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Photoelastic study of bi-material notches: Effect of mismatch parameters

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ABSTRACT

The effects of mismatch parameters on isochromatic fringe patterns were studied using the technique of photoelasticity. First, the mathematical equations of isochromatic fringes were derived for singular stress field near a bi-material notch. These equations were used to study the effects of mismatch parameters on the shape of isochromatic fringes theoretically. Analytical results indicated that the mismatch parameters have a significant effect on the shape of the isochromatic fringe patterns around the bi-material notch tip. In order to assess the accuracy of the analytical results, a photoelastic test program was conducted on the V-notched bi-material Brazilian disc specimens. A very good agreement was shown to exist between the experimental results and the analytical reconstructions.

1. Introduction

Bi-material joints are extensively employed in many engineering applications such as ceramic coatings, welded structures and adhesively bonded joints. Sharp notches and reentrant corners are also very often present at the interface joints, for which the generated stress concentration is not only due to a material discontinuity but also from a geometrical one at the interface of the bi-material notch. It becomes important to investigate the stress field near the bi-material notches because the failure of these joints often initiates at the bi-material notch tip under mechanical and/or thermal loading. Fig. 1 shows the geometrical configuration of a bi-material notch characterized by the two angles θ_1 and θ_2 and the polar coordinate component θ. A stress singularity may develop at the interface corner depending on the elastic properties of the two materials and the corner geometry (Williams, 1952; Bogy, 1968).

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According to Qian and Akisanya (1998), the stress field near the interface corner can be expressed as follows,

$$\sigma_{ij}^{m} = \sum_{k=1}^{N} H_k r^{\lambda_k - 1} f_{ijk}^{m} \quad (1)$$

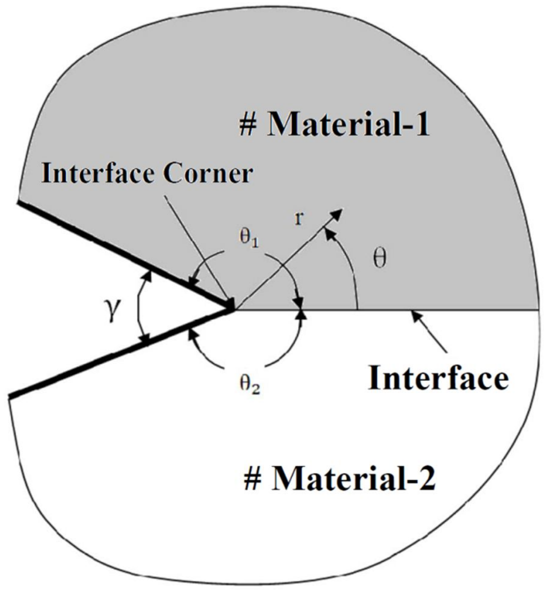


Fig.1. General configuration of a bi-material notch

where (i, j) ≡ (r, θ) are the polar coordinates with the origin at the bi-material notch tip, m=1, 2 denotes the material number and λ_k corresponds to the *k*th eigenvalue of the problem. H_k is the notch stress intensity factor associated with the eigenvalue λ_k. Also in Eq. (1), f_{ijk} and g_{ik} are functions of the eigenvalues λ_k, the local edge geometry characterized by the angles θ_1 and θ_2 and the mismatch parameters α and β expressed in Eq. (2).

$$\alpha = \frac{\mu_1(\kappa_2+1) - \mu_2(\kappa_1+1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)}$$
$$\beta = \frac{\mu_1(\kappa_2-1) - \mu_2(\kappa_1-1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)} \quad (2)$$

In this equation, E_m, υ_m and $\mu_m = E_m/2(1+\upsilon_m)$ are the Young's modulus, Poisson's ratio and shear modulus associated with the material m (m=1,2), respectively. The Kolosov constant κ_m is equal to $3-4\upsilon_m$ for the plane strain and $(3-\upsilon_m)/(1+\upsilon_m)$ for the plane stress conditions. When α=β=0, the stress field is similar to a homogeneous notch with identical materials. The parameter α is positive when material 2 is more compliant than material 1, while it is negative when material 2 is stiffer than material 1. Generally, the stress state near the bi-material notch is most likely to become singular due to one or two eigenvalues in the range of $0 \leq \lambda_k \leq 1$, depending on the interface notch configuration. There is an inherent interaction between the singular terms corresponding to mode I (opening) and II (shearing). The first singular term corresponds to mode I and the second one corresponds to mode II.
While very few investigations have been conducted in the past to study the photoelastic behavior of bi-material notches (e.g. Meguid & Tan, 2000; Ayatollahi et al., 2010, 2011), the effects of mismatch parameters on the photoelastic fringe patterns has not yet been studied by the researchers. In this research, first the effects of mismatch parameters (α and β) on the results of photoelasticity were studied theoretically in a general stress field problem. Then, In order to evaluate the analytical predictions, a photoelastic test program was conducted on the V-notched bi-material Brazilian disc specimens and the experimental results were compared with the analytical predictions.

2. Theoretical study of photoelastic fringes

Based on the classical concepts of photoelasticity, the mathematical equation for an isochromatic fringe is written generally as

$$2\tau_{\max} = \frac{Nf_{\sigma}}{h}, \tag{3}$$

where τ_{max} is the maximum in-plane shear stress; N is the fringe order; f_{σ} is the material fringe value and h is the specimen thickness. The relation between the maximum in-plane shear stress τ_{max} and the stress components in polar coordinate system is:

$$(2\tau_{\max})^2 = (\sigma_{rr} - \sigma_{\theta\theta}) + 4\tau^2_{r\theta}. \tag{4}$$

In order to study of the effects of mismatch parameters on the shape of the photoelastic fringes, it was assumed that the pure mode I conditions (corresponding to the first term of Eq. (1)) exists near the bi-material notch tip. Therefore, by substituting the first term of Eq. (1) into Eqs. (3) and (4), the radial distance r can be derived as:

$$r_m = \left[\frac{(\frac{Nf_{\sigma}}{h})_m^{\;2}}{(H_1)^2\left(\left[f_{rr}^{\;m}\right]^2 + \left[f_{\theta\theta}^{\;m}\right]^2 - 2f_{rr}^{\;m} f_{\theta\theta}^{\;m} + 4\left[f_{r\theta}^{\;m}\right]^2\right)} \right]^{\frac{1}{2(\lambda_1 - 1)}}. \tag{5}$$

Eq. (5) can be used for plotting r_m (locus of each fringe in each material) versus θ for different values of α and β .It was assumed in this study that the material 1 is stiffer than material 2. Therefore, the parameter α becomes a positive value. Figs. (2-4) show the isochromatic fringes around the bi-material notch tip for different mismatch parameters listed in Table 1.

Table 1. Different mismatch parameters

Cases	β	α
1	0, 0.1	0
2	0.2	0, 0.5, 0.95
3	0, 0.15, 0.3	0.6

Fig. 2 also shows the difference between the photoelastic fringe patterns related to homogeneous and bi-material notches. It is clear that the shape of isochromatic fringes and maximum fringe radius changes in each material.

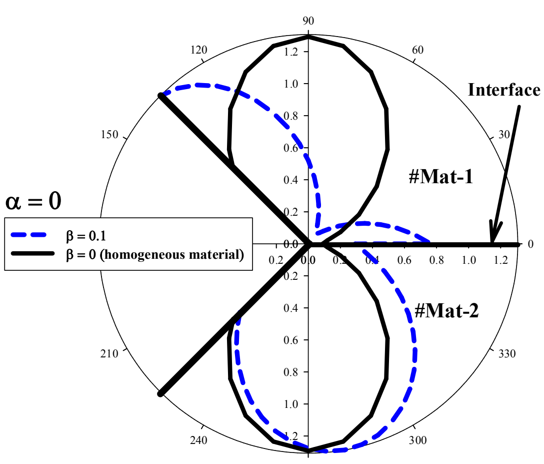


Fig. 2. Isochromatic friges near homogeneuos and bimaterial notch tip

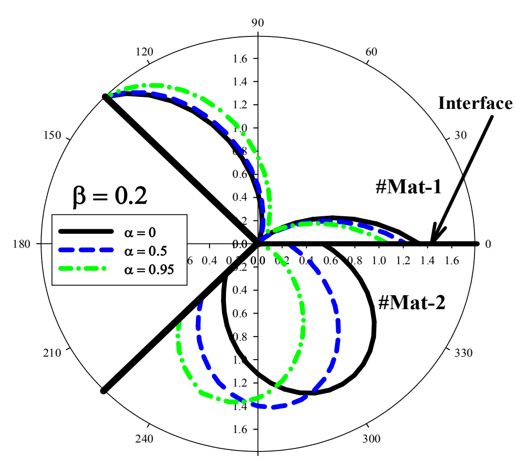


Fig. 3. Isochromatic fringes for different values of α (β = 0.2)

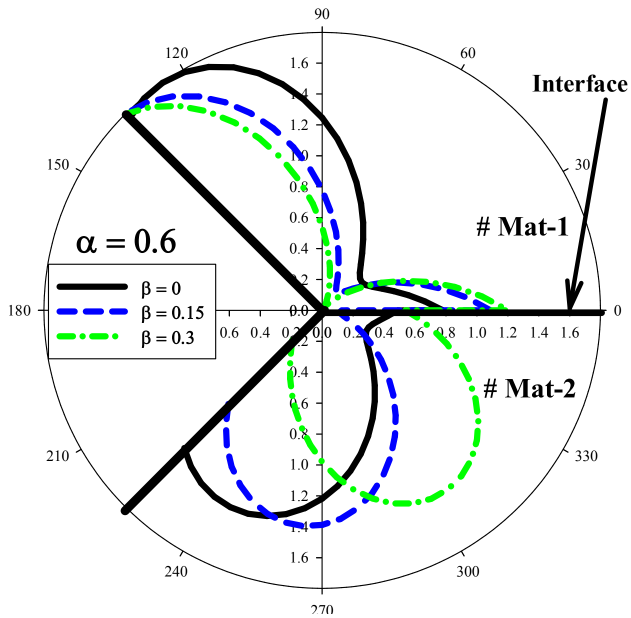


Fig. 4. Isochromatic fringes for different values of β (α = 0.6)

The effects of mismatch parameters α and β on the isochromatic fringes are shown in Figs. 3 and 4. It is seen from Fig. 3 that the maximum fringe radius around a bi-material notch tip rotates clockwise in each material by increasing α. It is also seen from Fig. 4 that by increasing β, the maximum fringe radius rotates counterclockwise in each material.

3. Experimental study of photoelastic fringes

To assess the accuracy of the theoretical study, photoelastic tests were conducted on two V-notched bi-material Brazilian disc specimens with a central notch, as shown in Fig. 5. The specifications of the test specimens are presented in Table 2. In Figs. 6 and 7, the photoelastic fringe patterns obtained experimentally under mode I conditions are illustrated and compared with the theoretical reconstruction. In specimen 2, the loading angle corresponding to pure mode I was obtained from finite element modeling of the test specimen. The angle θ' corresponding to mode I conditions was 132^0 which was determined from finite element analysis by finding the angle related to H_2=0 (The calculation details are explained by Meguid and Tan (2000) and Ayatollahi et al. (2010).

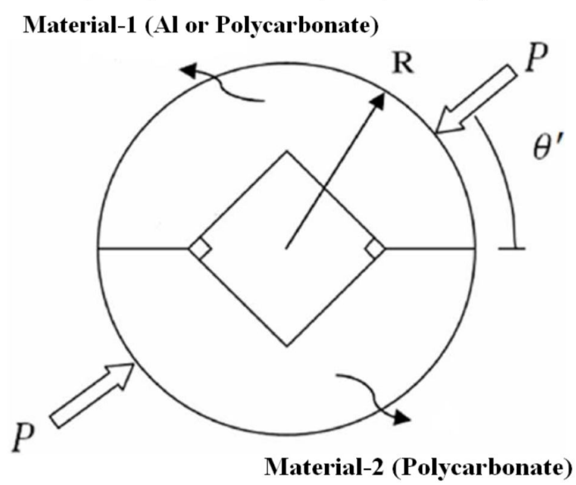
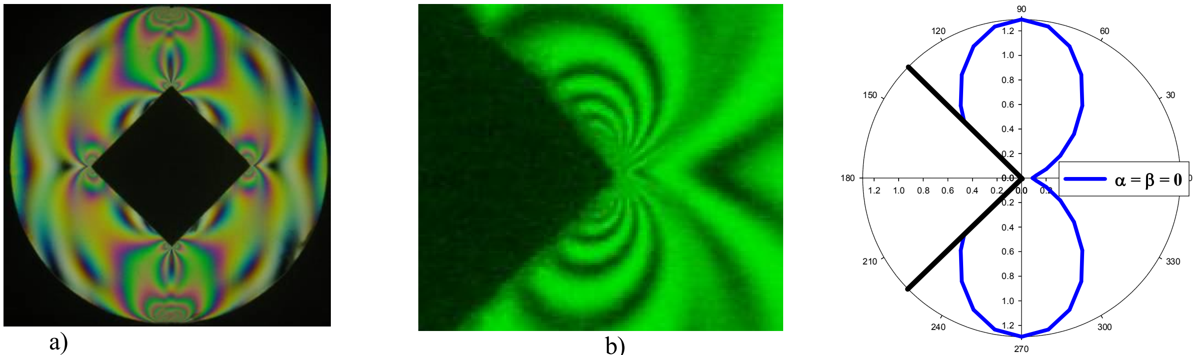


Fig. 5. Bi-material V-notched Brazilian disc

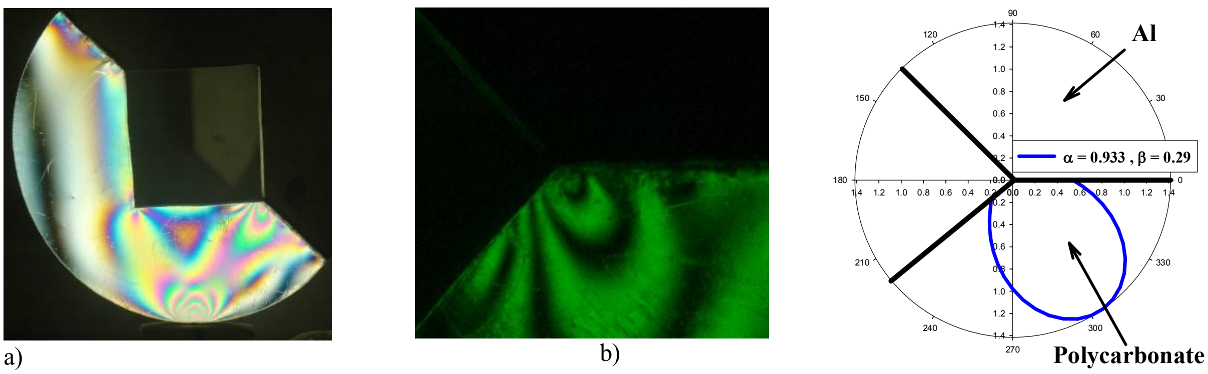
Table 2. Test specimens

Brazilian disc specimens	γ (Degrees)	Combination of materials	α	β
1	90^o	Polycarbonate(homogenous notch)	0	0
2	90^o	Al / Polycarbonate	0.93	0.29



a) b) c)

Fig. 6. Isochromatic fringe patterns in specimen 1. a) colored isochromatic fringes, b) monochromatic isochromatic fringes, c) reconstructed isochromatic fringes



a) b) c)

Fig. 7. Isochromatic fringe patterns in specimen 2. a) colored isochromatic fringes, b) monochromatic isochromatic fringes, c) reconstructed isochromatic fringes

It is seen from Figs. 6 and 7 that there is a very good correlation between the theoretical reconstruction and the experimental observations.

4. Conclusions

The effects of mismatch parameters on the shapes of photoelastic fringe patterns were studied in this research. It was shown that the mismatch parameter α rotates the isochromatic fringes clockwise and the mismatch parameter β rotates it counterclockwise in each material. The experimental results of photoelasticity were in a good agreement with the theoretical reconstructions.

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