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Wide range brittle fracture curves for U-notched components based on UMTS model

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ABSTRACT

Extensive brittle fracture curves are presented in the present paper for engineering components weakened by a U-shaped notch under different in-plane loading conditions from pure mode I to pure mode II. The curves were obtained in a computational manner on the basis of an appropriate brittle fracture model, namely the U-notched maximum tangential stress (UMTS) criterion, suggested and employed several times in the past by the author and his co-researchers to assess mixed mode fracture in numerous U-notched samples. Eight different notch tip radii were considered in the computations. Extensive brittle materials were also taken into consideration by using different values of the material critical distance in the calculations. By estimating theoretically the load-carrying capacity and the fracture initiation angle using solely the two basic material properties, namely the ultimate tensile strength and the plane-strain fracture toughness, engineers can design conveniently the U-notched brittle components and structures aiming to avoid abrupt fracture.

1. Introduction

Despite general targets exist in all kinds of mechanical design; a main goal is certainly to prevent failure in components with a desirable safety factor. For this purpose, several failure criteria have been suggested and employed regarding various material and structural failures like yielding, tearing, brittle fracture, fatigue fracture, creep rupture, buckling etc.

An important branch of mechanical engineering, namely the fracture mechanics, is focused on the design and analysis of the elements containing stress concentrators like cracks, flaws, scratches and notches. Such elements can fail in different manners depending on the material properties and also on the type of the loads applied. Since fracture occurs suddenly in brittle and quasi-brittle materials, the main attention in fracture mechanics is usually paid to the brittle fracture as a catastrophic failure mode.

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Unlike cracks and flaws which in the most cases are unfavorable to be detected in engineering components, notches, particularly U and V-shaped ones are utilized because of some special design requirements. A U-shaped notch concentrates stresses around its border and therefore can become prone to crack initiation. The likely cracks can grow in the notched body and finally result in fracture. From the view point of fracture mechanics, the mechanisms of crack initiation and propagation from a notch border are basically different for ductile and brittle materials. For brittle materials, the initiation of crack from the notch border consumes a large portion of the total fracture energy and the crack propagation is very less contributed in energy consumption. This is because the crack propagation is such a fast and unstable phenomenon that the final fracture occurs abruptly. Conversely, for ductile materials that exhibit moderate or large plastic deformations around notches, both crack initiation and propagation stages consume considerable amount of energy during ductile rupture.

Different failure concepts have been proposed in the open literature to estimate brittle fracture in engineering components and structures weakened by notches of various shapes. Lazzarin and Zambardi (2002) made use of the strain energy density (SED) approach to predict failure in sharp V-shaped notches under localized and generalized plasticity. The SED concept has also been employed by the same and the other researchers to predict brittle fracture in components containing U, V and key-hole notches under in-plane loading conditions (see for example Lazzarin and Zambardi, 2001; Berto and Lazzarin, 2009a; Gomez et al., 2009; Ayatollahi et al., 2011a; Berto and Lazzarin, 2009b; Lazzarin et al., 2009; Berto et al., 2012; Lazzarin et al., 2013). The fictitious notch tip radius is the other failure model utilized by Berto et al. (2008, 2009) for sharp V-shaped notches. Moreover, several fracture criteria have been previously proposed based on determining a critical value for the notch stress intensity factors, so-called the notch fracture toughness. In this field, one can refer to Gomez and Elices (2003a,b), Seweryn (1994), Leguillon and Yosibash (2003), Gomez and Elices (2004) and Ayatollahi and Torabi (2010a) for pure mode I and Ayatollahi and Torabi (2010b, 2011a,b), Ayatollahi et al. (2011b), Yosibash et al. (2006) and Priel et al. (2007) for mixed mode I/II and pure mode II loading conditions.

One of the most important concepts of failure in the context of brittle fracture is the maximum tangential stress (MTS) concept, proposed originally by Erdogan and Sih (1963) for investigating mixed mode brittle fracture in elements containing a sharp crack. The MTS concept has been utilized several times in the past by Ayatollahi and Torabi to predict mixed mode brittle fracture in the V-notched Brazilian disc (V-BD) specimens made of PMMA (Ayatollahi & Torabi, 2010b; Ayatollahi et al., 2011b), polycrystalline graphite (Ayatollahi & Torabi, 2011a) and soda-lime glass (Ayatollahi & Torabi, 2011b). The main conclusion obtained from those investigations was that the MTS model has been a suitable failure criterion with a very good accuracy in the context of mixed mode brittle fracture of V-notches. Ayatollahi and Torabi (2009) published a paper dealing with extending the MTS concept to U-notched domains in which the onset of brittle fracture and the fracture initiation angles for U-notched specimens of PMMA have been predicted by means of the mixed mode I/II fracture curves of the UMTS criterion. They have also used the UMTS model to estimate the fracture toughness and the fracture initiation angles in U-notched Brazilian disc specimens made of PMMA and soda-lime glass under pure mode II loading (see Ayatollahi & Torabi, 2010c). More recently, Torabi (2013) has successfully employed the UMTS model for predicting mixed mode brittle fracture in several graphite plates weakened by U-shaped notches.

In this paper, numerous failure curves are presented based on the UMTS model with the aim to predict the mixed mode brittle fracture in U-notched components in the entire domain from pure mode I to pure mode II loading conditions. The fracture curves were developed in a computational manner in terms of the notch stress intensity factors (NSIFs) by using the linear elastic stress distribution around the notch border. By using such curves, as an advanced engineering design package, one can predict rapidly and more conveniently the mixed mode I/II fracture in U-notched members made of brittle materials for various notch tip radii.

2. Linear elastic stress field around a U-notch border

Filippi et al. (2002) proposed some expressions for in-plane elastic stress distribution around a rounded-tip V-notch shown in Fig. 1. This stress distribution was an approximate formula because it satisfies the boundary conditions only in a finite number of points on the notch edge and not on the entire edge. In a reduced case where the notch angle is zero, the blunt V-notch becomes a U-notch (see Fig. 2).

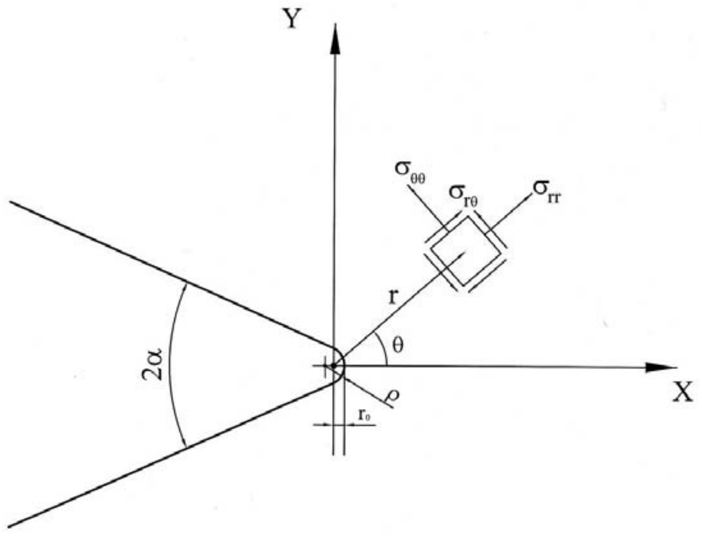


Fig. 1. Round-tip V-notch with its Cartesian and polar coordinate systems

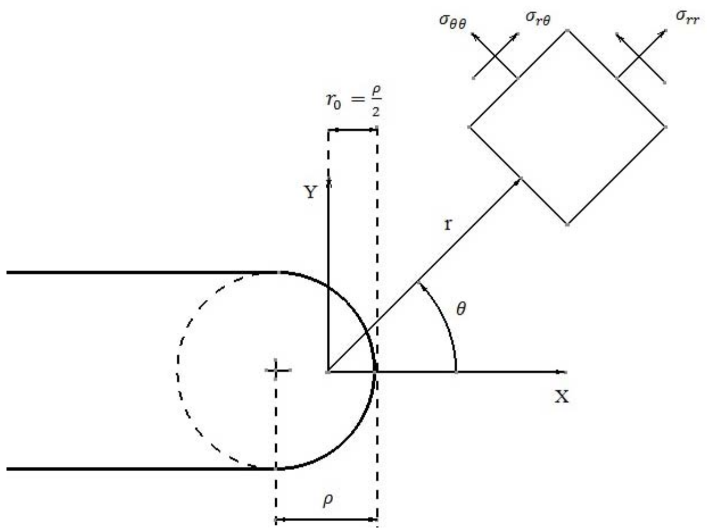


Fig. 2. U-notch with its Cartesian and polar coordinate systems

The mixed mode I/II stresses can be written as (Ayatollahi & Torabi, 2009)

$$\sigma_{\theta\theta} = \frac{1}{2\sqrt{2\pi r}}\left\{K_I^U\left[(\frac{3}{2}+\frac{\rho}{r})\cos\frac{\theta}{2}+\frac{1}{2}\cos\frac{3\theta}{2}\right]+K_{II}^U\left[(\frac{3}{2}-\frac{\rho}{r})\sin\frac{\theta}{2}+\frac{3}{2}\sin\frac{3\theta}{2}\right]\right\} \tag{1}$$

The parameters K_I^U and K_{II}^U are the mode I and mode II notch stress intensity factors (NSIFs), respectively. ρ is the notch tip radius and the parameters r and θ denote the local polar coordinate system located at the distance $r_0 = \rho/2$ behind the U-notch tip. The expressions for NSIFs are (Ayatollahi & Torabi, 2009; Torabi & Jafarinezhad, 2012):

$$K_I^U = \frac{\sqrt{\pi\rho}}{2}\sigma_{\theta\theta}(\frac{\rho}{2},0)\,, \tag{2}$$

$$K_{II}^U = Lim_{r\to\frac{\rho}{2}} \sqrt{2\pi\, r}\,\frac{(\sigma_{r\theta})_{\theta=0}}{(1-\frac{\rho}{2r})}\,. \tag{3}$$

In Eqs. 2 and 3, $\sigma_{\theta\theta}(\frac{\rho}{2},0)$ is the tangential stress at the U-notch tip and $(\sigma_{r\theta})_{\theta=0}$ is the in-plane shear stress along the notch bisector line. To compute the values of NSIFs associated with any load applied, first, a finite element (FE) model should be created for each notched component and the load should be applied to the model. Then, the tangential stress at the notch tip and the limit of the bisector line shear stress (when the distance from the notch tip tends to zero) are computed. By substituting the computed values into Eq. (2) and Eq. (3), the NSIFs are achieved.

In the next section, the U-notched maximum tangential stress (UMTS) fracture criterion, utilized several times in the past by the author and his co-researchers for predicting mixed mode I/II and mode II brittle fracture in U-notches (see Ayatollahi & Torabi, 2009; Ayatollahi & Torabi, 2010c; Torabi, 2013), is elaborated. Then, as an advanced engineering design package, extensive fracture curves of the UMTS model are presented in forthcoming sections for different brittle materials and various notch tip radii.

3. The UMTS fracture model

The traditional MTS model is a well-known failure criterion frequently utilized to study mixed mode brittle fracture in sharp crack problems (Erdogan & Sih, 1963). According to this criterion, fracture occurs along the direction of maximum tangential stress θ_0 when the tangential stress at the critical distance r_c from the crack tip attains a critical value $(\sigma_{\theta\theta})_c$. The critical parameters r_c and $(\sigma_{\theta\theta})_c$ are often considered to be independent of geometry and loading conditions. The suitability of the MTS and the generalized MTS (GMTS) (Smith et al., 2001) criteria in estimating the fracture instance in bodies containing a sharp crack have been evaluated in the past by several investigators for different brittle materials. A very good agreement has been found to exist between the theoretical and the experimental results (e.g. Erdogan & Sih, 1963; Ayatollahi & Aliha, 2008, 2009, Ayatollahi et al., 2011c).

In recent years, the classical MTS criterion has been extended to U and V-notched domains by Torabi and his co-researchers. Their first successful results has been published in a research article in which some mixed mode fracture test results reported in literature on U-notched PMMA plates have been predicted by means of the U-notched MTS (UMTS) criterion (Ayatollahi & Torabi, 2009). After that, they developed the sharp and the rounded-tip V-notched MTS fracture criteria (e.g. the SV-MTS (Ayatollahi et al., 2011b) and the RV-MTS (Ayatollahi & Torabi, 2010b, 2011a,b)) to predict sudden fracture in V-notched test specimens made of PMMA, polycrystalline graphite and soda-lime glass. Although the UMTS model has been elaborated by Ayatollahi and Torabi (2009), a brief description of this criterion is presented herein with the aim to formulate the model and reach the fracture curves.

According to the UMTS fracture criterion, the tangential stress at a critical distance around the notch border should be a maximum at the onset of fracture. Thus:

$$\frac{\partial\sigma_{\theta\theta}}{\partial\theta} = 0 \Rightarrow \theta = \theta_0 \tag{4}$$

The angle θ_0 is the fracture initiation angle (sometimes referred to as the notch bifurcation angle) which determines the location of crack initiation on the U-notch border with respect to the polar coordinate system (see Fig. 2).

Substituting Eq. (1) into Eq. (4) gives

$$K_I^U\left[-(\frac{3}{4}+\frac{\rho}{2r_{c,U}})\sin\frac{\theta_0}{2}-\frac{3}{4}\sin\frac{3\theta_0}{2}\right]+K_{II}^U\left[(\frac{3}{4}-\frac{\rho}{2r_{c,U}})\cos\frac{\theta_0}{2}+\frac{9}{4}\cos\frac{3\theta_0}{2}\right]=0\,. \tag{5}$$

Note that the parameter r in Eq. (1) is substituted with $r_{c,U}$ (i.e. the U-notch critical distance) according to the requirements of the UMTS criterion (see Ayatollahi & Torabi, 2009, 2010c; Torabi, 2013). In pure mode I loading conditions**,** crack initiates along the notch bisector line and the fracture initiation angle from the notch border (θ_0) is zero because both the geometry and loading are symmetric. In pure mode II loading, K_I^U is zero. Therefore, Eq. (5) is simplified to

$$(\frac{3}{4}-\frac{\rho}{2r_{c,U}})\cos\frac{\theta_0}{2}+\frac{9}{4}\cos\frac{3\theta_0}{2}=0\Rightarrow\theta_0=\theta_{0II}\,. \tag{6}$$

Eq. (6) implies that mode II fracture initiates from a point on the notch border that its angular position from the local polar coordinate system is recognized by the angle θ_{0II} which depends on the critical distance $r_{c,U}$ and the notch tip radius ρ. Another requirement of the UMTS criterion suggests that brittle fracture takes place when the tangential stress at the critical distance attains necessarily the critical value $(\sigma_{\theta\theta})_c$. Therefore, Eq. (1) in critical conditions can be written as

$$(\sigma_{\theta\theta})_c=\frac{1}{2\sqrt{2\pi r_{c,U}}}\left\{K_I^U\left[(\frac{3}{2}+\frac{\rho}{r_{c,U}})\cos\frac{\theta_0}{2}+\frac{1}{2}\cos\frac{3\theta_0}{2}\right]+K_{II}^U\left[(\frac{3}{2}-\frac{\rho}{r_{c,U}})\sin\frac{\theta_0}{2}+\frac{3}{2}\sin\frac{3\theta_0}{2}\right]\right\}. \tag{7}$$

A simple relationship has been reported by Ayatollahi and Torabi (2009) between $(\sigma_{\theta\theta})_c$ and the mode I notch fracture toughness K_{Ic}^U. It is

$$(\sigma_{\theta\theta})_c=\frac{(2+\frac{\rho}{r_{c,U}})}{2\sqrt{2\pi r_{c,U}}}K_{Ic}^U\,. \tag{8}$$

Substituting Eq. (8) into Eq. (7) gives

$$K_I^U\left[(\frac{3}{2}+\frac{\rho}{r_{c,U}})\cos\frac{\theta_0}{2}+\frac{1}{2}\cos\frac{3\theta_0}{2}\right]+K_{II}^U\left[(\frac{3}{2}-\frac{\rho}{r_{c,U}})\sin\frac{\theta_0}{2}+\frac{3}{2}\sin\frac{3\theta_0}{2}\right]=(2+\frac{\rho}{r_{c,U}})K_{Ic}^U\,. \tag{9}$$

Note that the parameter K_{Ic}^U (i.e. the mode I notch fracture toughness), which can be determined experimentally (see Ayatollahi & Torabi, 2009, 2010c), is not a material constant and depends not only on the material properties but also on the notch geometry. For known values of $r_{c,U}$ and K_{Ic}^U, one can divide both sides of Eq. (5) and Eq. (9) by K_{Ic}^U and solve simultaneously these two equations for any value of θ_0 between 0 and θ_{0II} and draw the variations of K_{II}^U/K_{Ic}^U (vertical axis) versus K_I^U/K_{Ic}^U (horizontal axis) in order to achieve the mixed mode fracture curves for U-notches of different tip radii (see Ayatollahi & Torabi, 2009; Torabi, 2013).

The notch critical distance $r_{c,U}$ which is measured from the origin of the coordinate system (not from the notch tip); see Fig. 2; can be considered as follows (Ayatollahi & Torabi, 2010a):

$$r_{c,U}=\frac{\rho}{2}+r_c=\frac{\rho}{2}+\frac{1}{2\pi}(\frac{K_{Ic}}{\sigma_c})^2\,, \tag{10}$$

where r_c, K_{Ic} and σ_c are the critical distance for sharp cracks, the plane-strain fracture toughness and the ultimate tensile strength of brittle material, respectively.

The procedure of plotting the mixed mode fracture curves for U-notched components and utilizing them in predicting the load-carrying capacity and the fracture initiation angles are presented in the forthcoming sections.

4. Fracture curves

Since use of notches, particularly U-shaped ones, in engineering components and structures are often inevitable; the main goal from the view point of mechanical design is to prevent failures in notched components which are more serious in comparison with un-notched ones because of stress concentration. As mentioned earlier, brittle fracture is a catastrophic failure by which the notched component may suddenly broken and lead to heavy damages. Therefore, the brittle fracture initiation in U-notches should be reliably predicted by means of suitable fracture models (e.g. the UMTS model). The procedure to find the load-carrying capacity of a U-notched component under mixed mode I/II loading by means of the UMTS fracture curve, can be simply explained as follows:

1. Apply a unit load to the FE model of the U-notched component and compute the ratio K_{II}^U / K_I^U for it by using the stress distribution around the U-notch tip (see Eq. (2) and Eq. (3)). Note that the ratio is independent of the magnitude of the load.

2. Draw a line with the slope of K_{II}^U / K_I^U in the plane of fracture curve. The first point of the line is the origin of the coordinate system and the second one is the intercept point between the curve and the line.

3. Read the horizontal and the vertical components of the intercept point (i.e. (K_I^U / K_{Ic}^U) and (K_{II}^U / K_{Ic}^U)).

4. Multiply both the obtained components by K_{Ic}^U to achieve critical K_I^U and K_{II}^U values (the parameter K_{Ic}^U can be determined experimentally or theoretically using some appropriate failure criteria like those presented by Gomez et al. (2006).

5. Increase the initially applied unit load to the greater values till NSIFs reach to the values computed in the step 4. The load associated with the critical NSIFs obtained in this step is, in fact, the load-carrying capacity of the U-notched component.

Beside the fracture load, the other important parameter in the fracture analysis of U-notched members is the fracture initiation angle (θ_0). This parameter plays usually an important role in determining the overall damage in notched structures. To plot the curves of fracture initiation angle, a useful parameter called the mode mixity parameter (M_U^e) is defined herein

$$M_U^e = \frac{2}{\pi}\tan^{-1}\left(\frac{K_I^U}{K_{II}^U}\right) . \tag{11}$$

The value of M_U^e varies from zero (for pure mode II) to one (for pure mode I). By extracting K_I^U / K_{II}^U from Eq. (5) and substituting into Eq. (11), one can obtain:

$$M_U^e = \frac{2}{\pi}\tan^{-1}\frac{\left(\frac{3}{4}-\frac{\rho}{2r_{c,U}}\right)\cos\frac{\theta_0}{2}+\frac{9}{4}\cos\frac{3\theta_0}{2}}{\left(\frac{3}{4}+\frac{\rho}{2r_{c,U}}\right)\sin\frac{\theta_0}{2}+\frac{3}{4}\sin\frac{3\theta_0}{2}} . \tag{12}$$

In order to plot the curves of fracture initiation angle (i.e. a graph consists of a horizontal axis for M_U^e and a vertical axis for θ_0) for given values of the notch tip radius ρ and the notch critical distance $r_{c,U}$, one can follow the steps below:

1- Choose an arbitrary value of M_U^e between zero and one.

2- Substitute M_U^e into Eq. (12).

3- Solve Eq. (12) and determine the fracture initiation angle θ_0.

4- Repeat steps 1 to 3 for other values of M_U^e.

5- Draw the curve of fracture initiation angle utilizing the points calculated in step 4.

In order to predict the fracture initiation angle in any engineering component containing a U-notch by using the UMTS curve under mixed mode I/II loading, one should follow the steps below, respectively:

1. Apply a unit load to the FE model of the U-notched component and compute the ratio K_I^U / K_{II}^U for it by using the stress distribution around the U-notch tip (see Eq. (2) and Eq. (3)).

2. Substitute the ratio into Eq. (11) and calculate M_U^e.

3. Read simply from the UMTS curve of fracture initiation angle the fracture angle associated with the M_U^e value obtained in the step 2 (note that the fracture angle does not depend on the load magnitude but depends only on the mode mixity ratio K_I^U / K_{II}^U).

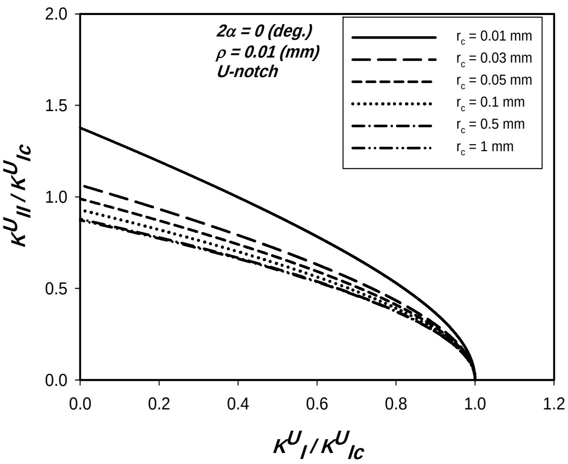
It should be noticed that by taking into account the notch coordinate system shown in Fig. 2, the sign of the fracture initiation angles (θ_0) and the mode II NSIFs (K_{II}^U) are negative, but the absolute values are shown in the forthcoming figures. In the next section, wide range failure curves are presented as a convenient design package to predict brittle fracture in U-notched components under mixed mode I/II loading. The curves depend highly on the values of the notch critical distance $r_{c,U}$ and the notch tip radius.

5. Results and discussion

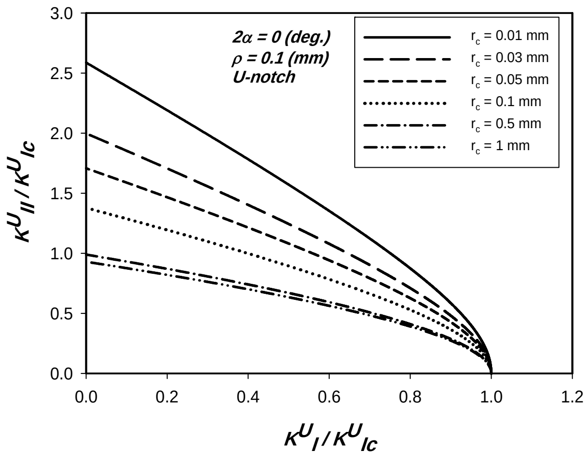
The most important parameter for engineers in designing mechanical components is perhaps the availability of a reliable, convenient and relatively rapid design package. The goal of this work is to prepare a design package for the engineers dealing with mixed mode fracture in mechanical components and structures made of brittle materials and weakened by a U-shaped notch.

The package consists of numerous fracture curves as well as the curves of fracture initiation angle obtained on the basis of the UMTS criterion as elaborated in the above sections. The notch tip radii included were ρ = 0.01, 0.1, 0.3, 0.5, 1, 3, 5 and 10 (mm). The generality of the curves was taken into account by considering different values of r_c parameter (i.e. $\frac{1}{2\pi}(\frac{K_{Ic}}{\sigma_c})^2$) related to various brittle materials. The values of r_c were 0.01, 0.03, 0.05, 0.1, 0.5 and 1 (mm). Total number of 96 curves (48 for the fracture curves and 48 for the curves of fracture initiation angle) were prepared.

Figs. 3 and Fig. 4 display respectively the mixed mode UMTS fracture curves and the curves of fracture initiation angle for various notch tip radii. Each figure consists of numerous curves related to different values of the critical distance r_c.



(a)



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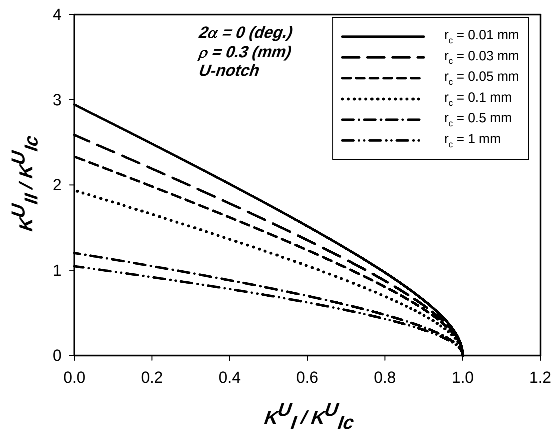

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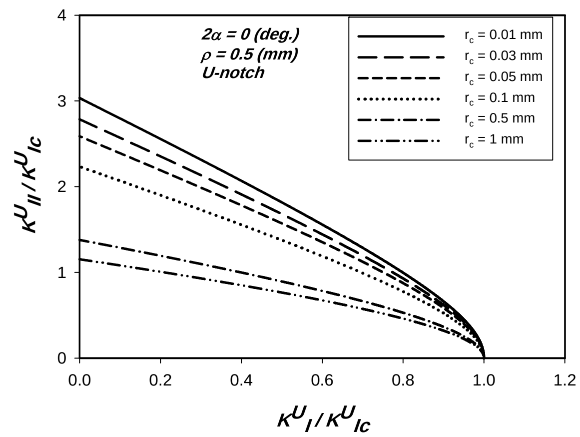

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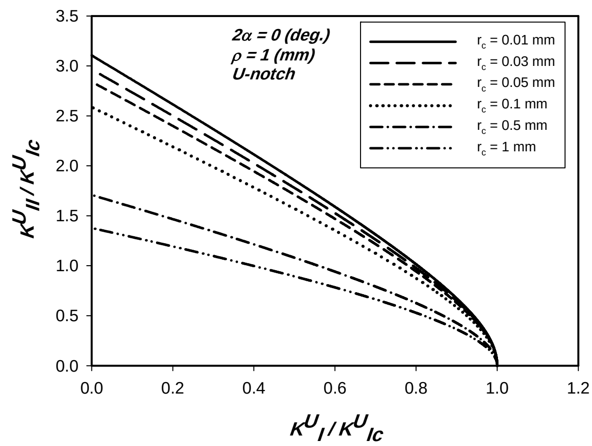

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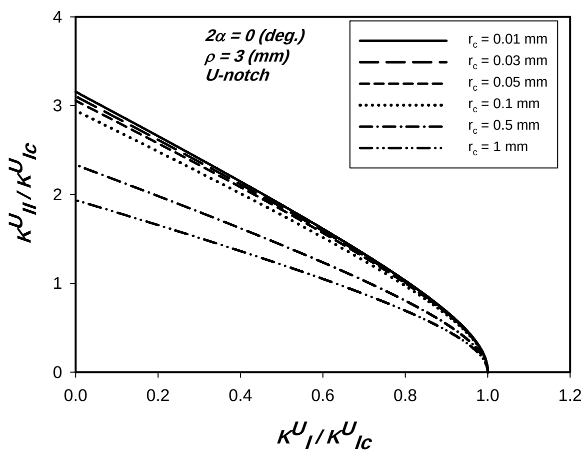

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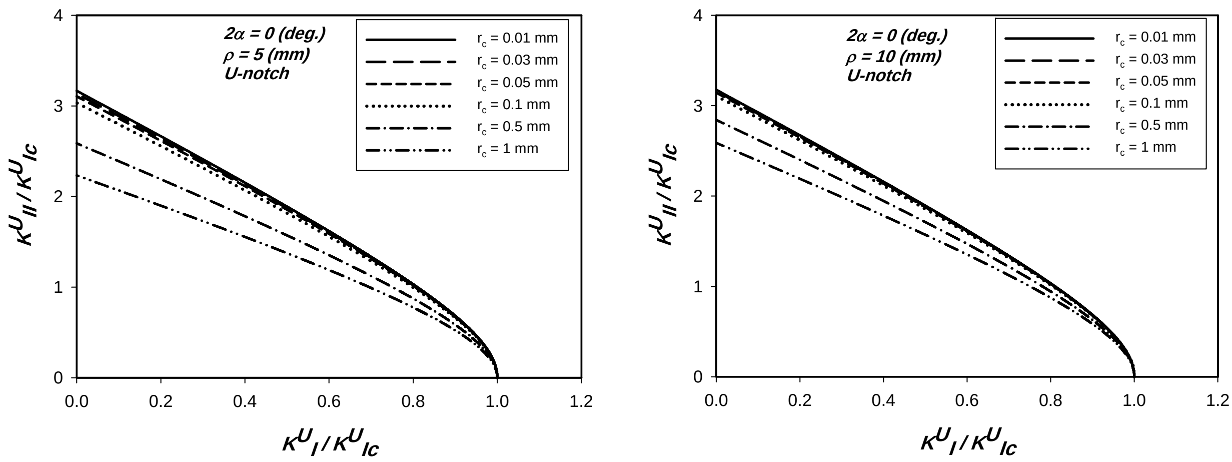
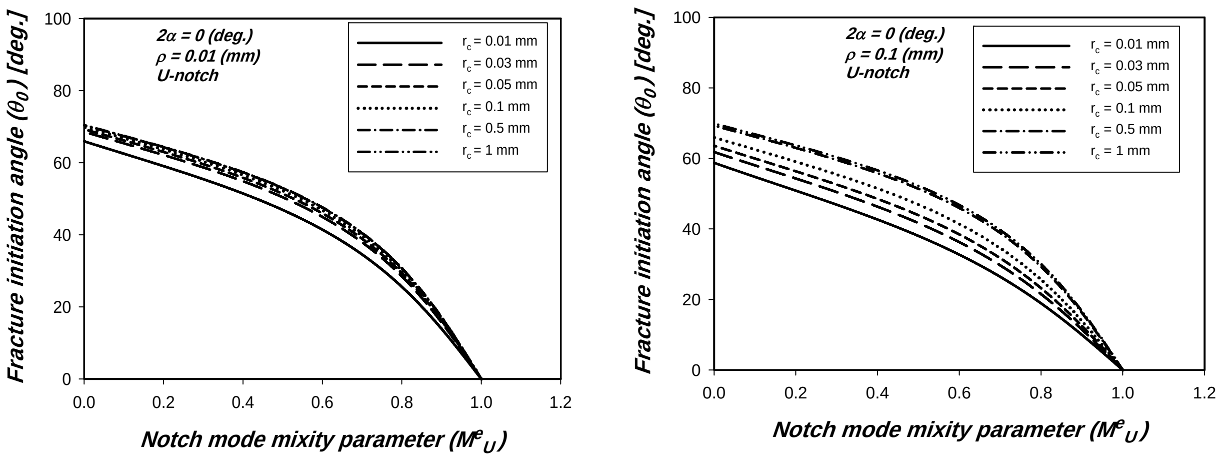
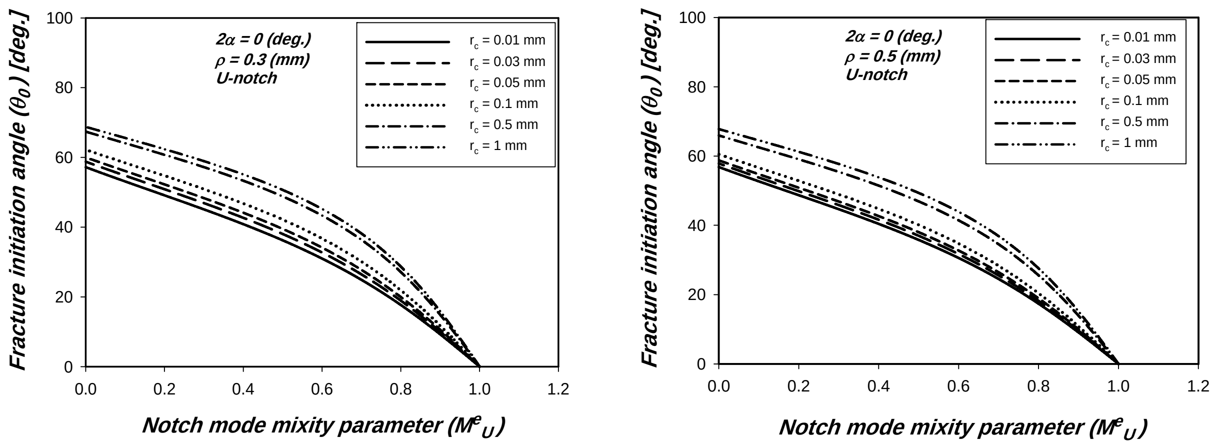


(g) *(h)*

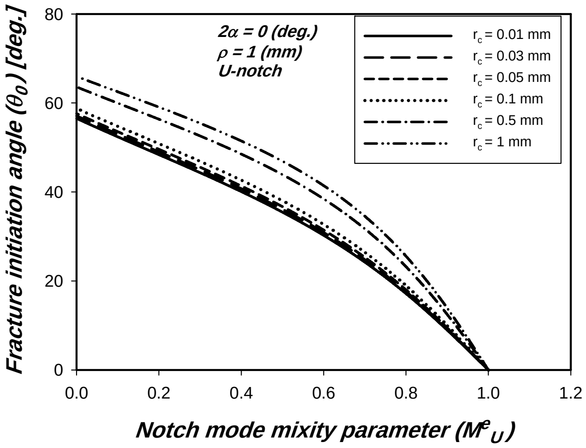
Fig. 3. Mixed mode UMTS fracture curves for different notch tip radii and various critical distances



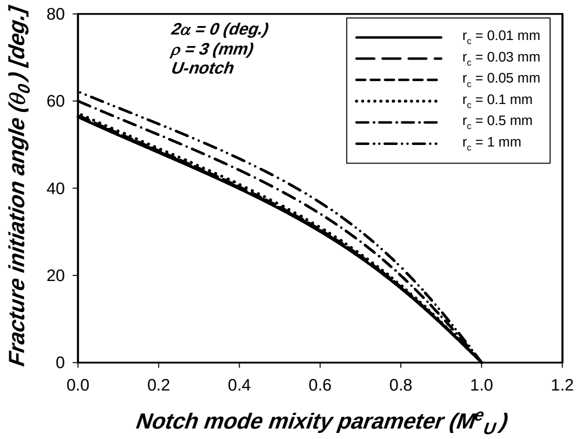
(a) *(b)*



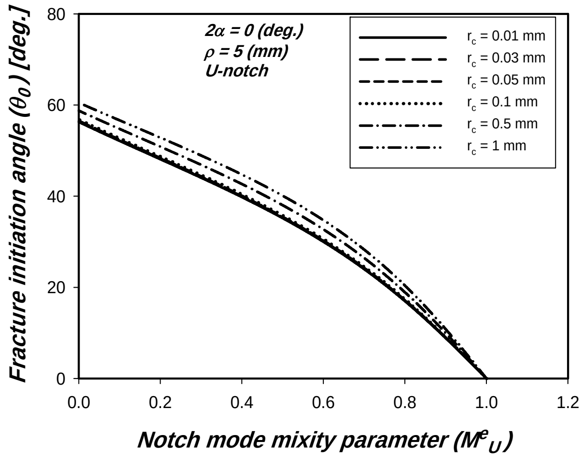
(c) *(d)*



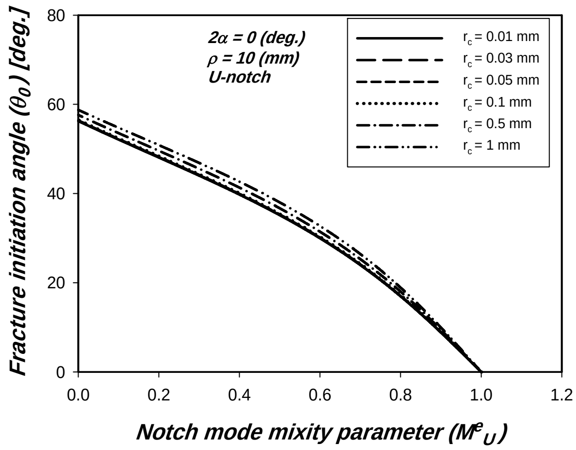
(e)



(f)



(g)



(h)

Fig. 4. Mixed mode UMTS curves of fracture initiation angle for different notch tip radii and various critical distances

It is worth mentioning that the curves presented above are recognized by their r_c values. Therefore, one should first calculate $r_c = \frac{1}{2\pi}(\frac{K_{Ic}}{\sigma_c})^2$ for the brittle material and then select the corresponding failure curves. Some important results obtained from Figs. 3 and 4 can be listed as follows:

1. For a constant notch tip radius, as r_c increases, the fracture curve travels downward. However, it is necessary to note that this behavior does not mean that fracture load decreases, because the fracture load depends on the critical values of K^U_I and K^U_{II} and not on the ratios K^U_I/K^U_{Ic} and K^U_{II}/K^U_{Ic}. Also, due to the dependence of r_c on both K_{Ic} and σ_c, for two notched members completely similar in geometry but made of different materials with various r_c values, it is not possible to make a comparison about the fracture loads. However, for such members with equal r_c, the member with lower σ_c would be broken at lower loads.

2. For a constant r_c, the fracture curve travels upward as the notch tip radius enhances meaning that the size of the safe zone increases. This is due to the decrease of the stress concentration.

3. For r_c values less than 0.1 mm, the fracture curves remains almost constant for the notch tip radii greater than 5 mm. This means that the curves are saturated and hence, the much enhancement of notch tip radius does not affect the level of stress concentration. For greater r_c values, the notch tip radius affects significantly the fracture curves for which the greater radius means greater safe zone.

4. Fig. 3h suggests that as the notch tip radius increases, the fracture curves for different r_c values would be very close together and a saturation would be taken place. In other words, for notch tip radii greater than a specific value, a single fracture curve would be obtained for different notch tip radii and various r_c values.

5. Fig. 4 shows that as r_c increases from 0.01 to 1 mm, the fracture initiation angle for the entire U-notches varies maximum of about 12 deg. The maximum is 12 deg. for the radii between 0.1 and 1 mm and the minimum is 3 deg. for the radius 10 mm.

6. For small notch tip radii, the curves of fracture initiation angles are less sensitive to great r_c values. Conversely, the curves related to great notch tip radii are less sensitive to small r_c values.

7. For the notch tip radii larger than 5 mm, a single curve of fracture initiation angle is achieved for the r_c values smaller than 0.1 mm.

8. Fig. 4h displays that for the notch tip radius equal to 10 mm, the fracture initiation angle varies about 3 deg. for the entire domain of variation of r_c which is negligible. Therefore, it is expected that for a specific notch tip radii larger than 10 mm, a constant curve would be achieved.

6. Conclusions

An advanced and comprehensive engineering design package was developed in the present work for predicting the load-carrying capacity and the fracture initiation angle of U-notched members made of brittle materials under mixed mode I/II loading. The package was resulted on the basis of the reliable and convenient UMTS fracture model and covers a wide range of notch tip radii and brittle materials. Using the package, one can design more rapidly and conveniently the brittle components and structures weakened by U-shaped notches without requiring formulating brittle fracture criterion.

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