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Bayesian semi-shared temporal modeling: A comprehensive approach to forecasting multiple stock prices

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A B S T R A C T

Stock prices of different companies frequently display similar temporal fluctuations because of common influencing factors. Accurate prediction of stock prices is of utmost importance for investors in determining their investment strategies. Utilizing multivariate forecasting, which involves analyzing multiple time series, has been shown to be highly effective and efficient when applied to stocks that exhibit similar temporal patterns. It is possible to model the relationship between shares by using a shared temporal model approach. Nevertheless, it is important to note that not all stocks selected for prediction demonstrate a strong correlation; certain stocks may deviate from expected patterns. Therefore, the direct implementation of a comprehensive shared temporal component model is not universally applicable. This study presents a new method called the Semi-Shared Temporal Model, which focuses on the correlation structure among variables that have similar patterns, while also modeling all stocks simultaneously. This methodology is applied to the three leading stocks of 2023: Amazon (AMZN), Alphabet (GOOG), and MercadoLibre (MELI). Based on monthly data collected from January 2010 to December 2023, the study forecasts the stock prices for the months of January to December 2024. The analysis findings suggest that the temporal patterns of AMZN and GOOG shares are highly similar, which supports the idea of modeling them together with shared temporality. Three forecasting methods are utilized: univariate models, full shared temporal models, and semi-shared temporal models. The analysis determines that the semi-shared temporal model approach produces the most precise forecasting outcomes, with a Mean Absolute Percentage Error (MAPE) of 17.97%, surpassing both univariate and full shared temporal models. The forecast for 2024 indicates a favorable trajectory for all three stocks.

1. Introduction

Time series data forecasting plays a crucial role across various domains such as health, weather, environment, and economics, aiding in the formulation of effective strategies and policies (Wang, Liu, Du, & Dong, 2023). In the realm of economics, a significant application is stock price forecasting, which holds paramount importance in economic studies. The accuracy of forecasting models hinges on their ability to discern systematic patterns from past data, patterns assumed to be repeated in the future. Time series models are broadly categorized into univariate and multivariate models, where the latter utilizes multiple input variables to provide a comprehensive understanding of the studied phenomenon (Chatfield, 2001).

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Numerous multivariate forecasting approaches, including classical methods like Vector-Autoregressive Moving Average (VARMA) (Reinsel, 1993), Support Vector Regression (SVR) (Sapankevych & Sankar, 2009), Gaussian Processes (GP) (Girard, Rasmussen, Candela, & Murray-S, 2002), and advanced techniques such as Recurrent Neural Networks (RNNs) (Salinas, Flunkert, Gasthaus, & Januschow, 2020) and Long Short-Term Memory (LSTM) networks (Hochreiter & Schmidhuber, 1997), have been developed. However, these approaches often overlook the explicit consideration of correlation structures between input variables, potentially limiting the optimal utilization of such structures for enhanced forecasting accuracy. An alternative to traditional multivariate modeling is the shared-temporal model, which not only addresses autocorrelation structures but also considers dependencies between input variables (Knorr-Held & Best, 2001). Shared component models, modelled using a joint likelihood approach, have gained prominence in multivariate modeling due to their efficacy in capturing strong correlations or similar temporal patterns across multiple variables. Initially introduced for modeling two diseases simultaneously, shared component models have evolved to handle increased complexity involving more input variables through hierarchical Bayesian approaches (Gomez-Rubio, Palmı-Perales, Lopez-Abente, Ramis-Prieto, & Fernandez-Navarro, 2019).

The Bayesian shared component model has found diverse applications, including forecasting COVID-19 cases (Jaya, Folmer, & Lundberg, 2022), predicting TB and HIV instances (Jaya et al., 2023a, 2023b), and estimating juvenile delinquency and violent crime simultaneously (Law & Abdullah, 2022). Despite the effectiveness of shared component models, fully implementing them may not always yield optimal results, as not all input variables exhibit similar patterns. Hence, a semi-shared temporal model is proposed, considering the unique patterns of each input variable without explicitly relying on correlation structures with other inputs. While shared component models are prevalent in disease modeling with both temporal and spatial dimensions, their application in economic studies, particularly in stock price forecasting, is relatively limited. Parameter estimation for shared model components typically involves a hierarchical Bayesian approach, with Integrated Nested Laplace Approximation (INLA) being a widely favored method due to its computational efficiency (Blangiardo & Cameletti, 2015).

This research endeavors to predict the performance of three major global stocks in 2023—Amazon (AMZN), Alphabet (GOOG), and MercadoLibre (MELI)—utilizing a semi-shared temporal model approach with Bayesian INLA estimation. Stock forecasting is a highly intricate process that necessitates the use of an appropriate model.

The remainder of this manuscript is structured as follows. In Section 2, we introduce the Bayesian semi-shared temporal model for time series modeling and forecasting. Section 3 details the application of this methodology in forecasting the stock prices of the top three companies: AMZN, GOOG, and MELI. Lastly, Section 4 comprises the discussion and conclusion of the study.

2. Bayesian Multivariate Semi-Shared Temporal Model

2.1 Model specification

This section discusses the multivariate time series model with a Bayesian approach for forecasting the three top global stocks: AMZN, GOOG, and MELI. Assume the input variables at time $\boldsymbol{t}$, for $\boldsymbol{t = 1, \dots, T}$ are $\boldsymbol{y_{1t}, y_{2t}, \text{and } y_{3t}}$ each follow a normal univariate distribution with means $\boldsymbol{\mu_1, \mu_2, \text{and } \mu_3}$ and variances $\boldsymbol{\sigma_1^2, \sigma_2^2, \text{and } \sigma_3^2}$ which can be modeled as follows:

$$\boldsymbol{y_{st} | \mu_s, \sigma_s^2 \sim N(\mu_s, \sigma_s^2) \text{ for } s = 1, 2, 3, \text{and } t = 1, \dots, T} \tag{1}$$

$$\boldsymbol{y_{st} = \mu_s + \varepsilon_{st};\ \varepsilon_{st} \sim N(0, \sigma_s^2)} \tag{2}$$

$$\boldsymbol{\mu_s = \alpha_s + v_{st} + \zeta_{st} + \gamma_{st}} \tag{3}$$

where $\boldsymbol{\alpha_s}$ signifies the intercept for the s-th input series, $\boldsymbol{v_{st}}$ denotes the temporally structured event component, $\boldsymbol{\zeta_{st}}$ represents the temporally unstructured component, and the final component $\boldsymbol{\gamma_{st}}$ accounts for seasonal effects. To facilitate Bayesian inference, we make the assumption that $\boldsymbol{\alpha_1, \alpha_2, \text{ and } \alpha_3}$ follow a Gaussian prior with a zero mean and $\boldsymbol{\sigma_\alpha^2}$, variance, i.e. $\boldsymbol{\{\alpha_1, \alpha_2, \alpha_3\} \sim N(0, \sigma_\alpha^2)}$. The temporally structured effect $\boldsymbol{v_{st}}$ follows a random walk of order one (RW1):

$$\boldsymbol{v_{s,t+1} - v_{st} | \sigma_v^2 \sim N(0, \sigma_v^2), \qquad \forall s \text{ and } t = 1, \dots, T-1,} \tag{4}$$

with variance hyperparameter σ_v^2. The temporally unstructured effects are modeled through exchangeable prior:

The seasonal component, γ_{st}, with periodicity q is defined as:

$$\boldsymbol{\gamma_{st} + \gamma_{st+1} + \cdots + \gamma_{s,t+q-1} | \sigma_\gamma^2 \sim N\left(0, \sigma_\gamma^2\right),\ \ \forall s \text{ and } t = 1, \dots, T-q+1,} \tag{5}$$

where σ_γ^2 is the variance hyperparameter of γ_{st} for $s = 1,2,3$

While the model outlined above involves multiple input variables, it is categorized as a univariate approach since the modeling is conducted independently, neglecting the relationships or correlation structure between the input variables during the forecasting process. To address the correlation among input variables, we adopt a shared-temporal component model by introducing weights for one or more random components. This involves if each random component, such as temporally structured and unstructured effects, and seasonal effects, shares the same prior distribution. For instance, assuming a common temporal trend for the three-input series, model (1) can be reformulated as follows:

$$\boldsymbol{\mu_s = \alpha_s + \beta_s v_t + \zeta_{st} + \gamma_{st}} \tag{6}$$

Here, $\boldsymbol{\beta_s}$ signifies the weight of the shared-temporal component, assumed to adhere to a normal distribution with a mean of zero and a variance of $\sigma^2_{\boldsymbol{\beta}}$. We refer to the model as a full shared-temporal model. However, employing the full-shared component model may not always be suitable, particularly when there are one or several input variables that do not exhibit the same temporal pattern. Therefore, we advocate for the utilization of a semi-shared temporal model approach. For instance, considering one stock, namely MELI, it demonstrates a pattern that is not entirely identical to other stocks. Consequently, the temporal trend of MELI cannot be assumed to be identical to that of AMZN and GOOG. The semi-shared temporal model can be expressed as follows:

$$\boldsymbol{\mu_s = \alpha_s + \beta_s v_t + \zeta_{st} + \gamma_{st}} \textbf{ for s} = \mathbf{1,2} \text{ and} \tag{7}$$

$$\boldsymbol{\mu_s = \alpha_s + v_{st} + \zeta_{st} + \gamma_{st}} \textbf{ for s} = \mathbf{3}$$

2.2 Bayesian inference using INLA

The parameters and hyperparameters estimation for the shared-temporal model described in equation (7) was performed using Integrated Nested Laplace Approximation (INLA). Let Φ represent the set $\mathbf{\Phi} = \{\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, v_{11}, \dots, v_{3T}, \zeta_{11}, \dots, \zeta_{3T}, \gamma_{11}, \dots, \gamma_{3T}\}$, which corresponds to the latent Gaussian field, and ψ is a hyperparameter vector denoted as $\boldsymbol{\psi} = \{\sigma^2_\alpha, \sigma^2_\beta, \sigma^2_v, \sigma^2_\zeta, \sigma^2_\gamma\}$. The posterior marginal of the parameters is given as:

$$\tilde{\boldsymbol{p}}(\boldsymbol{\Phi_l}|\mathbf{y}) \approx \sum_{\boldsymbol{j}} \tilde{\boldsymbol{p}}(\boldsymbol{\Phi_l}|\boldsymbol{\psi}^{(j)}, \mathbf{y}) \tilde{\boldsymbol{p}}(\boldsymbol{\psi}^{(j)}|\mathbf{y}) \boldsymbol{\Delta_j} \tag{8}$$

A wide range of numerical methodologies can be utilized to tackle Eq. (8), encompassing approaches such as central composite design and grid search.

2.3 Multivariate forecasting

To derive the multivariate forecast values for the stock prices of the top three global companies, namely AMZN, GOOG, and MELI, we employ their posterior predictive distribution in a multivariate context, defined as follows (Jaya, et al., 2023):

$$\boldsymbol{p}(\hat{\boldsymbol{y}}_{\boldsymbol{(T+h)}} \,|\mathbf{y}, \boldsymbol{\psi}) = \int \boldsymbol{p}(\hat{\boldsymbol{y}}_{\boldsymbol{(T+h)}} \,|\boldsymbol{\Phi}, \boldsymbol{\psi}) \mathbf{p}(\boldsymbol{\Phi}|\mathbf{y}, \boldsymbol{\psi}) \mathbf{d}\boldsymbol{\Phi} \tag{9}$$

where, $\hat{\boldsymbol{y}}_{(T+h)} = (\hat{\boldsymbol{y}}_{1(T+h)}, \hat{\boldsymbol{y}}_{2(T+h)}, \hat{\boldsymbol{y}}_{3(T+h)})$ represents the vector of forecasted values for AMZN, GOOG, and MELI at time $T + h$. In the Integrated Nested Laplace Approximation (INLA) method, forecasting is executed by inputting 'Not Available (NA)' for the $T + h$ period, where the forecasts are generated.

2.4 Model selection criteria

To forecast the prices of the three global stocks AMZN, GOOG, and MELI, we explored three models: the univariate model, full shared-temporal model, and semi-shared temporal model, as outlined below:

Univariate M1: $\eta_s = \alpha_s + v_{st} + \zeta_{st} + \gamma_{st}\,; s = 1,2,3$

Full-shared temporal M2: $\eta_s = \alpha_s + \boldsymbol{\beta_s} v_{st} + \zeta_{st} + \gamma_{st} + \delta_s \omega_t; s = 1,2,3$

Semi-shared temporal M3: $\eta_s = \alpha_s + \boldsymbol{\beta_s} v_t + \zeta_{st} + \gamma_{st}; s = 1,2$

$\eta_s = \alpha_s + v_{st} + \zeta_{st} + \gamma_{st}; s = 3$

To assess the accuracy of the forecasting models for the three global stocks AMZN, GOOG, and MELI, we examine the disparities between the actual and predicted values using metrics such as mean absolute error (MAE), root mean square error (MSE), mean absolute prediction error (MALE). Additionally, we gauge the suitability of the temporal pattern between actual and predicted data by calculating the Pearson correlation coefficient, as detailed below:

Mean absolute error (MAE)

$$MAE_s = \frac{\sum_{t'=T_1+1}^{T} |y_{st'} - \hat{y}_{st'}|}{T_2}; s = 1,2,3 \tag{10}$$

where notation |.| represents the absolute function.

Root means square error (RMSE)

$$RMSE_s = \sqrt{\frac{\sum_{t'=T_1+1}^{T} (y_{st'} - \hat{y}_{st'})^2}{T_2}}; s = 1,2,3 \tag{11}$$

Mean absolute prediction error (MAPE)

$$MAPE_s = \frac{1}{T_2} \sum_{t'=T_1+1}^{T} \left| \frac{y_{st'} - \hat{y}_{st'}}{y_{st'}} \right| \times 100\%; s = 1,2,3 \tag{12}$$

Pearson's correlation coefficient (r)

$$r_s = \frac{\sum_{t'=T_1+1}^{T} (y_{st'} - \bar{y}_s)(\hat{y}_{st'} - \bar{\hat{y}}_s)}{\sqrt{\sum_{t'=T_1+1}^{T} (y_{st'} - \bar{y}_s)^2 \sum_{t'=T_1+1}^{T} (\hat{y}_{st'} - \bar{\hat{y}}_s)^2}}; s = 1,2,3 \tag{13}$$

where, $\bar{y}_s$ and $\bar{\hat{y}}_s$ represent the averages of the number of incidences and the predicted number of cases, respectively. A model exhibiting lower MAE, RMSE, and MAPE values, coupled with a higher *r* value, signifies superior forecasting prowess.

3. Semi-Shared Temporal Model for modeling and forecasting of the closing stock prices of the three leading global stocks: Amazon (AMZN), Alphabet (GOOG), and MercadoLibre (MELI)

3.1 Exploratory Descriptive Analysis

Our primary objective in implementing the semi-shared temporal model is to predict the monthly closing stock prices of three prominent companies (Guberti, 2023): Amazon (AMZN), Alphabet (GOOG), and MercadoLibre (MELI) from January to December 2024. The dataset covers the period from January 2010 to December 2023 and was obtained from the website www.finance.yahoo.com. Table 1 provides a thorough and detailed descriptive analysis of these three stocks.

Table 1
Statistics descriptive Three Top Stock Prices AMZN, GOOG, and MELI

	Min	Q1	Me	Mean	Q3	Max	R	SD	CV
AMZN	5.900	14.375	40.800	63.289	98.525	180.800	174.900	54.798	86.584
GOOG	11.800	21.800	39.900	53.976	73.825	147.400	135.600	39.558	73.289
MELI	37.800	93.225	178.850	466.899	735.400	1853.700	1815.900	506.673	108.519

Note: Min=Minimum; Q1=Quartile one; Me=Median; Q3=Quartile three; R=Pearson's correlation; SD=Standard deviation; CV=Coefficient variation

Table 1 shows that MELI's share price is the most elevated in comparison to other share prices. This is highly apparent from the mean or median value. Nevertheless, it exhibits the most significant degree of variability as indicated by its coefficient of variation (CV). GOOG shares exhibit a comparatively lower degree of volatility. Between 2010 and 2013, the prices of the three stocks experienced significant increases (see Fig. 1), reaching their highest point in 2020 during the COVID-19 pandemic. The worldwide health emergency necessitated individuals to remain indoors, resulting in a heightened reliance on online platforms for their shopping needs (Gupta, 2021). Fig. 1 illustrates the temporal patterns of three stocks from January 2010 to December 2023. While the stocks exhibit relatively similar patterns, the most pronounced temporal similarities emerge between GOOG and AMZN. However, MELI shares show discernible differences across various time periods. In Figure 2, the scatter plot highlights the stock relationships. Notably, in 2010, AMZN and GOOG shares display a strong positive correlation, while MELI and GOOG exhibit a weaker correlation. In 2011, MELI appears to have a low correlation with AMZN and GOOGL. Subsequently, MELI demonstrates a negative correlation with AMZN and GOOG shares. These trends are explicitly presented in Table 2 (indicated by red letters)

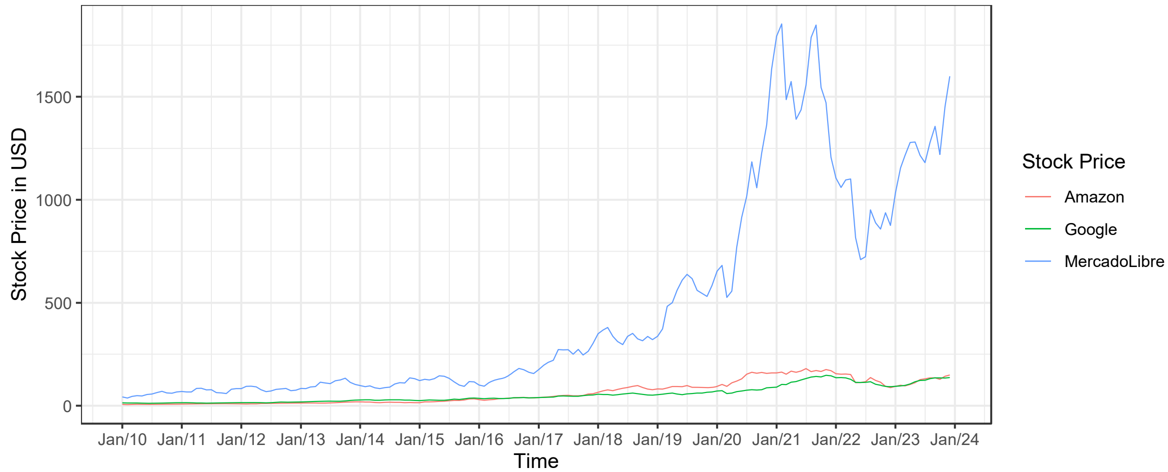
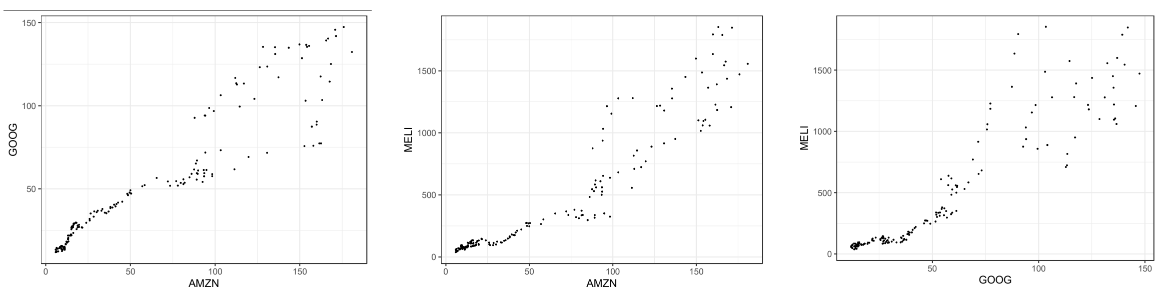


Fig. 1. Temporal Trends in the Stock Prices of Top 3 Companies: AMZN, GOOG, and MELI, January 2010 - December 2023



(A) AMZN vs. GOOG (B) AMZN vs. MELI (C) GOOG vs. MELI

Fig. 2. Scatter plot (A) AMZN vs. GOOG, (B) AMZN vs. GOOG, and (C) GOOG vs. MELI

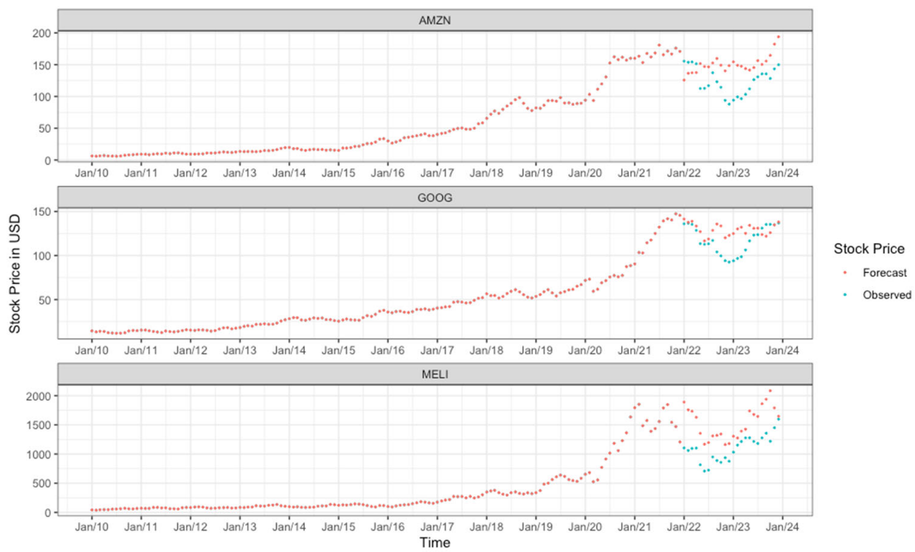
Table 2
Annual Pearson's Correlation of the Three Stock Prices: AMZN, GOOG, MELI (2010–2023)

Year	Stock	AMZN	GOOG	Year	Stock	AMZN	GOOG
2010	GOOG	0.68		2017	GOOG	0.97	
	MELI	0.61	-0.10		MELI	0.86	0.89
2011	GOOG	-0.38		2018	GOOG	0.65	
	MELI	-0.31	0.02		MELI	-0.35	0.05
2012	GOOG	0.72		2019	GOOG	0.15	
	MELI	-0.59	0.05		MELI	0.83	0.34
2013	GOOG	0.93		2020	GOOG	0.76	
	MELI	0.39	0.53		MELI	0.89	0.93
2014	GOOG	0.67		2021	GOOG	0.71	
	MELI	-0.26	-0.31		MELI	-0.15	-0.31
2015	GOOG	0.97		2022	GOOG	0.94	
	MELI	-0.49	-0.59		MELI	0.73	0.64
2016	GOOG	0.75		2023	GOOG	0.96	
	MELI	0.96	0.84		MELI	0.75	0.69

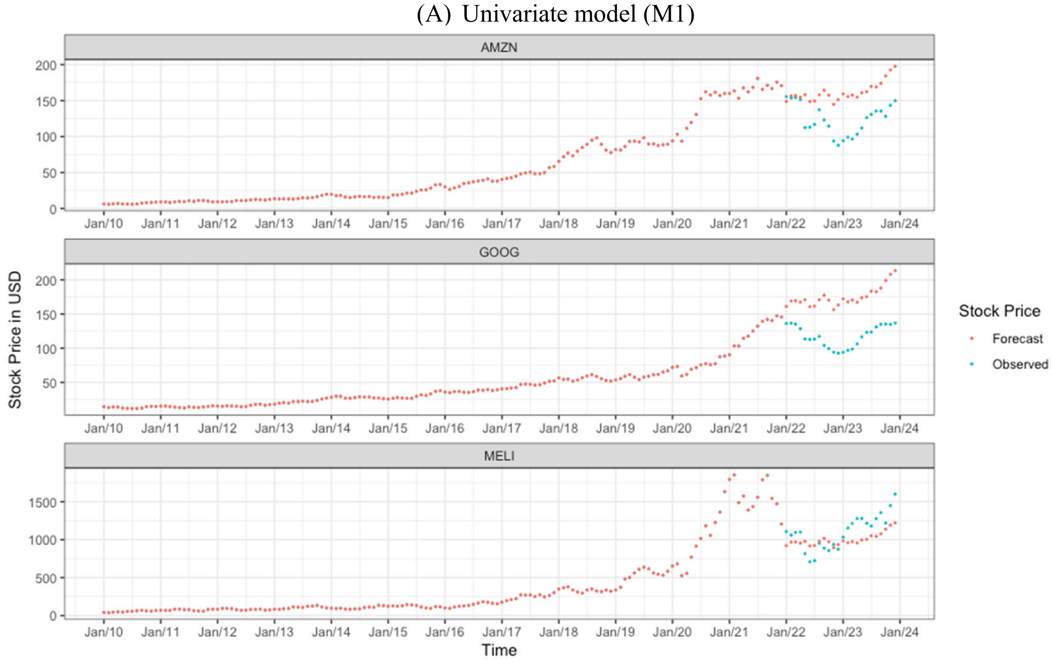
3.2 Models' comparison

Forecasting stock prices entails navigating a highly complex landscape, characterized by the influence of numerous factors and heightened volatility. Achieving accurate predictions requires the deployment of appropriate models. In this section, we introduce three models: M1, an univariate model; M2, a fully shared temporal model; and M3, a semi-shared temporal model. In the M1 model, we conduct univariate modeling three times, corresponding to the number of stock types. For M2, we perform multivariate modeling, assuming all stocks share the same temporal pattern. Conversely, in the M3 model, we carry out modeling by presuming that only AMZN and GOOG shares share a similar temporal pattern, as detailed in Section 3.1.

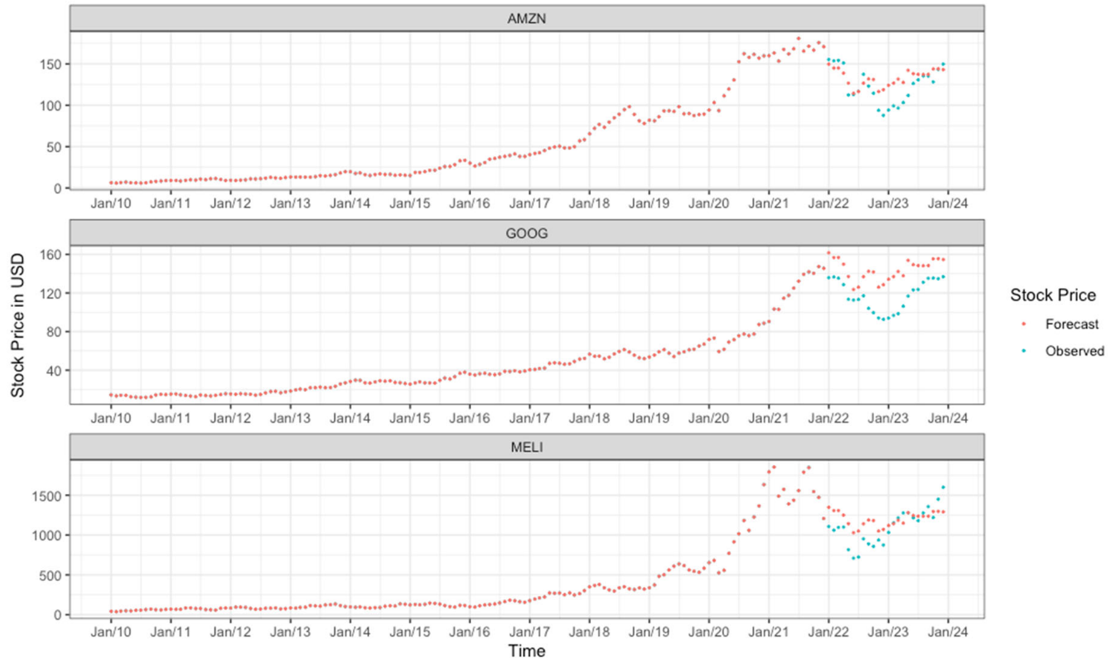
To identify the most suitable model, we employ an in-and-out sample prediction approach. Data is initially divided into training and testing sets, with the former used for model construction and the latter for performance evaluation. We utilize data spanning from January 2010 to December 2021 as our training dataset, while data from January 2022 to December 2023 serves as the testing dataset. We assess model performance using metrics such as Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Mean Absolute Prediction Error (MAPE) to gauge the disparities between actual and predicted values. Additionally, we calculate the Pearson correlation coefficient to evaluate the suitability of the temporal patterns in the forecast results. Comprehensive results of these statistical measures are presented in Fig. 3 and Table 2. Fig. 3 illustrates that Model M3 yields prediction results for the period January 2022 to December 2023 that closely align with the training data, surpassing the performance of models M1 and M2. The M1 model exhibits superior predictions for AMZN and MELI shares compared to M2, while M2 performs better in predicting MELI shares. A comprehensive overview of the models' goodness criteria is presented in Table 2. Generally, Model M3 attains the smallest Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Absolute Prediction Error (MAPE), and the largest R values. This pattern holds true for each stock, except for the RMSE and MAPE criteria, where the M2 model outperforms M3. Conversely, the M1 model demonstrates the least favorable overall performance. These findings suggest that leveraging the temporal patterns of AMZN and GOOG can enhance MELI predictions. However, incorporating MELI into a shared temporal component diminishes the accuracy of AMZN and GOOG predictions.



(A) Univariate model (M1)



(B) Full shared temporal model (M2)



(C) Semi-shared temporal model (M3)

Fig. 3. Evaluation of Training and Testing for (A) Univariate Model (M1), (B) Full Shared-Temporal Model (M2), and (C) Semi-Shared Temporal Model (M3).

Table 2
Models model comparison criteria Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Absolute Prediction Error (MAPE), and Pearson's correlation (R)

Stock Price	Model	MAE	RMSE	MAPE	R
AMZN	M1	33.501	36.213	29.505	0.093
	M2	38.849	43.108	34.469	0.402
	M3	13.948	17.567	12.832	0.762
GOOG	M1	14.677	18.808	14.007	0.407
	M2	57.679	59.448	51.068	0.540
	M3	26.521	28.285	23.911	0.785
MELI	M1	435.431	480.330	41.797	0.680
	M2	172.964	195.592	15.544	0.740
	M3	166.422	199.669	17.175	0.677

After evaluating the model comparison results, we have opted for the M3 model as the most favorable choice among the three. This model will be employed to predict the prices of the three stocks—AMZN, GOOG, and MELI—during the period from January to December 2024.

3.3 Forecasted Results for Three Stock Companies: AMZN, GOOG, and MELI, From January to December 2024

The semi-shared temporal model is employed for forecasting the performance of three stock companies: AMZN, GOOG, and MELI, spanning from January to December 2024. To mitigate the impact of data abnormalities and outliers, we apply a log transformation to the stock price response variable. The parameter estimation results for the log-linear model are presented in both Table 3 and Table 4.

Table 3
Posterior Mean of Fixed Effect Components with Their 95% Credible Intervals

Parameter	Mean	SD	$q_{(0.025)}$	$q_{(0.5)}$	$q_{(0.975)}$
Intercept AMZN	3.086	0.031	3.028	3.085	3.150
Intercept GOOG	3.146	0.028	3.094	3.144	3.204
Intercept MELI	4.991	0.032	4.931	4.990	5.057
Slope Time	6.16E-05	2.85E-06	5.57E-05	6.17E-05	6.69E-05

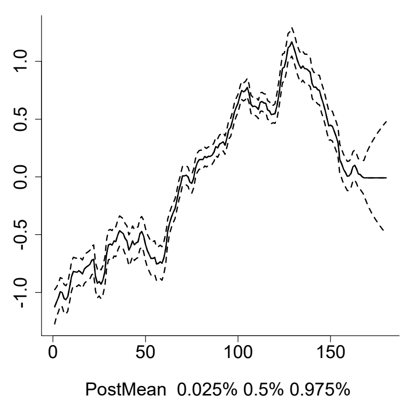
Table 3 presents the estimations of fixed effect parameters, encompassing the intercept (overall mean) of AMZN, GOOG, and MELI shares, along with the impact of time on share price variations. Given that the model employs a log transformation, interpretation of the model parameters involves exponentiating the parameter values. The baseline average values for AMZN, GOOG, and MELI stock prices, without the influence of the time component and random effect, are exp(3.086) = 21.883 USD, exp(3,146) = 23.232 USD, and exp(4.991) = 147.137 USD, respectively. Notably, MELI shares exhibit the highest average. Moreover, the temporal component contributes to the growth in share values, generally yielding an exponentiated factor of exp(6.16×10^{-5}) = 1.000061, equivalent to approximately 0.0061%.

Table 4

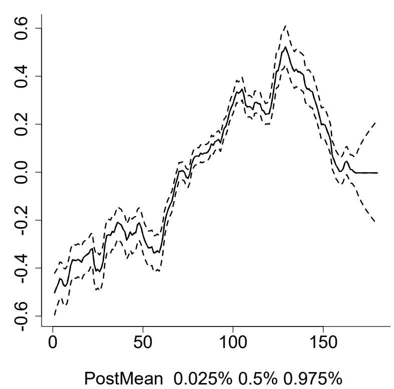
Posterior Mean of Random Effect Components with Their 95% Credible Intervals

Hyperparameter	Mean	SD	$q_{(0.025)}$	$q_{(0.5)}$	$q_{(0.975)}$	Fraction Var (%)
SD for Gaussian error AMZN	0.007	0.003	0.003	0.007	0.015	0.02
SD for Gaussian error GOOG	0.008	0.004	0.003	0.008	0.017	0.02
SD for Gaussian error MELI	0.008	0.004	0.003	0.007	0.017	0.02
SD for shared temporal AMZN- GOOG	0.164	0.016	0.135	0.163	0.196	9.41
SD for random walk MELI	0.482	0.034	0.419	0.481	0.554	81.73
SD for heterogeneity AMZN	0.007	0.004	0.002	0.007	0.018	0.02
SD for heterogeneity GOOG	0.110	0.007	0.097	0.110	0.124	4.25
SD for heterogeneity MELI	0.009	0.005	0.002	0.008	0.021	0.03
SD for seasonal	0.113	0.022	0.077	0.111	0.162	4.49
Beta	2.242	0.095	2.057	2.241	2.431	

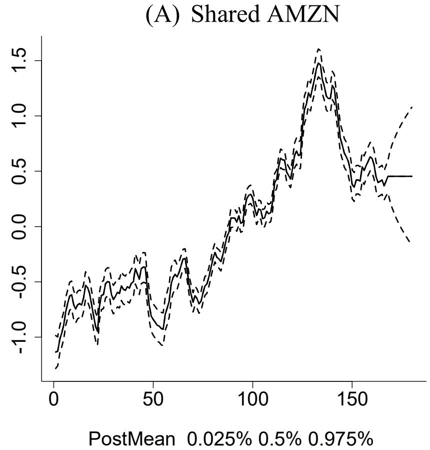
Table 4 shows the posterior mean for the random effect component which includes standard deviation Gaussian error for AMZN, GOOG shares, standard deviation shared temporal component for AMZN and GOOG, random effect temporal trend random walk for MELI, standard deviation heterogeneity for AMZN, GOOG, MELI and the standard deviation for the seasonal component. The final row represents the weights of the shared components. The results of the faction variance calculation found that the variability of the four components that were most dominant in explaining the semi-shared temporal model were sequentially the MELI temporal trend, AMZN-GOOG shared component, GOOG heterogeneity, and seasonal component. The shared components' weights, where $\beta = 2.242$, suggest that the stock price of AMZN is twice that of GOOG's stock price. The impact of the shared temporal component for AMZN-GOOG, random walk for MELI, spatial heterogeneity for AMZN, GOOG, and MELI, as well as the seasonal effects on the logarithmic scale, is illustrated in Fig. 5.



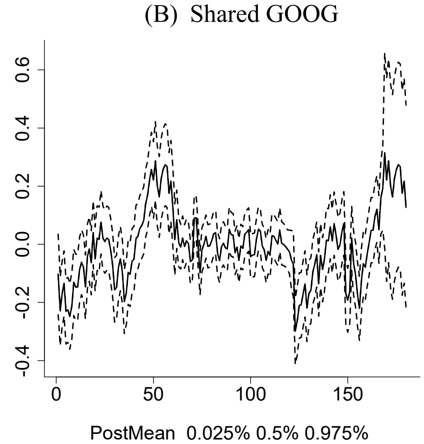
(A) Shared AMZN



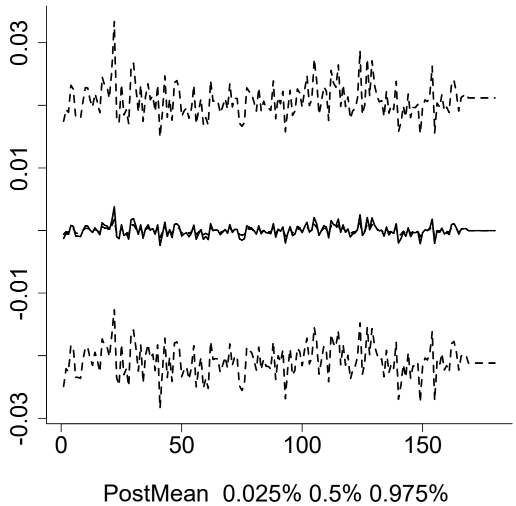
(B) Shared GOOG



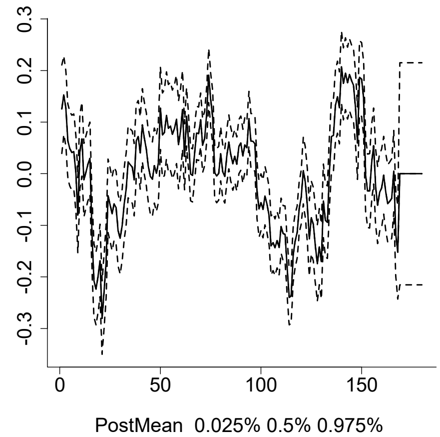
(C) Temporal trend RW1 MELI



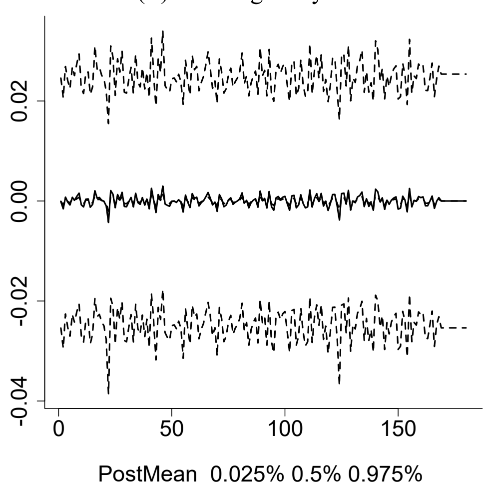
(D) Seasonal effect



(E) Heterogeneity AMZN



(F) Heterogeneity GOOG



(G) Heterogeneity MELI

Fig. 4. The impact of the shared temporal component for AMZN-GOOG, random walk for MELI, spatial heterogeneity for AMZN, GOOG, and MELI, as well as the seasonal effects on the logarithmic scale

In alignment with the insights discussed in Table 4, Fig. 4 provides a visual elucidation of the impact of each random component in explaining the temporal trends in AMZN, GOOG, and MELI stock prices. Graphs exhibiting a consistent pattern display an effect close to zero. In contrast, graphs displaying discernible patterns manifest effects greater than zero, particularly when observed on a logarithmic scale. Utilizing both fixed and random effect components, we conducted a forecast of AMZN, GOOG, and MELI stock prices from January to December 2024. The forecasted outcomes are detailed in Table 5 and visually presented in Figure 5. Overall, the forecast results reveal a positive trend in the stock prices of all three companies.

Table 5
Forecasted Results for Three Stock Companies: AMZN, GOOG, and MELI, From January to December 2024

Month	AMZN			GOOG			MELI		
	$q_{(0.025)}$	Mean	$q_{(0.975)}$	$q_{(0.025)}$	Mean	$q_{(0.975)}$	$q_{(0.025)}$	Mean	$q_{(0.975)}$
January	115.825	172.842	262.440	120.934	184.391	284.139	1215.287	1844.200	2847.125
February	105.973	160.671	244.807	111.610	171.409	263.300	1095.361	1714.330	2696.891
March	112.513	175.252	274.258	120.686	186.964	289.479	1148.272	1869.896	3059.051
April	102.787	164.062	263.071	112.119	175.025	272.903	1037.167	1750.485	2967.218
May	98.930	161.477	264.835	109.605	172.267	270.326	987.832	1722.895	3017.426
June	105.422	175.711	294.157	118.555	187.453	295.789	1042.953	1874.773	3384.643
July	108.966	185.233	316.130	124.297	197.612	313.416	1068.818	1976.369	3671.391
August	111.247	192.444	334.895	128.554	205.304	327.366	1080.301	2053.278	3916.026
September	111.052	195.463	346.211	130.000	208.525	334.031	1069.543	2085.482	4079.732
October	101.853	182.236	328.233	120.740	194.413	312.733	973.207	1944.343	3896.434
November	106.908	194.171	355.692	128.242	207.145	334.708	1012.401	2071.656	4246.509
December	98.408	181.323	336.966	119.511	193.439	313.381	925.237	1934.577	4051.846

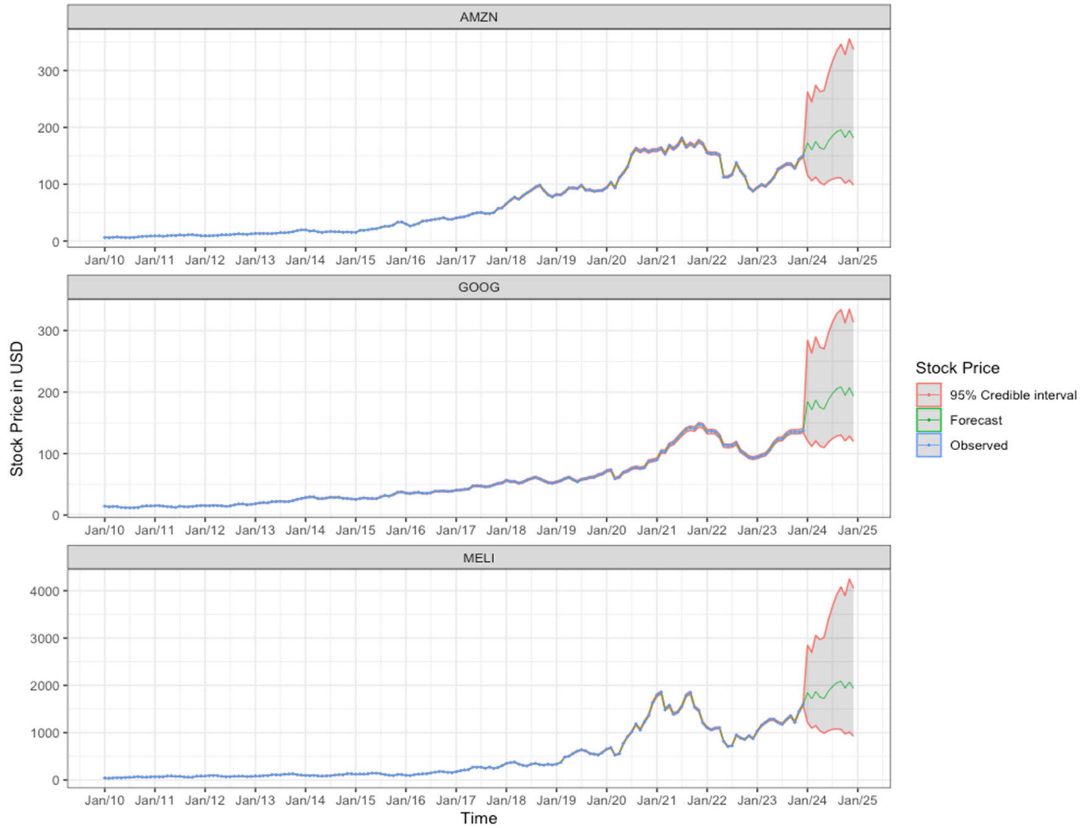


Fig. 5. Forecasted Results for Three Stock Companies: AMZN, GOOG, and MELI, From January to December 2024

4. Discussion and Conclusion

The utilization of multivariate analysis demonstrates superior efficiency in specific instances when compared to univariate analysis. The efficiency of a multivariate context is derived from the inherent interrelatedness of variables, which is influenced by various factors. When modeling, it is important to take into account the relationships between variables. Considering the covariance matrix for correlated variables is more efficient in estimating parameters than solely focusing on variance values and assuming mutual independence. Nevertheless, it is imperative to recognize that not all observed variables are inevitably correlated. The modeling process should possess sufficient flexibility to accommodate scenarios where certain variables exhibit correlation, while others do not. This flexibility does not suggest the act of modeling in a state of being separate or detached. Similarly, when conducting regression analysis with multiple groups, a simultaneous approach can be used by employing a dummy variable approach. This study focuses specifically on the modeling and forecasting of the closing stock prices of the three leading global stocks: Amazon (AMZN), Alphabet (GOOG), and MercadoLibre (MELI). The data exploration findings reveal that the thermal patterns of the three models are quite similar, especially between AMZN and GOOG. Nevertheless, MELI demonstrates a marginally distinct trend in recent years when compared to AMZN and GOOG. A joint likelihood model is used to simultaneously model these three stocks. Significantly, both AMZN and GOOG include a common temporal element, whereas MELI is represented without taking this element into account. The study utilizes a univariate methodology and a comprehensive shared component model to determine the most precise forecasting outcomes. Various accuracy measures, such as mean absolute error (MAE), root mean square error (RMSE), mean absolute prediction error (MAPE), and Pearson's correlation, are employed for the purpose of comparison. The forecast for January to December 2024 is based on monthly data spanning from January 2010 to December 2023. Based on the model comparison, it is evident that the semi-shared component model outperforms the other two models in terms of forecasting accuracy, while the univariate model performs the worst. The forecast results, based on the semi-shared component model, suggest a favourable upward trend for the three stocks (AMZN, GOOG, and MELI) in 2024. Nevertheless, it is crucial to acknowledge that this forecast is a technical analysis derived from statistical research, necessitating meticulous interpretation owing to the multitude of factors that impact stock price fluctuations.

To summarize, the research indicates that when there are multiple responses and strong correlations among the variables being studied, it is advisable to use multivariate modeling, specifically employing a full shared component model approach, rather than relying on univariate models. However, in cases where not all variables exhibit a strong correlation, it is more suitable to employ a semi-shared component model approach.

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References

Blangiardo, M., & Cameletti, M. (2015). *Spatial and Spatio-temporal Bayesian Models with R-INLA*. Chennai: John Wiley & Sons.

Chatfield, C. (2001). *Time-Series Forecasting*. USA: Chapman & Hall/CRC.

Girard, A., Rasmussen, C., Candela, J., & Murray-S, R. (2002). Gaussian process priors with uncertain inputs application to multiple-step ahead time series forecasting. *Advances in Neural Information Processing Systems, 15*, 1-8.

Gomez-Rubio, V., Palmı-Perales, F., Lopez-Abente, G., Ramis-Prieto, R., & Fernandez-Navarro, P. (2019). Bayesian joint spatio-temporal analysis of multiple diseases. *SORT, 43*(1), 51-74.

Guberti, M. (2023, November 28). 3 Top-Rated E-Commerce Stocks That Analysts Are Loving Now. Retrieved January 14, 2024, from InvestorPlace: https://investorplace.com/2023/11/3-top-rated-e-commerce-stocks-that-analysts-are-loving-now/

Gupta, S. (2021, February 18). Amazon vs. MercadoLibre: Which E-Commerce Stock is a Better Buy? Retrieved January 14, 2024, from StockNews: https://stocknews.com/news/amzn-meli-amazon-vs-mercadolibre-which-e-commerce-stock-is-a-better-buy/

Hochreiter, S., & Schmidhuber, J. (1997). Long short-term memory. *Neural Computing, 8*(9), 1735–1780.

Jaya, I., Chadidjah, A., Andriyana, Y., Setyanto, G., Supartini, E., & Kristiani, F. (2023a). Multiple endemic disease risk modeling using a Bayesian spatiotemporal shared components model. *Decision Science Letters*, *12*(2), 389-398.

Jaya, I., Handoko, B., Andriyana, Y., Chadidjah, A., Kristiani, F., & Antikasari, M. (2023b). Multivariate Bayesian Semiparametric Regression Model for Forecasting and Mapping HIV and TB Risks in West Java, Indonesia. *Mathematics, 11*(17), 3641.

Jaya, I., Folmer, H., & Lundberg, J. (2022). A joint Bayesian spatiotemporal risk prediction model of COVID-19 incidence, IC admission, and death with application to Sweden. The Annals of Regional Science, 1-45.

Knorr-Held, L., & Best, N. (2001). A Shared Component Model for Detecting Joint and Selective Clustering of Two Diseases. *Journal of the Royal Statistical Society. Series A (Statistics in Society), 164*(1), 73-85.

Law, J., & Abdullah, A. (2022). An Offenders-Offenses Shared Component Spatial Model for Identifying Shared and Specific Hotspots of Offenders and Offenses: A Case Study of Juvenile Delinquents and Violent Crimes in the Greater Toronto Area. Journal of Quantitative Criminology, 1-24.

Reinsel, G. (1993). *Element of Multivariate Time Series*. New York: Springer-Verlag.

Salinas, D., Flunkert, V., Gasthaus, J., & Januschowski, T. (2020). DeepAR: Probabilistic forecasting with autoregressive recurrent networks. *International Journal of Forecasting*, *36*(3), 1181-1191.

Sapankevych, N. I., & Sankar, R. (2009). Time series prediction using support vector machines: a survey. *IEEE computational intelligence magazine*, *4*(2), 24-38.

Wang, X., Liu, H., Du, J., & Dong, X. (2023). A long-term multivariate time series forecasting network combining series decomposition and convolutional neural networks. *Applied Soft Computing, 139*, 110214.