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Introducing resilience factor in the structure as a new control coefficient

Fereydoon Omidinasab[a*], Shima Najjar[b], Hossein Rezaei[c] and Peyman Beiranvand[c]

[a]*Assistant Professor, Department of Civil Engineering, Lorestan University, Khorram abad, Iran*
[b]*Department of Civil Engineering, Ferdowsi University of Mashhad, Mashhad, Iran*
[c]*Lecturer, Department of Civil Engineering, Lorestan University, Khorram abad, Iran*

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When a structure is influenced by the earthquake external force, some energy imposed to the structure is dissipated and remained energy causes structure displacements. Dissipated energy in the structure depends on the type of structure and its optimal engineering design. In any typical structure, the type of connections, stiffness of structure, dampers, place of windfall and damper and other factors play significant role in the amount of dissipated energy. This article introduces a new resilience factor which is a function of energy dissipation factors of input seismic energy. Mathematical equations are presented for this factor and its limits are determined for different periods. The applicability of the proposed factor is also investigated for two typical structural examples.

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1. Introduction

When a structure exposes to the seismic force, it sweeps with descending, ascending or monotonic movement. Free vibration of a structure equipped with the damper in an optimal place will approach to zero in descending path. But a structure without any external damper reduces due to vibrational movement in any time interval like $\left(j_{(i-1)}T_{(i-1)n}, j_iT_{in}\right)$ such that i=1: k and $j_0T_{0n} = 0$ are the structure stiffness. The amplitude of vibration in this case has ascending trend in general. But for a structure containing external damper, the amplitude of vibration cannot approach to zero because by reducing the stiffness of the structure in each time step, at the end of each step, there is a sudden increase in amplitude which we call it jump or resilience range. The governing movement equation of a structure with single degree of freedom in its first vibration step is written as:

$$u_1(t) = A\sin\omega_{1n}t + B\cos\omega_{1n}t \tag{1}$$

* Corresponding author.
E-mail addresses: peyman51471366@gmail.com (F. Omidinasab)

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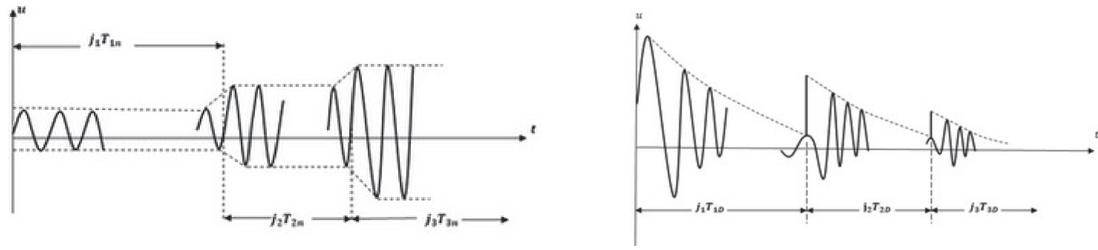
Assume that a structure in interval $(0, j_1 T_{1n})$ with stiffness constant $K_{1n} = M\omega_{1n}^2$ is vibrating such that at the end of $t_f = j_1 T_{1n}$ the stiffness of the structure suddenly reduces from K_{1n} to K_{2n}. In this case, motion Eq. (1) loses its validity and the following new vibration equation should be used for $t \geq t_f$

$$u_2(t) = A' \cos \omega_{2n} t + B' \sin \omega_{2n} t \quad (2)$$

In order to determine A' and B' from Eq. (2), continuity of time-displacement curve is studied such that $u_1(t_f) = u_2(t_f)$ and$u_1'(t_f) = u_2'(t_f)$. By determining factors A' and B', we see that amplitude of second phase vibration is:

$$\sqrt{[u_0]^2 + [\frac{u_0'}{\omega_{2n}}]^2} \quad (3)$$

As expected, the range of second phase vibration is dependent only on the natural frequency of second vibration. Regarding the reduction in the stiffness of the structure in this phase, the amplitude of vibration is increased. Fig. 1 shows the vibration of one dimensional system without and with damper.



(a) (b)

Fig. 1. Amplitude of one dimensional vibration system (a) without and (b) with damper (Hosseini et al., 2014)

According to the above discussion since the system under vibration loses its primary stiffness over the time, a suitable system for energy dissipation (caused by vibration in the structure) should be defined instead of reduced stiffness. Hence, a damper system is introduced here as an alternative for reduced stiffness in structure. A damping coefficient was obtained that could be a suitable alternative for reduced stiffness in different vibration phases such that in each vibration phase, a damper with specific damping coefficient is entered in the structure system and replaced with the reduced stiffness. This coefficient is defined such that it ignores the effects of reduced stiffness in the structure. Eq. (4) explains the relation which shows the equivalent damping coefficient with reduced stiffness (Hosseini et al., 2014).

$$A^* - A_D = 0$$

$$A^* = \sqrt{[u_0]^2 + [\frac{u_0'}{\omega_{2n}}]^2} \quad (4)$$

$$A_D = \sqrt{[u_0]^2 + [\frac{u_0' + \xi \omega_{1n} u_0}{\omega_{1D}}]^2}$$

where A^* is vibration amplitude of second phase and A_D is the amplitude of first phase for a system with damping coefficient of ξ.

Dissipated energy in any given structure under seismic loads induced by earthquakes depends on the type and design of structures and some researchers have focused on design of suitable dampers or energy absorbers for buildings and structures to resist against earthquake (Sugisawa & Saeki, 1995; Constantinou et al., 1998; Garcia, 2001; Lavan & Levy, 2006). Indeed, the type of connections, stiffness of structure, dampers, place of windfall and damper and other factors have significant role in the amount of dissipated energy. In this paper a new jump (resilience) factor (which is a function of energy dissipation factors of input seismic energy) is introduced and its practical applicability is investigated for two typical structural examples.

2. Introducing jump factor for elastic structure

Consider an elastic ball which is released from a certain height. When a ball hits to the ground, because of contact and clash with the earth, part of its active mechanical energy dissipates and the ball jumps again after collision (Fig. 2). If we show the relative speed of ball before clash with the ground with V_i and re-jump speed from the ground surface with V_j, in this case, the speed before and after clash is called jump factor, which is defined as:

$$e = \left|\frac{V_j}{V_i}\right| \tag{5}$$

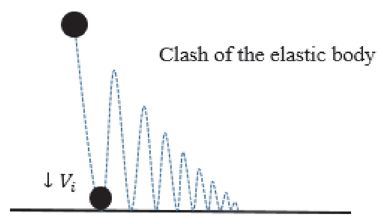


Fig. 2. Clash and jump of elastic body

If we assume that in these collisions, the nature of elastic body does not change, in all collisions the ratio of body speed after clash to its previous speed is *e*. in fact, if the body sweeps 2n times, jump (or resilience) factor is constant in n collision.

$$e_1 = e_2 = \cdots = e_n = e = \left|\frac{V_{jn}}{V_{in}}\right| \tag{6}$$

$$V_{in} = e^{n-1}\sqrt{2gh} \quad , \quad V_{jn} = e^{n}\sqrt{2gh}$$

where *h,* is the height of first fall. But why the speed of body reduces after hitting the ground and jump speed, is *e*. This is due to sudden reduction of mechanical energy in the clash moment. We can say that if the active energy of the body before clash was Π_i and after separating from the surface isΠ_j. In this case, dissipated energy in the clash moment is:

$$\Pi = \Pi_i - \Pi_j \tag{7}$$

Because releasing the energy is in kinetic form and by assuming constant nature of body, we can write:

$$\Pi = \frac{1}{2}m\left(V_i^2 - V_j^2\right), \tag{8}$$

where *m* is the mass of structure. Energy term in Eq. (8) is dissipated energy in the clash moment. This energy is dissipated as thermal and energy caused by contacts. For larger the contact surface of mass, more energy is expected to dissipate. This is because due to consistency of molecules, a body with larger contact surface needs more energy to separate its contact surface; therefore, more energy will be dissipated.

After brief introducing the jump factor, the optimization of this factor is explained here. since the effect of jump factor is on the after clash speed, therefore, if we assume that the ball stops after n clashes, in this case, sum of the amplitudes of the ball is:

$$\sum_{k=1}^{n} h_k = h_1 + h_2 + \cdots + h_n \tag{9}$$

where : $h_k = e^{2k-2}h$ (10)

$$F(V_i, V_j) = h[1 + e^2] = h[\frac{V_i^2 + V_j^2}{V_i^2}]$$

According to Eq. (10), sum of the passed amplitudes is function of speed before clash and the first speed after clash. Now, using Eq. (8) and Eq. (10), the dissipated energy is maximized such that sum of amplitudes passed by body in (h, 2h) interval. Using Lagrange theory, Lagrange function $L(\mu, V_i, V_j) = \Pi + \mu[F - \alpha h]$ is defined by Lagrange coefficient μ. By solving Eq. (11), the jump factor can be expressed as:

$$\begin{cases} \frac{\partial L}{\partial V_i} = 0 \\ \frac{\partial L}{\partial V_j} = 0 \\ \frac{\partial L}{\partial \mu} = 0 \end{cases} \tag{11}$$

As stated before, when an elastic body is in elastic clash with another body, a part of mechanical energy dissipates during contact and jump due to the contact of surfaces. The larger the contact surface or interface, dissipated mechanical energy will reduce more. Now, assume that elastic body has general surface form of $w = f(x, y, z)$ and contact surface is smooth with normal $\vec{N} = (a, b, c)$ (as shown in Fig. 3).

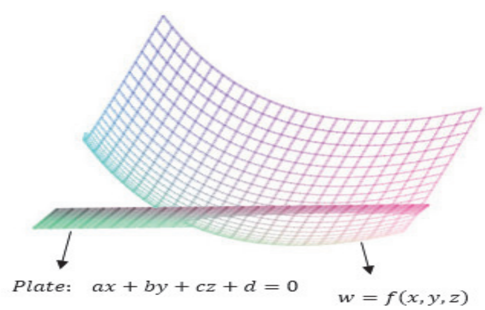


Fig. 3. Clash of elastic curved body with smooth surface (Hanson & Soong 2001)

According to Fig. 3, a contact surface is created during the clash and the larger contact surface, dissipates more mechanical energy. In fact, function Π will increase by increasing the contact surface. Eq. (14) shows the common clash plate in macroscopic dimensions.

$$\begin{aligned} &max \quad \Pi = \Pi_i - \Pi_j \\ &s.t. \quad \oiint dx\,dy = A_{max} \quad on\ cuntor\ \Gamma \end{aligned} \tag{12}$$

where Γ is contour of contact surface that is determined from:

$$\begin{cases} f(x, y, z) = 0 \\ ax + by + cz + d = 0 \end{cases}$$

$$curve\ \Gamma: f\left(x, y, \frac{-d - ax - by}{c}\right) = 0 \tag{13}$$

For example, if *f* is a homogenous sphere with volume density ρ ad radius R, the common contact surface of clash is:

$$x^2 + y^2 + (\frac{d + ax + by}{c})^2 = R^2 \tag{14}$$

$$\Gamma_k : e^k[x^2 + y^2 + (\frac{d + ax + by}{c})^2 - R^2] \tag{15}$$

As a practical application of above discussion for modeling the earthquake waves, spherical waves caused by earthquake in ground is considered. Since 1980s, reflective vibrational concepts have found widespread application in survey of depths below than 200m and especially, lower than 50m (Uang & Tsai, 2004). Huygens ' Principle is important for understanding the dissipation of seismic waves in layered lands and areas (Uang & Tsai, 2004; Yang et al., 200). According to this principle, each point on the wave front can be considered as a secondary source of spherical waves. New wave front is like a cover of these waves after an interval. Fig. (6) is a view of emission of spherical waves and formation of wave fronts. In fact, wavelength of these waves is equal with the radius of spherical waves and this concept can be explained in terms of Eqs. (12-15).

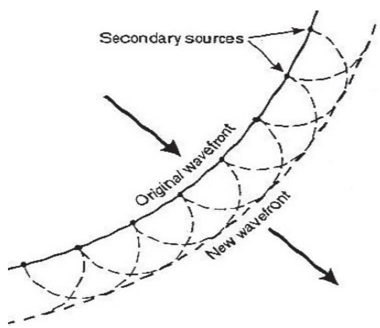


Fig. 4. Formation of spherical waves and wave fronts (Soong & Spencer 2002)

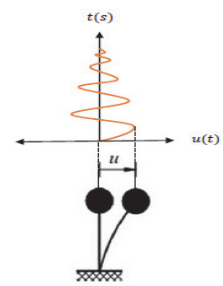


Fig. 5. Equivalent structure with a simple one-degree of freedom pendulum

When a wave clashes with the separating plate of two environments with different elastic characteristics, some of interface energy will reflect and the rest will pass from the interface and enters a new environment. Reflected and transferred energies depend on the speed of spherical waves and density of both environments. Now, if we consider spherical waves that release from one point as spherical body in clashing with the ground, in this case, impedance coefficient for seismic wave is equal with jump factor in elastic clashing of a spherical body. Also, in clashing spherical waves to interface of two layers, some of the wave energy reflects and the rest emits in the next layer. Dissipated energy is equal with dissipated mechanical energy of spherical body in clashing with the ground. We showed earlier that the larger the contact surface, the dissipated energy increases. Therefore, in order to reflect more energy, we should increase the area of clash plate.

3. Modelling jump factor in vibrational behavior of structures

Consider a structure which sweeps under free vibration. As shown schematically in Fig. 5 a pinned structure is assumed to have one degree of freedom in horizontal direction and the mass of the floor (or structure) is centralized at a single point. Accordingly, the vibration of structure is similar to the vibrational movement of a pendulum. The mass of pendulum is equal with the mass of floor and its basic stiffness is equal with the stiffness of the structure. Considering intrinsic damping in the structure, the pendulum vibrational motion will be as indicated in Fig.5.

The pendulum vibrational motion is similar to sweep and jump of spherical body in clash with the ground. By ignoring the stiffness reduction in the pendulum base during vibration, the vibrational response of pendulum can be expressed by the following differential equation:

$$M\ddot{u} + Ku = 0$$

$$\rightarrow u(t) = A\cos\sqrt{\frac{K}{M}}t + B\sin\sqrt{\frac{K}{M}}t \tag{16}$$

Regarding the concepts of jump factor for elastic factor, by assuming that the range of pendulum decreases in each $T = 2\pi\sqrt{\frac{M}{K}}$ the pendulum jump factor can be defined as:

$$e = \left|\frac{V_j}{V_i}\right| = \left|\frac{\dot{u}(T+t)}{\dot{u}(T-t)}\right| \tag{17}$$

$$e = \left|\frac{-A\sin\sqrt{\frac{K}{M}}(T+\tau) + B\cos\sqrt{\frac{K}{M}}(T-\tau)}{-A\sin\sqrt{\frac{K}{M}}(T-\tau) + B\cos\sqrt{\frac{K}{M}}(T-\tau)}\right| \tag{18}$$

$$e = \left|\frac{-A\sin(2\pi + \omega_n\tau) + B\cos(2\pi + \omega_n\tau)}{-A\sin(2\pi - \omega_n\tau) + B\cos(2\pi - \omega_n\tau)}\right|$$

$$e = \left|\frac{-A\sin\omega_n\tau + B\cos\omega_n\tau}{+A\sin\omega_n\tau + B\cos\omega_n\tau}\right|$$

The jump factor varies between 1 and zero and according to the Eq. (18), this factor depends on the factors A, B and ω_n. Based on previous discussions, for the lower jump factor values, the energy dissipation will be higher. Hence in order to obtain the maximum energy dissipation, the jump factor *e* should be minimized as follows:

$$\frac{\partial e}{\partial \omega_n} = 0$$

$$\rightarrow |u(0)\dot{u}(0)\sin 2\omega_n\tau| = 0 \rightarrow \omega_n\tau = \frac{k\pi}{2} \tag{19}$$

But in Eq. (18), for $\omega_n\tau = \frac{k\pi}{2}$, jump factor is always equal with maximum value i.e. 1. Figs. (6-9) show the variations of jump factor with different vibrational periods (T). As seen from this Figures, for larger periods, low jump factor and high energy absorption and dissipation will occur. Therefore, in order to reach low jump factor, we should reduce effective stiffness of the structure. This is consistent with flexibility or form of the structure.

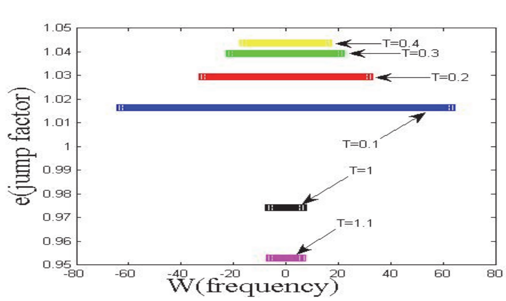


Fig. 6. Variations of jump factor for different periods (k=1)

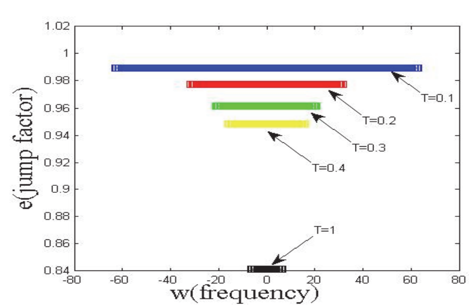


Fig. 7. Variations of jump factor for different periods (k=2)

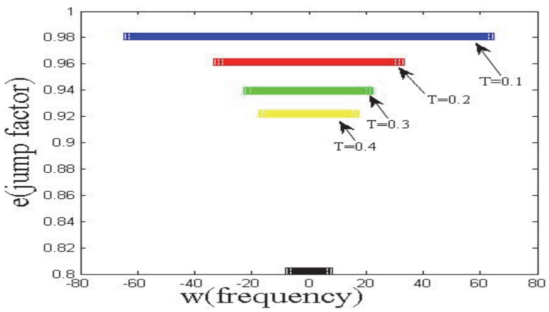


Fig. 8. Variations of jump factor for different periods (k=3)

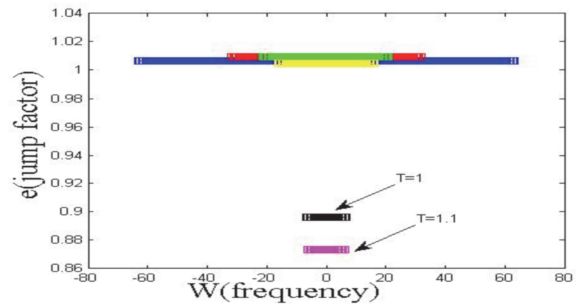


Fig. 9. Variations of jump factor for different periods (k=4)

For odd k values, jump factor value is a descending function of period and in comparison for *k=1*, this value is lower than *k=3*. But for even *k* values, there was not certain trend for the period. Therefore, regarding minimum jump factor for different periods in *k=1*, we could interpolate the jump factor as a Lagrange polynomial *T*. Fig. 10 shows the jump factor based on the period.

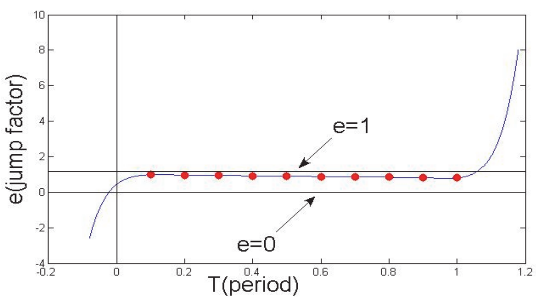


Fig. 10. Jump factor chart based on the period between two horizontal asymptotic

Fig. 11. Structural configuration of example 1

This Figure shows that the lower the period of the structure, the jump factor is close to 1 and the higher the period, this factor is low. Indeed, the jump factor is a descending function of period.

4. Results and discussion

After introducing the jump factor its practical application is studied for two typical structural examples in this section.

Example (1): Consider the frame shown in Fig. 11 having the weight of 10 ton. Steel elasticity module is $E_s = 2 \times 10^6 \frac{kg}{cm^2}$ and moment of inertia of columns is twice the beam moment of inertia. Height of columns is 3m and length of span is 6m. The structure has 3 degrees of freedom, two rotational degree of freedom in beam to column connection and one translational degree of freedom. Moment of inertia of columns is $I_C = 5700\ cm^{4}$' consequently, total stiffness matrix for this frame is:

$$10^9 \begin{bmatrix} 0.0001 & 0.0076 & 0.0076 \\ 0.0076 & 1.9000 & 0.1900 \\ 0.0076 & 0.1900 & 1.9000 \end{bmatrix}$$

Now, by dividing above matrix to matrices k_{tt}, k_{t0} and k_{00} and using equation (24) $I_C = 5700\ cm^4$, we can obtain equal stiffness of frame.

$$k = k_{tt} - k_{t0} k_{00}^{-1} k_{t0}^{T} \qquad (20)$$

$$k = \frac{120}{11} \frac{EI_C}{h^3} = 4606\ \frac{kg}{cm}$$

The period of system is therefore:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{w}{gk}} = 0.3\ sec \tag{21}$$

By assuming the speed and displacement and interval of 0.3s, jump factor is obtained equal to 0.9396. Therefore, speed ratio after and before 0.3s is:

$$e = \frac{V(\tau + 0.3)}{V(\tau - 0.3)} = 0.9396 \tag{22}$$

This figure reveals that before reducing the stiffness of structure, the speed of jump reduces by 6.04 percent. Also, kinetic energy of the structure has reduced 11.7 percent. Table 1 shows jump factor for different stiffness values in the investigated structure of example 1.

Table 1. Comparison of jump factor and speed reduction percent and dissipated energy in the frame of example 1

$I_C(cm^4)$	$I_b(cm^4)$	K(kg/cm)	T(sec)	e	% ΔV/V	% ΔΠ/Π
5700	2850	4606	0.30	0.9396	6.04	11.7
8090	4045	6537	0.25	0.9516	4.84	9.44
11260	5630	9099	0.20	0.9614	3.86	7.57
14920	7460	12056	0.18	0.9652	3.48	6.83
19270	9635	15571	0.16	0.9692	3.08	6.06

It can be concluded from this Table that by increasing in the moment of inertia of beam and column, magnitude of this factor increases and conversely speed and energy reduction becomes lesser. Therefore, for the frame of example 1, the weaker the columns and beam, results in higher energy absorption and speed reduction. Fig. 12 compares the variations of speed and energy reductions (i.e. dV/V and dU/U, respectively) for different column moment of inertia (i.e. IPB200 –IPB280) of example 1.

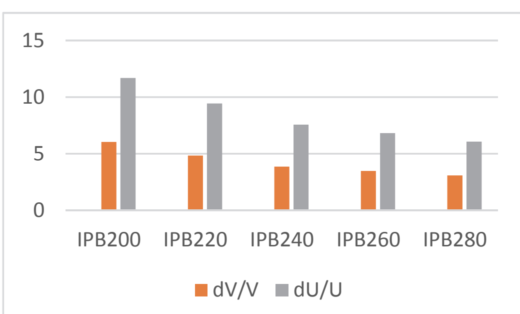


Fig. 12. Speed and energy reduction for different moment of inertia of columns in pinned ends

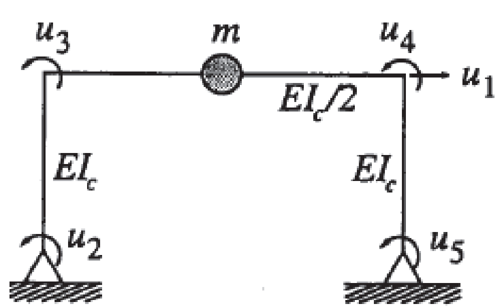


Fig. 13. Structural configuration of example 2.

Example (2): Consider the following joint frame with given characteristic of example (1). This frame has 4 rotational degrees of freedom and one translational degree of freedom as shown in Fig. 13. For this frame and different beam and column profiles, we calculated the jump factor, percent of speed reduction and dissipated energy and then compared them with corresponding values presented in Table 1 for the frame of example 1. The equivalent stiffness of this frame is:

$$K = \frac{2EI_C}{h^3} = 845\ \frac{kg}{cm} \tag{23}$$

Thus:

$$T = 2\pi\sqrt{\frac{w}{gk}} = 0.68\,sec \tag{24}$$

Consequently, the jump factor for this frame is determined as:

$$e = \frac{V(\tau + 0.68)}{V(\tau - 0.68)} = 0.8596 \tag{25}$$

Table 2 presents the variations of jump factor for the frame of example 2 with different moment of inertia.

Table 2. Variation of jump factor, speed reduction and dissipated energy comparison in frame with joint base (example 2)

$I_C(cm^4)$	$I_b(cm^4)$	K(kg/cm)	T(sec)	e	% ΔV/V	% ΔΠ/Π
5700	2850	845	0.68	0.8596	14.04	26.10
8090	4045	1198	0.57	0.8868	11.32	21.35
11260	5630	1668	0.48	0.9006	9.94	18.89
14920	7460	2210	0.42	0.9141	8.59	16.44
19270	9635	2854	0.37	0.9275	7.25	13.97

As seen from these results, energy reduction in joint frame and for profile IPB200, is twice the corresponding value of the frame with pinned end (i.e. example 1). By comparing Table 1 and Table 2, it is observed that energy absorption for joint frame with weak profile is 4 times of energy absorption for pinned frame and with strong profile. Fig. 14 compares the effect of base boundary condition and profile size for both examples.

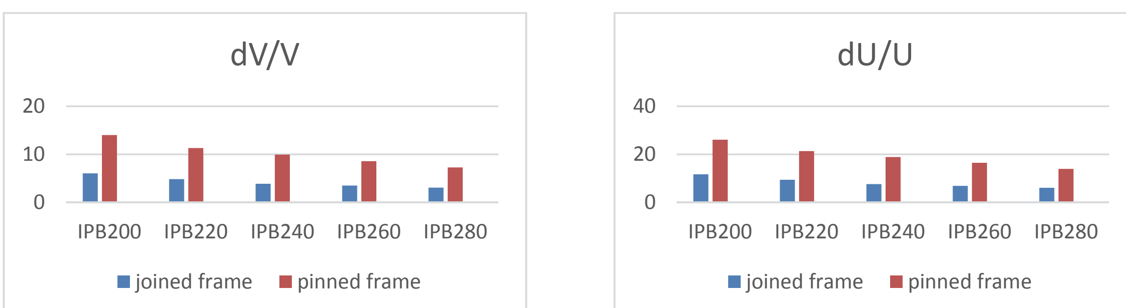


Fig. 14. Comparison of speed and energy reduction in pinned & joined frames

5. Conclusion

-A new jump factor for determining the speed and energy dissipation of structures was introduced.

-For a structure with high ductility, jump factor was low and energy dissipation was high.

- The applicability of suggested jump factor was investigated for two types of structural frames (with joined and pinned ends).

- It was shown that energy dissipation is more for joined frame and weaker cross-section of parts.

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