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Numerical modeling of nonlinear vibrations of viscoelastic shallow shells

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This paper presents a mathematical model of nonlinear supersonic flutter of viscoelastic shells. To describe the strain processes in shallow shells, the Boltzmann-Volterra integral model is used. Based on linear integral models in geometrically nonlinear formulations, equations of nonlinear oscillations of shallow shells are derived. The Koltunov-Rzhanitsyn kernel is used as a relaxation kernel. The equations of motion of shallow shells after applying the Bubnov-Galerkin method in axial coordinates are reduced to solve a system of nonlinear integro-differential equations (IDE) with variable coefficients relative to the time function. The IDE solution is found numerically using quadrature formulas.

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1. Introduction

The main body of aircraft (AC) is covered with a relatively thin skin. Usually in the supersonic regime of flight speeds, the aircraft skin loses stability and comes into oscillatory motion. If the process of oscillation lasts for long, or the amplitude of oscillations increases rapidly over time, the skin may be destroyed. The structural elements of aircraft covered with thin skin can be schematized in the form of cylindrical or shallow shells. The difference in the geometry of these elements is not a purely formal feature; it, to a large extent, determines the specific character of the aero-elastic behavior.

Significantly large numbers of theoretical studies were devoted to the study of plates and shells in a supersonic gas flow (Dowell, 1970; Fazilati, 2018; Merrett & Hilton, 2010; Vedeneev, 2012). In these references, flutter problems were mainly solved in an elastic statement. In contrast to the case of flat plates, the pattern of aero-viscoelastic behavior of curved shells is still not well understood (Khudayarov, 2019). In this paper, the flutter problems of viscoelastic shallow shells flowed about by a gas flow are studied. The main area of research is flutter of viscoelastic shallow shells at supersonic speeds.

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2. Statement of the problem

Let us consider a viscoelastic shallow rectangular shell in plan that is flowed from the outside by a supersonic gas flow with velocity V; its middle surface is an elliptical paraboloid (Fig. 1). The equation of this surface is written as follows:

$$z = f\left[\frac{f_1}{f}\left(2\frac{x}{a}-1\right)^2+\frac{f_2}{f}\left(2\frac{y}{b}-1\right)^2-1\right], \tag{1}$$

where $f = f_1 + f_2$ - is the shell boom lift. It is obvious that:

$$k_x \approx 8\frac{f_1}{a^2}; \qquad k_y \approx 8\frac{f_2}{b^2}, \qquad k_{xy} = 0. \tag{2}$$

The equation of motion of a viscoelastic shallow shell flowed about by a supersonic flow has the form:

$$\left.\begin{aligned} &\frac{D}{h}\left(1-R^*\right)\nabla^4 w = L(w,\Phi)+8\frac{f_1}{a^2}\frac{\partial^2\Phi}{\partial y^2}+8\frac{f_2}{b^2}\frac{\partial^2\Phi}{\partial x^2}+\frac{q}{h} \\ &\frac{1}{E}\nabla^4\Phi = -\left(1-R^*\right)\left(\frac{1}{2}L(w,w)+8\frac{f_1}{a^2}\frac{\partial^2 w}{\partial y^2}+8\frac{f_2}{b^2}\frac{\partial^2 w}{\partial x^2}\right)\end{aligned}\right\} \tag{3}$$

The following boundary conditions are set at the edges of the shell:
at $x=0$, $x=a$

$$w=0, \qquad \frac{\partial^2 w}{\partial x^2}=0, \qquad \frac{\partial^2\Phi}{\partial y^2}=0, \qquad \upsilon=0, \tag{4}$$

at $y=0$, $y=b$

$$w=0, \qquad \frac{\partial^2 w}{\partial y^2}=0, \qquad \frac{\partial^2\Phi}{\partial x^2}=0, \qquad u=0 \tag{5}$$

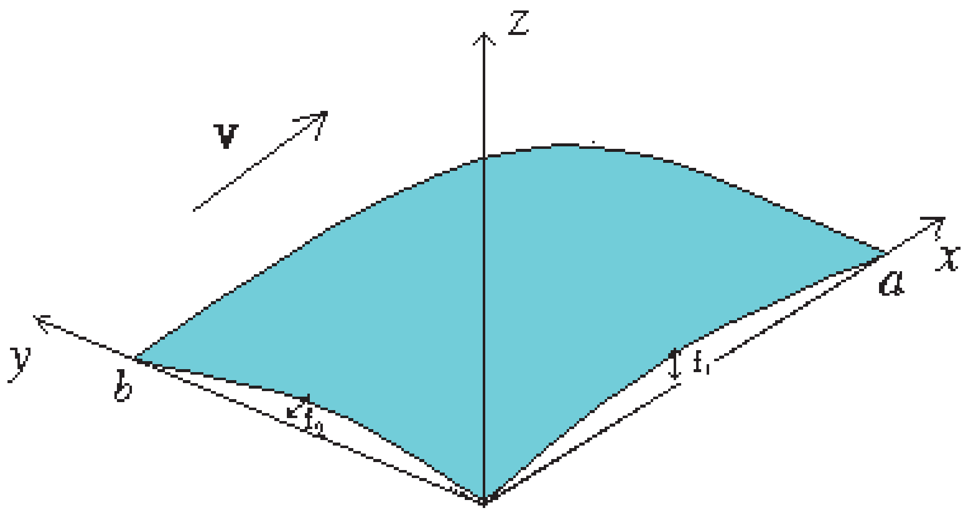

Fig. 1. Shallow shell

3. Solution method

We will seek a solution to Eq. (3), in the form:

$$w(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}w_{nm}(t)\zeta_n(x)\cdot\xi_m(y),\qquad(6)$$
$$\Phi(x,y,t)=\sum_{n=1}^{N}\sum_{m=1}^{M}\Phi_{nm}(t)\zeta_n(x)\cdot\xi_m(y),$$

where

$$\zeta_n(x)=\sin\frac{n\pi x}{a},\quad \xi_m(y)=\sin\frac{m\pi y}{b}.$$

Relationships (6) are substituted into equations (3) and, using the method of approximate solution of the boundary value problem for IDE, we obtain the IDE system from the coefficients w_{nm}, Φ_{nm}.:

$$\left.\begin{aligned}&\frac{D}{h}\left(1-R^*\right)\left(\left(\frac{\pi k}{a}\right)^2+\left(\frac{\pi l}{b}\right)^2\right)^2+\frac{8\pi^2}{a^2b^2}\left(k^2f_2+l^2f_1\right)\Phi_{kl}-\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}\frac{4\pi^4}{a^3b^3}B_{klnmir}w_{nm}\Phi_{ir}+\rho\ddot{w}_{kl}\\&+\frac{B}{h}\dot{w}_{kl}+\frac{4B}{hab}\sum_{n=1}^{N}\sum_{m=1}^{M}\beta_{klnm}w_{nm}+\frac{4B_1V^2}{hab}\sum_{n=1}^{N}\sum_{m=1}^{M}S_{klnmir}w_{nm}w_{ir}=0,\\&\Phi_{kl}(t)\left(\left(\frac{\pi k}{a}\right)^2+\left(\frac{\pi l}{b}\right)^2\right)^2=E\left(1-R^*\right)\left\{\frac{4\pi^4}{a^3b^3}\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}T_{klnmir}w_{nm}w_{ir}+\frac{8\pi^2}{a^2b^2}\left(k^2f_2+l^2f_1\right)w_{kl}\right\},\end{aligned}\right\}\qquad(7)$$

where

$$T_{klnmir}=\frac{a^2b^2}{\pi^4}\int_0^a\int_0^b\left(\zeta'_n\zeta'_i\xi'_m\xi'_r\zeta_k\zeta_l-\zeta''_n\xi''_m\zeta_i\xi_r\zeta_k\xi_l\right)dxdy,$$

$$B_{klnmir}=\frac{a^2b^2}{\pi^4}\int_0^a\int_0^b\left(\zeta''_n\xi_m\zeta_i\xi''_r-2\zeta'_n\xi'_m\zeta'_i\xi'_r+\zeta_n\xi_m\zeta''_i\xi''_r\right)\zeta_k\xi_l dxdy,$$

$$\beta_{klnm}=\int_0^a\int_0^b\zeta'_n\xi_m\zeta_k\xi_l dxdy,\quad S_{klnmir}=\int_0^a\int_0^b\zeta'_n\xi_m\zeta'_i\xi_r\zeta_k\xi_l dxdy.$$

Introducing dimensionless values into the system of IDE in Eq. (7)

$$\frac{x}{a},\quad\frac{y}{b},\quad\frac{f_1}{a},\quad\frac{f_2}{b},\quad\frac{w}{h},$$

and retaining the previous notations, the IDE system is reduced to an equation relative to deflection amplitude
w_{nm}:

$$\ddot{w}_{kl}+\left\{\lambda^4\Omega^2\left[\left(\frac{k}{\lambda}\right)^2+l^2\right]^2+8\lambda^2\pi^2M_E^2\left[f_1^2\,l^2E_{kl}^1+\frac{f_1f_2}{\lambda}\left(l^2E''_{kl}+k^2E_{kl}^1\right)+f_2^2\left(\frac{k}{\lambda}\right)^2k^2E_{kl}^2\right]\right\}\left(1-R^*\right)w_{kl}-$$
$$-\frac{12\lambda^2\left(1-\mu^2\right)}{\pi^2}\Omega^2\sum_{n,i,j=1}^{N}\sum_{m,r,s=1}^{M}a_{klnmirjs}w_{nm}\left(1-R^*\right)w_{ir}w_{js}-f_1\left(\frac{h}{a}\right)\lambda^2\pi^2M_E^2\sum_{n,i=1}^{N}\sum_{m,r=1}^{M}F_{klnmir}w_{nm}\left(1-R^*\right)w_{ir}-$$

$$+M_2 M_0 \dot{w}_{kl} - 2M_2 M^* \sum_{n=1}^{N} \gamma_{nk} w_{nl} + M_1 M^{*2} \sum_{n,i=1}^{N} \sum_{m,r=1}^{M} \Gamma_{k\ln mir} w_{nm} w_{ir}$$

$$+8f_2 \lambda \pi^2 \left(\frac{h}{a}\right) M_E{}^2 \sum_{n,i=1}^{N} \sum_{m,r=1}^{M} \mathrm{H}_{\mathrm{klnmir}} w_{nm} \left(1 - R^*\right) \; w_{ir} - 8f_1 \pi^2 \lambda^2 M_E{}^2 \left(\frac{h}{a}\right)^2 \sum_{n,i=1}^{N} \sum_{m,r=1}^{M} E_{k\ln mir} w_{nm} \left(1 - R^*\right) \; w_{ir} = 0, \tag{8}$$

where $\Omega^2 = \dfrac{\pi^4}{12(1-\mu^2)} M_E^2 / \lambda_1^2$, $M_2 = \aleph M_p^2 \lambda_1$, E_{kl}^1, E_{kl}^2, $E_{k\,ln\,mir}$,

$G_{k\,lnmir}$ are the dimensionless coefficients (Khudayarov et al., 2008, 2019).

4. Numerical results and discussion

Integration of system given in Eq. (8), was carried out using the computational method given in (Badalov et al., 2007; Khudayarov, 2008, 2019; Khudayarov & Turaev, 2016, 2019). The results of the numerical calculation are given in the Table 1. This Table shows the calculation results for a viscoelastic shallow shell flowed about by a supersonic gas flow with constant parameters $\aleph$ = 1.4; p_∞ = 1.004 *kg/cm²*; $V_\infty = 3.4 \cdot 10^4$ *cm/s*. It follows from the table that *Vcr* decreases with increase in *A*. For an elastic (A = 0) shell, *Vcr* has a value of 1089.5, for viscoelastic case (*A* = 0.1) *Vcr* = 508, the difference between them is 53.4%.

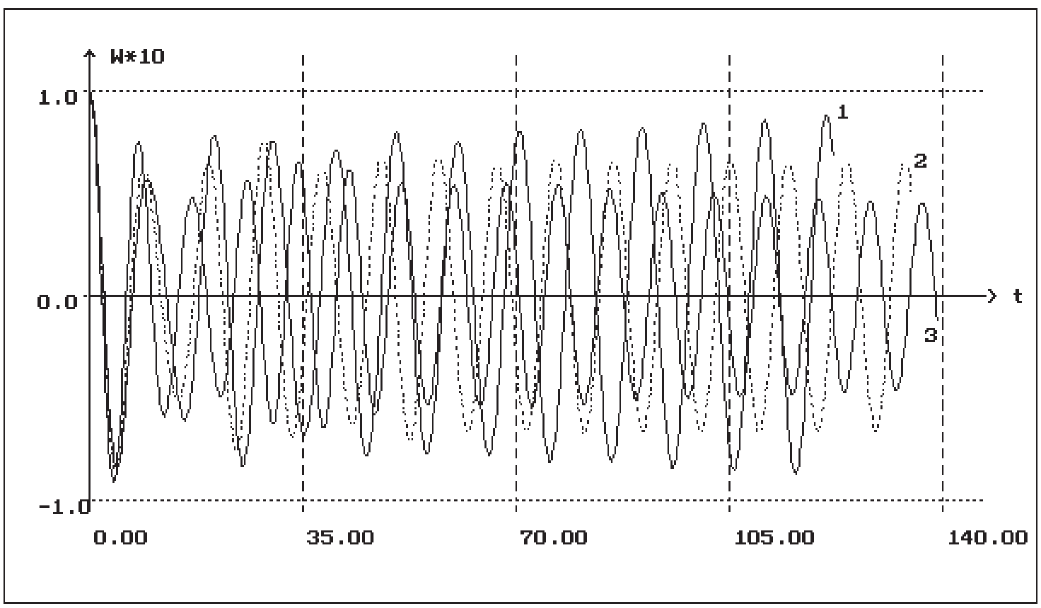

Fig. 2. Curves reflecting the deflection *w* for different f1 values f_1=0.0(1); 0.025(2); 0.05(3); *A*=0.05; α=0.25; β=0.05; f_2=0.01; λ=1; λ_1=250; *Vcr* =660.

The study of the effect of parameters f_1 and f_2 is given. Calculations show that the parameter f_2 has a greater influence on the critical velocity than f_1. An increase in f_1 and f_2 to 0.04 leads to an increase in *Vcr*, by 11.4% and 14% respectively. The effect of the parameter of relative thickness of shallow shell λ_1 on critical velocity *Vcr* of flutter was studied. With an increase in λ_1, the critical velocity of flutter decreases. The table also shows the influence of the parameters β and λ on the flutter velocity of the shells. Fig. 2 shows the curves reflecting the deflection *w* at f_1: 0 (1); 0.025 (2); 0.05 (3). Calculations were carried out with the following parameter values: λ = 1; $\aleph$ = 1.4; λ_1 = 250; p_∞ = 1.004 *kg/cm²*; $V\infty$ = 3.4·10⁴*cm/s*; A = 0.05; α = 0.25; β = 0.05; *V* = 660 (Fig. 2). Based on Fig. 2 in the case f_1

= 0 (curve 1), a rather noticeable increase in the amplitude of deflection of the shallow shell is observed. As f_1 increases, the critical velocity and frequency of flutter increase as well.

Table 1. Dependence of critical flutter velocity on the parameters of a shallow shell

A	α	β	f_1	f_2	λ	λ_1	Vcr (m/s)
0							1089.4
0.001							1060
0.01	0.25	0.05	0.01	0.01	1	250	780
0.05							645
0.1							508
	0.1						404
0.05	0.5	0.05	0.01	0.01	1	250	708
	0.7						748
0.05	0.25	0.01	0.01	0.01	1	250	638
		0.1					644
0.05	0.25	0.05	0	0	1	250	610
			0				615
0.05	0.25	0.05	0.02	0.01	1	250	655
			0.03				675
			0.04				683
				0			621
0.05	0.25	0.05	0.01	0.02	1	250	643
				0.03			683
				0.04			708
					0.5		375
0.05	0.25	0.05	0.01	0.01	0.8	250	505
					1.2		795
						200	1245
0.05	0.25	0.05	0.01	0.01	1	225	875
						275	475

5. Conclusion

The supersonic flutter of viscoelastic shells was studied. Dynamic model of a nonlinear supersonic flutter of viscoelastic shells is developed. The critical velocity of supersonic flutter of shallow shells was determined. By study of the flutter of shallow shells, a number of effects were revealed:

- It was found that an account for hereditary properties of the material of thin-walled aircraft structures leads to decreases 2.14 times in the velocity of flutter;
- It was established that taking geometric nonlinearities into account when solving the flutter problem for viscoelastic elements of an aircraft leads to an increase in the critical velocity by 15–20%.

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