International Journal of Data and Network Science 3 (2019) 119–136

Contents lists available at GrowingScience

International Journal of Data and Network Science

homepage: www.GrowingScience.com/ijds

Application of MADM methods as MOORA and WEDBA for ranking of FMS flexibility

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C H R O N I C L E

Article history:
Received: October 19, 2018
Received in revised format: October 25, 2018
Accepted: December 26, 2018
Available online:
December 26, 2018

Keywords:
FMS
Flexibility
Ranking
MADM
MOORA
AHP WEDBA
Entropy

A B S T R A C T

Flexibility has been cited as a key factor to enhance the performance of flexible manufacturing system (FMS). The main aim of this paper is to rank the flexibility of FMS. The ranking decisions are complex in the manufacturing field to analyze a number of alternatives based on a set of some attributes. In this research, two MADM methods i.e. MOORA (i.e. multi-objective optimization on the basis of ratio analysis) and weighted Euclidean distance based approach (WEDBA) are used for ranking of flexibility in FMS for new part development. MOORA approach can give decision with or without considering relative importance of attributes i.e. attribute weights. While in WEDBA, integrated attribute weights are used for evaluation which included the subjective and objective weights of attributes. Objective weights are calculated by entropy method and subjective weights are calculated by analytic hierarchy process. MOORA is applied in two ways i.e. ratio based and reference point analysis. Ranking of fifteen flexibility of FMS done on the basis fifteen variables which effect flexibility of FMS. The results of MOORA and WEDBA approach shows that product flexibility has the top most flexibility in fifteen flexibilities and programme flexibility has the least impact in fifteen flexibilities.

1. Introduction

FMS is a flexible manufacturing system. It is ‘flexible’ as it has the ability to produce a range of unalike part styles simultaneously at the different workstations, in reaction of sudden market demands (Jain & Raj, 2015b). Jain and Raj (2017) discussed that “flexible manufacturing system (FMS) is an integrated, computer-controlled complex arrangement of automated material handling devices and numerically controlled (NC) machine tools that can simultaneously process medium sized volumes of a variety of part types”. According to Chen and Chung (1996) “flexibility refers to the ability of the manufacturing system to respond quickly to changes in part demand and part mix” and according to Das (1996) “ability of a system or facility to adjust to changes in its internal or external environment”. In previous research, multiple-attribute-decision-making (MADM) methods were discussed for sorting of flexibility by AHP, TOPSIS, Improved PROMETHEE, Modified TOPSIS, VIKOR, SAW, WPM and GTMA (Jain & Raj,

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doi: 10.5267/j.ijdns.2018.12.003

2013b, 2013c, 2014a, 2015a, 2015b). Jain and Raj (2014a) used VIKOR method for evaluation of flexibility based on subjective weights as calculated by AHP method. Jain and Raj (2015b) used modified TOPSIS method based on subjective weights. Generally, subjective weights were used for MADM methods. But objective weights may be considered for taking any decision. So, an integrated weights are used in this paper which came from subjective and objective weights. Objectives weights are calculated from entropy method and subjective weights taken by AHP method.

In this research, ranking of flexibility of FMS is done by MADM methods as MOORA and WEDBA based on fifteen variables which effect the flexibility of FMS. These methods are easy to use. Chakraborty (2011) also stated that "in MOORA less computational time required for performing mathematical calculations". According to some researchers, (Jain & Raj, 2013a, 2013b, 2013c, 2014a, 2014b, 2015a, 2015b, 2015c, 2016; Jain & Raj, 2018; Raj et al., 2012), fifteen flexibilities and fifteen variables which effect flexibility of FMS are shown in Fig. 1 and Fig. 2 respectively.

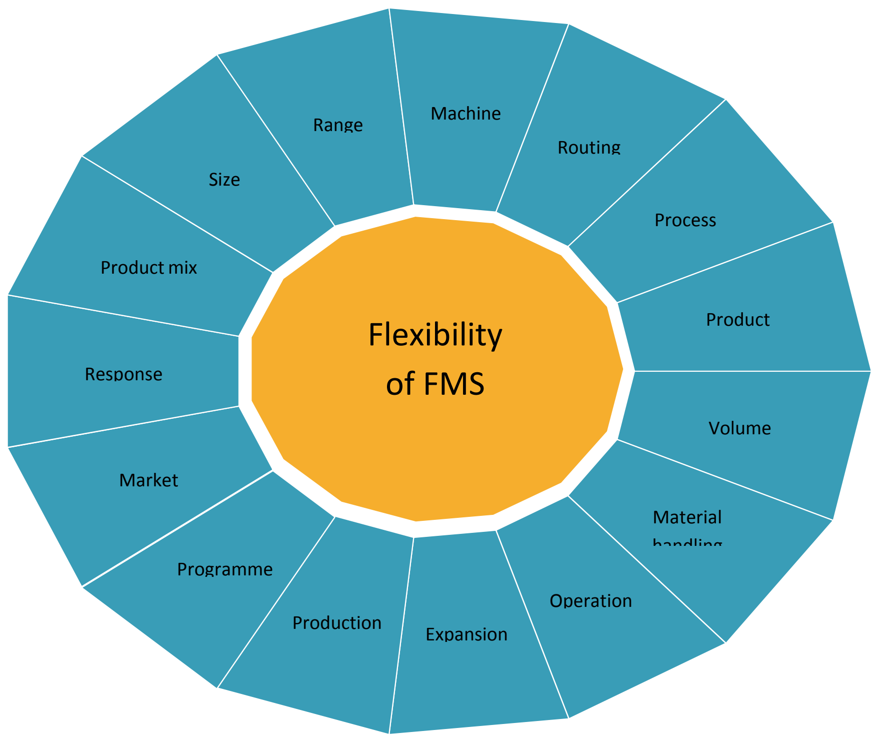

Fig. 1. Flexibility of FMS

The main objectives of this research are as follows:

- To find the objective and integrated weights of the attributes
- Ranking of flexibility based on variables by using weighted Euclidean distance based approach and MOORA methodologies.

In this paper, an overview of weighted Euclidean distance based approach methodology discussed in section 2. In section 3, an overview of MOORA MADM Method is presented. In section 4, analysis of ranking of flexibilities by weighted Euclidean distance based approach is discussed. Analysis of ranking of flexibility of FMS by MOORA is discussed in section 5. Discussion and conclusion are discussed in section 6.

Fig. 2. Variables which effect the flexibility of FMS

2. Weighted Euclidean distance based approach (WEDBA) methodology

Dattorro (2005) and Gower (1982) established the Euclidean distance concept in the field of Mathematics. This approach is based on the weighted distance of alternatives from the most and least favorable situations respectively. The ideal point is represented as the most favorable situation and the anti-ideal point is represented as the least favorable situation (Rao, 2013; Venkata Rao & Singh, 2012). Three types of attribute weights are considered in WEDBA method i.e. 1- objective weights, 2- subjective weights and 3- integrated weights. An overview of the WEDBA method is described below (Venkata Rao & Singh, 2011) :

Step 1: To find the Decision matrix

The collection of attribute data for each alternative is a decision matrix. In this decision matrix, M is the alternatives and N is the attributes, which is expressed as the i^{th} alternative i.e. i (i = 1,2,3,…., M) and for alternative j (j = 1,2,3,…., N).The standard form of decision matrix is given by Eq. (1).

$$
D_{MXN} = \begin{matrix} \\ D_1 \\ D_2 \\ D_3 \\ - \\ - \\ D_M \end{matrix} \begin{bmatrix} D_1 & D_2 & D_3 & - & - & D_N \\ d_{11} & d_{12} & d_{13} & - & - & d_{1N} \\ d_{21} & d_{22} & d_{23} & - & - & d_{2N} \\ d_{31} & d_{32} & d_{33} & - & - & d_{3N} \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ d_{M1} & d_{M2} & d_{M3} & - & - & d_{MN} \end{bmatrix} \tag{1}
$$

Chen and Hwang (1992) proposed an approach to solve more than 10 alternatives and they proposed first converts linguistic terms into fuzzy numbers and then the fuzzy numbers into crisp scores. The attributes are represented in linguistic terms and converted into corresponding crisp scores according to 11 point scale as given in Fig. 3 and Table 1 below (Jain & Raj, 2013c).

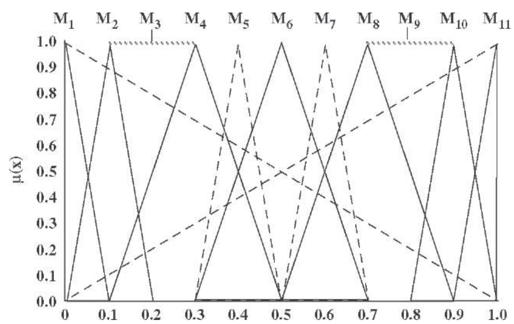

Fig. 3. Linguistic terms into their corresponding fuzzy (Jain & Raj, 2013)

Table 1

Conversion of linguistic terms into fuzzy scores (11-point scale) (Jain & Raj, 2013c)

Linguistic term	Fuzzy no.	Crisp no.
Exceptionally low	M_1	0.045
Extremely low	M_2	0.135
Very low	M_3	0.255
Low	M_4	0.335
Below average	M_5	0.410
Average	M_6	0.500
Above average	M_7	0.590
High	M_8	0.665
Very high	M_9	0.745
Extremely high	M_{10}	0.865
Exceptionally high	M_{11}	0.955

Step 2: To standardization of attribute data

In this step, standardization of attribute data is take place. The obtained values from the standardized process called as standard scores. It has value 0 and 1 i.e. standard score is mean of zero and a variance of 1 (i.e., standard deviation equals to 1). The standardized decision matrix $D^*_{M\times N}$ is given as follows:

$$D^*_{MXN} = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ - \\ - \\ D_M \end{array} \begin{bmatrix} D_1 & D_2 & D_3 & - & - & D_N \\ Z_{11} & Z_{12} & Z_{13} & - & - & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} & - & - & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} & - & - & Z_{3N} \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ Z_{M1} & Z_{M2} & Z_{M3} & - & - & Z_{MN} \end{bmatrix} \quad (2)$$

where

$$Z_{ij} = \frac{x_{ij} - \mu_j}{\sigma_j} \quad (3)$$

$$x_{ij} = \frac{d_{ij}}{\max_j \left(d_{ij}\right)}; \quad \left(if \text{ j}th \text{ attribute is beneficial}\right) \quad (4)$$

$$x_{ij} = \frac{\min_j \left(d_{ij}\right)}{d_{ij}}; \quad \left(if \text{ j}th \text{ attribute is non beneficial}\right) \quad (5)$$

$$\mu_j = \frac{1}{M}\sum_{i=1}^{M} x_{ij} \quad (6)$$

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{M}\left(x_{ij} - \mu_j\right)^2}{M}} \quad (7)$$

Z_{ij} is the standardized value of x_{ij}, μ_j is the expected value or mean value for jth attribute and σ_j is the standard deviation for the attribute j.

Step 3: To find the Ideal and anti-ideal points

The attribute values which is most desired is the ideal point and which is not desired i.e. least desirable is the anti- ideal point. These point are shown by a* and b* for ideal and anti-ideal point respectively.

$$a^* = \left\{a_j^*\right\} \text{ and } b^* = \left\{b_j^*\right\} \text{ where } j = \left\{1, 2, ...N\right\} \quad (8)$$

Step 4: To find the attribute weights

The attributes weights may be used by the researcher, depends upon the application condition. It may be subjective or objective or combination of subjective and objective i.e. integrated weights.

a) To find the objective weights of the attributes
To calculate objective weights, entropy method is suggested in this research work. Entropy is a measure of uncertainty in the information formulated using probability theory. It is based in information theory which assigns a small weight to an attribute if it has similar attribute values across alternatives, because such attribute does not help in differentiating alternatives.
The following are the steps to determine objective weight of attributes by entropy method as given below:

The Entopy value (E_j)

The amount of decision information contained in the decision matrix and associated with each attribute can be measured by the entropy value (E_j) as:

$$E_j = \frac{-\left(\sum_{i=1}^{M} p_{ij} \ln p_{ij}\right)}{\ln M}; (j = 1, 2, \ldots M) \quad (9)$$

Where

$$p_{ij} = \frac{d_{ij}}{\sum_{k=1}^{M} d_{kj}} \quad (10)$$

d_{ij} is the value of attribute j for alternative i and N is the number of alternatives.

The degree of divergence (dd_j)

The degree of divergence (dd_j) of the average intrinsic information contained by each attribute can be calculated as,

$$dd_j = 1 - E_j, (1 \leq j \leq N) \quad (11)$$

The more divergent the performance ratings p_{ij} (i = 1, 2, …, M) for the attribute j, the higher its corresponding dd_j, and the more important the attribute j for the decision making problem under consideration (R.V. Rao, 2007).

The objective weights

The objective weight for each attribute j is thus given by

$$w_{ij} = \frac{dd_j}{\sum_{j=1}^{N} dd_j} \quad (12)$$

b) To find the subjective weights of the attributes

The subjective weights may be calculated by analytic hierarchy process (AHP). So, these weights are calculated by AHP method (Saaty, 2000). A set of weights, w_j (for j= 1, 2, ..., N) such that $\sum w_j = 1$ may be decided upon. Jain and Raj (2013c) used AHP method to calculate these weights. So, these weights are taken here.

c) To find the integrated weights of the attributes

Integrated weights are the combination of subjective and objective weights. It is used when the researcher want to use both the weights i.e. objective and subjective weights of the attributes. The integrated weights are obtained using the formula given below.

$$w_j^I = \frac{w_j^o \times w_j^s}{\sum_{k=1}^{N} w_k^o \times w_k^s} \quad (13)$$

where w_j^o is the objective weight, w_j^s is the subjective weight and w_j^I is the integrated weight of the j^{th} attribute.

Step 5: To find the weighted Euclidean distance (WED), index score and ranking

The WEDBA method is based on the Euclidean distance i.e. the shortest distance between two points. The top ranked alternative is closest to the ideal solution and least ranked is the farthest from the anti-ideal solution. The overall performance index score of an alternative is determined by its Euclidean distance to ideal solution and anti- ideal solutions (Venkata Rao & Singh, 2011, 2012). WED between an alternative *i* and ideal point a* is denoted by WED_i^+ and between an alternative *i* and anti-ideal point b* is denoted by WED_i^-.

$$WED_i^+ = \left[\sum_{j=1}^{N} \left\{ w_j \left(Z_{ij} - a_j^* \right) \right\}^2 \right]^{1/2} \; for \left(i = 1, 2, ...M \right) \quad (14)$$

$$WED_i^- = \left[\sum_{j=1}^{N} \left\{ w_j \left(Z_{ij} - b_j^* \right) \right\}^2 \right]^{1/2} \; for \left(i = 1, 2, ...M \right) \quad (15)$$

The index score is calculated using following Eq. (16).

$$Index\ Score_i = \frac{WED_i^-}{WED_i^+ + WED_i^-} \quad (16)$$

The index score represents the relative closeness of a particular alternative to the ideal solution. The higher the index score for a particular alternative, the closer the alternative to the ideal solution. The alternative for which the value of index score is highest is the best choice for the considered decision making problem.

3. MOORA Methodology

In this section, MOORA MADM approach is presented for the ranking of flexibility of FMS. The MOORA approach refers to the ratio analysis and reference point approach. This approach is mainly used for multi-attribute optimization. This approach was presented by W. K. Brauers (2004). The researcher used MOORA in different ways and it is shown in Table 2 (Attri & Grover, 2014; Brauers, 2013; Brauers & Zavadskas, 2009, 2010, 2011; Chakraborty, 2011; Chand et al., 2014; Gadakh, Shinde, & Khemnar, 2013; Ginevičius et al., 2010; Karande & Chakraborty, 2012).

Table 2
MOORA applications found in literature

Sr. No.	Name of the authors	Application
1	Jain (2018)	FMS performance variables
2	Attri and Grover (2014)	Decision making in production system life cycle
3	Chand et al. (2014)	Risks assessment in supply chain
4	Brauers (2013)	Decision making of seaport
5	Gadakh et al. (2013)	Welding process parameters optimization
6	Karande and Chakraborty (2012)	Selection of material
7	Chakraborty (2011)	Decision making in manufacturing environment
8	Brauers and Zavadskas (2011)	Decision to buy property by a bank loan
9	Brauers and Zavadskas (2010)	Project management economics

10	Ginevičius et al. (2010)	Regional development in a country
11	Brauers and Zavadskas (2009)	To test the facilities sector

MOORA MADM method is applied according to the Karande and Chakraborty (2012), Attri and Grover (2014) and Jain (2018) as given below. Diagram of MOORA is shown in Fig. 4.

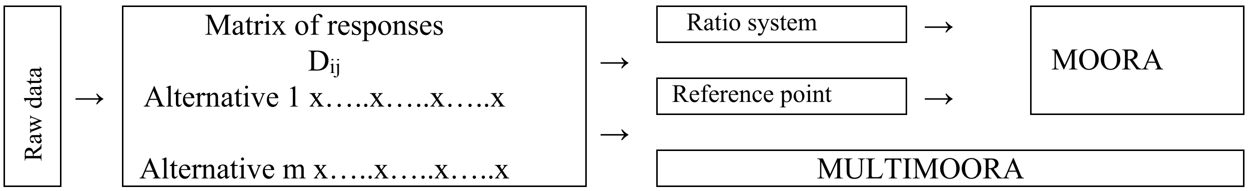

Fig. 4. Diagram of MOORA (Brauers 2013)

(a) Ratio System Approach

The following steps are involved in the ratio system approach as given below:

Step 1: Formulate the decision matrix:

MOORA approach starts with the creation of decision matrix comprising of value of various alternatives relating to different attributes as per equation 1.
The attributes are represented in linguistic terms and converted into corresponding crisp scores according to 11 point scale as shown in Fig. 3 and Table 1.

Step 2: Normalize the decision matrix:

In this step of MOORA approach, the decision matrix is normalized for making the decision matrix dimensionless. Normalization of the decision matrix is carried out by using the following equation (Karande & Chakraborty, 2012):

$$N_{ij} = \frac{D_{ij}}{\sum_{i=1}^{m} D_{ij}} \tag{17}$$

Here, N_{ij} = Normalized value i.e. dimensionless number (in 0, 1 interval), which describes the normalized performance value.

Step 3: Classification of attributes:

In this step, attributes are classified into two categories viz. beneficial attribute and non-beneficial attribute. Beneficial attributes are those attributes whose greater values are essential; while non-beneficial attributes are those attributes whose lower values are essential.

Step 4: Computation of assessment value:

The assessment value of each alternative is calculated by the following equation:

$$y_i = \sum_{j=1}^{g} N_{ij} - \sum_{j=g+1}^{n} N_{ij} \tag{18}$$

Here, y_i is the assessment value of i^{th} alternative with regard to all the attributes, g is the number of attribute to be maximized, *(n-g)* is the number of attribute to be minimized.

In Eq. (18), all the normalized performances are totalled for the beneficial attributes and deducted for the non-beneficial attributes. The assessment value can be positive or negative.

In some specific cases, attributes may be more significant than one another. To take this effect into consideration, weight of attributes is considered. The weights of attributes are generally computed by using the AHP approach. When the weights of attributes are taken into consideration, then equation 3 becomes:

$$y^*_i = \sum_{j=1}^{g} w_i N_{ij} - \sum_{j=g+1}^{n} w_i N_{ij} \tag{19}$$

where y^*_i is the assessment value of i^{th} alternative with regard to all the attributes which include weights of attributes, w_i is the weight of j^{th} attribute.

Step 5: *Determine the relative importance of the attributes i.e. weights of attributes.*

According to Saaty (2000) "construct a pair-wise comparison matrix using a scale of relative importance".
The attribute weights (w_j) by Eq. (20) and Eq. (21).

$$GM_J = \left[\prod_{i=1}^{N} d_{ij}\right]^{\frac{1}{N}} \tag{20}$$

and

$$W_J = GM_j / \sum_{j=0}^{N} GM_j \tag{21}$$

Step 5: Selection of best alternative:

The alternative having the maximum assessment value is considered as the best alternative.

(b) Reference Point Approach

The steps involved in the reference point approach are given below:

Step 1: Formulate the decision matrix:
In this step, decision matrix is constructed as in the ratio system approach.

Step 2: Normalize the decision matrix:
Here, the decision matrix is normalized to make it dimensionless. It is normalized in the similar manner as in ratio system approach.

Step 3: Classification of attributes:
In this step, attributes are categorized into two categories viz. beneficial attributes and non-beneficial attributes.

Step 4: Computation of reference point:
In this step, a reference point is deduced from the normalized matrix. The computation of reference point depends upon the beneficial and non-beneficial attributes.

The reference point in case of beneficial attribute is the maximum value while in case of non-beneficial attribute, it is minimum value.

Step 5: Computation of deviation from reference point:

The deviation of an attribute value from its set reference point *(r_i)* is calculated by following equation:

$$r_i - N_{ij} \tag{22}$$

Step 6: Computation of performance index:

The best alternative in the reference point approach will have maximum values in case of beneficial attributes along with the minimum values in case of non-beneficial attributes.
In practical situations, it is not likely all the times that a specific alternative will have all of the maximum values in the case of beneficial attributes and minimum values in the case of non-beneficial attributes. In such situations, there will be deviance from the reference point series. The deviation is computed by using the following equation (Brauers & Zavadskas, 2006):

$$P_i = \min_i \left\{ \max_j \left| r_i - N_{ij} \right| \right\} \tag{23}$$

where P_i = performance index

Step 7: Selection of best alternative:
The best alternative will have the total minimum deviance from the reference point series i.e. minimum value of $P_{i.}$

4. Ranking of flexibility by WEDBA

In this section, the ranking of flexibility is carried out by WEDBA is given below.

Step 1 Decision matrix

The measures of attributes for the flexibility ranking are given in Table 3. The values of are given in fuzzy linguistic terms. The conversion of linguistic data into corresponding crisp values is done using the eleven point fuzzy scale and the corresponding equivalent crisp values of attribute are shown in Table 4.

Table 3
Qualitative values of attributes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Extremely high	High	High	Average	Above average	Average	Below average	Above average	High	High	Above average	Low	Very low	Average	Below average
2	Below average	Below average	High	Average	Very low	Average	Above average	Above average	Below average	Below average	High	Above average	Average	Below average	Average
3	High	Average	Above average	Above average	Average	Average	Below average	Above average	Average	Average	Above average	Average	Average	Below average	Average
4	Very high	Extremely high	High	Above average	Below average	Average	Below average	Above average	Above average	Average	High	Below average	Below average	Average	Below average
5	Below average	Below average	Below average	Average	Average	Above average	Above average	Average	Average	High	Average	Below average	Below average	Below average	Below average
6	Very low	Very low	Below average	Below average	Average	Above average	Very high	Below average	Below average	Above average	Above average	Average	Below average	Low	Very low
7	Low	Very low	Below average	Average	Below average	Average	Below average	Below average	Above average	Average	Below average	Low	Low	Very low	Very low
8	Below average	Low	High	Average	Average	Below average	Average	High	Very high	Very high	Average	Low	Very low	Below average	Very low
9	High	Above average	Above average	Above average	Average	High	Above average	High	Extremely	Below average	Below average	Very low	Low	Average	Extremely
10	Very low	Very low	Low	Very low	Extremely	Average	Above average	Low	Below average	Low	Above average	Very low	Very low	Extremely	Extremely
11	Average	Above average	Average	Low	Very low	High	Extremely	Very low	Above average	Very low	Average	Very low	Extremely	Average	Extremely
12	Average	Above average	High	Above average	Low	Very high	Above average	Below average	Average	Low	Average	Very low	Below average	High	Average
13	Above average	Average	High	Average	Average	Above average	Average	Average	Average	Above average	Average	Above average	Below average	Average	Average
14	High	Above average	Average	Average	Average	Average	Above average	Average	Above average	High	Average	Below average	Low	Low	Very low
15	Average	Average	Above average	Average	Below average	Average	Above average	Below average	Average	Average	Above average	Low	Very low	Very low	Extremely

Table 4
Crisp score of attributes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.865	0.665	0.665	0.5	0.59	0.5	0.41	0.59	0.665	0.665	0.59	0.335	0.255	0.5	0.41
2	0.41	0.41	0.665	0.5	0.255	0.5	0.59	0.59	0.41	0.41	0.665	0.59	0.5	0.41	0.5
3	0.665	0.5	0.59	0.59	0.5	0.5	0.41	0.59	0.5	0.5	0.59	0.5	0.5	0.41	0.5
4	0.745	0.865	0.665	0.59	0.41	0.5	0.41	0.59	0.59	0.5	0.665	0.41	0.41	0.5	0.41
5	0.41	0.41	0.41	0.5	0.5	0.59	0.59	0.5	0.5	0.665	0.5	0.41	0.41	0.41	0.41
6	0.255	0.255	0.41	0.41	0.5	0.59	0.745	0.41	0.41	0.59	0.59	0.5	0.41	0.335	0.255
7	0.335	0.255	0.41	0.5	0.41	0.5	0.41	0.41	0.59	0.5	0.41	0.335	0.335	0.255	0.255
8	0.41	0.335	0.665	0.5	0.5	0.41	0.5	0.665	0.745	0.745	0.5	0.335	0.255	0.41	0.255
9	0.665	0.59	0.59	0.59	0.5	0.665	0.59	0.665	0.865	0.41	0.41	0.255	0.335	0.5	0.135
10	0.255	0.255	0.335	0.255	0.135	0.5	0.59	0.335	0.41	0.335	0.59	0.255	0.255	0.135	0.135
11	0.5	0.59	0.5	0.335	0.255	0.665	0.135	0.255	0.59	0.255	0.5	0.255	0.135	0.5	0.135
12	0.5	0.59	0.665	0.59	0.335	0.745	0.59	0.41	0.5	0.335	0.5	0.255	0.41	0.665	0.5
13	0.59	0.5	0.665	0.5	0.5	0.59	0.5	0.5	0.5	0.59	0.5	0.59	0.41	0.5	0.5
14	0.665	0.59	0.5	0.5	0.5	0.5	0.59	0.5	0.59	0.665	0.5	0.41	0.335	0.335	0.255
15	0.5	0.5	0.59	0.5	0.41	0.5	0.59	0.41	0.5	0.5	0.59	0.335	0.255	0.255	0.135

Step 2: Standardization

All attributes are the beneficial attributes. The standardization of attribute data is carried out using equations 3 to 7. The standardized decision matrix is shown in Table 5.

Table 5
Standardized decision matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2.00	1.07	0.97	0.10	1.41	-0.59	-0.73	0.81	0.87	1.11	0.66	-0.44	-0.94	0.73	0.63
2	-0.62	-0.47	0.97	0.10	-1.37	-0.59	0.58	0.81	-1.20	-0.73	1.64	1.82	1.55	0.02	1.26
3	0.85	0.08	0.31	1.08	0.67	-0.59	-0.73	0.81	-0.47	-0.08	0.66	1.02	1.55	0.02	1.26
4	1.31	2.27	0.97	1.08	-0.08	-0.59	-0.73	0.81	0.26	-0.08	1.64	0.23	0.64	0.73	0.63
5	-0.62	-0.47	-1.28	0.10	0.67	0.47	0.58	0.05	-0.47	1.11	-0.52	0.23	0.64	0.02	0.63
6	-1.52	-1.40	-1.28	-0.88	0.67	0.47	1.72	-0.72	-1.20	0.57	0.66	1.02	0.64	-0.58	-0.45
7	-1.06	-1.40	-1.28	0.10	-0.08	-0.59	-0.73	-0.72	0.26	-0.08	-1.71	-0.44	-0.13	-1.21	-0.45
8	-0.62	-0.92	0.97	0.10	0.67	-1.65	-0.07	1.45	1.52	1.69	-0.52	-0.44	-0.94	0.02	-0.45
9	0.85	0.62	0.31	1.08	0.67	1.35	0.58	1.45	2.50	-0.73	-1.71	-1.15	-0.13	0.73	-1.29
10	-1.52	-1.40	-1.94	-2.57	-2.37	-0.59	0.58	-1.36	-1.20	-1.27	0.66	-1.15	-0.94	-2.16	-1.29
11	-0.10	0.62	-0.48	-1.70	-1.37	1.35	-2.74	-2.04	0.26	-1.85	-0.52	-1.15	-2.15	0.73	-1.29
12	-0.10	0.62	0.97	1.08	-0.71	2.28	0.58	-0.72	-0.47	-1.27	-0.52	-1.15	0.64	2.03	1.26
13	0.42	0.08	0.97	0.10	0.67	0.47	-0.07	0.05	-0.47	0.57	-0.52	1.82	0.64	0.73	1.26
14	0.85	0.62	-0.48	0.10	0.67	-0.59	0.58	0.05	0.26	1.11	-0.52	0.23	-0.13	-0.58	-0.45
15	-0.10	0.08	0.31	0.10	-0.08	-0.59	0.58	-0.72	-0.47	-0.08	0.66	-0.44	-0.94	-1.21	-1.29

Step 3: Ideal and anti-ideal points

Ideal points and anti-ideal points are the highest and lowest values of the attributes which is obtained from standardized decision matrix and shown in Table 6.

Table 6
Ideal point

a*	2	2.27	0.97	1.08	1.41	2.28	1.72	1.45	2.5	1.69	1.64	1.82	1.55	2.03	1.26
b*	-1.52	-1.4	-1.94	-2.57	-2.37	-1.65	-2.74	-2.04	-1.2	-1.85	-1.71	-1.15	-2.15	-2.16	-1.29

Step 4: Attribute weights

The objective weights are calculated by using entropy method and the weights obtained are given in Table 7. The subjective weights of the attributes considered by Jain and Raj (2013c) are considered here for comparison purpose and these are given in Table 8.

Table 7
Objective weights

wj entropy, wo	0.095	0.099	0.036	0.032	0.077	0.018	0.072	0.05	0.041	0.063	0.018	0.068	0.072	0.086	0.176

Table 8
Subjective weights

wj AHP, ws	0.086	0.085	0.078	0.136	0.067	0.121	0.096	0.035	0.095	0.053	0.019	0.037	0.030	0.027	0.035

The integrated weights (w^I) are calculated using equation 13 and the integrated weights are shown in Table 9.

Table 9
Integrated weights

wI	0.135	0.14	0.046	0.072	0.085	0.036	0.114	0.029	0.064	0.056	0.006	0.041	0.036	0.038	0.102

Step 5: WED, index score and ranking
The WEDs of an alternative from ideal points and anti-ideal points are calculated using equation 14 and 15 respectively and the index score is calculated using equation 16. The index score values considering to all sets of weights are given in Table 10. The ranks of alternatives considering to different sets of weights i.e. subjective, objective and integrated are given in Table 11.

Table 10
Index score

WED+	0.538	0.766	0.633	0.549	0.714	0.889	0.936	0.795	0.613	1.088	0.992	0.621	0.605	0.583	0.748
WED-	0.628	0.406	0.496	0.658	0.370	0.374	0.218	0.358	0.520	0.232	0.392	0.494	0.471	0.455	0.301
Index Score	0.539	0.346	0.439	0.545	0.341	0.296	0.189	0.31	0.459	0.176	0.283	0.443	0.438	0.438	0.287

Table 11
Ranking of flexibility

Sr. No.	Flexibility	WI	Rank	WO	Rank	WS	Rank
1	Machine	0.539	**2**	0.531	**4**	0.47	**4**
2	Routing	0.346	**8**	0.483	**6**	0.347	**10**
3	Process	0.439	**5**	0.539	**3**	0.43	**6**
4	Product	0.545	**1**	0.542	**2**	0.497	**3**
5	Volume	0.341	**9**	0.43	**7**	0.375	**9**
6	Material handling	0.296	**11**	0.313	**11**	0.322	**13**
7	Operation	0.189	**14**	0.213	**13**	0.265	**14**
8	Expansion	0.31	**10**	0.327	**10**	0.379	**8**
9	Production	0.459	**3**	0.348	**9**	0.594	**1**
10	Programme	0.176	**15**	0.151	**15**	0.186	**15**
11	Market	0.283	**13**	0.237	**12**	0.327	**12**
12	Respone	0.443	**4**	0.521	**5**	0.549	**2**
13	Product mix	0.438	**6**	0.552	**1**	0.441	**5**
14	Size	0.438	**6**	0.371	**8**	0.402	**7**
15	Range	0.287	**12**	0.206	**14**	0.333	**11**

In this method, from Table 11, according to integrated weight found that product flexibility (4) has got the rank one and programme flexibility (10) got fifteen rank by using integrated weights.

5. Ranking of flexibility of FMS by MOORA Method:

(a) By Ratio system approach:

To rank the flexibility of FMS the following steps are taken as given below:

Step 1: Decision matrix for flexibility of FMS is presented in Table 4.

Step 2: Decision matrix is normalized by using equation (17) as presented in Table 12.

Step 3: The attributes are classified into beneficial and non-beneficial attribute. Here, all the attribute are the beneficial attributes.

Step 4: The assessment value (y_i) for each considered alternative is computed by using Eq. (18) as presented in Table 12.

Table 12
Normalized decision matrix

Sr. No	Alternatives (Flexibilities)	Attributes														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Machine	0.111	0.091	0.080	0.068	0.094	0.061	0.054	0.080	0.079	0.087	0.073	0.058	0.049	0.082	0.086
2	Routing	0.053	0.056	0.080	0.068	0.040	0.061	0.077	0.080	0.049	0.053	0.082	0.102	0.096	0.067	0.104
3	Process	0.086	0.068	0.071	0.080	0.079	0.061	0.054	0.080	0.060	0.065	0.073	0.087	0.096	0.067	0.104
4	Product	0.096	0.118	0.080	0.080	0.065	0.061	0.054	0.080	0.071	0.065	0.082	0.071	0.079	0.082	0.086
5	Volume	0.053	0.056	0.049	0.068	0.079	0.071	0.077	0.067	0.060	0.087	0.062	0.071	0.079	0.067	0.086
6	Material handling	0.033	0.035	0.049	0.056	0.079	0.071	0.097	0.055	0.049	0.077	0.073	0.087	0.079	0.055	0.053
7	Operation	0.043	0.035	0.049	0.068	0.065	0.061	0.054	0.055	0.071	0.065	0.051	0.058	0.064	0.042	0.053
8	Expansion	0.053	0.046	0.080	0.068	0.079	0.050	0.065	0.090	0.089	0.097	0.062	0.058	0.049	0.067	0.053
9	Production	0.086	0.081	0.071	0.080	0.079	0.081	0.077	0.090	0.103	0.053	0.051	0.044	0.064	0.082	0.028
10	Programme	0.033	0.035	0.040	0.035	0.021	0.061	0.077	0.045	0.049	0.044	0.073	0.044	0.049	0.022	0.028
11	Market	0.064	0.081	0.060	0.046	0.040	0.081	0.018	0.034	0.071	0.033	0.062	0.044	0.026	0.082	0.028
12	Response	0.064	0.081	0.080	0.080	0.053	0.090	0.077	0.055	0.060	0.044	0.062	0.044	0.079	0.109	0.104
13	Product mix	0.076	0.068	0.080	0.068	0.079	0.071	0.065	0.067	0.060	0.077	0.062	0.102	0.079	0.082	0.104
14	Size	0.086	0.081	0.060	0.068	0.079	0.061	0.077	0.067	0.071	0.087	0.062	0.071	0.064	0.055	0.053
15	Range flexibility	0.064	0.068	0.071	0.068	0.065	0.061	0.077	0.055	0.060	0.065	0.073	0.058	0.049	0.042	0.028

Step 5: The relative importance of attributes is analyze with respect to the objective and it is taken as in AHP methodology. Weight of attributes are taken from (Jain & Raj, 2013c) and shown in Table 13.

Table 13
Ranking of flexibility (without considering weights of attributes)

Sr. No	Alternatives (Flexibilities)	y_i	Ranking
1	Machine	1.151	2.000
2	Routing	1.069	7.000
3	Process	1.130	4.000
4	Product	1.168	1.000
5	Volume	1.032	9.000
6	Material handling	0.948	11.000
7	Operation	0.833	13.000
8	Expansion	1.006	10.000
9	Production	1.070	6.000
10	Programme	0.656	15.000
11	Market	0.769	14.000
12	Response	1.082	5.000
13	Product mix	1.141	3.000
14	Size	1.041	8.000
15	Range flexibility	0.904	12.000

Step 6: The assessment value (y_i) for each considered alternative with weight of attributes (Table 8) are computed by using equation (19) as presented in Table 14.

Table 14
Ranking of flexibility (considering weights of attributes)

Sr. No	Alternatives (Flexibilities)	y_{i*}	Ranking
1	Machine	0.081	2.000
2	Routing	0.071	9.000

3	Process	0.076	6.000
4	Product	0.081	1.000
5	Volume	0.071	8.000
6	Material handling	0.065	12.000
7	Operation	0.059	13.000
8	Expansion	0.070	10.000
9	Production	0.079	3.000
10	Programme	0.048	15.000
11	Market	0.057	14.000
12	Response	0.076	5.000
13	Product mix	0.077	4.000
14	Size	0.074	7.000
15	Range flexibility	0.067	11.000

From Table 13 and 14, product flexibility is the top flexibility.

(b) By Reference point approach:

Step 1: Decision matrix for flexibility of FMS is presented in Table 4 as in the ratio system approach.
Step 2: Decision matrix (Table 4) is normalized by using equation (17) as presented in Table 12 as in the ratio system approach.
Step 3: The attributes are classified into beneficial and non-beneficial attribute. Here, all the attribute are the beneficial attributes.
Step 4: Set the reference point as maximum value because all attributes are beneficial. The reference point of attributes are shown in Table 15. The deviation from reference point are calculated by using equation 22 and shown in table 16.

Table 15
Reference point of attributes

Attributes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Reference point	0.111	0.118	0.080	0.080	0.094	0.090	0.097	0.090	0.103	0.097	0.082	0.102	0.096	0.109	0.104

Table 16
Deviation from reference point of the attributes

Sr. No	Alternatives (Flexibilities)	Attributes														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	Machine	0.000	0.027	0.000	0.012	0.000	0.030	0.044	0.010	0.024	0.010	0.009	0.044	0.047	0.027	0.019
2	Routing	0.059	0.062	0.000	0.012	0.053	0.030	0.020	0.010	0.054	0.044	0.000	0.000	0.000	0.042	0.000
3	Process	0.026	0.050	0.009	0.000	0.014	0.030	0.044	0.010	0.044	0.032	0.009	0.016	0.000	0.042	0.000
4	Product	0.015	0.000	0.000	0.000	0.029	0.030	0.044	0.010	0.033	0.032	0.000	0.031	0.017	0.027	0.019
5	Volume	0.059	0.062	0.031	0.012	0.014	0.019	0.020	0.022	0.044	0.010	0.020	0.031	0.017	0.042	0.019
6	Material handling	0.079	0.083	0.031	0.024	0.014	0.019	0.000	0.034	0.054	0.020	0.009	0.016	0.017	0.054	0.051
7	Operation	0.068	0.083	0.031	0.012	0.029	0.030	0.044	0.034	0.033	0.032	0.031	0.044	0.032	0.067	0.051
8	Expansion	0.059	0.073	0.000	0.012	0.014	0.041	0.032	0.000	0.014	0.000	0.020	0.044	0.047	0.042	0.051
9	Production	0.026	0.038	0.009	0.000	0.014	0.010	0.020	0.000	0.000	0.044	0.031	0.058	0.032	0.027	0.076
10	Programme	0.079	0.083	0.040	0.046	0.072	0.030	0.020	0.044	0.054	0.053	0.009	0.058	0.047	0.087	0.076
11	Market	0.047	0.038	0.020	0.035	0.053	0.010	0.080	0.055	0.033	0.064	0.020	0.058	0.070	0.027	0.076
12	Response	0.047	0.038	0.000	0.000	0.040	0.000	0.020	0.034	0.044	0.053	0.020	0.058	0.017	0.000	0.000
13	Product mix	0.035	0.050	0.000	0.012	0.014	0.019	0.032	0.022	0.044	0.020	0.020	0.000	0.017	0.027	0.000
14	Size	0.026	0.038	0.020	0.012	0.014	0.030	0.020	0.022	0.033	0.010	0.020	0.031	0.032	0.054	0.051
15	Range flexibility	0.047	0.050	0.009	0.012	0.029	0.030	0.020	0.034	0.044	0.032	0.009	0.044	0.047	0.067	0.076

Step 5: Computation of performance index is done by using equation 23 and shown in Table 17.

Table 17
Ranking of flexibility by using performance index

Sr. No	Alternatives (Flexibilities)	P_i	Ranking
1	Machine	0.0470	2
2	Routing	0.0622	7
3	Process	0.0499	3
4	Product	0.0438	1
5	Volume	0.0622	7
6	Material handling	0.0834	14
7	Operation	0.0834	13
8	Expansion	0.0725	9
9	Production	0.0762	10
10	Programme	0.0866	15
11	Market	0.0797	12
12	Response	0.0581	6
13	Product mix	0.0499	3
14	Size	0.0539	5
15	Range flexibility	0.0762	10

Step 6: Best alternative and ranking is shown in Table 17.
From Table 17, product flexibility has got rank one.

6. Discussion and Conclusion

The objective of this research is to focus on the ranking of fifteen flexibility which is identified in a flexible manufacturing system so that management may effectively deal with these flexibilities. In this research, ranking of flexibilities is taken by a WEDBA and MOORA approaches i.e. MADM methods. These approaches represents the qualitative attribute on a conversion scale using fuzzy logic to a quantitative attributes values. Objective weight of attributes have taken by entropy method and integrated weights have taken from both i.e. subjective and objective weights. Subjective weight of attributes have taken same as AHP method. By WEDBA got the ranking is 4-1-9-12-3-13-14-2-5-8-6-15-11-7-10. In this, product flexibility has the top ranking and programme flexibility has lower most ranking.
While MOORA as a MADM method is also applied for ranking of flexibility of FMS by two ways i.e. simple ratio analysis and reference point. Further ratio analysis is applied in two ways i.e. considering attributes and without considering attribute weights. Weights of attributes are taken as per AHP methodology. Ranking without considering the attributes weights product flexibility is the top most flexibility and programme flexibility is the last one. While considering the weights of attributes results are same as product flexibility is the top most flexibility and programme flexibility is the last one. After this reference point analysis is taken and the results are same as top one is product flexibility and programme flexibility is the last.

These methods are easy to use and simple for calculation purposes. So, computational time will be less. Special thing is that this method is not required any special parameters as required in VIKOR. So, finally concluded that these methods can be utilized for decision making environments effectively.
Ranking of flexibilities is shown in Fig. 5.

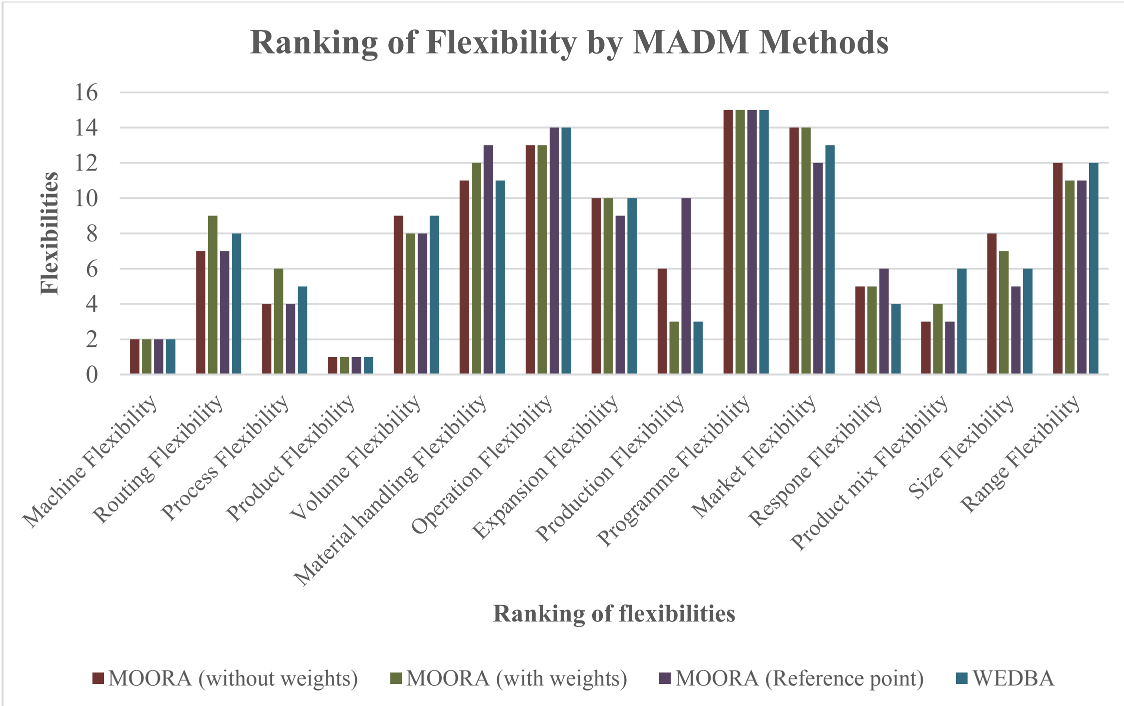

Fig. 5. Ranking of flexibility of FMS by MOORA and WEDBA method

It is concluded that product flexibility is the main flexibility in FMS. So, manager should focus on the product flexibility to increase the increase the flexibility of FMS and finally performance of FMS will increased as the flexibility is one of the factors of performance of FMS. Furthermore, there is a very high Spearman's rank correlation between the methods used. The Spearman's rank correlation coefficient between the methods are shown in Fig. 6.

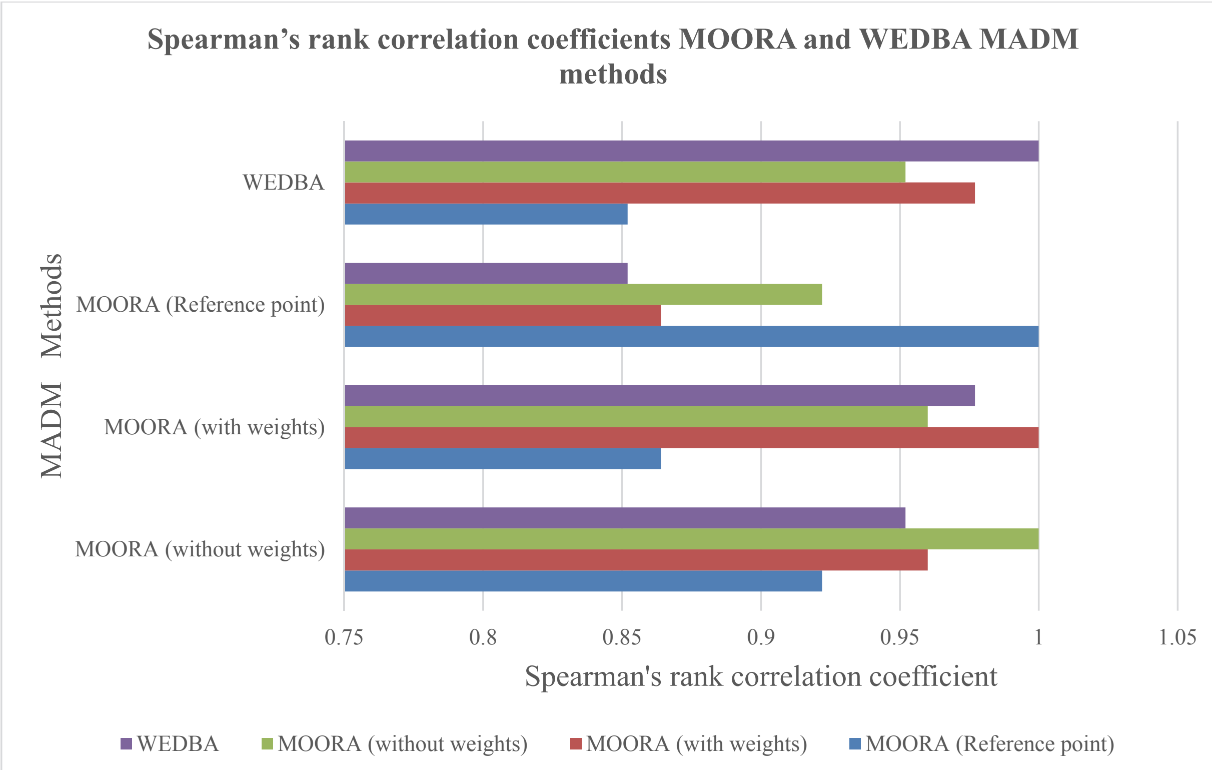

Fig. 6. Spearman's rank correlation coefficients between MOORA and WEDBA MADM methods for ranking of flexibility of FMS

There is no consistency in the result. The approaches are logical, simple and convenient to implement. These approaches can be extended to any other decision making situations of the manufacturing environment.

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