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Pricing and inventory control policy for non-instantaneous deteriorating items with time- and price-dependent demand and partial backlogging

Hiwa Farughi[a*], Narges Khanlarzade[b] and Babak Yousefi Yegane[c]

[a]*Assistant Professor of Industrial Engineering, Faculty of Engineering, University of Kurdistan, Iran*
[b]*MS.c graduate of Industrial Engineering, Tarbiat Modares University*
[c]*Department of Industrial Engineering, Islamic Azad University ,Malayer Branch, Malayer, Iran*

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A B S T R A C T

Determining the optimal inventory control and selling price for deteriorating items is of great significance. In this paper, a joint pricing and inventory control model for deteriorating items with price- and time-dependent demand rate and time-dependent deteriorating rate with partial backlogging is considered. The objective is to determine the optimal price, the replenishment time, and economic order quantity such that the total profit per unit time is maximized. After modeling the problem, an algorithm is proposed to solve the resulted problem. We also prove that the problem statement is concave function and the optimal solution is indeed global.

1. Introduction

Nowadays, most business units are faced with increasingly volatile business environments, characterized by shorter product life cycles and more rapid technological developments. In order to obtain competitive margins, new products must be introduced into the market, frequently. In this case, life cycles of old and new products overlap and they coexist in a considerable period of time (Chew et al., 2014). Deterioration can be defined as the loss of marginal value of commodity, which yields in decreased usefulness. Under this definition, many goods such as clothing and electronic devices can be considered as perishable items. Today, competition in the market has led all the competitors to increase the quality of their products, so a producer's success is determined by the price of his/her products. Pricing and inventory control policy are two important factors for the success of business owners. In recent years, many researchers have studied the pricing and inventory control issues simultaneously for deteriorating items. Most physical goods such as drugs, vegetables deteriorate over time (Wee, 1993). Pricing and inventory control of deteriorating items have been

* Corresponding author. Tel: +989188736075
E-mail addresses: h.farughi@uok.ac.ir (H. Farughi)

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extensively studied by many researchers. Deteriorating inventory analysis began with the work of Ghare and Schrader (1963), who established the classical no-shortage inventory model with a constant rate of decay. However, it has been empirically observed that failure and life expectancy of many items can be expressed in Weibull distribution items. This empirical observation has prompted researchers to present the products' deterioration time by Weibull distribution. Covert and Philip (1973) extended Ghare and Schrader's model and obtained an economic order quantity model for variable rate of deterioration by assuming a two-parameter Weibull distribution. Researchers such as Philip (1974), Misra (1975), Tadikamalla (1978), Wee (1997), Chakrabarty et al. (1998), and Mukhopadhyay et al. (2004) developed economic order quantity models by concentrating on this type of products. Abad (1996) considered a pricing and lot sizing problem for a product with variable rate of deterioration and partial backlogging. Aggarwal and Jaggi (1995) explored the ordering policy for deteriorating items under permissible delay in payments. Hwang and Shinn (1997) dealt with pricing and lot sizing decisions for exponentially deteriorating products, with also permissible delay in payments. Jamal et al. (1997) generalized Aggarwal and Jaggi's model to allow for shortages. Chang and Dye (2001) extended Jamal et al.'s model. Chang et al. (2002) considered the linear demand for deteriorating items over time and partial backlogging rate.

Chang et al. (2006) established an EOQ model for deteriorating items for a retailer to determine its optimal selling price and lot sizing policy with partial backlogging. Dye et al. (2007) presented a pricing and inventory policy for deteriorating items with shortage. Most studies assume that deterioration begins from the moment of a product's arrival in the stock. In fact, most of the goods are thought to have a quality maintenance or original condition span in which no deterioration occurs. In the real world, this phenomenon exists commonly among goods such as fresh fruits and vegetables. Wu et al. (2006) defined the *non-instantaneous* phenomenon and developed a replenishment policy for non-instantaneous deteriorating items with stock-dependent demand to minimize the total inventory cost per unit time.

Geetha and Uthayakumar (2010) proposed an EOQ-based model for non-instantaneous deteriorating items with permissible delay in payments. In this model, demand and price are constant and shortages are allowed and are partially backlogged. Cai et al. (2011) studied pricing and ordering policy problems in two-stage supply chains by considering the partial lost sales based on the game theory. Musa and Sani (2012) developed a mathematical model for inventory control of non-instantaneous deteriorating items with permissible delay in payments. Maihami and Nakhai (2012) developed a mathematical model for joint pricing and inventory control of non-instantaneous deteriorating item with partial backlogging, the unsatisfied demand being backlogged and the fraction of shortage backordered considered as $\beta(x) = k_0 e^{-\delta x}$. Avinadav et al. (2013) employed a price-and time-dependent function and developed a mathematical model to calculate the optimal price, the order quantity and the replenishment period for perishable items.

Pricing is a major strategy for a seller to achieve the maximum profit. Consequently, in this paper, Maihami and Nakhai's proposed model is developed and a different backlogging function for unsatisfied demand and time-dependent deterioration rate is used. The rest of the paper is organized as follows. In section 2, we describe the assumption and notation employed throughout this study is described; therein, the mathematical model and the necessary considerations for finding an optimal solution are established. Furthermore, it is demonstrated that the total profit is a concave function of selling price when the replenishment schedule is given. In section 3, we provide a simple algorithm to find the optimal replenishment schedule and selling price for the proposed model. In section 4, we use a numerical example to illustrate the algorithm. Finally, we make a summary and provide some suggestions for future research in section 5.

2. Assumptions and notations

2.1. Assumptions

The mathematical model is based on the following assumptions:

1. The mathematical model is proposed for a non-instantaneous deterioration item.
2. The lead time is zero.
3. The demand rate $D(t,p) = (a - bp)e^{\lambda t}$, $(a > 0, b > 0)$ is a linearly decreasing function of the price and decreases (increases) exponentially with time when $\lambda < 0$ $(\lambda > 0)$.
4. Shortages are allowed; only a fraction of the demand is assumed to be backlogged. Following Chang and Dye (1999) we take $\beta(x) = 1/(1 + \delta x)$ $(\delta > 0)$; Note that if $\beta(x) = 1$ (or 0) for all x, and then the shortage is completely backlogged (or lost).
5. The on-hand inventory deteriorates at a rate θ.

There is no replacement or repair of deteriorated items and they are withdrawn immediately from store.

2.2. Notations

c	the constant purchasing cost per unit
h	the holding cost per unit per unit time
s	The backorder cost per unit per time
o	The cost of lost sales per unit
t_d	The length of time in which the product exhibit no deterioration
t_1	The length of time in which there is no inventory shortage
T	The length of replenishment cycle time
Q	The order quantity
P^*	The optimal selling price per unit
t_1^*	The optimal length of time in which there is no inventory shortage
T^*	The optimal length of replenishment cycle time
Q^*	The optimal order quantity
$I_1(t)$	the inventory level at time $t \in [0, t_d]$
$I_2(t)$	the inventory level at time $t \in [t_d, t_1]$
$I_3(t)$	the inventory level at time $t \in [t_1, T]$
I_0	the maximum inventory level
S	The maximum amount of demand backlogged
$TP(p,t_1,T)$	The total profit per unit time of the inventory system
TP^*	The optimal total profit per unit time of the inventory system

2.3. Mathematical formulation

Based on the represented notations, the inventory level follows the pattern depicted in Fig.1. In order to establish the total profit function, the following time intervals are considered separately, $[0, t_d]$ the inventory level is assumed to decrease only by demand, the interval $[t_d, t_1]$ in which the inventory level is affected by both demand and deterioration and drops to zero and the interval $[t_1, T]$ where the shortage occurs. Hence, the inventory level is governed by the following differential equation during the first interval:

$$\frac{d\,I_1(t)}{dt} = -D(p,t) \qquad 0 \le t \le t_d \tag{1}$$

With the boundary condition $I_1(0) = I_0$, solving the differential Eq. (1) for the inventory yields,

$$I_1(t) = \frac{a-bp}{\lambda}\left(1-e^{\lambda t}\right) + I_0 \quad 0 \le t \le t_d \tag{2}$$

At the next interval $[t_d, t_1]$, the inventory level is affected by demand and deterioration simultaneously, so the inventory status can be presented by solving the equation below:

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D(p,t) \qquad t_d \le t \le t_1 \tag{3}$$

and the boundary condition $I_2(t_1) = 0$ the inventory level is follows,

$$I_2(t) = \frac{a-bp}{\lambda+\theta}\, e^{-\theta t}\left[e^{(\lambda+\theta)t_1} - e^{(\lambda+\theta)t}\right] \quad t_d \le t \le t_1 \tag{4}$$

Considering the continuity of $I(t)$ at $t = t_d$, the maximum inventory level for each cycle is as follows,

$$I_0 = \frac{a-bp}{\lambda+\theta}\, e^{-\theta t_d}\left[e^{(\lambda+\theta)t_1} - e^{(\lambda+\theta)t_d}\right] - \frac{a-bp}{\lambda}\left[1-e^{\lambda t_d}\right] \tag{5}$$

During the interval $[t_1, T]$, the inventory level only depends on demand, shortage occurred and demand is partially backlogged according to the fraction $\beta(T-t)$. That is, the inventory level at time t is governed by the following differential equation:

$$\frac{dI_3(t)}{dt} = -D(p,t)\beta(T-t) = \frac{D(p,t)}{1+\delta(T-t)} \quad t_1 \le t \le T \tag{6}$$

With the condition $I_3(t_1) = 0$ the solution of Eq. (6) is as follows,

$$I_3(t) = -(a-bp)\int_{t_1}^{t} \frac{e^{-\lambda x}}{1+\delta(T-x)}dx = \frac{e^{\left(T+\frac{1}{\delta}\right)}}{\delta}\left[Ei\left(t_1 - T - \frac{1}{\delta}\right) - Ei\left(t - T - \frac{1}{\delta}\right)\right] \; t_1 \le t \le T \tag{7}$$

where $Ei(z) = \int_1^{\infty} \frac{e^{-zt}}{t^n}dt$. The maximum shortage is as follows,

$$SS = -I_3(T) = \frac{e^{\left(T+\frac{1}{\delta}\right)}}{\delta}\left[Ei\left(t_1 - T - \frac{1}{\delta}\right) - Ei\left(-\frac{1}{\delta}\right)\right] \tag{8}$$

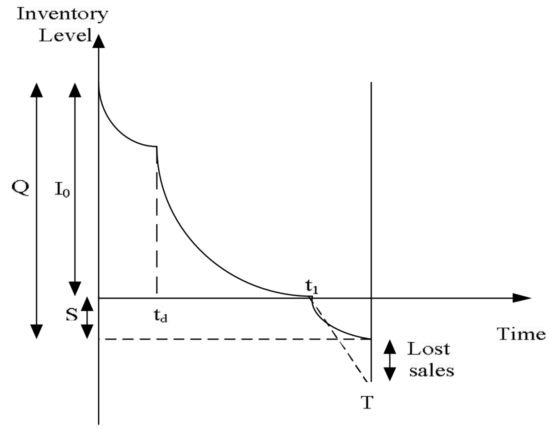

Fig. 1. Graphical representation of the inventory system

The order quantity per cycle is the sum of I_0 and SS , i.e.

$$Q = I_0 + SS = \frac{a-bp}{\lambda+\theta}\, e^{-\theta t_d}\left[e^{(\lambda+\theta)t_1} - e^{(\lambda+\theta)t_d}\right] - \frac{a-bp}{\lambda}\left[1 - e^{\lambda t_d}\right] + \frac{e^{\left(T+\frac{1}{\delta}\right)}}{\delta}\left[Ei\left(t_1 - T - \frac{1}{\delta}\right) - Ei\left(-\frac{1}{\delta}\right)\right] \tag{9}$$

Next, the total relevant inventory cost per cycle consists of the following elements:

i. the ordering cost per cycle is A .
ii. The inventory holding cost that is denoted by HC is given by

$$HC = h\left[\int_0^{t_d} I_1(t)dt + \int_{t_d}^{t_1} I_2(t)dt\right] = h\left[\begin{array}{c} \dfrac{(a-bp)e^{-\theta t_d}\left(\theta e^{(\lambda+\theta)t_d} + \lambda e^{(\lambda+\theta)t_1} - (\lambda+\theta)e^{(\lambda t_1 + \theta t_d)}\right)}{\lambda\theta(\lambda+\theta)} \\ + \dfrac{(a-bp)e^{-\theta t_d}\left(\dfrac{-((e^{-\theta t_d}(e^{\lambda t_d}-1)(\lambda+\theta))}{\lambda} + \theta t_d e^{(\lambda+\theta)t_d} + \lambda t_d e^{(\lambda+\theta)t_1}\right)}{\lambda(\lambda+\theta)} \end{array}\right] \tag{10}$$

iii. The shortage cost per cycle due to backlog that is denoted by SC is given by

$$SC = s\int_{t_1}^{T}[-I_3(t)]dt = s(a-bp)\frac{(a-bp)}{\delta^2\lambda}\left[\left(\delta e^{\lambda t_1} - \lambda e^{\left(T+\frac{1}{\delta}\right)\lambda}(\delta(t_1 - T) - 1)Ei\left[\lambda\left(t_1 - \left(T + \frac{1}{\delta}\right)\right)\right]\right) - \delta e^{\lambda T} + \lambda e^{\left(T+\frac{1}{\delta}\right)\lambda} Ei\left(-\frac{1}{\delta}\right)\right] \tag{11}$$

iv. The opportunity cost due to lost sales which is denoted by OC is given by

$$OC = o\int_{t_1}^{T} D(p,t)(1 - \beta(T-t))dt = o(a-bp)\left[\left(\frac{e^{\lambda T}}{\lambda} + \frac{e^{\left(T+\frac{1}{\delta}\right)}Ei\left[-\frac{1}{\delta}\right]}{\delta}\right) - \left(\frac{e^{\lambda t_1}}{\lambda} + \frac{e^{\left(T+\frac{1}{\delta}\right)}Ei\left[\left(t_1 - T - \frac{1}{\delta}\right)\lambda\right]}{\delta}\right)\right] \tag{12}$$

v. the purchase cost per cycle is as follows,

$$PC = CQ = C\left[\frac{a-bp}{\lambda+\theta}\left(e^{(\lambda+\theta)t_1} - 1\right) + \frac{e^{\left(T+\frac{1}{\delta}\right)}}{\delta}\left[Ei\left(t_1 - T - \frac{1}{\delta}\right) - Ei\left(-\frac{1}{\delta}\right)\right]\right] \tag{13}$$

vi. SR : The sales revenue

$$SR = \frac{e^{\lambda t_1} - 1}{\lambda}(a-bp)p + p\left(\frac{e^{\left(T+\frac{1}{\delta}\right)}}{\delta}\left(Ei\left(t_1 - T - \frac{1}{\delta}\right) - Ei\left(-\frac{1}{\delta}\right)\right)\right) \tag{14}$$

Therefore, the total profit per unit time of proposed model is obtained as follows,

$$\boldsymbol{TP(p,t_1,T)}=\frac{1}{T}(\textit{Sales Revenue} - \textit{ordering cost} - \textit{purchace cost} - \textit{shortage cost} - \textit{opportunity cost} - \textit{inventory holding cost}) \tag{15}$$

$TP_{(p,t_1,T)}$ is function of t_1, T, p; so for any given p The necessary conditions for the total relevant profit per unit time to be maximized are $\frac{\partial TP(p,t_1,T)}{\partial t_1}=0$ and $\frac{\partial TP(p,t_1,T)}{\partial T}=0$ simultaneously. That is:

$$\begin{aligned}\frac{\partial \boldsymbol{TP(p,t_1,T)}}{\partial \boldsymbol{t_1}} &= \\ &\frac{e^{\lambda t_1}p(a-bp)}{T}+\frac{e^{t_1}p}{\delta\left(-T-\frac{1}{\delta}+t_1\right)}-C\left(e^{(\theta+\lambda)t_1}(a-bp)+\frac{e^{t_1}}{\delta\left(-T-\frac{1}{\delta}+t_1\right)}\right)-o(a-bp)\left(-e^{\lambda t_1}-\frac{e^{\lambda(t_1)}}{\delta\left(-T-\frac{1}{\delta}+t_1\right)}\right)- \\ &\frac{(a-bp)^2 s(e^{\lambda t_1}\delta\lambda-e^{(T+\frac{1}{\delta})\lambda}\delta\lambda\mathrm{Ei}[\lambda(-T-\frac{1}{\delta}+t_1)]-\frac{\lambda}{\delta}e^{\lambda t_1})}{\delta^2\lambda}-h\left(\frac{e^{-\theta t_d}(a-bp)\left(e^{(\theta+\lambda)t_1}\lambda(\theta+\lambda)-e^{\lambda t_1+\theta t_d}\lambda(\theta+\lambda)\right)}{\theta\lambda(\theta+\lambda)}-e^{(\theta+\lambda)t_1-\theta t_d}(a-bp)t_d\right)\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial \boldsymbol{TP(p,t_1,T)}}{\partial \boldsymbol{T}} = &-\frac{(-1+e^{\lambda t_1})p(a-bp)}{T^2\lambda}+\frac{e^{T+\frac{1}{\delta}}p\left(-\mathrm{Ei}\left[-\frac{1}{\delta}\right]+\mathrm{Ei}\left[-T-\frac{1}{\delta}+t_1\right]\right)}{\delta}-\frac{e^{t_1}p}{\delta\left(-T-\frac{1}{\delta}+t_1\right)}-C\left(\frac{e^{T+\frac{1}{\delta}}(-\mathrm{Ei}[-\frac{1}{\delta}]+\mathrm{Ei}[-T-\frac{1}{\delta}+t_1])}{\delta}-\frac{e^{t_1}}{\delta(-T-\frac{1}{\delta}+t_1)}\right)- \\ &o(a-bp)\left(e^{T\lambda}+\frac{e^{(T+\frac{1}{\delta})\lambda}\lambda\mathrm{Ei}[-\frac{\lambda}{\delta}]}{\delta}-\frac{e^{(T+\frac{1}{\delta})\lambda}\lambda\mathrm{Ei}[\lambda(-T-\frac{1}{\delta}+t_1)]}{\delta}+\frac{e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}}{\delta(-T-\frac{1}{\delta}+t_1)}\right)- \\ &\frac{\left\{(a-bp)^2 s\left(-e^{T\lambda}\delta\lambda-e^{\left(T+\frac{1}{\delta}\right)\lambda}\lambda^2\mathrm{Ei}[-\frac{\lambda}{\delta}]+e^{\left(T+\frac{1}{\delta}\right)\lambda}\delta\lambda\mathrm{Ei}[\lambda(-T-\frac{1}{\delta}+t_1)]-e^{\left(T+\frac{1}{\delta}\right)\lambda}\lambda^2\mathrm{Ei}\left[\lambda\left(-T-\frac{1}{\delta}+t_1\right)\right]\left(-1+\delta(-T+t_1)\right)+\frac{e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}\lambda(-1+\delta(-T+t_1))}{-T-\frac{1}{\delta}+t_1}\right)\right\}}{\delta^2\lambda}\end{aligned} \tag{17}$$

Theorem 1.

(a) The system of (16) and (17) has a unique solution.
(b) The solution in (a) satisfies the second-order conditions for maximization.

Proof. See Appendix A for details.

Solving the Eq. (16) and Eq. (17), the optimum value for T^* and t_1^* is obtained, so the selling price can be determined from the Eq. (18). For this purpose, it is sufficient to solve the following equation:

$$\begin{aligned}\frac{\partial \boldsymbol{TP(p,t_1^*,T^*)}}{\partial \boldsymbol{p}} = &\frac{b(-1+e^{\lambda t^*{}_1})p}{T^*\lambda}+\frac{(-1+e^{\lambda t^*{}_1})(a-bp)}{T^*\lambda}+\frac{bC(-1+e^{(\theta+\lambda)t^*{}_1})}{\theta+\lambda}+\frac{e^{T^*+\frac{1}{\delta}}(-\mathrm{Ei}[-\frac{1}{\delta}]+\mathrm{Ei}[-T-\frac{1}{\delta}+t^*{}_1])}{\delta}+ \\ &bo\left(\frac{e^{T^*\lambda}}{\lambda}-\frac{e^{\lambda t^*{}_1}}{\lambda}+\frac{e^{(T^*+\frac{1}{\delta})\lambda}\mathrm{Ei}[-\frac{\lambda}{\delta}]}{\delta}-\frac{e^{(T^*+\frac{1}{\delta})\lambda}\mathrm{Ei}[\lambda(-T^*-\frac{1}{\delta}+t^*{}_1)]}{\delta}\right)+\frac{\left\{2b(a-bp)s(-e^{T^*\lambda}\delta+e^{\lambda t^*{}_1}\delta-e^{\left(T^*+\frac{1}{\delta}\right)\lambda}\lambda\mathrm{Ei}[-\frac{\lambda}{\delta}]-e^{\left(T^*+\frac{1}{\delta}\right)\lambda}\lambda\mathrm{Ei}[\lambda(-T^*-\frac{1}{\delta}+t^*{}_1)](-1+\delta(-T^*+t^*{}_1)))\right\}}{\delta^2\lambda}- \\ &h\left(-\frac{be^{-\theta t_d}(e^{(\theta+\lambda)t_d}\theta+e^{(\theta+\lambda)t^*{}_1}\lambda-e^{\lambda t_1+\theta t_d}(\theta+\lambda))}{\theta\lambda(\theta+\lambda)}+\frac{be^{-\theta t_d}(-\frac{e^{\theta t_d}(-1+e^{\lambda t_d})(\theta+\lambda)}{\lambda}+e^{(\theta+\lambda)t_d}\theta t_d+e^{(\theta+\lambda)t^*{}_1}\lambda t_d)}{\lambda(\theta+\lambda)}\right)\end{aligned} \tag{18}$$

The second order derivation of $TP_{(P,t_1^*,T^*)}$ with respect to P is given by the following equation:

$$\frac{\partial^2 \boldsymbol{TP(p,t_1^*,T^*)}}{\partial \boldsymbol{p^2}}=-\frac{2b(-1+e^{\lambda t_1})}{T\lambda}-\frac{2b^2 s(-e^{T\lambda}\delta+e^{\lambda t_1}\delta-e^{(T+\frac{1}{\delta})\lambda}\lambda\mathrm{Ei}[-\frac{\lambda}{\delta}]-e^{(T+\frac{1}{\delta})\lambda}\lambda\mathrm{Ei}[\lambda(-T-\frac{1}{\delta}+t_1)](-1+\delta(-T+t_1)))}{\delta^2\lambda}<0 \tag{19}$$

3. The algorithm

We propose a simple algorithm to obtain the optimal solution of the problem.

Step 1. Start with j=0 and the initial value of $p_j = p_1$.

Step 2. Find the optimal value of T^* and t^* for a given price p_j.

Step 3. Use the result in step 2 and then determine the optimal p_{j+1} by Eq. (18).

Step 4. If the difference between p_j and p_{j+1} is sufficiently small, set $p^* = p_{j+1}$, otherwise set $j = j+1$ and go to step 2.

By using above algorithm, we obtain the optimal solution p^*, t_1^*, T^*, we can obtain TP^* using Eq. (15).

4. Numerical example

To illustrate the solution procedure, we solve the following numerical example; the results can be obtained by applying the Mathematica 8.0.

4.1. Example.

We adopt the same example of Maihami and Nakhai (2012) to see the optimal inventory control policy and optimal selling price. The example is based on the following parameters and functions:

$A = \$250/\,perorder, C = \$200/\,perunit$, $h = \$40/\,perunittime, sc = \$80/\,perunit/\,pertime$,

$oc = \$120/\,perunit$, $\theta = 0.08$, $t_d = 0.04$ $D(t,p) = (500 - 0.5p)e^{-0.98t}$, $\beta(x) = \dfrac{1}{1+0.2x}$

First we set $p_1 = \dfrac{a+bc}{2b}$, after five iteration we have $p^* = 1880.64$, $t_1^* = 0.06321$, $T^* = 0.08547$ $TP^* = 9.64617 \times 10^6$. Fig.2. shows that TP^* is strictly concave in p.

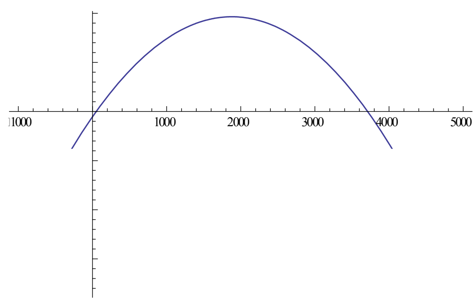

Fig. 2. Graphical representation of $TP(p \mid t_1^*, T^*)$

6. Sensitivity analysis

In this section, we focus on the effects of changes in the parameters of the system on p^*, t_1^*, T^*, and TP^*. The sensitive analysis is performed by changing each value of the parameters by +50%, +25%, -25% and -50%, taking one parameter at a time end keeping the remaining parameter values unchanged. The computational results are shown in Table 1.

The sensitive analysis shown in Table 1 indicates the following observations:

1- When the value of parameters increases, the optimal selling rate will increase. p^* is too much positively sensitive to change in parameter c. This result is reasonable because the purchase cost has a strong and positive effect on the optimal selling rate.

2- When the values of A, s, and o increase, the optimal value of t_1^* increases and it decreases as the value parameters h and θ increase.

3- When the value of parameter A increases, the optimal length of T^* increases and as the values of parameters h, s, o, and θ increases, it would decrease.

4- When the values of all the above parameters increase, the optimal profit per unit time will decrease; this implies that the increase in costs and deterioration rate have a negative effect on the total profit per unit time.

Table 1
Sensitive analysis with respect to the model parameters

Parameter	Value	p^*	t_1^*	T^*	TP^*
A	125	1873.00	0.05182	0.06732	9.62021×10^6
	188	1876.16	0.05687	0.07341	9.61300×10^6
	313	1886.38	0.06601	0.08796	9.57023×10^6
	375	1891.07	0.06936	0.08984	9.21869×10^6
c	100	1842.82	0.07401	0.08162	9.97000×10^6
	150	1861.24	0.06872	0.08221	9.86203×10^6
	250	1908.00	0.06767	0.08602	9.20153×10^6
	300	1937.21	0.07337	0.08917	9.00205×10^6
h	20	1877.53	0.07901	0.08392	9.67899×10^6
	30	1881.05	0.07464	0.08016	9.64865×10^6
	50	1884.61	0.07258	0.07936	9.62001×10^6
	60	1885.04	0.07209	0.07786	9.61054×10^6
s	40	1878.59	0.06723	0.08429	9.66253×10^6
	60	1880.00	0.06907	0.08391	9.65713×10^6
	100	1881.04	0.07045	0.07975	9.63190×10^6
	120	1883.19	0.07100	0.07816	9.61732×10^6
o	60	1880.35	0.06509	0.08625	9.66631×10^6
	90	1880.51	0.06896	0.08390	9.65163×10^6
	150	1881.01	0.06973	0.08238	9.64502×10^6
	180	1881.11	0.07013	0.08547	9.64265×10^6
θ	0.04	1878.28	0.07901	0.08982	9.67953×10^6
	0.06	1879.86	0.07316	0.08801	9.66385×10^6
	0.1	1882.32	0.06016	0.08391	9.63509×10^6
	0.12	1883.36	0.05736	0.08072	9.629876×10^6

7. Conclusion

In this paper, an appropriate model for a retailer to determine its optimal selling price and replenishment schedule for deteriorating item has been established. The demand is deterministic and depend on time and price, simultaneously. In addition, shortage is allowed and can be partially backlogged, where the backlogging rate is variable and dependent on the time of waiting for the next replenishment. In this study, some useful theorems, which characterize the optimal solution have been mentioned and an algorithm has been presented for determining the optimal price and optimal inventory control parameters. Finally, a numerical example is provided to illustrate the algorithm and solution procedure.

Appendix A

(a) Because of high complication in Eqs. (15) and (16), a straightforward proof does not exist. So, we only explain the proof procedure. First we must obtain $t_1 (or\, T)$ based on $T (or\, t_1)$ from Eq. (15) and Eq. (18), (call this function $F(x)$). For $F(x)$, we take the first-order derivative with respect to x and show that $F(x)$ is a strictly decreasing or increasing function. Next we use the intermediate value theorem and complete the proof. A simple and similar proof can be found in Yang et al. (2009).

(b) Let (t_1^*, T^*) be the solution of Eq.(15) and Eq. (16), we obtain

$$\frac{\partial TP_{(P,t_1,T)}}{\partial t_1^2} = \left[\frac{e^{\lambda t_1} p(a-bp)\lambda}{T}\right] - \left[e^{t_1} p\left(\frac{1}{\delta\left(T+\frac{1}{\delta}-t_1\right)^2} + \frac{1}{\delta\left(T+\frac{1}{\delta}-t_1\right)}\right)\right] - \left[C\left(e^{(\theta+\lambda)t_1}(a-bp)(\theta+\lambda) - e^{t_1}\left[\frac{1}{\delta\left(T+\frac{1}{\delta}-t_1\right)^2} + \frac{1}{\delta\left(T+\frac{1}{\delta}-t_1\right)}\right]\right)\right] +$$

$$\left[o\lambda(a-bp)e^{\lambda t_1}\left(1-\frac{1}{\delta\lambda\left(T+\frac{1}{\delta}-t_1\right)^2}-\frac{\lambda}{\delta\left(T+\frac{1}{\delta}-t_1\right)}\right)\right]-\left[\frac{(a-bp)^2 s e^{\lambda t_1}(\delta\lambda+\frac{2\delta}{T+\frac{1}{\delta}-t_1}-\frac{1}{\delta(T+\frac{1}{\delta}-t_1)}-\frac{\lambda}{\delta})}{\delta^2}\right]+\left[he^{-\theta t_d}(a-bp)(\frac{\left(e^{\lambda t_1+\theta t_d}\lambda(\theta+\lambda)+e^{(\theta+\lambda)t_1}(\theta+\lambda)^2\right)}{\theta(\theta+\lambda)}+e^{(\theta+\lambda)t_1}(\theta+\lambda)t_d)\right].$$

By assuming $\lambda < 0$, $|\lambda| > \theta$, $|\lambda| > \delta$, $bp < a$, we have $\frac{\partial TP_{(P,t_1^*,T^*)}}{\partial t_1^2} < 0$, because the first, second, fourth, and fifth brackets are always negative, and the summation of these brackets is greater than the third bracket.

$$\frac{\partial TP_{(P,t_1,T)}}{\partial T^2} =$$

$$\frac{2(-1+e^{\lambda t_1})p(a-bp)}{T^3\lambda}+\frac{e^{T+\frac{1}{\delta}}p(-\mathrm{Ei}[-\frac{1}{\delta}]+\mathrm{Ei}[-T-\frac{1}{\delta}+t_1])}{\delta}-\frac{e^{t_1}p}{\delta(-T-\frac{1}{\delta}+t_1)^2}-\frac{e^{t_1}p}{\delta(-T-\frac{1}{\delta}+t_1)}-C(\frac{e^{T+\frac{1}{\delta}}(-\mathrm{Ei}[-\frac{1}{\delta}]+\mathrm{Ei}[-T-\frac{1}{\delta}+t_1])}{\delta}-$$
$$\frac{e^{t_1}}{\delta(-T-\frac{1}{\delta}+t_1)^2}-\frac{e^{t_1}}{\delta(-T-\frac{1}{\delta}+t_1)})-o(a-bp)(e^{T\lambda}\lambda+\frac{e^{(T+\frac{1}{\delta})\lambda}\lambda^2\mathrm{Ei}[-\frac{\lambda}{\delta}]}{\delta}-\frac{e^{(T+\frac{1}{\delta})\lambda}\lambda^2\mathrm{Ei}[\lambda(-T-\frac{1}{\delta}+t_1)]}{\delta}+\frac{e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}}{\delta(-T-\frac{1}{\delta}+t_1)^2}+$$
$$\frac{e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}\lambda}{\delta(-T-\frac{1}{\delta}+t_1)})-\frac{1}{\delta^2\lambda}(a-bp)^2 s(-e^{T\lambda}\delta\lambda^2-e^{(T+\frac{1}{\delta})\lambda}\lambda^3\mathrm{Ei}[-\frac{\lambda}{\delta}]+2e^{(T+\frac{1}{\delta})\lambda}\delta\lambda^2\mathrm{Ei}[\lambda(-T-\frac{1}{\delta}+t_1)]-$$
$$\frac{2e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}\delta\lambda}{-T-\frac{1}{\delta}+t_1}-e^{(T+\frac{1}{\delta})\lambda}\lambda^3\mathrm{Ei}[\lambda(-T-\frac{1}{\delta}+t_1)](-1+\delta(-T+t_1))+\frac{e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}\lambda(-1+\delta(-T+t_1))}{(-T-\frac{1}{\delta}+t_1)^2}+$$
$$\frac{e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}\lambda^2(-1+\delta(-T+t_1))}{-T-\frac{1}{\delta}+t_1})$$

$$\frac{\partial TP_{(P,t_1,T)}}{\partial t_1\partial T} = \frac{e^{\lambda t_1}p(a-bp)}{T^2}-\frac{Ce^{t_1}}{\delta(-T-\frac{1}{\delta}+t_1)^2}+\frac{e^{t_1}p}{\delta(-T-\frac{1}{\delta}+t_1)^2}+\frac{e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}o(a-bp)}{\delta(-T-\frac{1}{\delta}+t_1)^2}$$
$$-\frac{(a-bp)^2 s(-e^{(T+\frac{1}{\delta})\lambda}\delta\lambda^2\mathrm{Ei}[\lambda(-T-\frac{1}{\delta}+t_1)]+\frac{2e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}\delta\lambda}{-T-\frac{1}{\delta}+t_1}-\frac{e^{(T+\frac{1}{\delta})\lambda+\lambda(-T-\frac{1}{\delta}+t_1)}\lambda(-1+\delta(-T+t_1))}{(-T-\frac{1}{\delta}+t_1)^2})}{\delta^2\lambda}$$

By assuming $\lambda < 0, |\lambda| > \theta, |\lambda| > \delta, \quad bp < a$ we have $\frac{\partial^2 TP(p,t_1,T)}{\partial t_1^2} < 0$, $\frac{\partial^2 TP(p,t_1,T)}{\partial T^2} < 0$, $\frac{\partial^2 TP(p,t_1,T)}{\partial T^2} > \frac{\partial^2 TP(p,t_1,T)}{\partial t_1 \partial T}$ & $\frac{\partial^2 TP(p,t_1^*,T^*)}{\partial t_1^{*2}} > \frac{\partial^2 TP(p,t_1^*,T^*)}{\partial t_1 \partial T}$

Using the example provided by Maihami and Nakhai (2012), we find the optimum value for t_1^* and T^*, thus, the determinant of hessian matrix at the stationary point (t_1^*, T^*) is:

$$Det(H) = \frac{\partial^2 TP}{\partial t_1^2} . \frac{\partial^2 TP}{\partial T^2} - \left(\frac{\partial^2 TP}{\partial t_1 \partial T}\right)^2 > 0$$

Hence, the Hessian matrix at point $\left(T^*, t_1^*\right)$ is negative definite and this completes the proof.

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