools is assessed by the knowledge and perception of the experts. Therefore the matrix expressing the degree of manufacturers' reputation is required to be corrected/ modified by the respective weight of the expert. This modification is carried out in the following matrix.

$$R_M = \begin{array}{c} \\ M_1 \\ \vdots \\ M_i \\ \vdots \\ M_q \end{array} \begin{array}{c} \begin{array}{ccccc} E_1 & \cdots & E_j & \cdots & E_p \end{array} \\ \begin{bmatrix} y_{11}^{M}\overline{e}_1 & \cdots & y_{1j}^{M}\overline{e}_j & \cdots & y_{1p}^{M}\overline{e}_p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{i1}^{M}\overline{e}_1 & \cdots & y_{ij}^{M}\overline{e}_j & \cdots & y_{ip}^{M}\overline{e}_p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{q1}^{M}\overline{e}_1 & \cdots & y_{qj}^{M}\overline{e}_j & \cdots & y_{qp}^{M}\overline{e}_p \end{bmatrix} \end{array} \tag{14}$$

Here y_{ij}^{M} represents normalized weight for i^{th} manufacturer's reputation awarded by j^{th} expert.

Step 13: Calculation of relative weight of manufacturers by heterogeneous experts has been accomplished by the following formula. This ensures the weight of each manufacturer lies between 0 to 1 but the sum of the weights is not necessarily 1.

$$w_j^{M} = \left[\prod_{j=1}^{p}\left(y_{ij}^{M}\overline{e}_j\right)\right]^{\frac{1}{p}} = \frac{\left[\prod_{j=1}^{p}\left(y_{ij}^{M}\overline{e}_j\right)\right]^{\frac{1}{p}}}{\max\left[\left[\prod_{j=1}^{p}\left(y_{ij}^{M}\overline{e}_j\right)\right]^{\frac{1}{p}}\right]} \tag{15}$$

where, w_j^{M} is the relative weight of manufacturers assessed by experts.

Step 14: The normalized performance rating is combined with normalized weight of criteria for the accurate assessment of alternatives. The combined value thus computed is termed as weighted normalized performance rating which is incorporated in the normalized weighted decision matrix. The calculation procedure is accomplished by the following formula.

$$z_{ij} = w_j \times y_{ij}^{r} \tag{16}$$

$$N_w = \begin{array}{c} \\ M_1 \\ \vdots \\ M_i \\ \vdots \\ M_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & \cdots & C_j & \cdots & C_p \end{array} \\ \begin{bmatrix} z_{11} & \cdots & z_{1j} & \cdots & z_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ z_{i1} & \cdots & z_{ij} & \cdots & z_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ z_{n1} & \cdots & z_{nj} & \cdots & z_{mn} \end{bmatrix} \end{array} \tag{17}$$

Step 15: Matrix of exponential deviation performance index

$$
\begin{array}{c} \\ \\ \\ P_E = \\ \\ \\ \end{array}
\begin{array}{c} \\ A_1 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array}
\begin{array}{c} \begin{array}{ccccc} C_1 & \cdots & C_j & \cdots & C_n \end{array} \\
\begin{bmatrix} \left(EXP(z_{11})-1\right) & \cdots & \left(EXP(z_{1j})-1\right) & \cdots & \left(EXP(z_{1n})-1\right) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \left(EXP(z_{i1})-1\right) & \cdots & \left(EXP(z_{ij})-1\right) & \cdots & \left(EXP(z_{in})-1\right) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \left(EXP(z_{m1})-1\right) & \cdots & \left(EXP(z_{mj})-1\right) & \cdots & \left(EXP(z_{mn})-1\right) \end{bmatrix} \end{array} \tag{18}
$$

Step16: The present algorithm proposes nonlinear exponential function for assessment of relative performance index of the alternatives. Computation of relative performance index (RPI) for i^{th} alternative is given by the following equation.

$$
RPI_i = \sum_{j=1}^{n} \left[EXP(z_{ij} - 1 \right] \tag{19}
$$

Step 17: The relative performance index is modified by the impact of relative weight of manufacturers. The following equation is proposed for the computation of modified performance index (MPI) for each alternative.

$$
MPI_j = \sum_{j=1}^{n} \left[EXP(z_{ij} - 1 \right] \times \frac{\left[\prod_{j=1}^{p} (y_{ij}^{M} \overline{e}_j \right]^{\frac{1}{p}}}{\max \left[\left[\prod_{j=1}^{p} (y_{ij}^{M} \overline{e}_j \right]^{\frac{1}{p}} \right]} \tag{20}
$$

Step 18: The relative performance index is normalized to control the values within the prescribed range 0 to 1. This objective is fulfilled by the application of the equation for computation of normalized performance index (NPI) for alternative A_i as follows.

$$
NPI_i = \frac{\sum_{j=1}^{n} \left[EXP(z_{ij} - 1) \right] \times \frac{\left[\prod_{j=1}^{p} (y_{ij}^{M} \overline{e}_j) \right]^{\frac{1}{p}}}{\max \left[\left[\prod_{j=1}^{p} (y_{ij}^{M} \overline{e}_j) \right]^{\frac{1}{p}} \right]}}{\max \left[\sum_{j=1}^{n} \left[EXP(z_{ij} - 1) \right] \times \frac{\left[\prod_{j=1}^{p} (y_{ij}^{M} \overline{e}_j) \right]^{\frac{1}{p}}}{\max \left[\left[\prod_{j=1}^{p} (y_{ij}^{M} \overline{e}_j) \right]^{\frac{1}{p}} \right]} \right]} \tag{21}
$$

Step 19: Arrange the alternatives in the increasing order of their normalized performance indices. Select the alternative having the highest normalized performance index value as the best alternative.
The flow chart depicting the proposed methodology is clearly presented in Fig. 1. The proposed technique is demonstrated in three different examples to ascertain its applicability and effectiveness in industrial applications.

3. Numerical examples and Illustrations

In this section, the proposed approach is illustrated on three different examples uunder subsections 3.1, 3.2 and 3.3.

An automotive spare parts manufacturing company in Delhi national capital region in India is engaged in manufacturing a high variety of piston, piston rings and crank shafts for multi-axle automotive vehicles. To address the competitive manufacturing standard and to satisfy the needs of the fierce competitive market the company has decided to purchase suitable CNC vertical machining centers, CNC horizontal machining centers and CNC horizontal turning centers to meet the supply demand.

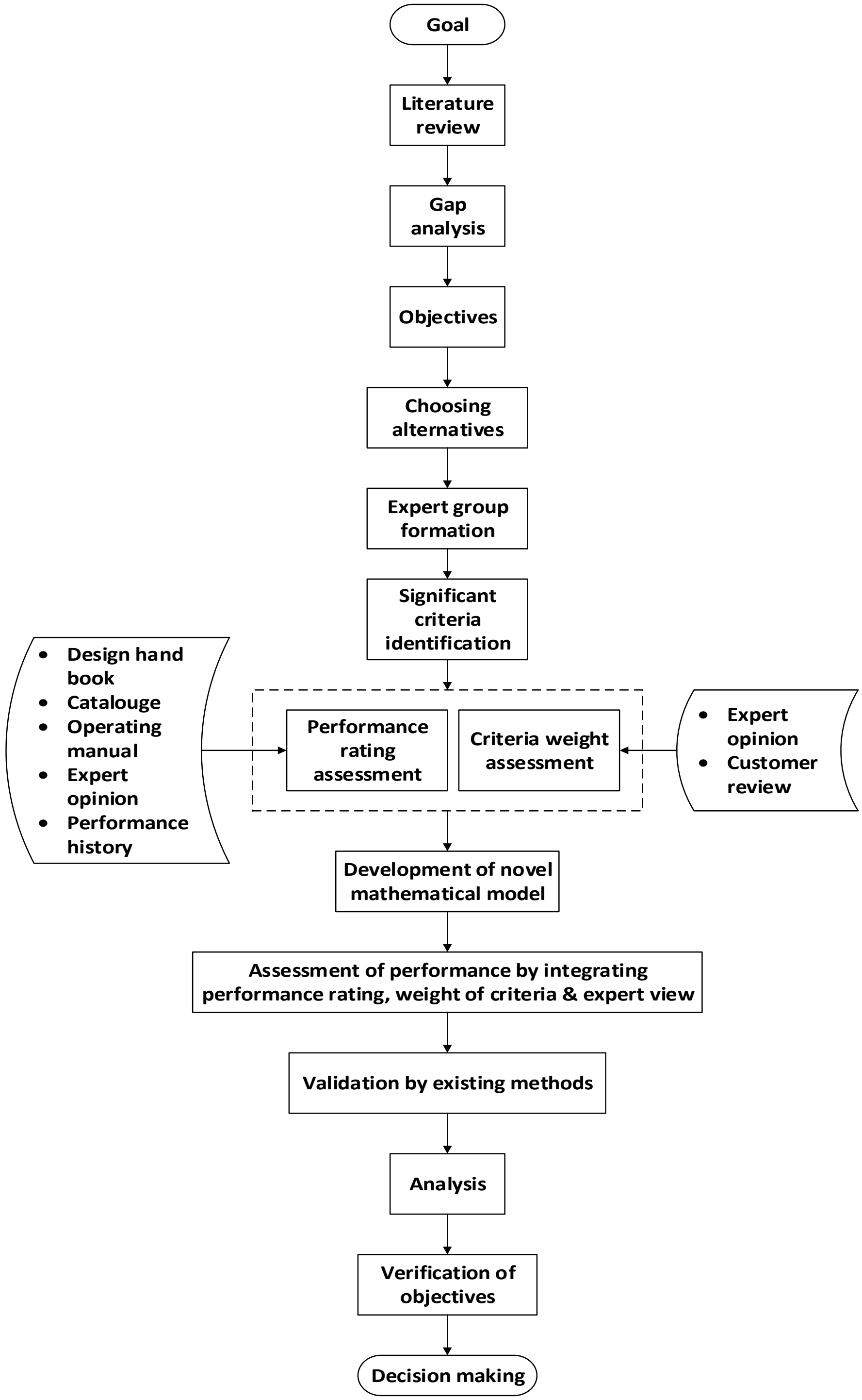

Fig. 1 Scheme of the proposed decision framework

Furthermore, based on the collective opinion of a heterogeneous group of six experts, the manufacturing company has suggested several relevant diverse conflicting criteria including green criteria for the respective machine tools to be purchased. In this evaluation process some top CNC manufacturers are chosen such as Jyoti CNC automation Ltd., Philips machine tool India private Ltd., Ace manufacturing Systems Ltd., Yamazaki, Mazak, Mori Seiki, Hyundai, Hitachi, Makino and Okuma. In this study the entire decision process is supervised by six decision making experts whose qualifications and job accountabilities are shown in Table 2.

In the decision analysis all qualitative criteria are assigned with triangular fuzzy numbers against the linguistic values. Linguistic-TFN interpretation is presented in Tables 3-4.

Table 2
Detailed expert information

Expert	Educational qualification	Experience	Gender	Designation	Job accountability
E1	Master of Commerce	More than 30 years	Female	Customer relationship manager	To Manage Customer-Business Communications
E2	Master of Management	More than 20 years	Female	Marketing and Sales manager	Market research and cost analysis
E3	Diploma in Mechanical Engineering	More than 15 years	Male	CNC Operator	To operate various CNC machine tools
E4	Bachelor of Engineering (Production)	More than 17 years	Female	Production planning and control engineer	Scheduling, programming & Inventory control
E5	Bachelor of Engineering (Manufacturing)	More than 20 years	Male	Production supervisor	Supervising CNC machining & allied works
E6	Master of Engineering (Mechanical)	More than 25 years	Male	Maintenance engineer	Maintenance of Computer controlled equipment

Table 3
Linguistic variables and triangular fuzzy numbers for performance rating of CNC machine tools

Description	Symbol	Corresponding Triangular Fuzzy Number
Outstanding	O	(8,9,10)
Excellent	E	(7,8,9)
Very Good	VG	(6,7,8)
Good	G	(5,6,7)
Medium	M	(4,5,6)
Fair	F	(3,4,5)
Satisfactory	S	(2,3,4)
Poor	P	(1,2,3)
Very Poor	VP	(0,1,2)

Table 4
Linguistic variables and TFN for weights of the criteria, experts and reputation of manufacturers

Description	Abbreviation	Corresponding Triangular Fuzzy Number
Extremely high	EH	(8,9,10)
Very high	VH	(7,8,9)
High	H	(6,7,8)
Slightly high	SH	(5,6,7)
Medium	M	(4,5,6)
Slightly low	SL	(3,4,5)
Low	L	(2,3,4)
Very low	VL	(1,2,3)
Extremely low	EL	(0,1,2)

3.1 Example 1: Performance assessment of CNC Vertical machining centers

In this sub-section the proposed methodology is illustrated with a decision problem on CNC vertical machining centers. The computer controlled vertical machining centers employs a vertically oriented spindle from the tool holder that approaches the work piece mounted on their table to perform machining operations across the top of a work piece. It performs metal removal operations for complex curved machine parts. Through this illustrative example it can be shown that choosing a suitable vertical machining center is really a tricky one for decision makers.

Table 5
Decision matrix of CNC vertical machining centers

Alternatives	C1 (+)	C2 (+)	C3 (+)	C4 (+)	C5 (+)	C6 (+)	C7 (+)	C8 (+)	C9 (+)	C10 (+)	C11 (+)	C12 (+)	C13 (-)	C14 (-)	C15 (-)
MC1	650× 400 × 500	400	8000	7.5	16	120	14	3	30	VG	SH	M	VL	H	H
MC2	600× 400 × 460	400	8000	13.5	20	80	18	4	24	VG	SH	H	L	VH	L
MC3	610× 460 × 510	400	8000	7.5	16	100	18	3	36	F	H	M	SL	H	M
MC4	600× 450 × 500	300	8000	11	24	80	18	4	30	VG	VH	L	M	H	L
MC5	760 ×500 × 450	400	8000	7.5	18	80	16	4	32	S	SH	SL	M	VH	H
MC6	610× 460 × 510	600	8000	7.5	18	80	18	5	36	G	H	L	VL	SH	M
MC7	600× 400 × 460	850	12000	18.5	20	80	18	8	26	E	L	SH	M	L	VH
MC8	762 × 406 ×508	1361	8100	14	20	89	16	3	25.4	M	VH	SL	H	H	H
MC9	610× 508 × 610	1360	9000	17.2	20	90	18	5	38	G	EH	M	VL	H	M
MC10	560× 460 × 460	500	8000	11	20	90	16	5	40	F	H	VL	M	H	L
MC11	650× 480 × 510	400	8000	5.5	16	85	18	4	16	VG	SH	L	H	SH	SH
MC12	600× 410 × 460	300	10000	7.5	24	90	18	3	48	S	M	VH	L	VH	SH

In the evaluation process of alternatives decision variables are considered as C1: X-Y-Z tool travel (mm mm mm), C2: Maximum load capacity (kg.), C3: Spindle speed (rpm), C4: Motor power (kW), C5: Number of tool stations, C6: Maximum tool diameter (mm), C7: T-slot width (mm), C8: Number of T-slots, C9: X-Y-Z Rapid tool traverse (m/min), C10: CNC controller, C11: Use of local materials, C12: Life expectancy, C13: Carbon emission, C14: Work fluid contamination and C15: Noise effect. Here C13, C14 and C15 criteria are considered to be lower the better (Non-beneficial criteria) whereas others criteria C1 to C12 are considered to be higher the better (Beneficial criteria). For computation purposes, alternatives, criteria and manufacturers are coded as MC1, MC2…MC12; C1, C2…C15 and M1, M2…M12 respectively.

Step 1: After finally determining all decision variables and feasible alternatives, a heterogeneous expert group establishes a decision matrix for the numerical example 1 as shown in Table 5 according to Eq.(1). The linguistic weight of performance rating is put using the described set of nine degrees of linguistic variables as shown in Table 3.

Step 2: In this step the criteria weight matrix is formed in terms of linguistic variables according to each of experts' experience and perception according to Eq. (2). The linguistic weight of the criteria is put in Table 6 using the described set of nine degrees of linguistic variables.

Table 6

Criteria weight matrix in linguistic variables assessed by heterogeneous experts

Experts	C1 (+)	C2 (+)	C3 (+)	C4 (+)	C5 (+)	C6 (+)	C7 (+)	C8 (+)	C9 (+)	C10 (+)	C11 (+)	C12 (+)	C13 (-)	C14 (-)	C15 (-)
E1	VH	M	M	M	M	VH	M	L	H	H	M	H	VH	L	M
E2	M	M	VH	VH	VH	L	VL	H	M	VH	SL	SL	L	H	VH
E3	L	L	SH	M	M	SL	H	L	H	M	M	M	SL	L	M
E4	M	M	VH	SH	L	M	L	M	VH	H	H	SL	M	M	SH
E5	VH	VL	M	VH	H	L	SL	M	M	VH	M	H	L	M	VH
E6	M	M	VH	L	VH	M	L	SL	H	SL	VH	M	M	SL	L

Table 7

Experts' weights matrix measured by mutual judicious capability of the experts

Expert	E1	E2	E3	E4	E5	E6
E1	H	M	H	H	M	M
E2	M	VH	M	M	H	L
E3	L	L	H	M	L	H
E4	H	M	M	VH	M	H
E5	M	H	M	M	H	VL
E6	H	L	H	L	H	VH

Step 3: In this step the experts' weights matrix is formed according to Eq.(3) and are presented in Table 7. The linguistic weight of experts by assigning mutual judicious capability of the experts is put using the described set of nine degrees of linguistic variables as shown in Table 4.

Step 4: In this step the degree/level of reputation of the manufacturer's matrix is formed according to Eq.(4) and is represented in Table 8. The linguistic weight of the manufacturers as assigned by experts is put using the described set of nine degrees of linguistic variables as shown in Table 4.

Table 8

Matrix of degree/level of reputation of manufacturers assessed by heterogeneous experts

Manufacturers	E1	E2	E3	E4	E5	E6
M1	VH	H	VH	M	M	SL
M2	H	VH	H	L	H	VH
M3	M	M	H	M	H	M
M4	M	H	VH	M	L	H
M5	H	L	H	L	H	VH
M6	VH	H	VH	M	L	H
M7	H	L	H	M	VH	H
M8	M	H	H	VH	M	L
M9	H	L	V	M	H	H
M10	H	H	VH	H	SL	L
M11	VH	VH	L	SL	M	H
M12	VL	M	H	VL	M	H

Step 5: For computation purpose all linguistic variables as assigned by experts are converted into pure crisp values according to Eq.(5) and are presented in Table 9.

Step 6: All crisp values in Table 9 are normalized according to Eq.(6). For example in column C1 and row MC1 in Table 10 the normalized value is 0.25 which is calculated as y_{ij}^{c} = [(130000000-110400000) / (189026800-110400000)] =0.25 and thus remaining values are presented in Table 10.

Table 9
Decision matrix in terms of pure crisp numbers

Alternatives	C1 (+)	C2 (+)	C3 (+)	C4 (+)	C5 (+)	C6 (+)	C7 (+)	C8 (+)	C9 (+)	C10 (+)	C11 (+)	C12 (+)	C13 (-)	C14 (-)	C15 (-)
MC1	130000000	400	8000	7.5	16	120	14	3	30	7	6	5	2	7	7
MC2	110400000	400	8000	13.5	20	80	18	4	24	7	6	7	3	8	3
MC3	143106000	400	8000	7.5	16	100	18	3	36	4	7	5	4	7	5
MC4	135000000	300	8000	11	24	80	18	4	30	7	8	3	5	7	3
MC5	171000000	400	8000	7.5	18	80	16	4	32	3	6	4	5	8	7
MC6	143106000	600	8000	7.5	18	80	18	5	36	6	7	3	2	6	5
MC7	110400000	850	12000	18.5	20	80	18	8	26	8	3	6	5	3	8
MC8	157160976	1361	8100	14	20	89	16	3	25.4	5	8	4	7	7	7
MC9	189026800	1360	9000	17.2	20	90	18	5	38	6	9	5	2	7	5
MC10	118496000	500	8000	11	20	90	16	5	40	4	7	2	5	7	3
MC11	159120000	400	8000	5.5	16	85	18	4	16	7	6	3	7	6	6
MC12	113160000	300	10000	7.5	24	90	18	3	48	3	5	8	3	8	6
Max	189026800	1361	12000	18.5	24	120	18	8	48	8	9	8	7	8	8
Min	110400000	300	8000	5.5	16	80	14	3	16	3	3	2	2	3	3

Table 10
Normalized decision matrix of performance rating of alternatives

Alternatives	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
MC1	0.25	0.09	0.00	0.15	0.00	1.00	0.00	0.00	0.44	0.80	0.50	0.50	1.00	0.20	0.20
MC2	0.00	0.09	0.00	0.62	0.50	0.00	1.00	0.20	0.25	0.80	0.50	0.83	0.80	0.00	1.00
MC3	0.42	0.09	0.00	0.15	0.00	0.50	1.00	0.00	0.63	0.20	0.67	0.50	0.60	0.20	0.60
MC4	0.31	0.00	0.00	0.42	1.00	0.00	1.00	0.20	0.44	0.80	0.83	0.17	0.40	0.20	1.00
MC5	0.77	0.09	0.00	0.15	0.25	0.00	0.50	0.20	0.50	0.00	0.50	0.33	0.40	0.00	0.20
MC6	0.42	0.28	0.00	0.15	0.25	0.00	1.00	0.40	0.63	0.60	0.67	0.17	1.00	0.40	0.60
MC7	0.00	0.52	1.00	1.00	0.50	0.00	1.00	1.00	0.31	1.00	0.00	0.67	0.40	1.00	0.00
MC8	0.59	1.00	0.03	0.65	0.50	0.23	0.50	0.00	0.29	0.40	0.83	0.33	0.00	0.20	0.20
MC9	1.00	1.00	0.25	0.90	0.50	0.25	1.00	0.40	0.69	0.60	1.00	0.50	1.00	0.20	0.60
MC10	0.10	0.19	0.00	0.42	0.50	0.25	0.50	0.40	0.75	0.20	0.67	0.00	0.40	0.20	1.00
MC11	0.62	0.09	0.00	0.00	0.00	0.13	1.00	0.20	0.00	0.80	0.50	0.17	0.00	0.40	0.40
MC12	0.04	0.00	0.50	0.15	1.00	0.25	1.00	0.00	1.00	0.00	0.33	1.00	0.80	0.00	0.40
SUM	4.52	3.46	1.78	4.78	5.00	2.60	9.50	3.00	5.92	6.20	7.00	5.17	6.80	3.00	6.20

Step 7: In this step criteria weight matrix (Table 6), Experts' weights matrix measured by mutual judicious capability of the experts (Table 7) and Matrix of degree/level of reputation of manufacturers assessed by heterogeneous experts (Table 8) are standardized according to Eqs.(7)-(8)-(9) respectively and are shown in Tables 11-13 respectively**.** For example, the corresponding TFN for VH (weight of criteria C1 in linguistic terms) in row E1 and column C1 in Table 6 is (7, 8, 9). The crisp value is

Table 11
Matrix of standardization of criteria weights assessed by heterogeneous experts

Experts	C1 +	C2 +	C3 +	C4 +	C5 +	C6 +	C7 +	C8 +	C9 +	C10 +	C11 +	C12 +	C13 -	C14 -	C15 -
E1	0.8	0.5	0.5	0.5	0.5	0.8	0.5	0.3	0.5	0.7	0.5	0.9	0.8	0.3	0.5
E2	0.5	0.5	0.8	0.8	0.8	0.3	0.1	0.7	0.5	0.8	0.3	0.3	0.3	0.7	0.8
E3	0.3	0.3	0.6	0.5	0.5	0.4	0.5	0.3	0.7	0.5	0.5	0.5	0.4	0.3	0.5
E4	0.5	0.5	0.8	0.6	0.9	0.5	0.3	0.5	0.8	0.7	0.7	0.4	0.5	0.5	0.6
E5	0.8	0.1	0.5	0.8	0.7	0.3	0.4	0.5	0.5	0.8	0.5	0.7	0.3	0.5	0.8
E6	0.5	0.5	0.8	0.9	0.8	0.5	0.3	0.4	0.7	0.3	0.8	0.5	0.5	0.4	0.3

Table 12
Matrix of standardization of experts' weights measured by mutual judicious capability of the experts

Experts	E1	E2	E3	E4	E5	E6
E1	0.7	0.5	0.7	0.7	0.5	0.5
E2	0.5	0.8	0.5	0.5	0.7	0.3
E3	0.3	0.3	0.7	0.5	0.3	0.7
E4	0.7	0.5	0.5	0.8	0.5	0.7
E5	0.5	0.7	0.5	0.5	0.7	0.2
E6	0.7	0.3	0.7	0.3	0.7	0.8

Table 13

Matrix of standardization of degree/level of reputation of manufacturers assessed by experts

Manufacturers	E1	E2	E3	E4	E5	E6
M1	0.8	0.7	0.8	0.5	0.5	0.4
M2	0.7	0.8	0.7	0.3	0.7	0.8
M3	0.5	0.5	0.7	0.5	0.7	0.5
M4	0.5	0.7	0.8	0.5	0.3	0.7
M5	0.7	0.3	0.7	0.3	0.7	0.8
M6	0.8	0.7	0.8	0.5	0.3	0.7
M7	0.7	0.3	0.7	0.5	0.8	0.7
M8	0.5	0.7	0.7	0.8	0.5	0.3
M9	0.7	0.3	0.8	0.5	0.7	0.7
M10	0.7	0.7	0.8	0.7	0.4	0.3
M11	0.8	0.8	0.3	0.4	0.5	0.7
M12	0.1	0.5	0.7	0.2	0.5	0.7

$x_{ij} = \dfrac{7+4\times 8+9}{6} = 8 = \omega_i^C$. Now standardized value of criteria weight is $\omega_i^{CS} = \dfrac{8}{10} = 0.8$ in Table 11. Thus reaming standardized values are presented in Tables 11-13.

Step 8: In this step aggregate value of the weight of expert (e_j) is computed using Table 12 and according to Eq. (10). For example, $e_j = (0.7\times0.5\times0.7\times0.7\times0.5\times0.5)^{0.166} = 0.593$ (Table 12) and thus remaining values of aggregate values of weight of expert (e_j) are put in Table 14.

Step 9: In his step the normalized weight of experts ($\overline{e}_j$) are calculated. For example, $\overline{e}_j$ = (0.593/0.606)=0.978 and thus remaining values of normalized weight are put in Table 14.

Table 14

Normalized weight of heterogeneous experts

Experts	GM (e_j)	Normalized weight $(\overline{e}_j)$
E1	0.593	0.978
E2	0.527	0.868
E3	0.435	0.717
E4	0.606	1.000
E5	0.482	0.794
E6	0.541	0.892
Max	0.606	

Step 10: In this step modified criteria weight matrix is formed using Eq.(12). For example, 0.8 (y_{ij}^c) in C1 column and E1 row in standardization of criteria weight matrix (Table 11) multiplied with corresponding element 0.978 ($\overline{e}_j$) in Table 14 makes 0.78($y_{ij}^c \overline{e}_j$) as shown in Table 15 and thus remaining elements are presented in Table 15. Step 11: In this step the integrated weight of criteria by heterogeneous experts is calculated according to Eq.(13). For example, integrated weight of criteria w_j = (0.47/6.75)=0.07 in Table 15 and so obtained remaining values are presented in Table 15.

Table 15

Modified weight of the criteria influenced by the heterogeneous experts

Experts	C1 +	C2 +	C3 +	C4 +	C5 +	C6 +	C7 +	C8 +	C9 +	C10 +	C11 +	C12 +	C13 -	C14 -	C15 -	Sum
E1	0.78	0.49	0.49	0.49	0.49	0.78	0.49	0.29	0.49	0.68	0.49	0.88	0.78	0.29	0.49	
E2	0.43	0.43	0.70	0.70	0.70	0.26	0.09	0.61	0.43	0.70	0.26	0.26	0.26	0.61	0.70	
E3	0.22	0.22	0.43	0.36	0.36	0.29	0.36	0.22	0.50	0.36	0.36	0.36	0.29	0.22	0.36	
E4	0.50	0.50	0.80	0.60	0.90	0.50	0.30	0.50	0.80	0.70	0.70	0.40	0.50	0.50	0.60	
E5	0.64	0.08	0.40	0.64	0.56	0.24	0.32	0.40	0.40	0.64	0.40	0.56	0.24	0.40	0.64	
E6	0.45	0.45	0.71	0.80	0.71	0.45	0.27	0.36	0.62	0.27	0.71	0.45	0.45	0.36	0.27	
GM	0.47	0.31	0.57	0.58	0.59	0.38	0.27	0.38	0.53	0.52	0.46	0.45	0.38	0.38	0.48	6.75
Weight	0.07	0.05	0.08	0.09	0.09	0.06	0.04	0.06	0.08	0.08	0.07	0.07	0.06	0.06	0.07	

Step 12: In this step the matrix of modified degree of reputation of manufacturers is formed according to Eq.(14) and is presented in Table 16. For example, 0.8 (y_{ij}^M) in C1 column and E1 row in standardization of criteria weight matrix (Table

13) multiplied with corresponding element 0.978 ($\overline{e}_j$) in Table 14 makes 0.782 ($y_{ij}^M \overline{e}_j$) and thus remaining elements are presented in Table 16.

Step 13: In this step the relative weight of manufacturers by heterogonous experts (w_j^M) is computed according to Eq.(15). For example, w_j^M = (0.519/0.553)=0.938, and thus remaining weights of manufacturers are shown in Table 16.

Table 16
Modified degree/level of reputation of manufacturers as measured by the heterogeneous experts

Manufacturers	E1	E2	E3	E4	E5	E6	GM	Relative weight of manufacturers (w_j^M)
M1	0.782	0.608	0.573	0.500	0.397	0.357	0.519	0.938
M2	0.684	0.695	0.502	0.300	0.556	0.714	0.553	1.000
M3	0.489	0.434	0.502	0.500	0.556	0.446	0.487	0.880
M4	0.489	0.608o	0.573	0.500	0.238	0.624	0.484	0.874
M5	0.684	0.260	0.502	0.300	0.556	0.714	0.470	0.849
M6	0.782	0.608	0.573	0.500	0.238	0.624	0.523	0.945
M7	0.684	0.260	0.502	0.500	0.635	0.624	0.512	0.924
M8	0.489	0.608	0.502	0.800	0.397	0.267	0.484	0.874
M9	0.684	0.260	0.573	0.500	0.556	0.624	0.512	0.924
M10	0.684	0.608	0.573	0.700	0.317	0.267	0.493	0.891
M11	0.782	0.695	0.215	0.400	0.397	0.624	0.477	0.861
M12	0.097	0.434	0.502	0.200	0.397	0.624	0.320	0.579
Max	---	---	---	---	---	---	0.553	---

Step 14: In this step weighted normalized performance rating matrix is computed by multiplying normalized performance rating (y_{ij}^r) in Table 10 with normalized weight of criteria (w_j) in Table 15 according to Eq.(16). For example, the element of weighted normalized performance rating matrix =0.25 0.07=0.017 and thus other elements of the matrix are put accordingly in Table 17 according to Eq.(17).

Step 15: In this step, using Table 17 the matrix of exponential deviation performance index for the alternatives is formed according to Eq. 18. For example, the element in MC1 row and C1 column (Table 18) =0.0175= [EXP (0.017) -1] and so obtained other exponential deviations are shown in Table 18.

Step 16: In this step the relative performance index (RPI) of each alternative is calculated according to Eq.(19) and values of RPI are presented in Table 19.

Step 17: In this step the modified performance index (MPI) is calculated according to Eq. 20. For example, MPI=0.34 0.938=0.32 which is obtained by multiplying RPI value in Table 19 with corresponding value of relative weight of manufacturers (w_j^M) and thus remaining values of MPI are presented in Table 19.

Step 18: In this step the normalized performance index (NPI) is calculated using Eq. 21. For example, NPI=0.32/0.61=0.5273 which is determined by NPI_i=[MPI_i / $max(MPI_i)$] and thus remaining values are put in Table 19.

Step 19: In this step the ranking of alternatives are computed based on the values of NPI and is presented in Table 19.

Table 17
Modified weighted decision matrix by heterogeneous expert's preferences

Alternatives	C1 +	C2 +	C3 +	C4 +	C5 +	C6 +	C7 +	C8 +	C9 +	C10 +	C11 +	C12 +	C13 -	C14 -	C15 -
MC1	0.01	0.00	0.00	0.01	0.00	0.05	0.00	0.00	0.03	0.06	0.03	0.03	0.05	0.01	0.01
MC2	0.00	0.00	0.00	0.05	0.04	0.00	0.04	0.01	0.02	0.06	0.03	0.05	0.04	0.00	0.07
MC3	0.02	0.00	0.00	0.01	0.00	0.02	0.04	0.00	0.04	0.01	0.04	0.03	0.03	0.01	0.04
MC4	0.02	0.00	0.00	0.03	0.08	0.00	0.04	0.01	0.03	0.06	0.05	0.01	0.02	0.01	0.07
MC5	0.05	0.00	0.00	0.01	0.02	0.00	0.02	0.01	0.03	0.00	0.03	0.02	0.02	0.00	0.01
MC6	0.02	0.01	0.00	0.01	0.02	0.00	0.04	0.02	0.04	0.04	0.04	0.01	0.05	0.02	0.04
MC7	0.00	0.02	0.08	0.08	0.04	0.00	0.04	0.05	0.02	0.07	0.00	0.04	0.02	0.05	0.00
MC8	0.04	0.04	0.00	0.05	0.04	0.01	0.02	0.00	0.02	0.03	0.05	0.02	0.00	0.01	0.01
MC9	0.06	0.04	0.02	0.07	0.04	0.01	0.04	0.02	0.05	0.04	0.06	0.03	0.05	0.01	0.04
MC10	0.00	0.00	0.00	0.03	0.04	0.01	0.02	0.02	0.05	0.01	0.04	0.00	0.02	0.01	0.07
MC11	0.04	0.00	0.00	0.00	0.00	0.00	0.04	0.01	0.00	0.06	0.03	0.01	0.00	0.02	0.02
MC12	0.00	0.00	0.04	0.01	0.08	0.01	0.04	0.00	0.07	0.00	0.02	0.06	0.04	0.00	0.02

Table 18
Exponential deviation performance index for the alternatives

Alternatives	C1 +	C2 +	C3 +	C4 +	C5 +	C6 +	C7 +	C8 +	C9 +	C10 +	C11 +	C12 +	C13 -	C14 -	C15 -
MC1	0.0175	0.0043	0.0000	0.0133	0.0000	0.0585	0.0000	0.0000	0.0348	0.0641	0.0345	0.0339	0.0585	0.0112	0.0144
MC2	0.0000	0.0043	0.0000	0.0543	0.0450	0.0000	0.0411	0.0112	0.0197	0.0641	0.0345	0.0572	0.0465	0.0000	0.0742
MC3	0.0293	0.0043	0.0000	0.0133	0.0000	0.0288	0.0411	0.0000	0.0501	0.0156	0.0463	0.0339	0.0347	0.0112	0.0439
MC4	0.0220	0.0000	0.0000	0.0370	0.0921	0.0000	0.0411	0.0112	0.0348	0.0641	0.0582	0.0112	0.0230	0.0112	0.0742
MC5	0.0550	0.0043	0.0000	0.0133	0.0223	0.0000	0.0203	0.0112	0.0398	0.0000	0.0345	0.0225	0.0230	0.0000	0.0144
MC6	0.0293	0.0129	0.0000	0.0133	0.0223	0.0000	0.0411	0.0225	0.0501	0.0477	0.0463	0.0112	0.0585	0.0225	0.0439
MC7	0.0000	0.0238	0.0878	0.0896	0.0450	0.0000	0.0411	0.0572	0.0247	0.0807	0.0000	0.0455	0.0230	0.0572	0.0000
MC8	0.0421	0.0465	0.0021	0.0577	0.0450	0.0129	0.0203	0.0000	0.0232	0.0315	0.0582	0.0225	0.0000	0.0112	0.0144
MC9	0.0719	0.0465	0.0213	0.0803	0.0450	0.0143	0.0411	0.0225	0.0552	0.0477	0.0702	0.0339	0.0585	0.0112	0.0439
MC10	0.0072	0.0086	0.0000	0.0370	0.0450	0.0143	0.0203	0.0225	0.0604	0.0156	0.0463	0.0000	0.0230	0.0112	0.0742
MC11	0.0439	0.0043	0.0000	0.0000	0.0000	0.0071	0.0411	0.0112	0.0000	0.0641	0.0345	0.0112	0.0000	0.0225	0.0290
MC12	0.0024	0.0000	0.0430	0.0133	0.0921	0.0143	0.0411	0.0000	0.0813	0.0000	0.0229	0.0690	0.0465	0.0000	0.0290

Table 19
Ranking order of competitive alternatives CNC vertical machining centers

Alternatives	Relative performance index (RPI)	Modified performance index (MPI)	Normalized performance index (NPI)	Rank
MC1	0.34	0.32	0.5273	8
MC2	0.45	0.45	0.7366	3
MC3	0.35	0.31	0.5058	9
MC4	0.48	0.42	0.6840	4
MC5	0.26	0.22	0.3609	12
MC6	0.42	0.40	0.6494	5
MC7	0.576	0.53	0.8678	2
MC8	0.39	0.34	0.5528	7
MC9	0.66	0.61	1.0000	1
MC10	0.39	0.34	0.5601	6
MC11	0.27	0.23	0.3777	11
MC12	0.45	0.26	0.4293	10
Max	---	0.61	---	---

3.2 Example 2: Performance assessment of CNC Horizontal machining centers

In this subsection the proposed methodology is illustrated with a decision problem on CNC Horizontal machining centers. Computer controlled horizontal machining center employs spindle in a horizontal orientation that favors continuous production work high surface finish. Horizontal centers don't have to be cleared from the table as it allows chips to fall away effectively. Horizontal centers enable loading work on one pallet while machining operation on the other pallet resulting in saving manufacturing cost and time considerably.

Table 20
Decision matrix of CNC Horizontal machining centers

Alternatives	C1 (+)	C2 (+)	C3 (+)	C4 (+)	C5 (+)	C6 (+)	C7 (+)	C8 (+)	C9 (+)	C10 (+)	C11 (+)	C12 (-)	C13 (-)	C14 (-)
HC1	600 × 500 × 500	600 × 700	40	20	28	300	7000	12.5	6	7	6	3	5	7
HC2	560 × 640 × 640	630 × 800	60	50	40	280	10000	17	7	5	3	7	6	6
HC3	630 × 600 × 600	650 × 750	50	30	35	250	9000	15.5	6	6	4	5	7	6
HC4	560 × 640 × 640	630 × 780	60	50	45	350	11000	16.5	7	4	5	3	6	7
HC5	650 × 600 × 650	700 × 600	75	45	50	410	9000	18	6	5	3	7	4	5
HC6	700 × 560 × 500	630 × 520	70	55	30	300	10000	13.5	6	7	4	4	5	5
HC7	660 × 660 × 660	700 × 610	50	40	25	350	9000	12	6	5	4	5	7	7
HC8	610 × 610 × 610	610 × 520	60	48	50	300	8000	10.5	7	4	5	3	6	5
HC9	750 × 610 × 660	800 × 710	50	35	60	400	9000	10.5	6	7	7	5	7	6
HC10	710 × 610 × 610	680 × 550	50	40	50	390	11500	11.5	7	5	4	4	5	7

In the assessment process of CNC Horizontal machining centers, decision criteria are considered as C1: X-Y-Z tool travel (mm mm mm) , C2: Max workpiece size (mm mm), C3: X-Y-Z rapid feed (m/min), C4: Cutting speed (m/min), C5: Number of tools, C6: Maximum Tool length (mm), C7: Spindle speed (rpm). C8: Motor power (kW), C9: CNC controller, C10: Maintainability, C11: Life expectancy, C12: Impact on air quality, C13: Noise effect and C14: Power saving. Here C12, C13 and C14 criteria are considered to be lower the better (Non-beneficial criteria) whereas others criteria C1 to C11 are

considered to be higher the better (Beneficial criteria). For calculation purpose coded used are as C1, C2…C14 for criteria, as M1, M2…M10 for manufacturers and as HC1, HC2…HC10 for alternatives against model no HX4000, A40, HSX540, A51nx, MH500, J6, FF5000/40, HX400iG, HX500iTGA, HX400i, FTGA respectively.
Now finally a heterogeneous expert group establishes a decision matrix with crisp values and are presented in Table 20.

The steps are similar to example no. 1 to calculate the ranking order of the alternatives. The ranking order of the alternatives is presented in Table 21.

Table 21
Ranking order of competitive alternatives CNC Horizontal machining centers

Alternatives	Relative performance index	Modified performance index	Normalized performance index	Rank
HC1	0.3521	0.3302	0.6153	9
HC2	0.5169	0.5169	0.9631	3
HC3	0.3945	0.3673	0.6844	8
HC4	0.6043	0.4199	0.7823	6
HC5	0.6317	0.5368	1.0000	1
HC6	0.6084	0.5322	0.9915	2
HC7	0.3478	0.3217	0.5993	10
HC8	0.4957	0.4336	0.8078	5
HC9	0.5441	0.3998	0.7449	7
HC10	0.5274	0.4701	0.8757	4
Max	---	0.5368	---	---

3.3 Example 3: Performance assessment of CNC Horizontal turning centers

In this subsection the proposed methodology is illustrated with a decision problem on CNC Horizontal turning centers. CNC horizontal turning center is an advanced computer numerically controlled multi-axes machine tool along with a multitude of cutting capabilities including milling, drilling and tapping. The entire arrangement is housed within an enclosed setup to ensure safety to operators. CNC turning is a subtractive manufacturing operation in which a cylindrical workpiece rotates while cutting tool is fed to the work piece, thus removing material to give a high surface finish of the workpiece with cost effectiveness. Automotive manufacturers opt for this CNC machine tool as it can remove material from the work piece in large quantities while retaining a consistent finished product.

In this example alternatives (Horizontal turning centers) are coded as TC1, TC2…..TC11 against manufacturer 'model no DX60, DX100, NLX2000, DX135nvu, DX200/5Anvu, DX250nvu, NLX1500, DX200-3, CTX2500, AX200, SM-16G respectively and criteria are coded as C1, C2, C3…C12 against the decision variables such as C1: Swing diameter over bed (mm), C2: Maximum turning length (mm), C3: Cross longitudinal travel (mm mm), C4: X-Z rapid feed (m/min), C5: Spindle speed (rpm), C6: Spindle motor power (kW), C7: Maximum bar capacity (mm), C8: CNC controller, C9: Use of local materials, C10: Power saving, C11: Iron dust pollution and C12: Noise effect. Here C10, C11 and C12 criteria are considered to be lower the better (Non-beneficial criteria) whereas others criteria C1 to C9 are considered to be higher the better (Beneficial criteria).
Now as in examples 1 and example 2, the heterogeneous expert group establishes a decision matrix with crisp value and is presented in Table 22.

Now computation of the ranking order of alternatives is carried out according to the same proposed algorithm used in example 1 and example 2 and are presented in Table 23.

Table 22
Decision matrix of CNC Horizontal turning centers

Alternatives	C1 (+)	C2 (+)	C3 (+)	C4 (+)	C5 (+)	C6 (+)	C7 (+)	C8 (+)	C9 (+)	C10 (-)	C11 (-)	C12 (-)
TC1	360	220	140 × 260	30 × 45	6.6	3800	35	8	5	3	6	5
TC2	410	180	210 × 270	24 × 35	9.8	3000	44	6	6	6	3	3
TC3	450	250	210 × 360	35 × 48	7.5	3200	45	7	5	3	5	6
TC4	400	240	120 × 240	28 × 40	8.5	4200	40	8	6	5	3	5
TC5	500	280	200 × 320	25 × 45	9.5	3500	55	6	5	3	5	3
TC6	475	310	150 × 230	26 × 42	8.2	2800	52	8	6	6	3	5
TC7	386	315	190 × 280	35 × 50	6.5	3500	48	6	5	3	6	5
TC8	470	320	175 × 320	24 × 35	8.5	3020	45	7	5	6	5	6
TC9	430	370	160 × 290	30 × 45	10.5	4000	48	6	6	6	3	5
TC10	480	325	200 × 300	24 × 35	9.15	4500	52	6	5	3	6	6
TC11	400	350	18 × 0275	25 × 45	8.25	4500	45	7	6	6	5	5

Table 23
Ranking order of competitive alternatives CNC Horizontal turning centers

Alternatives	Relative performance index	Modified performance index	Normalized performance index	Rank
TC1	0.320	0.266	0.428	11
TC2	0.487	0.403	0.649	6
TC3	0.499	0.371	0.597	8
TC4	0.529	0.506	0.814	3
TC5	0.621	0.621	1.000	1
TC6	0.564	0.485	0.781	5
TC7	0.401	0.361	0.582	9
TC8	0.341	0.277	0.447	10
TC9	0.619	0.516	0.831	2
TC10	0.496	0.496	0.799	4
TC11	0.500	0.402	0.648	7
	Max	0.621		

4. Results and discussion

In this section the proposed MCDM method is discussed to demonstrate its efficacy, reliability and compatibility in present industrial scenario.

4.1 Discussion on numerical example 1

According to the proposed heterogeneous decision makers' priority based MCDM method the best alternative is one which has the highest NPI value and the lowest NPI value is the worst alternative. From Table 19 it is clear that MC9 has the highest NPI value (1.0000) and MC5 has lowest NPI value (0.3609). So MC9 is the best alternative and MC5 is the worst one amongst the feasible alternatives as determined by the proposed MCDM method. The ranking order of the alternatives obtained by the proposed method is MC9>MC7>MC2>MC4>MC6>MC10>MC8>MC1>MC3>MC12>MC11>MC5. To validate the proposed method, some well-established methods such as Grey relational analysis, SAW, TOPSIS and PROMETHEE have been suggested by competent experts and corresponding ranking orders are shown in Table 24.

Table 24
Validation of the proposed method by comparison with existing methodologies

Alternatives	GRA		SAW		TOPSIS		PROMETHEE		Proposed method
	GRG	Rank	Composite score	Rank	Relative closeness	Rank	Net flow	Rank	Rank
MC1	0.032	7	0.337	10	0.40	7	-0.095	10	8
MC2	0.036	5	0.440	5	0.42	6	0.036	5	3
MC3	0.031	10	0.346	9	0.37	12	-0.065	9	9
MC4	0.037	4	0.466	3	0.46	5	0.060	3	4
MC5	0.027	12	0.256	12	0.39	8	-0.183	12	12
MC6	0.033	6	0.413	6	0.37	11	0.003	6	5
MC7	0.043	2	0.558	2	0.56	2	0.161	2	2
MC8	0.032	9	0.380	7	0.50	3	-0.027	7	7
MC9	0.044	1	0.646	1	0.58	1	0.261	1	1
MC10	0.032	8	0.377	8	0.38	9	-0.036	8	6
MC11	0.029	11	0.263	11	0.38	10	-0.160	11	11
MC12	0.038	3	0.441	4	0.49	4	0.045	4	10

From Table 24 and Fig. 2 it is clear that alternatives MC 9 and MC7 are positioned rank 1 and rank 2 respectively by proposed method and well established methods. The rank values of other alternatives as obtained by well-established methods deviate slightly to moderate from proposed method's rank value. For example MC4 has rank 4 for two times, rank 3 for two times and rank 5 for one time as shown in Fig. 2. Similarly MC6 has rank 5 for one time, rank 6 for three times and rank 11 for one time as shown in Table 24. Since the sole purpose of the manufacturing company is to choose the best alternative, the heterogeneous experts find the proposed method as a preeminent option for such type of decision making.

4.1.1 Sensitivity analysis

Sensitivity analysis is an imperative part of decision making. It shows the sensitivity to ranking order of the alternatives if change is made in machine selection index (MSI) with respect to coefficient of decision making attitude. However a very few researchers have focused on this ranking analysis. Here

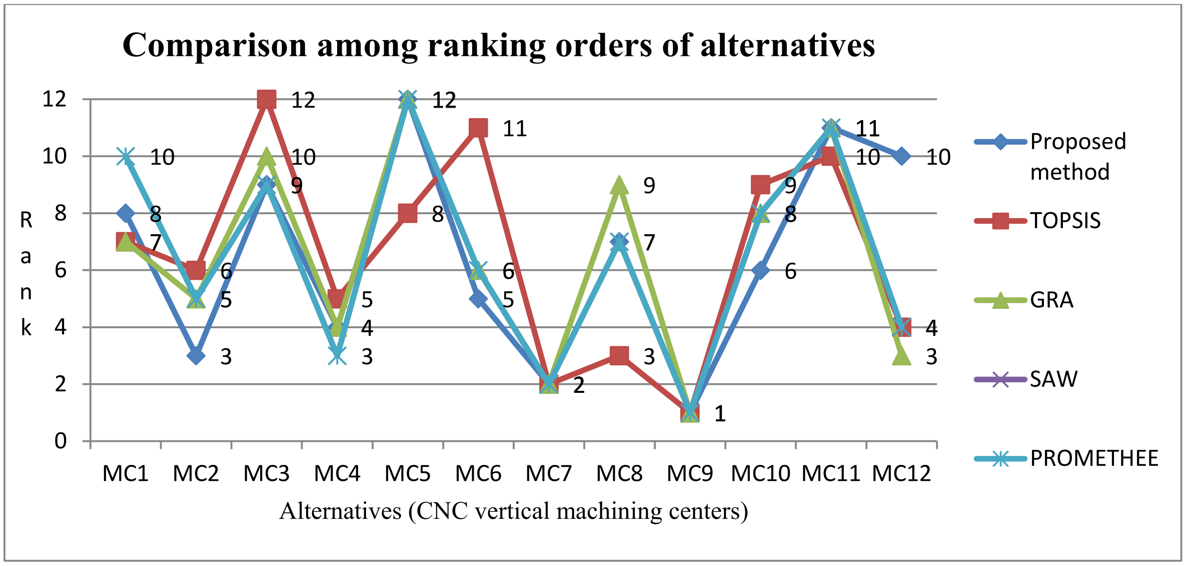

Fig. 2 Comparison among the ranking order of the alternatives by various methods

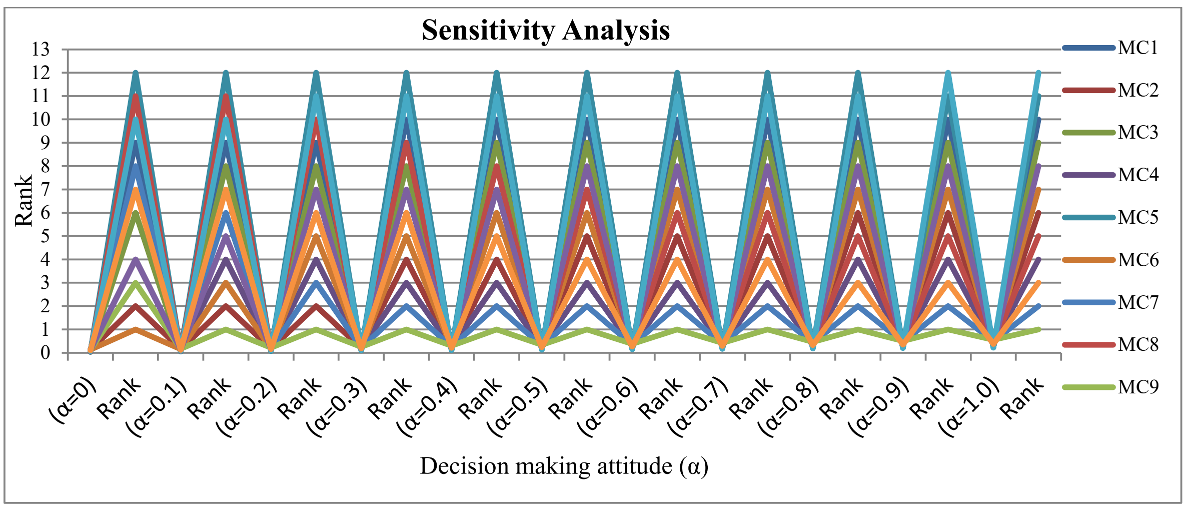

Fig. 3 Relationship between alternative rankings and decision making attitudes

Table 25
Sensitivity analysis to ranking order of the alternatives on variation of decision making attitude

Alternatives	Relative performance (+) w_1	Relative performance (-) w_2	Rank	Rank	Rank	Rank	Rank	Rank	Rank	Rank	Rank	Rank	Rank
MC1	0.261	0.084	7	9	9	9	10	10	10	10	10	10	10
MC2	0.331	0.121	2	3	3	4	4	5	5	5	6	6	6
MC3	0.263	0.090	6	7	8	8	9	9	9	9	9	9	9
MC4	0.371	0.108	4	4	4	3	3	3	3	3	3	4	4
MC5	0.223	0.037	11	12	12	12	12	12	12	12	11	11	11
MC6	0.297	0.125	1	2	5	5	6	6	6	7	7	7	7
MC7	0.495	0.080	8	6	2	2	2	2	2	2	2	2	2
MC8	0.362	0.026	12	11	10	10	8	7	7	6	5	5	5
MC9	0.550	0.114	3	1	1	1	1	1	1	1	1	1	1
MC10	0.277	0.108	4	5	6	7	7	8	8	8	8	8	8
MC11	0.217	0.052	10	10	11	11	11	11	11	11	12	12	12
MC12	0.379	0.076	9	8	7	6	5	4	4	4	4	3	3
Decision making attitude (α)			α=0.0	α=0.1	α=0.2	α=0.3	α=0.4	α=0.5	α=0.6	α=0.7	α=0.8	α=0.9	α=1.0

Table 25 and Fig.3 show that slight intermittent changes in rank places of alternatives occur when changes are made in the coefficient of decision making attitude (α) which ranges from 0.0 to 1.0. In this analysis mathematical expression used to compute machine selection index is $MSI = [\alpha w_1 + (1\text{-}\alpha) w_2]$, where w_1,w_2 represent relative performance weight based on benefit criteria and relative performance weight based on non-benefit criteria respectively. For example, w_1=0.261, w_2=0.084 and α=0.0 and substituting these values in the mathematical expression it gives MSIs 0.084, 0.121, 0.090, 0.0108, 0.037, 0.125, 0.080, 0.026, 0.114, 0.108, 0.052 and 0.076 for MC1, MC2, MC3, MC4, MC5, MC6, MC7, MC8, MC9, MC10, MC11 and MC12 respectively. Thus remaining values of MSI are calculated followed by rank calculations which are presented in Table 25. *4.1.2 Statistical analysis*

The authors find this analysis quite indispensable to measure the effectiveness of this proposed method as it hasn't been applied previously as far as literature review is concerned. In this study the Rank Correlation Coefficient has been incorporated to evaluate the strength of the relationship between two ranking variables of alternatives.

From Table 26 it is clear that Spearman's correlation coefficients increases as 0.53<0.78<0.83. Here the Proposed method-PROMETHEE and the Proposed method-SAW have same coefficient 0.83. The Proposed method-GRA has coefficient 0.78. The Proposed method-TOPSIS has coefficient 0.53. From these it is evident that a moderate to stronger positive linear relationship exists between ranking orders of alternatives obtained by proposed method and well established methods.

From sensitivity and statistical analysis it is evident that the proposed method is reasonably practicable and reliable to decision makers.

Table 26
Spearman's rank correlation coefficient of the ranking orders

Sl. No.	Methods in pair	Ranking variables of alternatives												Rank correlation coefficient
		MC1	MC2	MC3	MC4	MC5	MC6	MC7	MC8	MC9	MC10	MC11	MC12	
1.	Proposed Method	8	3	9	4	12	5	2	7	1	6	11	10	0.83
	PROMETHEE	10	5	9	3	12	6	2	7	1	8	11	4	
2.	Proposed Method	8	3	9	4	12	5	2	7	1	6	11	10	0.78
	GRA	7	5	10	4	12	6	2	9	1	8	11	3	
3.	Proposed Method	8	3	9	4	12	5	2	7	1	6	11	10	0.53
	TOPSIS	7	6	12	5	8	11	2	3	1	9	10	4	
4.	Proposed Method	8	3	9	4	12	5	2	7	1	6	11	10	0.83
	SAW	10	5	9	3	12	6	2	7	1	8	11	4	

4.2 Discussion on numerical example 2

According to the proposed heterogeneous decision based MCDM method the ranked one alternative is one which has highest NPI value and the lowest NPI value is the worst alternative. From Table 21 it is clear that HC5 has the highest NPI value (1.0000) and HC7 has lowest NPI value (0.5993). So HC5 is the best alternative and HC7 the worst amongst the selected alternatives determined by the proposed MCDM decision framework. The ranking order of the alternatives as obtained by the method is HC5>HC6>HC2>HC10>HC8>HC4>HC9>HC3>HC1>HC7. To ensure its effectiveness the proposed method is validated similarly as in example 1 by PROMETHEE, SAW, GRA and TOPSIS methods and is shown in Table 27. The comparison of ranking orders by well-established method and proposed method are presented in Fig.4.

Table 27
Validation of the proposed method by comparison with existing methodologies

Alternatives	TOPSIS		GRA		SAW		PROMETHEE		Propose method
	Relative closeness	Rank	GRG	Rank	Composite score	Rank	net flow	Rank	Rank
HC1	0.406	9	0.037	8	0.342	9	-0.164	9	9
HC2	0.508	5	0.039	7	0.504	6	0.017	6	3
HC3	0.421	8	0.033	9	0.387	8	-0.114	8	8
HC4	0.553	3	0.043	4	0.589	3	0.111	3	6
HC5	0.583	1	0.048	1	0.612	1	0.137	1	1
HC6	0.554	2	0.046	2	0.591	2	0.114	2	2
HC7	0.386	10	0.033	10	0.341	10	-0.165	10	10
HC8	0.473	7	0.041	5	0.483	7	-0.007	7	5
HC9	0.527	4	0.045	3	0.527	4	0.042	4	7
HC10	0.492	6	0.040	6	0.515	5	0.029	5	4

From Table 27 and Fig.4 it is clear that the alternative HC5, HC6 and HC7 are placed rank 1, rank 2 and rank 10 respectively by proposed method and well established methods. Although the rank places of other alternatives obtained by well-established methods deviate slightly to moderate from proposed method's rank place. Since the sole purpose of the manufacturing company is to choose the best alternative, the heterogeneous experts find the proposed method as an effective and reliable option for such type of decision making.

4.2.1 Sensitivity analysis

As carried out in example 1, the sensitivity analysis here also indicates the slight deviation in ranking order of the alternatives by proposed method with respect to changes made in coefficients of decision making attitude i.e. from α=0.0 to α=1.0 as shown in Fig. 5.

4.2.2 Statistical analysis

As carried out in example 1, Table 28 indicates that Spearman's correlation coefficients increases as 0.75<0.81<0.83 which means that a stronger positive linear relationship exists between ranking orders of alternatives obtained by proposed methods and well established methods. The correlation coefficient is 0.81for two times in case of proposed method-PROMETHEE and Proposed method-SAW. The correlation coefficient is 0.82 for one time in case of proposed method-TOPSIS and 0.75 for only one time in case of proposed method-GRA.

From sensitivity and statistical analysis it is obvious that the proposed method is highly effective in deciding the most suitable alternative.

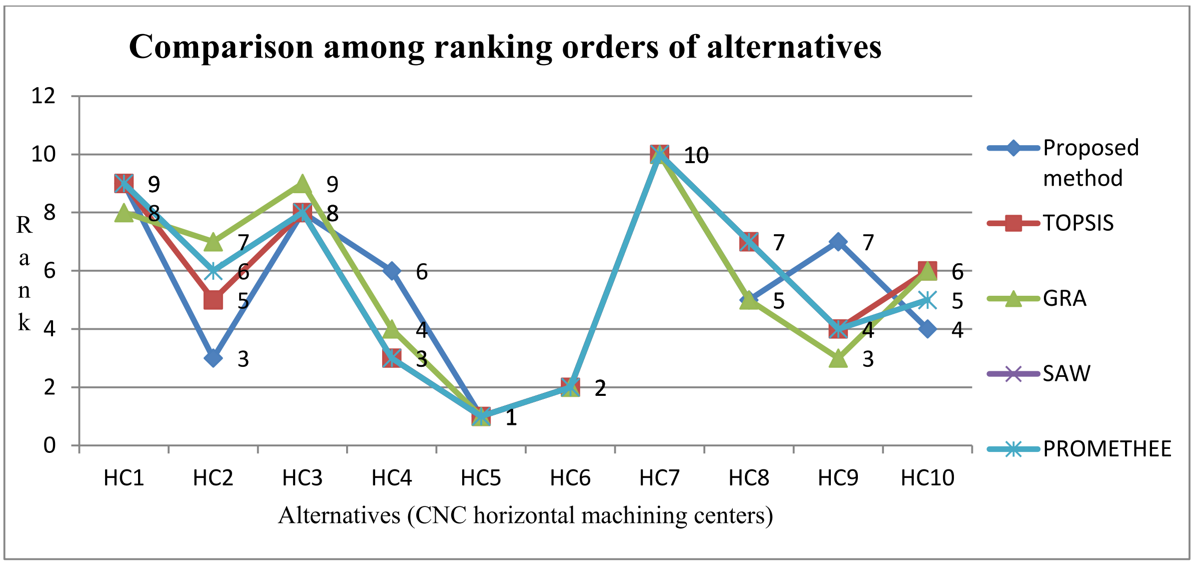

Fig. 4 Comparison among the ranking order of the alternatives by various methods

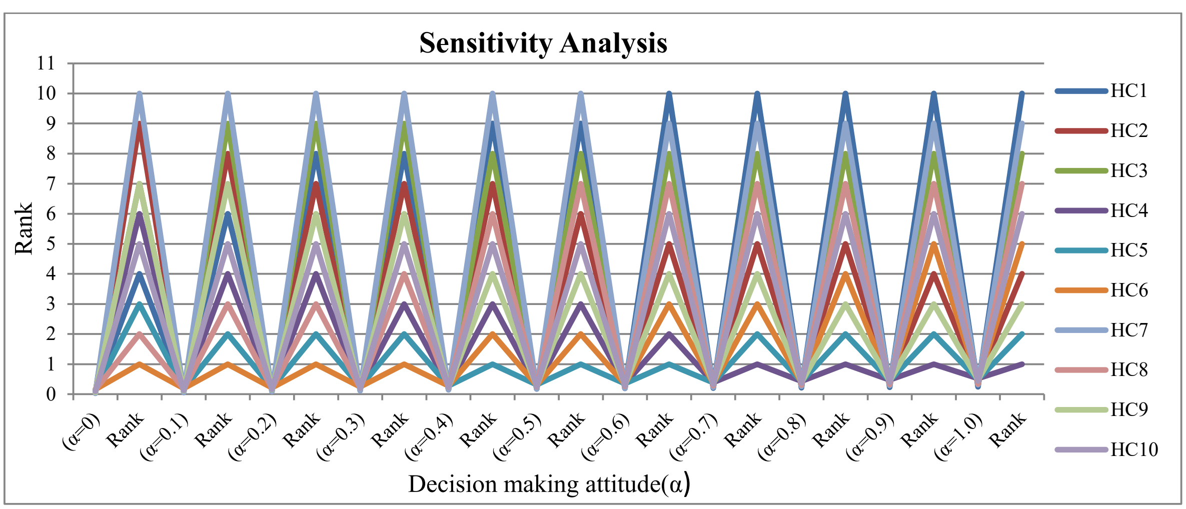

Fig. 5 Relationship between alternative rankings and decision making attitudes

Table 28
Spearman's rank correlation coefficients of the ranking orders

Sl. No.	Methods in pair	HC1	HC2	HC3	HC4	HC5	HC6	HC7	HC8	HC9	HC10	Rank correlation coefficient
1	Proposed Method	9	3	8	6	1	2	10	5	7	4	0.81
	PROMETHEE	9	6	8	3	1	2	10	7	4	5	
2	Proposed Method	9	3	8	6	1	2	10	5	7	4	0.75
	GRA	8	7	9	4	1	2	10	5	3	6	
3	Proposed Method	9	3	8	6	1	2	10	5	7	4	0.82
	TOPSIS	9	5	8	3	1	2	10	7	4	6	
4	Proposed Method	9	3	8	6	1	2	10	5	7	4	0.81
	SAW	9	6	8	3	1	2	10	7	4	5	

4.3 Discussion on numerical example 3

The priority order of the alternatives determined by the proposed method as obtained from Table 23 is TC5>TC9>TC4>TC10>TC6>TC2>TC11>TC3>TC7>TC8. According to the proposed decision framework the ranked one alternative is one which has highest NPI value and lowest NPI value is the worst. From Table 23 it is clear that TC5 has the highest NPI value (1.0000) and TC1 has lowest NPI value (0.428). So TC5 is the best alternative and TC1 is the worst one amongst the chosen alternatives determined by the proposed MCDM method. Here validation of the proposed algorithm is carried out similarly as in example 1 and example 2 and is presented in Table 29. The comparison of the ranking orders of alternatives by PROMETHEE, SAW, GRA, TOPSIS and proposed methods are shown in Fig. 6.

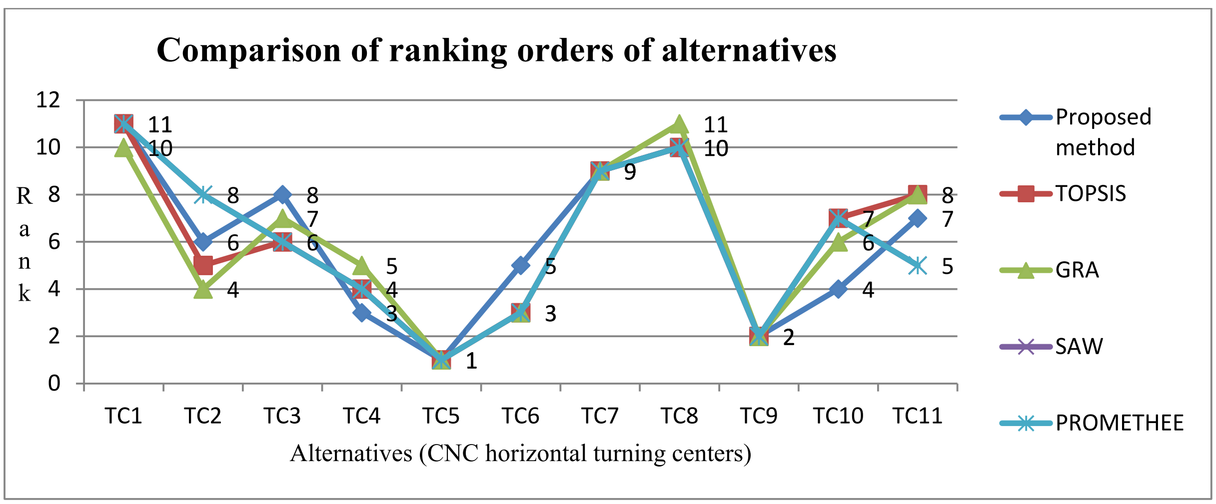

Fig. 6 Comparison among the ranking order of the alternatives by various methods

From Table 29 and Fig. 6 it is clear that the alternatives TC5, TC9, TC6 and TC7 are placed rank 1, rank 2, rank 3 and rank 9 respectively by proposed method and well established methods. Since the sole purpose of the manufacturing company is to choose the best alternative, the heterogeneous experts find the proposed method as an effective and reliable technique for such type of decision making.

4.3.1 Sensitivity analysis

As carried out in example 1and example 2, the sensitivity analysis here also shows the slight deviation in ranking order of the alternatives by proposed method with respect to changes made in coefficients of decision making attitude i.e. from α=0.0 to α=1.0 as shown in Fig. 7.

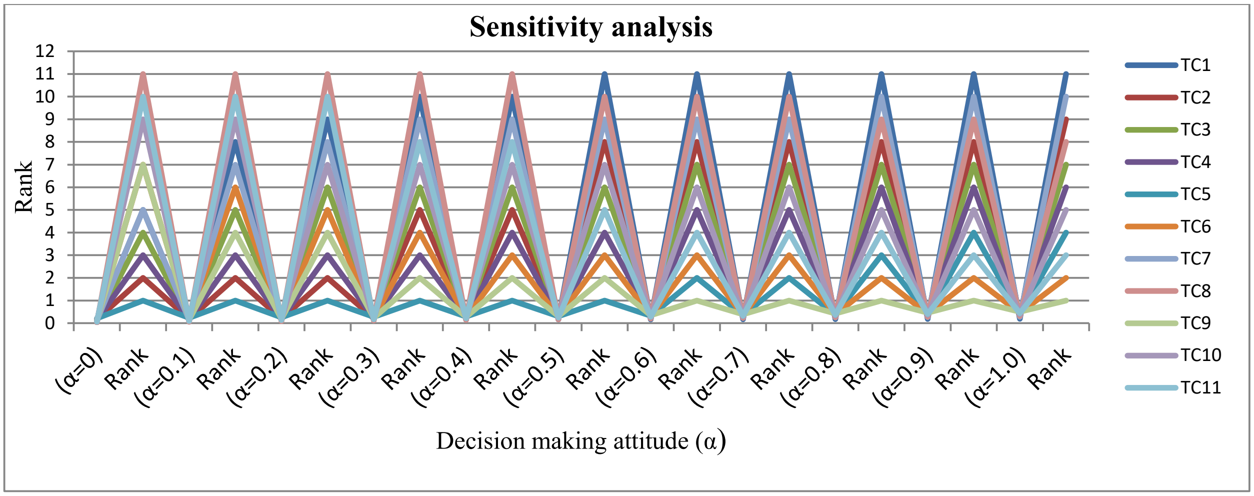

Fig. 7 Relationship between alternative rankings and decision making attitudes

Table 29
Validation of the proposed method by comparison with existing methodologies

Alternatives	TOPSIS		GRA		SAW		PROMETHEE		Proposed method
	Relative closeness	Rank	GRG	Rank	Composite score	Rank	Net flow	Rank	Rank
TC1	0.372	11	0.041	10	0.310	11	-0.179	11	11
TC2	0.494	5	0.049	4	0.469	8	-0.004	8	6
TC3	0.491	6	0.048	7	0.483	6	0.0113	6	8
TC4	0.509	4	0.049	5	0.512	4	0.0435	4	3
TC5	0.568	1	0.055	1	0.599	1	0.1384	1	1
TC6	0.531	3	0.052	3	0.544	3	0.0788	3	5
TC7	0.422	9	0.044	9	0.388	9	-0.093	9	9
TC8	0.377	10	0.038	11	0.333	10	-0.154	10	10
TC9	0.567	2	0.055	2	0.598	2	0.1373	2	2
TC10	0.487	7	0.049	6	0.478	7	0.006	7	4
TC11	0.480	8	0.046	8	0.487	5	0.0152	5	7

4.3.2 Statistical analysis

As carried out in example 1 and example 2, statistical analysis here also indicates a stronger positive linear relationship (Spearman's rank correlation coefficient 0.88 and 0.91) exists as shown in Table 30. This stronger positive linear relationship ensures that the proposed method is highly efficient. From sensitivity and statistical analysis it is obvious that the heterogeneous experts may consider the proposed method as a reliable option for such type of alternative selections.

Table 30
Spearman's rank correlation coefficient of the ranking orders

Sl. No.	Methods in pair	TC1	TC2	TC3	TC4	TC5	TC6	TC7	TC8	TC9	TC10	TC11	Rank correlation coefficient
1	Proposed Method	11	6	8	3	1	5	9	10	2	4	7	0.88
	PROMETHEE	11	8	6	4	1	3	9	10	2	7	5	
2	Proposed Method	11	6	8	3	1	5	9	10	2	4	7	0.91
	GRA	10	4	7	5	1	3	9	11	2	6	8	
3	Proposed Method	11	6	8	3	1	5	9	10	2	4	7	0.91
	TOPSIS	11	5	6	4	1	3	9	10	2	7	8	
4	Proposed Method	11	6	8	3	1	5	9	10	2	4	7	0.88
	SAW	11	8	6	4	1	3	9	10	2	7	5	

5. Conclusions

The performance assessment of CNC machine tools for manufacturing establishments is a multifaceted task as improper assessment may bring an adverse effect on manufacturing quality and productivity. Numerical examples and illustrations in this paper reflect some managerial aspects associated with proposed method i.e. flexibility and better informed decision which enable heterogeneous experts to express their ratings or opinions over the alternatives, criteria and expert judicious capability in linguistic terms as well as to assess the potential strengths and weaknesses of a decision problem. The main contributions of this paper are summarized as follows.

- This investigation proposes a new multi criteria decision making technique for the performance assessment of alternatives.

➢ This technique is effective and useful in ranking and identifying the best alternative.

➢ This study involves a group of six experts having different academic backgrounds, experience, domains and possibly inconsistent interests that lead to judicious decision towards choosing the best alternative unanimously.

➢ The heterogeneous decision makers' priority based MCDM method considers some distinctive green attributes such as CO_2 emission, toxicity, use of local materials, and impact on air quality in the performance analysis of the alternatives.

➢ The application of statistical analysis aims at proving the efficacy of the proposed method by showing a stronger positive linear relationship between two ranking variables of alternatives obtained by the proposed method and well established methods.

Moreover Sensitivity analysis indicates that no matter how decision making attitude (α) does vary, the ranking orders of the competitive alternatives remain more or less stable that makes the proposed method more reliable and practical.

While the proposed heterogeneous decision makers' priority based MCDM method provides some significant direction for choosing the best machine tool, there are some confines such as incompleteness of evaluation criteria, the integration of other MCDM approaches and the static decision assessment.
Therefore, in view of the above, the future research could be directed in a numerous ways:

- More robust version in group decision making approach for unanimous decision,
- Hybrid approach
- Artificial intelligence and machine learning based MCDM model.
- Finally, the proposed method can be implemented into other pioneer fields such as non-traditional machine tool evaluations, 3-D printer evaluations, facility assessment, construction equipment evaluations, CNC machine supplier selection etc.

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