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Free vibration analysis of moderately thick functionally graded skew plates

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Finite element method is used to study the free vibration analysis of functionally graded skew plates. The material properties of the skew plates are assumed to vary continuously through their thickness according to a power-law distribution of the volume fractions of the plate constituents. The first order shear deformation theory is used to incorporate the effects of transverse shear deformation and rotary inertia. Convergence study with respect to the number of nodes has been carried out and the results are compared with those from past investigations available in the literature. Two types of functionally graded skew plates - Al/ZrO2 and Al/Al2O3 are considered in this study and the effects of the volume fraction, different external boundary conditions and thickness ratio on the natural frequencies are studied in detail.

1. Introduction

Functionally graded materials are a class of composites that have continuous variation of material properties from one surface to another and thus eliminate the stress concentration found in laminated composites. A typical functionally graded material is made from a mixture of ceramic and metal. These materials are often isotropic but nonhomogeneous. The gradation of properties in an FGM reduces the thermal stresses, residual stresses, and stress concentrations found in traditional composites. The reason for interest in functionally graded materials (FGMs) is that it may be possible to create certain types of FGM structures capable of adapting to operating conditions. The increase in FGM applications requires accurate models to predict their responses.

There are many approaches used to describe the material gradient of FGMs which are manufactured from two phases of materials. In general, most of the approaches are based on the volume fraction distribution rather than developed from actual graded microstructures (Bao and

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Wang 1995, Frostig and Shenhar 1995). Reddy (2000) presented a theoretical formulation and finite element models for functionally graded plates (FGPs) based on the third-order shear deformation theory. The formulations accounted for the thermo mechanical coupling, time dependency, and von Ka´rma´ n-type geometric nonlinearity of the plates. A review on the stress and vibration analysis of composite plates is studied by Sharma and Mittal (2010). Free vibration analysis of laminated composite plates with elastically restrained edges using FEM is studied by Sharma and Mittal (2013). The governing equations employed are based on the first order shear deformation theory including the effects of rotary inertia. Several combinations of translational and rotational elastic edge constraints are considered.

Fukui and Yamanaka (1992) examined the effects of the gradation of components on the strength and deformation of thick-walled functionally gradient material tubes under internal pressure. Fukui et al. (1993) further extended their previous work by considering a thick-walled FGM tube under uniform thermal loading, and investigated the effect of graded components on residual stresses. They generated an optimum composition of the FGM tube by minimizing the compressive circumferential stress at the inner surface. Neves et al. (2013) developed a higher order shear deformation theory (HSDT) with cubic and parabolic variations for in-plane and transverse displacements, respectively, based on Carrera's unified formulation. With the use of polynomial functions in aforementioned works, trigonometric functions are also employed in the development of HSDTs. The frequency characteristics of thick annular FGPs of variable thickness were analyzed by Efraim and Eisenberger (2007), who utilized the first-order shear deformation theory and exact element method to derive the stiffness matrix.

Recently, Matsunaga (2008) carried out an analysis of the free vibration and stability of FGPs using the two-dimensional higher-order deformation theory. Xiang et al. (2011) and Xiang and Kang (2013) proposed a n-order shear deformation theory in which Reddy's theory can be considered as a specific case. The methods employed in the paper included a higher order shear deformation theory and two novel solutions for FGM structures. According to this paper, the application of the normal deformation theory may be justified if the in-plane size to thickness is equal to or smaller than 5. Researchers have also turned their attention to the vibration and dynamic response of FGM's structures (Yang and Shen 2003, Huang and Shen 2004). Wu et al (2007) presented exact solutions for free vibration analysis of rectangular plates using Bessel functions with three edges conditions. Matsunaga (2008) presented in his paper, the analysis of natural frequencies and buckling of FGM's plates by taking into account the effects of transverse shear and normal deformations and rotary inertia. For plates with cutouts, Chai (1996) presented finite element and some experimental results on the free vibration of symmetric composite plates with central hole. Thus, needs exist for the development of shear deformation theory which is simple to use. From the review of the above literature it is observed that very little work has been done yet on the natural frequencies of the functionally graded skew plates.

The aim of this paper is to develop a simple first order shear deformation theory for the free vibration analysis of functionally graded skew plates. The first order shear deformation theory is used to incorporate the effects of transverse shear deformation and rotary inertia. Numerical examples are presented to verify the accuracy of the present theory. This work, thus, aims to study the free vibration problem of functionally graded skew plates which have not been studied in detail as yet. An FGM's gradation in material properties allows the designer to tailor material response to meet design criteria. The developed formulation is validated by extensive convergence and comparison studies of functionally graded - Al/ZrO_2 and Al/Al_2O_3 skew plates. The variation of natural frequencies is studied with respect to the volume fraction exponent, different external boundary conditions and thickness ratio. These results are presented through graphical plots.

2. Functionally Graded Material Properties

A functionally graded material plate as shown in Fig. 1 and Fig. 2 is considered to be a plate of uniform thickness that is made of ceramic and metal. The material property is to be graded through the thickness according to a Power-Law distribution. On the basic of the rule of mixture, the effective material properties, P, can be written as

$$P = P_m V_m + P_c V_c \tag{1}$$

where P_m, P_c, V_m and V_c are the material properties and the volume fraction of the metal and ceramic, respectively, the compositions represent in relation to

$$V_c + V_m = 1. \tag{2}$$

The power law distribution based on the rule of mixture was introduced by Wakashima et al. (1990) in order to define the effective material properties of FGMs. The volume fraction of ceramic (V_c) can then be written as follows:

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n \qquad (n \geq 0), \tag{3}$$

where the positive number n $(0 \leq n \leq \infty)$ is the power law or the volume fraction index. z is a distance parameter along the graded direction, while, h is the total length of the direction. To find out the results of material properties according to the power law distribution, this can be achieved by substituting the equations of material volume fractions Eq. (2) and Eq. (3) into Eq. (1).

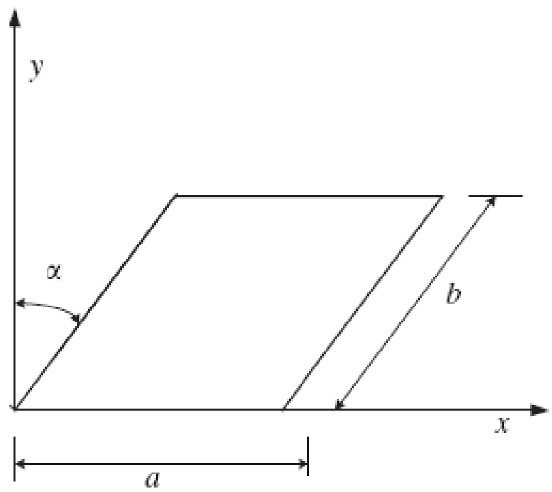

Fig. 1. Geometry of a skew plate

Fig. 2. Functionally Graded Plate

2.1 Functionally Graded Plate Elements

SOLID187 element is a higher order 3-D, 10-node element. SOLID187 has a quadratic displacement behavior and is well suited to modeling irregular meshes (such as those produced from various CAD/CAM systems). The element is defined by 10 nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastic-plastic materials, and fully incompressible hyperelastic materials.

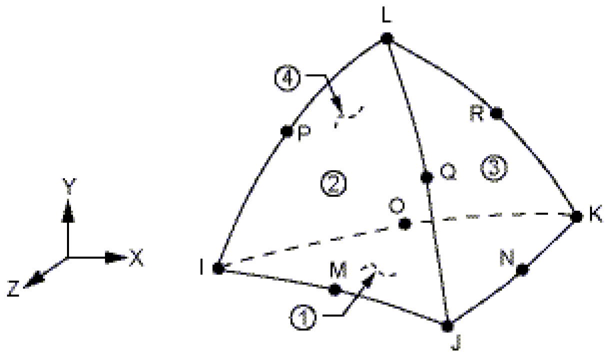

Fig. 3. SOLID 187 element

The geometry, node locations, and the coordinate system for this element are shown in Fig. 3. In addition to the nodes, the element input data includes the orthotropic or anisotropic material properties.

3. Mathematical Formulation

Fig. 1 shows the geometry of a Functionally Graded Plates plate. Considering the first order shear deformation theory, the displacement fields are expressed as follows (Reddy, 1997).

$$\left.\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\phi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\phi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned}\right\} \quad (4)$$

where $(u_0, v_0, w_0, \phi_x, \phi_y)$ are unknown functions to be determined. As before, (u_0, v_0, w_0) denote the displacements of a point on the plane z = 0; Note that

$$\frac{\partial u}{\partial z} = \phi_x, \quad \frac{\partial v}{\partial z} = \phi_y, \quad (5)$$

which indicate that ϕ_x and ϕ_y are the rotations of a transverse normal about the y and x axes, respectively. The strain displacement relations can be expressed as follows.

In-plane strains at the mid-plane are:

$$\in_x^0 = \frac{\partial u_0}{\partial x} \quad (6)$$

$$\in_y^0 = \frac{\partial v_0}{\partial y} \quad (7)$$

$$\in_{xy}^0 = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \quad (8)$$

$$k_x^0 = \frac{\partial \phi_x}{\partial x} \quad (9)$$

$$k_y^0 = \frac{\partial \phi_y}{\partial y} \quad (10)$$

$$k_{xy}^0 = \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \quad (11)$$

The shear strains in xz and yz planes are:

$$\varepsilon_{xz} = \phi_x + \frac{\partial w_0}{\partial x} \quad (12)$$

$$\varepsilon_{yz} = \phi_y + \frac{\partial w_0}{\partial y} \tag{13}$$

The strain components at any point can thus be expressed as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \varepsilon_{yz}^{(0)} \\ \varepsilon_{xz}^{(0)} \\ \varepsilon_{xy}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} k_{xx}^{(0)} \\ k_{yy}^{(0)} \\ 0 \\ 0 \\ k_{xy}^{(0)} \end{Bmatrix} \tag{14}$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \phi_y + \frac{\partial w_0}{\partial y} \\ \phi_x + \frac{\partial w_0}{\partial x} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ 0 \\ 0 \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix} \tag{15}$$

4. Numerical Results and Discussion

The present study gives the free vibration results of moderately thick functionally graded skew plates. The effects of volume fraction index, boundary conditions and length to thickness ratio are studied. To verify the results, the convergence study of functionally graded skew plates is first examined with respect to the mesh dimensions (M×N). Plates with the length-to-thickness ratios (*a/h* =10) and the values of the volume fraction exponent, n = 0, 0.5,1, 3, 5, 10, 200 are considered. The default parameter values of the functionally graded plates are as given in Table 1.

Table 1. Properties of the FGM components:

Material	Properties		
	E (N/m^2)	ν	ρ (Kg/m^3)
Aluminum (Al)	70.0x10^9	0.30	2707
Alumina(Al_2O_3)	380x10^9	0.30	3800
Zirconia(ZrO_2)	151x10^9	0.30	3000

The accuracy and convergence behaviors of the first eight frequency parameters are tested in Tables 2 and 3 for the functionally graded skew plates with clamped edges. In order to show the accuracy of methodology used for free vibration analysis of FG skew plates, the fundamental natural frequencies of different plates are compared with the solutions presented by Zhao et al. (2009). To validate the isotropic skew plate with respect to the volume fraction index, n=0 and skew angle, α=30, the convergence study is as given in Table 2, and to validate the isotropic skew plate with respect to the volume fraction index n=0 and α=15, the convergence study is as given in Table 3. It can be seen in these Tables that convergence is achieved at the mesh size of (20 x 20). It is obvious that by increasing the number of grid points, the accuracy of the results is also increases. It is found that the results of this study show a trend of monotonic convergence trend, and the solutions are slightly larger than those given in the literature. The difference ranges from 1 % to 4% for the plates with *a/h*=10. These discrepancies may be due to the different types of plate theories and the solution strategies adopted.

Table 2. Convergence study with respect to the results given by Zhao et al. (2009) for a isotropic skew plate with the volume fraction index *n*=0, skew angle *α*=30 and *a/h*=10 (fully clamped for external boundaries)

M=N	$\overline{\omega} = \omega a^2/h\sqrt{\rho_c/E_c}$							
	$\overline{\omega}$ =1	2	3	4	5	6	7	8
4	13.8579	24.0821	30.4328	34.4091	38.8752	45.6571	46.1185	47.856
6	13.0626	22.0612	27.7627	31.0500	38.7255	41.3194	41.4549	45.3463
8	12.8571	21.5661	27.0754	30.1671	38.6724	39.9043	39.9583	44.4235
10	12.7633	21.3855	26.7761	29.8456	38.6494	39.3915	39.4154	43.6804
12	12.7243	21.2898	26.6592	29.6906	38.6387	39.1232	39.1435	43.4103
14	12.6960	21.2305	26.5972	29.6021	38.6316	38.9832	39.0142	43.2864
16	12.6561	21.1676	26.5202	29.4985	38.6237	38.8229	38.8398	43.1340
18	12.4896	20.7912	26.0118	28.8945	37.9311	37.9621	38.6157	42.0890
20	12.4790	20.7744	25.9835	28.8697	37.8895	37.9232	38.6122	42.0341
Zhao et al. (2009)	12.2116	20.349	25.452	28.226				

Table 3. Convergence of non-dimensional fundamental frequencies of isotropic skew plate with the volume fraction index *n*=0 and *α*=15, for (*n*=0, *a/h*=10) (fully clamped for external boundaries)

	$\overline{\omega} = \omega a^2/h\sqrt{\rho_c/E_c}$							
M=N	i=1	2	3	4	5	6	7	8
4	11.9264	22.0603	25.531	32.2828	37.305	40.433	43.452	45.604
8	10.822	19.647	21.9753	28.2684	34.7757	36.252	37.019	38.8327
10	10.7494	19.4664	21.768	27.9389	34.2798	35.7331	36.9924	38.2898
12	10.7051	19.3433	21.6511	27.7423	34.0124	35.4869	36.9729	37.9276
14	10.6892	19.2955	21.6131	27.6396	33.9043	35.3664	36.9632	37.7425
16	10.6378	19.2034	21.4678	27.474	33.6882	35.0512	36.9534	37.3998
18	10.5271	18.9138	21.1614	27.0205	33.0187	34.4481	36.7719	36.9446
20	10.528	18.92	21.1712	27.0232	33.0355	34.4507	36.7799	36.9419
Zhao et al. (2009)	10.308	18.539	20.75	26.398				

The comparison of the results for the non dimensional fundamental frequency for clamped Al/ZrO_2 FG skew plates with length-to-thickness ratio, *a/h* =10 and skew angle, *α* =30 and the volume fraction exponent (*n*=0 and *n*=3.0) are shown in Table 4. To compare the solutions the results of Zhao et al. (2009) is cited.

Table 4. Comparison of the non dimensional fundamental frequency $\overline{\omega}$ for clamped skew Al/ZrO_2 FG plates (*a/b* =1, *a/h* =10, *α* =30)

	n=0		*n* =3.0	
Mode	Present	Zhao et al. (2009)	Present	Zhao et al. (2009)
1	12.4790	12.2116	9.2953	9.9388
2	20.7744	20.349	15.4732	16.5315
3	25.9835	25.452	19.3539	20.659
4	28.8697	28.226	21.5033	22.902
5	37.8895		28.2214	
6	37.9232		28.2462	
7	38.6122		28.7599	
8	42.0341		31.3086	

Table 5 shows the variation of the non-dimensional frequency parameter with the volume fraction exponent for the Al/ZrO_2 FG skew plates (*a/b*=1, *a/h*=10, *α*=15). Only the results for the first eight modes are computed. For the plates with the CFCF, CFFF and CCCC boundary conditions (where C and F denote Clamped and Free, respectively), the frequencies in all eight modes decreaseas as the volume fraction exponent *n* increases. This is clear that a larger volume fraction exponent means that a plate has a smaller ceramic component and that its stiffness is thus reduced.

Table 5. Non-dimensionalized frequencies of the skew plate for a fully clamped Al/ZrO_2 plate (*a/b*=1, *a/h*=10, *α*=15)

Boundary condition	*n*	$\bar{\omega} = \omega a^2/h\,\sqrt{\rho_c/E_c}$							
		$\bar{\omega}$ =1	2	3	4	5	6	7	8
CFCF	0	6.6073	7.5543	12.0619	16.8716	18.1070	18.2859	20.5937	23.7252
	0.5	5.9463	6.7985	10.8557	15.1836	16.2950	16.4571	18.5330	21.3509
	1	5.5459	6.3407	10.1242	14.1607	15.1978	15.3492	17.2851	19.9127
	5	4.7843	5.4692	8.7355	12.2239	13.1157	13.2256	14.9197	17.1939
	10	1.4998	1.7144	2.7384	3.8320	4.1116	4.1458	4.6769	5.3899
CFFF	0	1.0745	2.5000	6.3454	6.6325	7.4851	9.3634	14.0412	15.4936
	0.5	0.9671	2.2499	5.7105	5.9694	6.7364	8.4266	12.6366	13.9438
	1	0.9020	2.0983	5.3260	5.5674	6.2828	7.8592	11.7856	13.0050
	5	0.7774	1.8091	4.5920	4.7969	5.4170	6.7779	10.167	11.2055
	10	0.2437	0.5671	1.4395	1.5037	1.6981	2.1247	3.1872	3.5126
CCCC	0	10.5085	18.8695	21.1145	26.9417	32.9195	34.3383	36.6426	36.9366
	0.5	9.4573	16.9823	18.9988	24.2460	29.6251	30.8968	32.9753	33.2419
	1	8.8201	15.8390	17.7200	22.6129	27.6298	28.8165	30.7551	31.0031
	5	7.6133	13.679	15.3067	19.5372	23.8873	24.9119	26.5910	26.7177
	10	7.5474	13.5612	15.1739	19.3681	23.680	24.6958	26.3608	26.4856

Table 6 and Table 7 show the frequencies of the first eight modes for clamped functionally graded Al/ZrO2 and Al/Al_2O_3 skew plates (*a/h* =10, *a/b* = 1). The volume fraction exponent *n* varies between 0 and 3, and the skew angle ranges from 15^0 to 60^0. It is observed that, for plates with a fixed volume fraction exponent, the non-dimensional frequencies in all eight modes increase with increasing the skew angle, whereas for plates with a fixed skew angle, the non-dimensional frequencies gradually decreases as the volume fraction exponent increases.

Table 6. Non-dimensionalized frequencies with the skew angle *α* for a fully clamped Al/ZrO2 plate (*a/b*=1, *a/h*=10)

		$\bar{\omega} = (\omega a^2/h)\sqrt{\rho_c/E_c}$							
N	*α*	*i*=1	2	3	4	5	6	7	8
0	15	10.5085	18.8695	21.1145	26.9417	32.9195	34.3383	36.6426	36.9366
	30	12.4790	20.7744	25.9835	28.8697	37.8895	37.9232	38.6122	42.0341
	45	17.0815	25.9366	34.3524	36.1750	43.3563	43.5166	48.8586	52.3168
	60	28.5190	38.6945	47.9597	55.4766	57.4560	59.2493	66.9009	73.7599
0.5	15	9.4741	17.0230	19.0475	24.3124	29.7252	31.0004	33.0896	33.2472
	30	11.2312	18.6959	23.3843	25.9817	34.1000	34.1301	34.7501	37.8302
	45	15.3652	23.3285	30.9004	32.5033	38.9939	39.1648	43.9337	47.0608
	60	25.6665	34.8253	43.1615	49.9275	51.7075	53.3237	60.2084	66.3925
1	15	8.8361	15.8770	17.7651	22.6749	27.7228	28.9131	30.8614	31.0084
	30	10.4749	17.4375	21.8097	24.2327	31.8037	31.8311	32.4094	35.2823
	45	14.3308	21.7574	28.8192	30.3141	36.3680	36.5275	40.9749	43.8912
	60	23.9378	32.4803	40.2549	46.5648	48.2253	49.7326	56.1541	61.9212
3	15	7.8409	14.0890	15.7646	20.1217	24.6011	25.6567	27.3854	27.5156
	30	9.2953	15.4732	19.3539	21.5033	28.2214	28.2462	28.7599	31.3086
	45	12.7163	19.3079	25.5735	26.9010	32.2721	32.4138	36.3610	38.9487
	60	21.2420	28.8227	35.7216	41.3212	42.7940	44.1321	49.8301	54.9479

In addition to the observations made from Tables 6 and 7, it is clear that the variation in the non dimensional frequencies is less when the skew angle varies from 0^0 to 30^0, but the variation in the non dimensional frequencies is more when the skew angle rises from 30^0 to 60^0. The variation in frequencies in the FG skew plates with different volume fraction exponents also increase as the skew angle increases.

Table 7. Non-dimensionalized frequencies with the skew angle *α* for a fully clamped Al/Al_2O_3 plate (*a/b*=1, *a/h*=10)

		$\overline{\omega} = (\omega a^2/h)\sqrt{\rho_c/E_c}$							
N	*α*	*i*=1	2	3	4	5	6	7	8
0	15	10.5273	18.9153	21.1653	27.0148	33.0290	34.4465	36.7674	36.9427
	30	12.4795	20.7744	25.9838	28.8702	37.8902	37.9235	38.6128	42.0352
	45	17.0762	25.9398	34.3679	36.1360	43.3660	43.5180	48.8354	52.3200
	60	28.5196	38.6970	47.9595	55.4773	57.4552	59.2515	66.9024	73.7699
0.5	15	7.8696	14.1395	15.8215	20.1939	24.6901	25.7494	27.4848	27.6155
	30	9.3285	15.5293	19.4236	21.5812	28.3242	28.3487	28.8639	31.4224
	45	12.7654	19.3903	25.6910	27.0123	32.4170	32.5308	36.5054	39.1104
	60	21.3192	28.9268	35.8514	41.4703	42.9494	44.2921	50.0103	55.1468
1	15	6.8555	12.3174	13.7826	17.5921	21.5083	22.4313	23.9430	24.0567
	30	8.1266	13.5281	16.9204	18.8003	24.6737	24.6957	25.1443	27.3730
	45	11.1198	16.8915	22.3804	23.5315	28.2400	28.3387	31.8013	34.0707
	60	18.5716	25.1990	31.2308	36.1266	37.4146	38.5839	43.5658	48.0400
3	15	5.6567	10.1642	11.3730	14.5165	17.7479	18.5094	19.7572	19.8508
	30	6.7060	11.1632	13.9623	15.5136	20.3604	20.3780	20.7487	22.5877
	45	9.1758	13.9384	18.4679	19.4173	23.3028	23.3844	26.2414	28.1144
	60	15.3251	20.7939	25.7714	29.8108	30.8733	31.8384	35.9494	39.6413

Fig. 4 shows a comparison of the fundamental natural frequency parameters of two Al/Al_2O_3 and Al/ZrO_2 clamped functionally graded skew plates. It can be seen that both the curves shows a similar behavior. It is clear that as the volume fraction exponent increases, the frequency parameter starts decreasing. The curves for the plates made of a combination of Al/Al_2O_3 and Al/ZrO_2 shows that the Al/ZrO2 FG skew plate has the higher values of frequencies than the Al/Al_2O_3 FG skew plates. A prominent drop in frequency occurs when the volume fraction exponent varies between 0 and 2, but beyond the values of the volume fraction exponent 5, both the curves become flatter.

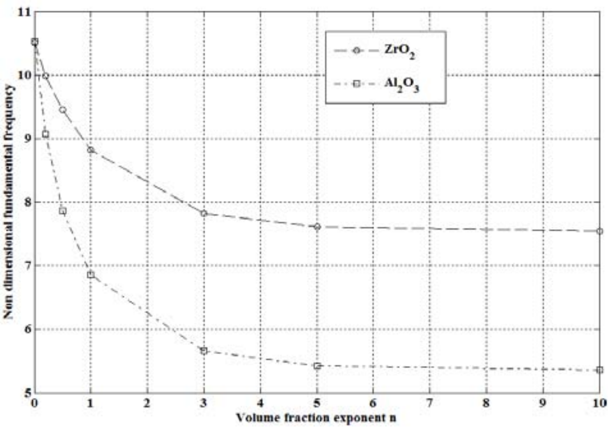

Fig. 4. Variation of the fundamental natural frequency parameter with the volume fraction exponent for fully clamped plates (CCCC)

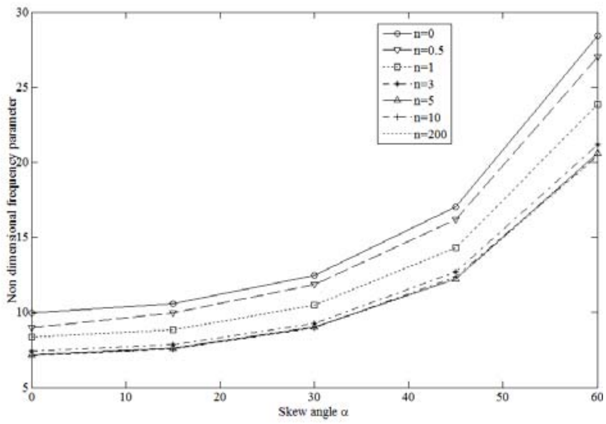

Fig. 5. Variation of the frequency parameter $\overline{\omega}$ with the skew angle for fully clamped Al/ZrO2 skew plates

Fig. 5 shows the variation of the non dimensional fundamental frequencies with the skew angle for the clamped plates. In addition to the observations made from Tables 7 and 8, it is clearly noticed that the frequencies gradually increases as the skew angle varies from 0 ^{o} to 30^o, but the variations in the frequencies is more when the skew angle varies from 30^o to 60 ^{o}. The frequency discrepancies among the plates with different volume fraction exponents also increase as the skew angle grows. Fig. 6 shows the effects of the volume fraction exponent and length-to-thickness ratio on the fundamental natural frequency parameter of functionally graded clamped skew Al/ZrO_2 plates. It shows that, for plates with a certain volume fraction, the frequency rises as the length-to-thickness ratio increases up to around 25, but when it increases further no variation in the frequency occurs. It is therefore concluded that the effects of the length-to-thickness ratio on the frequency of plates is independent of the variation in the volume fraction.

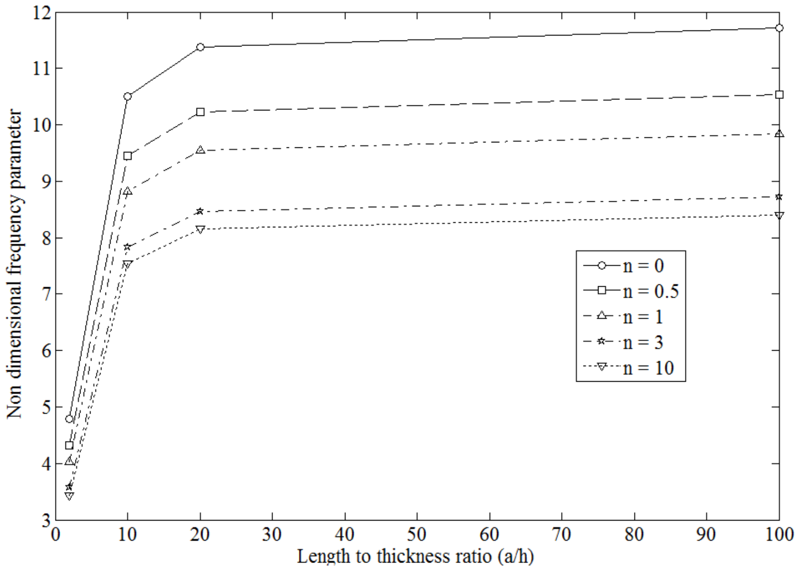

Fig. 6. Variation of the fundamental natural frequency parameter ($\bar{\omega}$) with the length-to-thickness ratio for Clamped skew Al/ZrO2 plates (α=15) FG Plates.

5. Conclusion

The free vibration analysis of functionally graded skew plates is carried out using the finite element method. The first-order shear deformation plate theory is used to consider the transverse shear effect and rotary inertia. The properties of functionally graded skew plates are assumed to vary through the thickness according to a power law. The results derived with this method are compared with the solutions available in the literature to validate the accuracy. It is found that when the length-to-thickness ratio of functionally graded skew plates is increases beyond 25, the variation in the frequency parameter is very negligible and also found that a volume fraction exponent that ranges between 0 and 5 has a significant influence on the frequency.
From this study, it is clear that the effects of the length-to-thickness ratio on the frequency of a FG plate are independent of the volume fraction. For a skew plate, a fast frequency increment trend is observed when the skew angles are greater than 30^o.

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