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# Prediction the Charpy impact energy of functionally graded steels

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**ABSTRACT**

Functionally Graded Steels (FGSs) are possible solutions to improve the properties of steels made by Martensite and Bainite brittle phases. These phases are usually present in the interface between the carbon ferritic steel and the stainless austenitic steel. FGSs materials are widely investigated in the recent literature but only few works have been devoted to investigate the impact energy in the case of crack arresters. To partially fill this gap, the effect of the distance between the notch tip and the position of the median phase on the Charpy impact energy is investigated in the present paper. The results show that when the notch apex is close to the median layer the impact energy reaches its maximum value due to the increment of the absorbed energy by plastic deformation ahead of the notch tip. On the other hand, when the notch apex is far from the median layer, the impact energy strongly decreases. Keeping into account the relationship between the Charpy impact energy and the plastic volume size, a new theoretical model has been developed to link the composite impact energy with the distance from the notch apex to the median phase. The results of the new model show a sound agreement with previous results taken from the literature.



## 1. Introduction

A number of cracking criteria applicable to brittle or quasi-brittle materials under monotonic loading have been published, with the aim of evaluating the critical static load of a notched component subjected to mode I loading and weakened by sharp and blunt notches (Gómez et al. 2000, Gómez & Elices, 2003a,b, 2004, 2006; Gómez et al. , 2005).

Torabi and Aliha (2013) developed a guideline for approving the railway axles made of C35 steel and containing surface and/or in-body defects after manufacturing. First, they modeled several circular cracks on the surface and in the body of the axle at its critical cross-section. Then, they determined the admissible size of such cracks utilizing the fracture mechanics. Finally, they

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compared the theoretical result for the semi-circular surface crack with the allowable size obtained by the international railway standard. In another work, Torabi (2013a) investigated the components weakened by U-notch under different in-plane loading conditions from pure mode I to pure mode II. He presented the brittle fracture curves based on the maximum tangential stress criterion, to evaluate the mixed mode fracture in numerous U-notched samples and also obtained the critical load of the components. Moreover, Torabi (2013b) investigated the sudden fracture in V-notched brittle materials under combined tension-shear loading conditions. He used the maximum tangential stress theory for predicting the mixed mode brittle fracture in the specimens weakened by notch. Finally, in that work the obtained results have been compared with the experimental ones. Mirlohi and Aliha (2013) investigated the mixed mode I/II crack growth path of an angled crack biaxial plate utilizing the higher-order terms of Williams's series expansion and the maximum tangential stress criterion. They compared their results with the experimentally reported trajectories for the angled crack plate specimen and obtained a very good agreement. Ayatollahi et al. (2006), Aliha et al. (2008), Aliha and Ayatollahi (2009) and Ayatollahi and Aliha (2011) have shown that considering the first non-singular stress term in addition to the conventional singular terms can improve significantly the theoretical predictions of brittle fracture results obtained from different materials and test specimens.
Torabi (2013c) predicted the critical load of O-notched ductile steel plates under pure tension utilizing the equivalent material concept together with the mean stress and the point stress failure concepts. Functionally graded steels (FGSs) may be produced during welding of alloy steels. Jang et al. (2008) studied experimentally and numerically the effects of notch position on the failure behavior and energy absorption when the Charpy impact test was performed at -1°C. Carbon steel plate with a thickness of 25 mm was welded and specimens were fabricated from the welded plate. The Charpy impact tests were then performed on specimens having different notch positions varying within HAZ. A series of 3-D FE analysis which simulate the Charpy test were also carried out.

Recently, FGSs have been produced by electroslag remelting (ESR) (Aghazadeh et al., 2005). Studies on transformation characteristics of FGSs produced from austenitic stainless steel and plain carbon steel have shown that as chromium, nickel, and carbon atoms diffuse at remelting stage, alternating regions with different transformation characteristics are created in the material. By selecting appropriate arrangement and thickness of original ferritic and original austenitic steels as electrodes, composites with graded ferrite, and austenite regions together with bainite or martensite layers may be made (Nazari et al., 2009).

In the works performed by Nazari and Aghazadeh Mohandesi (2009) and Nazari (2012) Charpy impact energy of crack divider specimens was measured experimentally and the obtained results show that the impact energy of the specimens depends on the type and the volume fraction of the present phases. A theoretical model based on the rule of mixtures, which correlates the impact energy of FGSs to the impact energy of the individual layers through Vickers microhardness of the layers, was obtained in the study of Nazari and Aghazadeh Mohandesi (2009). Following parallel tracks, Charpy impact energy of FGSs produced by electroslag remelting in the form of crack arrester configuration has been investigated in Nazari et al. (2009).

Nazari (2012) obtained the impact energy for all layers in the case of crack divider of FGSs utilizing the relation between the impact energy of each layer and the surrounded area of stress-strain diagram. The results obtained in that study indicate that the notch tip position with respect to bainite or martensite layer significantly affects the impact energy. The closer the notch tip to the tougher layer, the higher the impact energy of the composite due to increment of energy absorbed by plastic deformation zone ahead of the notch and vice versa. Empirical relationships have been determined to correlate the impact energy of FGSs to the morphology of each layer (Nazari et al., 2010).

As stated in a research work reported by Nazari (2012) for crack arrester configuration, no accurate mathematical modeling was presented except that done by finite element simulation.

The aim of the present work is to develop a new analytical model for the assessment of the Charpy impact energy of FGSs in the form of a crack arrester configuration. The outputs of the proposed model are compared with the experimental results taken from the recent literature showing a sound agreement.

## 2. Theoretical model

### 2.1 Initial Analysis

The experimental Charpy impact energy of FGSs in the form of crack arrester configuration has been obtained by Nazari and Aghazadeh (2010). That work showed that the impact energy of γβα composite when the notch is in the austenitic region is higher than in the case in which the notch is in the ferritic region. This is due to the fact that the plastic region at the notch tip, and then the capability to absorb energy, is larger when the notch is in the austenitic region. When the notch tip is close to the baintic layer but remaining in the austenitic region the impact energy decreases while, on the other hand, the impact energy increases if the notch is placed in the ferritic region. Furthermore, variation of impact energy in austenitic region of γMγ composite is the same of austenitic region of γβα composite.

A link between the impact energy and the size of the plastic region has been obtained in (Aghazadeh Mohandesi et al., 2010). The equation is as follows:

$$CV = Nr_y + M, \tag{1}$$

where CV is the impact energy, $r_y$ is the plastic region radius, *N* and *M* are constants which depend on the material. Considering the Von-Mises yield criterion, the following relation is obtained for the plastic region size of a homogenous material under plane strain conditions and mode I loading (Aghazadeh Mohandesi et al., 2010).

$$r_y = \alpha\left(\frac{K_{IC}}{\sigma_y}\right)^2 \times \cos^2\left(\frac{\theta}{2}\right) \times \left[(1-2\nu)^2 + 3\sin^2\left(\frac{\theta}{2}\right)\right], \tag{2}$$

where ν is Poisson's ratio, $K_{IC}$ is the material's fracture toughness, $\sigma_y$ is the yield stress and α is a constant. Eq. (2) shows that the plastic region size varies as a function of the angle, θ. In order to neglect the effect of the angle on the size of the plastic region, the following new equation is proposed to assess the impact energy in the case of a crack arrester:

$$CV_{FG(d)} = M + \left[\frac{A_{rFG(d)}}{A_{rH(d)}}\right]^{\frac{1}{2}} \times (CV_{H(d)} - M). \tag{3}$$

Eq. (3) depends on the distance of the notch tip from median phase of FGS (d) and on the type of the considered composite.

$CV_{H(d)}$ is the impact energy of the layer containing the notch tip. $A_{rH(d)}$ is the area of the plastic region for a homogenous specimen with the properties of the layer containing the notch tip. $A_{rFG(d)}$ is the area of the plastic region for a FG specimen in which the mechanical properties vary along the direction of the notch bisector line.

In crack divider specimen, the mechanical properties for all layers have been obtained from ferritic/austenitic layer to median baintic/martensitic layer (which denotes with distance *X* from ferritic/austenite layer by Nazari, (2012). In crack arrester specimen, the distance between notch tip and median bainite/martensite position is considered with $d$. Moreover, the mechanical properties of notch tip in crack arrester specimen could be obtained with determining its position in crack divider specimen.

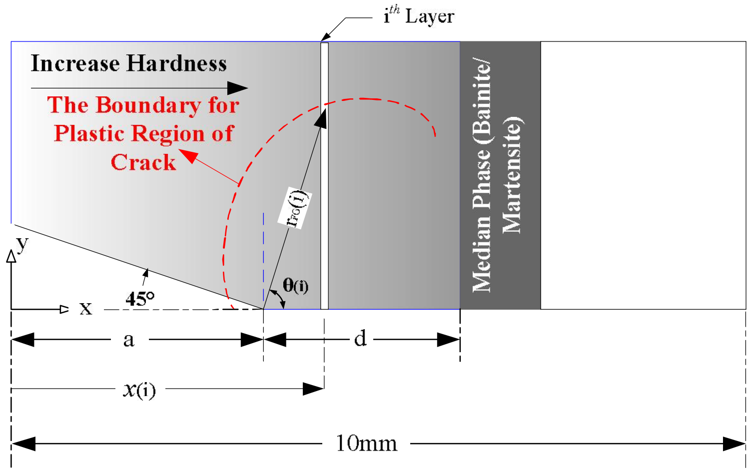



**Fig. 1.** Profile of the plastic region in front of the notch tip in the graded region of a FGS (crack arrester configuration)

In order to determine the mechanical properties of the layer containing the notch tip, it has been supposed that the fracture toughness ($K_{IC}$) and yield stress ($\sigma_y$) vary exponentially with respect to the direction $X$:

$$K_{IC}(X) = D_1 \exp(D_2 X) \quad (4)$$

$$\sigma_y(X) = D_3 \exp(D_4 X) \quad (5)$$

where $X$ is the distance of the layer from the crack divider specimen edge, $D_i$ are the constants that can be obtained by imposing the appropriate boundary conditions shown in Table 1.

The fracture toughness ($K_{IC(d)}$) and the yield stress ($\sigma_{y(d)}$) corresponding with the layer containing the notch tip could be obtained by substituting $X=X^*-d$ in the Eqs. (4-5).
By considering Eq. (2), $A_{rh}$ can be obtained as follows:

$$A_{rH(d)} = 2\left[\frac{1}{2}\int_{\theta=0}^{\theta^*} r_H^{\,2}(\theta)d\theta\right] = \alpha^2\left(\frac{K_{IC(d)}}{\sigma_{y(d)}}\right)^4 \int_{\theta=0}^{\theta^*} f(\theta)^2 d\theta = \alpha^2 \times \left(\frac{D_1}{D_3}\right)^4 \times \exp\left[4(D_2-D_4)(X^*-d)\right] \times I_1, \quad (6)$$

where

$$I_1 = \int_{\theta=0}^{\pi}\left\{\cos^2\frac{\theta}{2}\left[(1-2\nu)^2 + 3\sin^2\frac{\theta}{2}\right]\right\}^2 d\theta. \quad (7)$$

Eq. 7 gives values independent of the type of the composite and the values of the distance $d$. $I_1$ is equal to 2.2594 for $\nu$=0.33.

For determining $A_{rFG}$, since $(K_{IC}/\sigma_y)^2$ varies at different layers ahead of the notch, the influence of the angle, θ, and the radial distance have to be considered simultaneously.

In order to evaluate $A_{rFG}$, a layer ahead of the notch is considered in the graded region, being $X'_{(i)}$ the distance from the notch tip. In Fig. 1, $r_{FG(i)}$ and $\theta_{(i)}$ indicate the vectorial radius and vectorial angle of the plastic region with respect to the $i^{th}$ layer. By considering Eq. (2), $r_{FG(i)}$ can be expressed as follows:

$$r_{FG(i)} = \alpha \times H(X'_{(i)}) \times f(\theta_{(i)}), \tag{8}$$

$$H(X'_{(i)}) = \left(K_{IC}(X'_{(i)}) / \sigma_y(X'_{(i)})\right)^2, \tag{9}$$

where $K_{IC}(X'_{(i)})$ and $\sigma_y(X'_{(i)})$ are the fracture toughness and the yield stress corresponding to the $i^{th}$ layer. The relationship between the radius and the angle of the plastic region and the position of the considered layer can be expressed as follows:

$$X'_{(i)} = r_{FG(i)} \cos(\theta_{(i)}) \tag{10}$$

By combining Eqs. 8 and 10 the following expressions can be derived:

$$\frac{X'_{(i)}}{H(X'_{(i)})} = \alpha f(\theta_{(i)}) \cos(\theta_{(i)}) \tag{11}$$

$$H(X'_{(i)}) = \left(\frac{K_{IC}(X'_{(i)})}{\sigma_y(X'_{(i)})}\right)^2 = \left(\frac{D_1}{D_3}\right)^2 \exp\left[2(D_2 - D_4)(X^* - d + X'_{(i)})\right] \tag{12}$$

By using Taylor's series expansion (exp(x)=1+x+O($x^2$)) and neglecting the higher order terms, $X'_{(i)}$ assumes the following form:

$$X'_{(i)} = \frac{\alpha \times A \times f(\theta_{(i)}) \cos(\theta_{(i)})}{1 + \alpha \times B \times f(\theta_{(i)}) \cos(\theta_{(i)})}, \tag{13}$$

$$A = \left(D_1 / D_3\right)^2 \left[1 + 2(D_2 - D_4)(X^* - d)\right], \tag{14}$$

$$B = -2(D_2 - D_4)\left(D_1 / D_3\right)^2. \tag{15}$$

By substituting $X'_{(i)}$ into Eq. 8, the radius $r_{FG}(i)$ is derived as follows:

$$r_{FG}(\theta_{(i)}) = \alpha \left(\frac{D_1}{D_3}\right)^2 \times \left[1 + 2(D_2 - D_4)\left(X^* - d + \frac{\alpha \times A \times f(\theta_{(i)}) \cos(\theta_{(i)})}{1 + \alpha \times B \times f(\theta_{(i)}) \cos(\theta_{(i)})}\right)\right] \times f(\theta_{(i)}) \tag{16}$$

By integrating Eq. (16), $A_{rFG}$ is obtained as follows:

$$\begin{aligned} A_{r_{FG}} = \int_{\theta=0}^{\theta=\pi} r_{FG}^2(\theta) d\theta = \alpha^2 \times \left(\frac{D_1}{D_3}\right)^4 \times \Big\{ \left[1 + 4(D_2 - D_4)(X^* - d) + 4(D_2 - D_4)^2 (X^* - d)^2\right] \times I_1 \\ + 4\alpha \times A \times \left[2(D_2 - D_4)^2 (X^* - d) + (D_2 - D_4)\right] \times I_3 + 4\alpha^2 . A^2 . (D_2 - D_4)^2 \times I_2 \Big\}, \end{aligned} \tag{17}$$

where $I_2$ and $I_3$ are defined according to the following expressions

$$I_2 = \int_{\theta=0}^{\theta=\pi} \frac{f(\theta)^4 \times \cos^2(\theta)}{\left[1 + \alpha \times B \times f(\theta) \cos(\theta)\right]^2}, \tag{18}$$

$$I_3 = \int_{\theta=0}^{\theta=\pi} \frac{f^3(\theta) \cos(\theta)}{1 + \alpha \times B \times f(\theta) \cos(\theta)} d\theta. \tag{19}$$

These integrals contain parameter $B$ which depends on the composite type (boundary conditions and $X^*$) according to Eq. (15). Therefore, $I_2$ and $I_3$ have to be calculated for each graded region. These integrals are independent of $d$. $M$ is calculated as follows taking advantage of Eq. (1):

$$M = \left[ r_{y(1)} CV_{(2)} - r_{y(2)} CV_{(1)} \right] / \left[ r_{y(1)} - r_{y(2)} \right]. \tag{20}$$

Subscripts 1 and 2 correspond to the mechanical properties of the austenite-ferrite layers (located at the outer edge region, in the composite) and baynite-martensite (median phase in the composite) from graded regions, respectively.

*2.2 Application of model for αβγ and γMγ functionally graded steel*

The mechanical properties of the different phases of FGSs are reported in Table 1.

**Table 1.** Mechanical properties of single phase steels in the composite (Nazari et al., 2009; Aghazadeh Mohandesi, 2010)

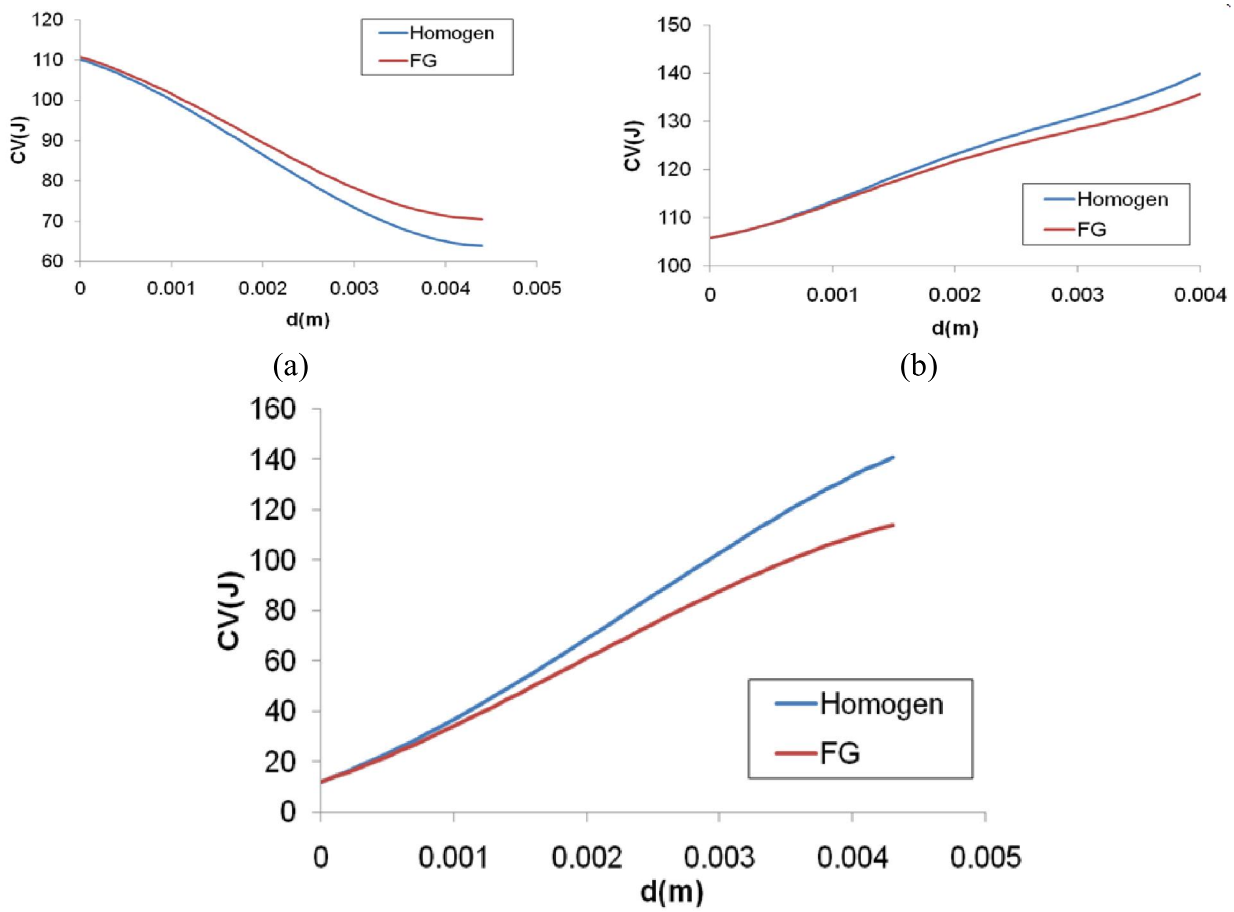
| Single phase | Yield Strength [MPa] | Impact Energy CV [J] | $K_{IC}$ [MPa.m$^{0.5}$] | Poisson's Ratio ν |
|---|---|---|---|---|
| Ferritic | 245 | 64 | 45.72 | 0.33 |
| Austenitic | 200 | 140 | 107.77 | 0.33 |
| Bainitic | 1025 | 108 | 82.08 | 0.33 |
| Martensitic | 1440 | 11 | 6.09 | 0.33 |

By using as boundary conditions the values reported in the Table 1 the constants of Eqs. (6), (17) and (20) have been obtained. By substituting Eq. (6), Eq. (17) and Eq. (20) into Eq. (3), the impact energy for different types of FGSs has been derived as a function of the notch tip position, $d$.

$CV_{H(d)}$ is the impact energy of a layer which is at the distance ($X^*$-$d$) from the crack divider specimen edge. For different distances from the notch tip to the median phase of the composite, $CV_{H(d)}$ has been evaluated following the method proposed in Ref. (Nazari and Aghazadeh Mohandesi, 2010). By substituting $CV_{H(d)}$ in Eq. (3), the impact energy of the FGS, $CV_{FG(d)}$, has been obtained. The variation of the impact energy both for the homogeneous material and for the FG steel as a function of the distance from the notch tip to the median layer are shown in Fig. 2. Fig. 2 shows that the impact energy when the notch is in the ferritic region of the composite is always greater than the value obtained from the homogenous material characterized by the mechanical properties of the layer corresponding to the notch tip.

On the other hand, in the other cases, the impact energy of graded materials is always lower than that corresponding to the homogenous specimen. In order to investigate the accuracy of the model, the results are compared with the experimental data taken from Nazari and Aghazadeh Mohandesi (2010). The impact energy for three values of parameter $d$ are reported in Table 2.

The comparison shows that Eqs. (21-23) allow predicting the impact energy with good accuracy in the case of *αβγ* composite. The deviation between theoretical predictions and experimental results increases for the case of *γMγ* steel. For that material, in fact, the two phases with high variation in brittleness and ductility are close each other. In this case, the influence of the strain rate should be considered to improve the accuracy of the predictive model.




(c)

**Fig. 2.** Variation of the impact energy of Functionally Graded (FG) steel and homogenous ones made of the Adhesive layer of the notch tip versus distance d : a) ferritic region in *αβγ* composite ($X^*$=4mm), b) austenitic region in *αβγ* composite ($X^*$=4.4mm), c) austenitic region in *γMγ* composite ($X^*$=4.25mm)

**Table 2.** Comparison between impact energy of the model and experimental data for different distances of the notch tip from median phase in FGSs

| | d (mm) | | | | | |
|---|---|---|---|---|---|---|
| | 1 | | 2 | | 3 | |
| **FGS Type** | $CV_{exp}$(Nazari and Aghazadeh Mohandesi, 2010) | $CV_{Eq. (3)}$ | $CV_{exp}$(Nazari and Aghazadeh Mohandesi, 2010) | $CV_{Eq. (3)}$ | $CV_{exp}$(Nazari and Aghazadeh Mohandesi, 2010) | $CV_{Eq. (3)}$ |
| **αβγ (α)** | 99 | 101.22 | 85 | 86.43 | 75 | 73.40 |
| **αβγ (γ)** | 111 | 113.51 | 119 | 123.27 | 125 | 131.36 |
| **γMγ** | 25 | 23.15 | 44 | 50.50 | 83 | 87.91 |

## 3. Conclusion

1. A new analytical model is proposed to evaluate the Charpy impact energy of functionally graded steels with a crack arrester configuration. Different distances of the notch tip from the median phase of the composite are considered as well as the size of plastic region ahead of the notch tip.

2. The results from the model are discussed for two types of FGSs ( αβγ and γMγ). The minimum average error has been obtained when the notch is in the ferritic region of αβγcomposite . The maximum average error is obtained for γMγ composite.

3. The impact energy of the FGSs is compared with the specimen made of the homogeneous material corresponding to the notch tip layer. The results show that the impact energy in the ferritic region of αβγ composite is higher than that corresponding to the homogeneous material. On the other hand, in the other cases, the impact energy of graded materials is always lower than that corresponding to the homogenous specimen.

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