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Scheduling stochastic two-machine flow shop problems to minimize expected makespan

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C H R O N I C L E

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A B S T R A C T

During the past few years, despite tremendous contribution on deterministic flow shop problem, there are only limited number of works dedicated on stochastic cases. This paper examines stochastic scheduling problems in two-machine flow shop environment for expected makespan minimization where processing times of jobs are normally distributed. Since jobs have stochastic processing times, to minimize the expected makespan, the expected sum of the second machine's free times is minimized. In other words, by minimization waiting times for the second machine, it is possible to reach the minimum of the objective function. A mathematical method is proposed which utilizes the properties of the normal distributions. Furthermore, this method can be used as a heuristic method for other distributions, as long as the means and variances are available. The performance of the proposed method is explored using some numerical examples.

1. Introduction

Scheduling has become a well-studied problem and there are literally tremendous efforts on providing solution strategies for various kinds of modeling formulations such as job shop and flowshop. There are literally many applications for flowshop problem (e.g. Defersha, 2010; Mahavi Mazdeh et al., 2010; Braglia et al., 2011). Zammori et al. (2011), for instance, investigated CONWIP card setting in a flow-shop system with a batch-processing machine. In addition, there are different types of flowshop problem. Araújo and Nagano (2010), for instance, studied a new effective heuristic method for the no-wait flowshop with sequence-dependent setup times problem. Mgwatu (2011) studied an integration of part selection, machine loading and machining optimization decisions for balanced workload in flexible manufacturing system.

For years, all input parameters in scheduling such as processing times were considered deterministic. For instance, Johnson (1954) studied deterministic flow shop scheduling and proposed an algorithm called Johnson's Rule. His study led to so many researches analyzing the same problem under various conditions and using different techniques to reach some optimal solutions. Reisman et al. (1997)

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reviewed 170 articles on scheduling flow shop problems, all of which were associated with the problems with contributions to the “subdiscipline”. Moreover, Farminan et al. (2004) studied 76 articles with the identical topic on scheduling deterministic flow shop problems . Saghafian and Hejazi (2005) reviewed 176 articles on flow shop problems with makespan objective function and various heuristic methods. Ruiz and Maroto (2005) performed a review on 53 heuristics presented for permutation flow shop with makespan objective function. Allahverdi and Mittenthal (1995), for instance, proposed a solution procedure for scheduling on a two-machine flowshop subject to random breakdowns with a makespan objective function.

Allahverdi and Fatih Tatari (1997), in another assignment, performed an empirical investigation on stochastic machine dominance in flowshop problem. Ruiz and Allahverdi (2007) presented some effective heuristics for no-wait flowshop with setup times to minimize total completion time. Ruiz and Stützle (2008) suggested an iterated greedy heuristic for the sequence dependent setup times flowshop problem with makespan and weighted tardiness objectives. Sayadi et al. (2010), in another assignment, presented new meta-heuristics called discrete firefly with an adaptation of local search for makespan minimization in permutation flow shop scheduling problems. Ancău (2010) discussed some weakness and strength of stochastic search in solving flowshop scheduling problem. Naderi-Benia et al. (2012) presented a two-phase fuzzy programming model for a complex bi-objective no-wait flow shop scheduling. Wang and Tang (2012) offered a discrete particle swarm optimization algorithm with self-adaptive diversity control for the permutation flowshop problem with blocking.

In stochastic flow shop problem, processing times are supposed to be stochastic and it is considered as random variable with certain probability distribution. This simple difference between stochastic and deterministic problems leads to many complexities in stochastic problems. Flowshop problem with uncertain processing time has been an open research for the past few years. Therefore, a challenging task of the previous researches is that the less attention has been paid to find methods for solving stochastic flow shop problems. If stochastic flow shop problem were limited to two-machine version and exponential processing times, then the expected makespan value would be minimized in the case that the jobs are sorted non-increasingly in terms of parameter $(1/\mu_{i1} - 1/\mu_{i2})$. This method was proposed by Talwar (1967) which is known as Talwar's Rule. Later, Cunningham and Dutta (1973) proved its optimality.

Adiri and Frostig (1984) studied a stochastic permutation-flowshop scheduling problem minimizing in distribution the schedule length. Ku and Niu (1986) obtained a sufficient condition for stochastic dominance and showed that Talwar’s Rule yields a stochastically minimal makespan. Sethi et al. (1993) offered feedback production planning in a stochastic two-machine flowshop based on asymptotic analysis and computational results. Elmaghraby and Thoney (1999) studied the two-machine stochastic flowshop problem with arbitrary processing time distributions. Moreover, Kalczynski and Kamburowski (2004) generalized Johnson and Talwar’s scheduling rules for two-machine stochastic flow shops. Soroush and Allahverdi (2005) presented a stochastic two-machine flowshop scheduling problem with total completion time criterion. Kalczynski and Kamburowski (2006) tried to apply Talwar’s Rule for Weibull distribution. Portougal and Trietsch (2006) applied Johnson's Rule for solving stochastic problems. They utilized mean processing time of every job as job processing time in Johnson's Rule and produced a response with asymptotically optimal expected makespan values in order to minimize the makespan.

Baker and Trietsch (2010) also explored three heuristic methods for two-machine stochastic model with a general distribution function. They compared Johnson’s Heuristic method and Talwar’s Heuristic method, applying mean processing time instead of job processing time in these two methods, and heuristic method of changing neighboring pairs where two neighboring jobs are separately considered and are displaced if their order can be optimized and figured out that none of these methods dominate the others. In another study, Kenneth and Dominik (2012) used three heuristic methods for flow shop problem with m machines, supposing general distributions for processing times. They investigated the performance of these algorithms in a set of problems using simulation and noticed that these algorithms had near-optimal performance.

In spite of the various researches in the last decades, few studies have been performed on flow shop problems with stochastic processing times. This paper investigates the stochastic scheduling problem in two-machine flow shop environment. The purpose of this study is to give a stochastic method to solve such problems using a stochastic mathematical model. The logic applied in this research is to emphasize on minimizing waiting times in the second machine. In this method, decisions about prioritization and scheduling of the jobs are made based on the waiting time produced in the second machine. In this regard, first, the considered mathematical model are presented in deterministic mode and then the model is customized for the stochastic version. The objective function in the proposed model is to minimize expected weighted sum of free times in the second machine and thus to minimize expected makespan. The value of objective function would be stochastic in this problem. In this research, we assume that the job processing time is a random variable with normal distribution and processing times of different jobs are independent from each other. A mathematical model is proposed for two-machine stochastic flow shop problem and its performance is examined using some numerical examples. Furthermore, this model is applicable for situations where the processing time distribution is not normally distributed or the distributions are totally unknown.

Research assumptions and definitions are presented in Section 2. In Section 3, the deterministic mathematical model are presented. Stochastic mathematical model are developed in Section 4. Then, the algorithm's performance are examined using some numerical examples in Section 5. Finally, the results are presented in the conclusion section.

2. Research assumptions and definitions

Let i be job index, j be machine index and k represent job priority index. It is supposed that $i = \{1, 2, 3, \dots, n\}$ is the set of jobs, which have to be processed in two-machine flow shop environment. Every job has stochastic processing time, $t_{ij} \sim N\left(\mu_{ij}, \sigma_{ij}^2\right)$, and jobs are independent of each other. The objective function is to minimize expected makespan. The applied symbols are as follows:

$$x_{ik} = \begin{cases} 1 & if \;\; job\, i\; has\, the\; priority\; of\;\; k \\ 0 & otherwise \end{cases} \tag{1}$$

t_{ij} : Processing time of job i on machine j ,

μ_{ij} : Mean of processing time of job i on machine j ,

σ_{ij}^2 : Variance of processing time of job i on machine j ,

C_{kj} : Completion time of job with priority k on machine j ,

$\mu_{C_{kj}}$: Mean of completion time of the job with priority k on machine j ,

$\sigma_{C_{kj}}^2$: Variance of completion time of the job with priority k on machine j ,

T_k : Waiting time of the second machine from completing the job with priority $k-1$ until starting the job with priority k ,

α : The probability of producing waiting time in the second machine is compared to this coefficient. This coefficient is called “confidence percentage”.

$$y_k = \begin{cases} 1 & \text{if } \; p(C_{k1} \geq C_{(k-1)2}) > \alpha \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The considered assumptions in this article are as follows:

- Machines have constant speed, which cannot be varied.
- The order of processing jobs is the same on the first and second machines.
- Machines are ready to be utilized at zero time.
- Every machine can operate at most one job at a time.
- Initiation of any job on the second machine would be after completion of the job on the first machine.
- Preemption of the jobs is not allowed.

3. Developing deterministic mathematical model

Since the objective function of this study is expected makespan minimization and considering the fact that makespan increase is due to increase in waiting time in the second machine, this paper focuses on minimizing sum of expected waiting times of the second machine. In this section, the problem in deterministic (non-stochastic) mode is investigated. In addition, the appropriate mathematical model is presented.

One of two S_1 and S_2 scenarios would occur in the second machine from the completion of one job to start of the next one. These scenarios are demonstrated in Fig. 1 and Fig. 2.

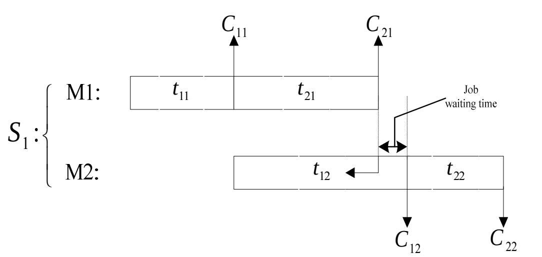

Fig. 1. First scenario and the second job waiting time

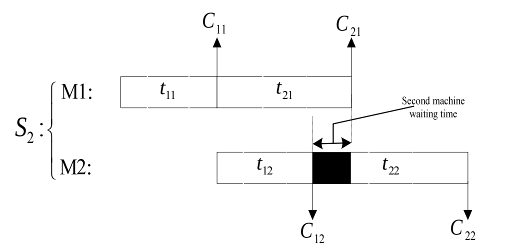

Fig. 2. Second scenario and creation of waiting time in the second machine

values of C_{k1} and C_{k2} are calculated as follows:

$$C_{k1} = \sum_{k=1}^{k}\sum_{i=1}^{n} t_{i1} x_{ik}$$

$$C_{12} = \sum_{j=1}^{2}\sum_{i=1}^{n} t_{ij} x_{i1} \tag{3}$$

$$C_{k2} = \max(C_{k1}, C_{(k-1)2}) + \sum_{i=1}^{n} t_{i2} x_{ik} \qquad for \;\; k = 2,3,\dots,n$$

Therefore, waiting time of the second machine from the completion of the job with *k-1* priority to start of the job with *k* priority is given by:

$$T_k = \max(C_{k1} - C_{(k-1)2}, 0) \tag{4}$$

Thus, for instance, waiting time of the second machine from the completion of the job with the first priority to start of the job with second priority is as follows:

$$T_1 = \max(0, C_{21} - C_{12}) \tag{5}$$

Finally, the mathematical model of the problem in deterministic mode is as follows:

$$\min \sum_{k=1}^{n} T_k$$

subject to

$$C_{12} = \sum_{j=1}^{2}\sum_{i=1}^{n} t_{ij} x_{i1}$$

$$T_1 = \sum_{j=1}^{n} t_{i1} x_{i1}$$

$$C_{k1} = \sum_{k=1}^{k}\sum_{i=1}^{k} t_{i1} x_{ik} \qquad \text{For } k = 2,3,...,n$$

$$C_{k2} = y_k.C_{k1} + (1-y_k).C_{(k-1)2} + \sum_{i=1}^{n} t_{i2} x_{ik} \qquad \text{For } k = 2,3,...,n \tag{6}$$

$$\left.\begin{aligned} C_{k1} - C_{(k-1)2} &\le M.y_k \\ C_{k1} - C_{(k-1)2} &\ge M.(y_k - 1) \end{aligned}\right. \qquad \text{For } k = 2,3,...,n$$

$$T_k = \max(C_{k1} - C_{(k-1)2}, 0) \qquad \text{For } k = 2,3,...,n$$

$$\sum_{i=1}^{n} x_{ik} = 1 \qquad \text{For } k = 1,2,3,...,n$$

$$\sum_{k=1}^{n} x_{ik} = 1 \qquad \text{For } i = 1,2,3,...,n$$

4.The problem in stochastic mode

In stochastic mode, waiting time in the second machine is calculated by the difference between C_{k1} and $C_{(k-1)2}$. Since the processing time of jobs is stochastic, so completion times will be stochastic too. Parameter α, as mentioned earlier, is confidence percentage and is defined to compare completion times. The probability of producing waiting time in the second machine is compared with this coefficient. If $p(C_{k1} > C_{(k-1)2}) > \alpha$, initiation time of the job with priority k in the second machine is supposed to be equal to the completion time of the same job in the first machine (C_{k1}) and the second machine will have waiting time (Fig. 1 shows this scenario). Furthermore, if this probability is less than α, then the initiation time of the job with priority k in the second machine is equal to completion time of the job with priority $k-1$ on the same machine. Therefore, after processing operations in the first machine, the job with priority k will wait for the second machine to complete the previous job, and there would be no waiting in the second machine. Thus, the expected value of waiting time in the second machine and consequently value of expected makespan can be obtained. Since the processing time is normally distributed, thus, C_{k1} and $C_{(k-1)2}$ will be normally distributed, as well.

$t_{ij} \sim N(\mu_{ij}, \sigma_{ij}^2)$

$$C_{12} \sim N(\mu_{c_{12}}, \sigma_{c_{12}}^2) \Rightarrow \mu_{c_{12}} = \sum_{j=1}^{2}\sum_{i=1}^{n} \mu_{ij} x_{i1}\ ,\ \sigma_{c_{12}}^2 = \sum_{j=1}^{2}\sum_{i=1}^{n} \sigma_{ij}^2 x_{i1}$$

$$C_{k1} \sim N(\mu_{C_{k1}}, \sigma_{C_{k1}}^2) \Rightarrow \mu_{C_{k1}} = \sum_{k=1}^{k}\sum_{i=1}^{n} \mu_{i1} x_{ik}\ ,\ \sigma_{C_{k1}}^2 = \sum_{k=1}^{k}\sum_{i=1}^{n} \sigma_{i1}^2 x_{ik} \tag{7}$$

$$\mu_{C_{k2}} = y_k.\mu_{C_{k1}} + (1-y_k).\mu_{C_{(k-1)2}} + \sum_{i=1}^{n} \mu_{i2} x_{ik}$$

$$\sigma^2_{C_{k2}} = y_k.\sigma^2_{C_{k1}} + (1-y_k).\sigma^2_{C_{(k-1)2}} + \sum_{i=1}^{n} \sigma^2_{i2} x_{ik}$$

$$y_k = \begin{cases} 1 & if \;\; p(C_{k1} > C_{(k-1)2}) > \alpha \\ 0 & else \end{cases}$$

Both C_{k1} and $C_{(k-1)2}$ follow normal distribution, thus:

$$C_{k1} - C_{(k-1)2} \sim N(\mu_{C_{k1}} - \mu_{C_{(k-1)2}}, \sigma^2_{C_{k1}} + \sigma^2_{C_{(k-1)2}}) \tag{8}$$

Therefore, expected sum of waiting time of the second machine from the completion of the job with priority $k-1$ until the initiation of the job with priority k is equal to:

$$p(C_{k1} > C_{(k-1)2}) = p(C_{k1} - C_{(k-1)2} > 0) = p(z > \frac{\mu_{C_{(k-1)2}} - \mu_{C_{k1}}}{\sqrt{\sigma^2_{C_{(k-1)2}} + \sigma^2_{C_{k1}}}})$$

$$= 1 - p(z \le \frac{\mu_{C_{(k-1)2}} - \mu_{C_{k1}}}{\sqrt{\sigma^2_{C_{(k-1)2}} + \sigma^2_{C_{k1}}}}) = 1 - \Phi(\frac{\mu_{C_{(k-1)2}} - \mu_{C_{k1}}}{\sqrt{\sigma^2_{C_{(k-1)2}} + \sigma^2_{C_{k1}}}}) \qquad for \;\; k = 2,3,...,n \tag{9}$$

$$T_k = (1 - \Phi(\frac{\mu_{C_{(k-1)2}} - \mu_{C_{k1}}}{\sqrt{\sigma^2_{C_{(k-1)2}} + \sigma^2_{C_{k1}}}})).\max(\mu_{C_{k1}} - \mu_{C_{(k-1)2}}, 0) \qquad for \;\; k = 2,3,...,n$$

where Φ is cumulative distribution function (cdf) of standard normal. Moreover, second machine's waiting time during the processing of job with priority 1 is equal to completion time of that job on machine 1 (C_{11}). Accordingly, the mathematical model of the problem in the stochastic mode would be as follows:

$$\min \sum_{k=1}^{n} T_k$$

subject to

$$\mu_{C_{12}} = \sum_{j=1}^{2} \sum_{i=1}^{n} \mu_{ij} x_{i1}$$

$$\sigma^2_{C_{12}} = \sum_{j=1}^{2} \sum_{i=1}^{n} \sigma^2_{ij} x_{i1}$$

$$T_1 = \sum_{i=1}^{n} \mu_{i1} x_{i1} \tag{10}$$

$$\mu_{C_{k1}} = \sum_{k=1}^{k} \sum_{i=1}^{n} \mu_{i1} x_{ik}$$

$$\sigma^2_{C_{k1}} = \sum_{k=1}^{k} \sum_{i=1}^{n} \sigma^2_{i1} x_{ik} \qquad \text{For } k = 2,3,...,n$$

$$\mu_{C_{k2}} = y_k.\mu_{C_{k1}} + (1-y_k).\mu_{C_{(k-1)2}} + \sum_{i=1}^{n} \mu_{i2} x_{ik} \qquad \text{For } k = 2,3,...,n$$

$$\sigma^2_{C_{k2}} = y_k.\sigma^2_{C_{k1}} + (1-y_k).\sigma^2_{C_{(k-1)2}} + \sum_{i=1}^{n} \sigma^2_{i2} x_{ik} \qquad \text{For } k = 2,3,...,n$$

$$1-\Phi\left(\frac{\mu_{C_{(k-1)2}}-\mu_{C_{k1}}}{\sqrt{\sigma^2_{C_{(k-1)2}}+\sigma^2_{C_{k1}}}}\right)-\alpha \le M.y_k$$

$$1-\Phi\left(\frac{\mu_{C_{(k-1)2}}-\mu_{C_{k1}}}{\sqrt{\sigma^2_{C_{(k-1)2}}+\sigma^2_{C_{k1}}}}\right)-\alpha \ge M.(y_k-1)$$

$$T_k=\left(1-\Phi\left(\frac{\mu_{C_{(k-1)2}}-\mu_{C_{k1}}}{\sqrt{\sigma^2_{C_{(k-1)2}}+\sigma^2_{C_{k1}}}}\right)\right).\max(\mu_{C_{k1}}-\mu_{C_{(k-1)2}},0) \qquad \text{For } k=2,3,...,n$$

$$\sum_{i=1}^{n} x_{ik}=1 \qquad \text{For } k=1,2,3,...,n$$

$$\sum_{k=1}^{n} x_{ik}=1 \qquad \text{For } i=1,2,3,...,n$$

This mathematical model uses the properties of normal distribution; however, it is applicable as a heuristic when the distributions are non-normal, as long as their means and variances are available.

5. Computational experiment

In this section, the performances of three methods for solving stochastic scheduling problem are compared with each other in order to minimize the expected makespan. Johnson's Heuristic method solved the two-machine stochastic flow shop problem using mean of the processing time. In fact, this method applied Johnson's Rule to the mean time of processing jobs in stochastic terms. Talwar's method also solved two-machine stochastic flow shop problem by sorting the jobs using non-increasing differences of the mean processing rates.

The algorithm proposed in this paper is called HMB and is based on minimization of the sum of all expected delays in the second machine. Between the two sequences, the one, which resulted in less delay in the second machine and finally smaller expected makespan, would be selected. Here we explain the issue with a numerical example.

Example1: Consider a stochastic two-machine flow shop problem with three jobs and specifications given in Table 1 as follows,

Table 1
The mean and variance of jobs on machine one and two for example 1

i	μ_{i1}	σ^2_{i1}	μ_{i2}	σ^2_{i2}
1	67	15	42	10
2	48	14	51	16
3	45	12	59	14

Considering sequence 3-1-2, weighted sum of expected waiting times in the second machine is computed as follows:

$$\begin{aligned} &C_{11}\sim N(45,12) \quad \mu_{C_{12}}=\mu_{31}+\mu_{32} \quad \sigma^2_{C_{12}}=\sigma^2_{31}+\sigma^2_{32} \quad \Rightarrow C_{12}\sim N(104,26)\\ &\mu_{C_{21}}=\mu_{31}+\mu_{11} \quad \sigma^2_{C_{21}}=\sigma^2_{31}+\sigma^2_{11} \quad \Rightarrow \quad C_{21}\sim N(112,27)\end{aligned} \tag{11}$$

If we assume that $\alpha=0.8$, then we have to calculate $p(C_{21}>C_{12})$ and compare it with α:

$$(C_{21}-C_{12}) \sim N(\mu_{21}-\mu_{12}, \sigma_{21}^2+\sigma_{12}^2) \Rightarrow (C_{21}-C_{12}) \sim N(8,53)$$
$$p(C_{21}>C_{12}) = p(C_{21}-C_{12}>0) = 1-\Phi(-1.099) = 0.86 \quad (12)$$

Since $p(C_{21}>C_{12})>\alpha$, initiation time of the job with priority 2 (job 1) in the second machine is supposed to be equal to the completion time of the same job in the first machine (C_{21}) and the second machine will have waiting time. C_{22} is calculated as follows:

$$\mu_{C_{22}} = \mu_{C_{21}} + \mu_{12} \quad \sigma_{C_{22}}^2 = \sigma_{C_{21}}^2 + \sigma_{12}^2 \quad \Rightarrow C_{22} \sim N(154,37) \quad (13)$$

The completion time of third priority job (job 2) will be calculated as follows:

$$\mu_{C_{31}} = \mu_{31}+\mu_{11}+\mu_{21} \quad \sigma_{C_{31}}^2 = \sigma_{31}^2+\sigma_{11}^2+\sigma_{21}^2 \quad \Rightarrow C_{31} \sim N(160,41)$$
$$(C_{31}-C_{22}) \sim N(\mu_{C_{31}}-\mu_{C_{22}}, \sigma_{C_{31}}^2+\sigma_{C_{22}}^2) \Rightarrow (C_{31}-C_{22}) \sim N(6,78) \;\; p(C_{31}>C_{22}) = 1-\Phi(-0.68) = 0.75 \quad (14)$$

So $p(C_{31}>C_{22})<\alpha$ and the initiation time of the job with priority 3 in the second machine is equal to completion time of the job with priority 2 on the same machine:

$$\mu_{C_{32}} = \mu_{C_{22}} + \mu_{22} \quad \sigma_{C_{32}}^2 = \sigma_{C_{22}}^2 + \sigma_{22}^2 \Rightarrow C_{32} \sim N(205,53) \quad (15)$$

Based on the above calculations mean and variance of completion time of jobs on machines 1 and 2 are given in Table 2 as follows,

Table 2
Mean and variance of completion time of jobs on machines 1 and 2

k (*priority*)	$\mu_{C_{k1}}$	$\sigma_{C_{k1}}^2$	$\mu_{C_{k2}}$	$\sigma_{C_{k2}}^2$
1	45	12	104	26
2	112	27	154	37
3	160	41	205	53

Fig. 3 shows this sequence situation:

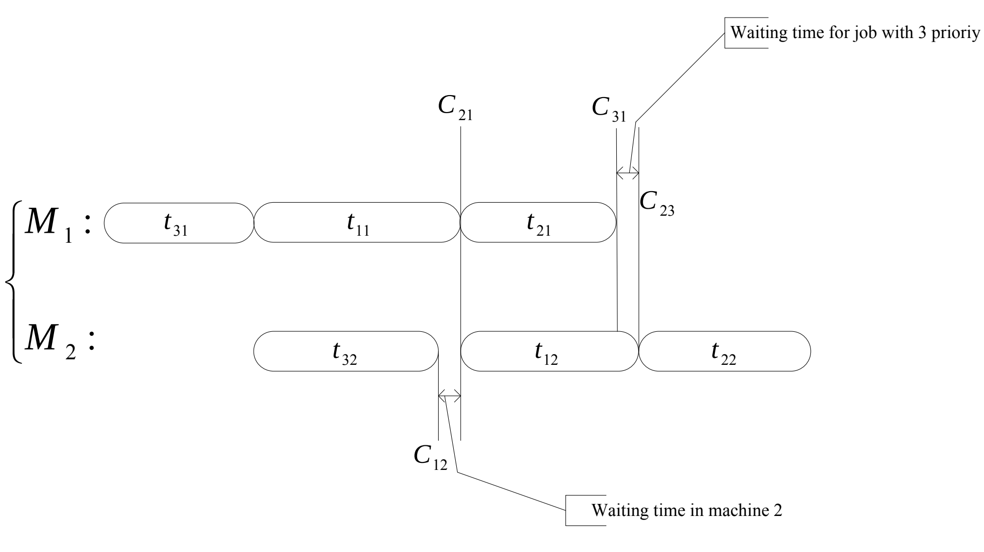

Fig. 3. Example 1 with 3-1-2 sequence and $\alpha = 0.8$

Expected sum of waiting time of the second machine is equal to:

$$T = \sum_{k=1}^{3} T_k$$

$$T_1 = \mu_{C_{11}} = 45$$

$$T_2 = (1 - \Phi(\frac{\mu_{C_{12}} - \mu_{C_{21}}}{\sqrt{\sigma^2_{C_{12}} + \sigma^2_{C_{21}}}})).\max(\mu_{C_{21}} - \mu_{C_{12}}, 0) = 6.88 \quad (16)$$

$$T_3 = (1 - \Phi(\frac{\mu_{C_{22}} - \mu_{C_{31}}}{\sqrt{\sigma^2_{C_{22}} + \sigma^2_{C_{31}}}})).\max(\mu_{C_{31}} - \mu_{C_{22}}, 0) = 4.5 \Rightarrow T = 56.38$$

For a specific problem of two-machine stochastic flow shop, we need to verify which of these three algorithms provides a better function, which is accomplished through simulation technique. Gourgand et al. (2003) made comparisons in cases where Markovian analysis is applicable to assess the accuracy of simulation. In addition, they concluded that the sample sizes of 5000 were adequate for testing procedures. They provided answers to some other questions about evaluating the efficiency of simulation method.

The purpose of this paper is to compare the performance of the three mentioned methods by this technique. For this purpose, a set of two-machine stochastic flow shop problems are designed. Number of jobs, and mean and variance of their processing times should be determined for the first and the second machines. The values of mean and variance of the jobs are generated randomly in the specified range. When these values are determined, the three mentioned methods are used to evaluate the sequence of jobs. It is evident that these three methods may result in completely different sequences depending on the problem. In the next step, 5000 samples for deterministic problem are produced (based on mean and variance of jobs in that problem). Then, based on the sequence presented by these three methods, real makespan value for all the problems will be computed. The performances of these methods are compared based on the following index, *improvement index*, and denoted by *IM*.

IM is defined as "the ratio of the cases in which the performance of one method is better than or equal to other methods for all cases"

For example, if HMB is better than or equal to two other methods in *m* cases, we have:

$$IM_{HMB} = \frac{m}{n}, \quad (17)$$

where *n* is the total number of samples. Moreover, if this index takes greater value for one method than for the other methods, it indicates that the method provides better performance in most of the cases.
Table 3 shows the comparison of the performance results of the three methods, namely HMB, the Talwar (T) and Johnson (J).

- The first row of the table shows the number of jobs and machines in each problem.
- Second and third rows are the ranges in which the mean and variance of the jobs are generated.
- The following rows show *IM* index, which compares the performance of one method to another.

Table 3
Comparison of the *IM* for constant variance (The number of machines = 2, The number of Jobs = 10)

Means	145-155	140-160	130-170	120-180	110-190
σ	30	30	30	30	30
J	52%	41%	43%	38%	35%
T	53%	43%	46%	42%	41%
HMB	54%	43%	47%	45%	42%

As can be seen from Table 3, in most of the cases, HMB algorithm provided a better response compared with other two methods. For the next case, we assume that the same calculation is performed when the variance is not constant. The results of this case are presented in Table 4.

Table 4

Comparison of the *IM* for variable variance (The number of machines = 2, The number of Jobs = 10)

Means	145-155	140-160	130-170	120-180	110-190
σ	15-35	15-35	15-35	15-35	15-35
J	43%	39%	43%	41%	49%
T	47%	41%	47%	44%	52%
HMB	47%	42%	47%	48%	55%

Comparison of the values in Table 4 indicates that the performance of HMB algorithm is better than Johnson's and Talwar's methods. Furthermore, the results of simulation with more jobs, which have been computed with supposition $\alpha = 0.8$, also confirmed the better performance of this algorithm. However, sensitivity analysis can be performed for the different values of α. For example, if we assume that the mean and variance of processing time of jobs are considered as fourth column of Table 3, then the performance of the HMB algorithm, in comparison with two other methods, for different values of α would be those shown in Table 5:

Table 5

Comparison of the *IM* for variable variance base on α in constant variance mode (The number of machines = 2, The number of Jobs = 10, Means 130-170, Variance = 900)

α	0.4	0.5	0.6	0.7	0.8	0.9
J	43%	48%	47%	41%	43%	39%
T	52%	52%	49%	46%	46%	41%
HMB	41%	51%	49%	47%	47%	40%

As shown in the Table 5, the performance of the HMB algorithm is different for different values of α. In addition, the algorithm has better performance when α is between 0.6 and 0.8. However, it may be different for other cases. In addition, if the variance is not constant, it will be a different situation as shown in Table 6:

Table 6

Comparison of the *IM* for variable variance base on α in constant variance mode (The number of machines = 2, The number of Jobs = 10, Means 130-170, Variance = 500-1000)

α	0.4	0.5	0.6	0.7	0.8	0.9
J	52%	38%	49%	44%	38%	38%
T	54%	39%	53%	46%	42%	42%
HMB	55%	40%	55%	46%	37%	35%

Therefore, the performance of HMB algorithm will change by changing the value of α. For instance, in Table 6 when α is between 0.5 and 0.7, it provides better performance for the algorithm. Therefore, an optimal value of α should be used in order to solve the stochastic scheduling problem using HMB algorithm which provides the most appropriate performance.

6. Concluding remarks

In this paper, scheduling problem was investigated for two-machine flow shop environment with stochastic processing time and the objective function of expected makespan minimization. First, the mathematical model of this problem was developed in deterministic mode considering the approach of minimizing waiting times in the second machine. Then, this model was customized for the stochastic mode. This customized model can be considered as the first studies, which are presented to solve stochastic flow shop problem with a mathematical model. Moreover, results of the proposed algorithm were obtained, and indicate a significant improvement in performance compared to the two previous heuristic methods. Development of this algorithm in flow shop problem with more machines version can be recommended for the future studies.

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