International Journal of Industrial Engineering Computations 10 (2019) 349–360

Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

Firefly algorithm based upon slicing structure encoding for unequal facility layout problem

G. La Scalia[a*], R. Micale[a], A. Giallanza[a] and G. Marannano[a]

[a]*Department of Engineering, University of Palermo, Italy*

C H R O N I C L E

Article history:
Received December 18 2018
Received in Revised Format January 26 2019
Accepted February 3 2019
Available online February 4 2019

Keywords:
Unequal Area-Facility Layout Problem
Firefly Algorithm
Slicing Structure

A B S T R A C T

Finding the locations of departments or machines in a workspace is classified as a Facility Layout Problem. Good placement of departments has a relevant influence on manufacturing costs, work in process, lead times and production efficiency. This paper analyses the problem of allocating departments with restrictions in terms of unequal area and rectangular shape within a facility, in order to minimize the sum of material handling costs taking into account the satisfaction of the aspect ratio requested. In particular, we propose for the first time a Firefly Algorithm based on the slicing structure encoding. The proposed method was tested comparing the results obtained from other authors on the same literature instance. The results confirm the effectiveness of the Firefly Algorithm in solving the Facility Layout Problem by generating the best solutions with respect to those provided by previous researches.

1. Introduction

Facility layout problems (FLPs) deal with finding the locations of departments in a given area in order to minimize the sum of material handling costs. In Unequal Area FLPs (UA-FLPs) each department has a different length and width and the departments should be totally placed in the given layout area without overlapping. Layout problems are known to be complex and are generally NP-Hard (Garey & Johnson, 1979) and as a consequence, a tremendous amount of research has been carried out in this area during the last decades. In particular, several heuristic and meta-heuristic algorithms have been developed in order to find optimal solutions in a reasonable computational time. Researchers have developed different meta-heuristics methodologies such as Firefly Algorithm (Tavakkoli-Moghaddam et al., 2015; Sonmez & Baray, 2013), Simulated Annealing (Şahin et al., 2010), Tabu Search (McKendall & Jaramillo, 2006; Scholz et al., 2009), Genetic Algorithm (Palomo-Romero et al., 2017; Aiello et al., 2013), Harmony search (Chang & Ku, 2013; Kang &Chae, 2017) and Ant Colony (Komarudin & Wong, 2010; Ulutas & Kulturel-Konak, 2012). Although abundant literature for solving the FLP by using the above-mentioned solution methodologies exists, to the best of our knowledge, the Firefly Algorithm (FA) based upon the slicing structure encoding is not implemented on unequal area FLPs. The FA is one of the newly introduced non-traditional optimization techniques (Yang, 2010). This algorithm allows us to obtain optimal solutions and it is adequate to solve hard combinatorial optimization problems. In

* Corresponding author Tel.: +393479150738
E-mail: giada.lascalia@unipa.it (G. La Scalia)

2019 Growing Science Ltd.
doi: 10.5267/j.ijiec.2019.2.003

particular, the FA is a population-based optimization approach which aims at finding the optimum value of the given objective function. The population consists of a certain number of fireflies, each one is typified by its light intensity, which represents one solution of the objective function. To obtain the optimal solution, the fireflies should move in the direction of the most attractive ones and the new position is calculated based on the different light intensity. In most articles about layout problems, the main objective is to minimize a function related to the total material handling cost, however, to be more realistic, we have also considered more than a single aspect. This article presents a Firefly Algorithm to minimize the sum of the material handling costs maintaining the satisfaction of the department's aspect ratio requested. There are two main formulations for unequal area facility layout problems: the Flexible Bay Structure (FBS) and the more recent slicing tree representation. In the FBS formulation (Tate & Smith, 1995; Konak et al., 2006), the assignment of departments generates columns or bays with different widths and number of departments. The width of a bay is automatically corrected according to the number of departments contained. The main advantage of the FBS is that the bays can be easily transformed in aisles helping designers to convert the model into a real facility. However, the FBS divides the floor only in one direction (vertically or horizontally). The slicing structure represents an alternative layout fractionation (Tam, 1992) in which an initial rectangular is divided either in horizontal or vertical direction and the procedure is recursively applied to the submatrices generated until the whole area is fully represented by rows or columns (Scholz et al., 2010; Aiello et al., 2012). In this paper, in order to explore a wider space of solutions, we propose to solve the facility layout problem by using a Firefly algorithm encoded by a slicing structure. The remainder of this article is organized as follows. The slicing structure and the firefly optimization procedure is formulated in Section 2. Section 3 provides an explanation of the computational models that are used in our proposed method by means of a numerical example. In Section 4, the methodology is discussed and the obtained results are analysed through a benchmarking procedure and a performance analysis. Finally, concluding remarks and future works are presented in Section 5.

2. Slicing layout generation and firefly optimization procedures

2.1 Random layout generation

The proposed FA is based on the referenced slicing structure, where a solution is represented by an $n \times m$ matrix E, called location matrix, which contains information about the relative locations of the departments on the floor. In our representation, in order to obtain a uniform encoding scheme, only quadratic matrices ($n = m$) are considered. Consequently, given N departments, the rank (r) of the corresponding location matrix is determined as the ceiling function of the square root of N:

$$r = \lceil \sqrt{N} \rceil. \tag{1}$$

The number of elements in the matrix is thus greater or equal to the number of departments. In this case, Dummy departments *($D = r^2 - N$)* with the null area are introduced. These Dummy departments have null material fluxes from/to other departments and are indexed as zero. The demonstration steps of the generation of the random layouts based on the slicing structure are reported below using an illustrative example of 20 departments.

The **first step** of the proposed optimization procedure consists in randomly generating an initial vector $A = (a_{1,1} \ldots a_{1,j})$ corresponding to the first firefly. This firefly contains a set of random numbers equal to the sum of the departments and the dummy departments *($j = D+N$)* belonging to the range [0-1].
This procedure is reported in the following example, in which there are 20 departments and 5 dummy facilities.

$$A = (0.259\ 0.142\ 0.276\ 0.226\ 0.107\ 0.282\ 0.154\ 0.166\ 0.663\ 0.702\ 0.474\ 0.433\ 0.771\ 0.370\ 0.832\ 0.744\ 0.777\ 0.755\ 0.321\ 0.319\ 0.522\ 0.825\ 0.534\ 0.877\ 0.351)$$

When $r^2 > N$, the dummy departments D are assigned to the smaller values of the string and they are substituted by *0* values in the same position.

$$A = (0.259\ 0\ 0.276\ 0\ 0\ 0.282\ 0\ 0\ 0.663\ 0.702\ 0.474\ 0.433\ 0.771\ 0.370\ 0.832\ 0.744\ 0.777\ 0.755\ 0.321\ 0.319\ 0.522\ 0.825\ 0.534\ 0.877\ 0.351)$$

The **second step** consists in the creation of a vector $B = (b_{1,1} \dots b_{1,k})$ in which the random numbers other than zero of the A vector are ordered *(k=20)*.

$$B = (0.259\ 0.276\ 0.283\ 0.319\ 0.321\ 0.352\ 0.370\ 0.433\ 0.474\ 0.522\ 0.534\ 0.663\ 0.702\ 0.744\ 0.755\ 0.772\ 0.777\ 0.825\ 0.832\ 0.877)$$

The **third step** substitutes at each element $b_{1,k}$ of the vector B, a number in the range between 1 to 20 and these numbers are inserted in a vector $C = (c_{1,1} \dots c_{i,k})$ (k=20) in which their position corresponds to the ones reported in the main vector A, excluding the dummy departments from this procedure:

$$C = (1\ 2\ 3\ 15\ 14\ 20\ 9\ 7\ 6\ 16\ 18\ 4\ 5\ 11\ 13\ 8\ 12\ 17\ 10\ 19)$$

The **fourth step** generates a vector $D = (d_{1,1} \dots d_{1,j})$ (j=25) obtained from the A vector, in which, except the 0 values, each random value is substituted by the value $c_{1,k}$.

$$D = (1\ 0\ 2\ 0\ 0\ 3\ 0\ 0\ 15\ 14\ 20\ 9\ 7\ 6\ 16\ 18\ 4\ 5\ 11\ 13\ 8\ 12\ 17\ 10\ 19)$$

In the **fifth step,** the D vector is reshaped in a quadratic matrix $E = \begin{pmatrix} e_{11} & \cdots & e_{1r} \\ \vdots & \vdots & \vdots \\ e_{r1} & \dots & e_{rr} \end{pmatrix}$

where r is the rank of the E matrix (r=5).

In this matrix, the slicing structure is applied and the corresponding layout is generated.

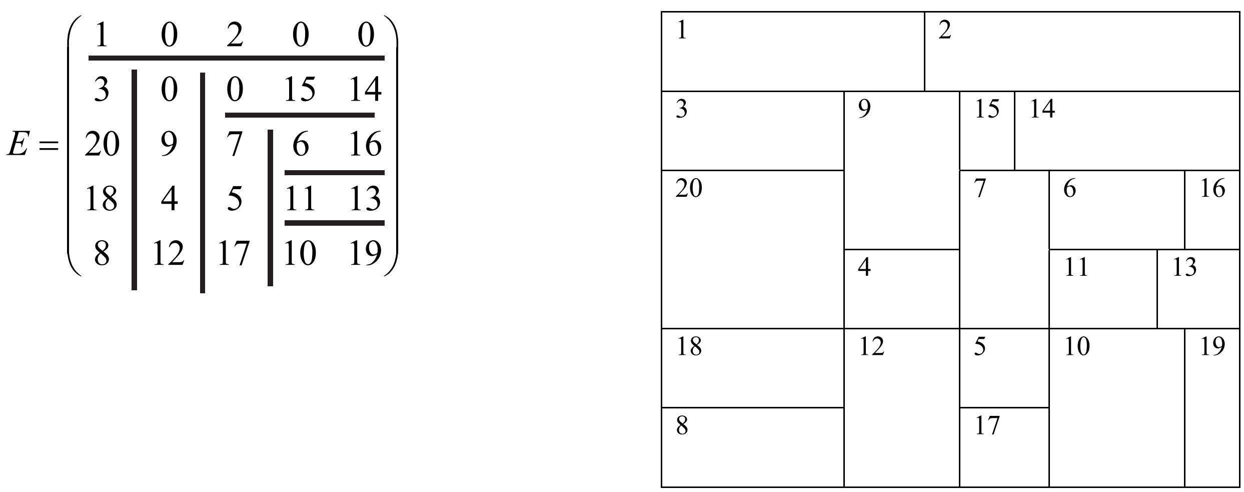

Fig. 1. Location matrix, slicing structure and the corresponding layout

The proposed algorithm generates random values 0 or 1 where 0 indicates a sequence of horizontal–vertical cuts, whereas 1 a sequence of vertical-horizontal cuts. Considering every possible alternative for the decomposition, the maximum number of cuts in which the matrix can be divided, is calculated by the equation reported in Aiello et al., (2012).

In the **sixth step,** the objective function can be calculated in terms of Material Handling Cost (MHC) and a control on the Aspect Ratio Satisfaction (ARS) is effectuated (section 2.3). Layouts are considered feasible only if the condition $\prod ARS \neq 0$ is reached.

The whole procedure starts from 2,000 iterations and it is reiterated until a population of at least ten feasible solutions is obtained. Each feasible layout represents a firefly and the optimization procedure is reported in the section below. Moreover, the software memorizes the sequence of the cuts used to generate the layouts.

2.2. Firefly optimization procedure

In the FA, the variation of the light intensity and attractiveness are main concerns. This attractiveness is determined by brightness, which is associated to the objective function. After generating an initial number of fireflies or solutions of the problem, the light intensity of firefly is updated. Assuming the absorption coefficient γ, the light intensity of firefly varies depending on the square of the distance d, as in the following equation:

$$L = L_0 e^{-\gamma d^2}, \tag{2}$$

where L_0 denotes the light intensity of the source. The attractiveness of fireflies is proportional to their light intensity L. Thus, Eq. (3) is given, in order to describe the attractiveness.

$$\beta = \beta_0 e^{-\gamma d^2}, \tag{3}$$

where β_0 is the attractiveness at $d = 0$. The distance between any two fireflies p_i and p_j is taken as the Euclidean distance. Considering each firefly as a sequence of $D+N$ departments, the distance between two fireflies can be formulated as follows:

$$d_{ij} = \|p_i - p_j\| = \sqrt{\sum_{k=1}^{D+N} \left(p_{i,k} - p_{j,k}\right)^2}. \tag{4}$$

The i-th firefly is attracted by another brighter firefly j. The movement of the firefly from one position to another is expressed by the following equation:

$$p_{inew} = p_{iold} + \beta\left(p_j - p_{iold}\right) + \alpha\varepsilon \tag{5}$$

in which $\alpha=0.2$ and ε is a random number in the range [0,1]. The parameter γ has a crucial effect on the convergence speed of the algorithm. The value of this parameter depends on the problem which needs optimization. Typically, its value ranges from 0.1 to 10 (Yang, 2010). The FA is controlled by three parameters: the randomization parameter, the attractiveness, and the absorption coefficient. By adjusting these parameters, we can obtain good results from an optimization problem. The flowchart of the FA is shown in Fig. 2.

In the **first step** the vectors $A = \left(a_{1,1} \ldots a_{1,j}\right)$ corresponding to the feasible layouts (fireflies) are sorted on the basis of the objective function and afterwards the matrix $F = \begin{pmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \vdots & \vdots \\ a_{z1} & \ldots & a_{zj} \end{pmatrix}$ is built, in which for the example considered $j=25$ and z as the number of feasible layouts. The first line corresponds to the firefly with the minimum material handling cost and it is considered as a “Firefly Queen” (FQ) of the initial population.

In the **second step** the distance of each firefly from the best (FQ) is calculated using Eq. (4).

In the **third step** the position of fireflies is updated by putting the distance and the intensity values in Eq. (5).

In the **fourth step** the software creates a matrix G= $\begin{pmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \vdots & \vdots \\ p_{z1} & \cdots & p_{zj} \end{pmatrix}$

which memorizes the new values obtained by applying Eq. (5). In order to maintain the FQ, the first line of the G matrix is calculated using $\varepsilon = 0$ and the same cut sequence of the first population.

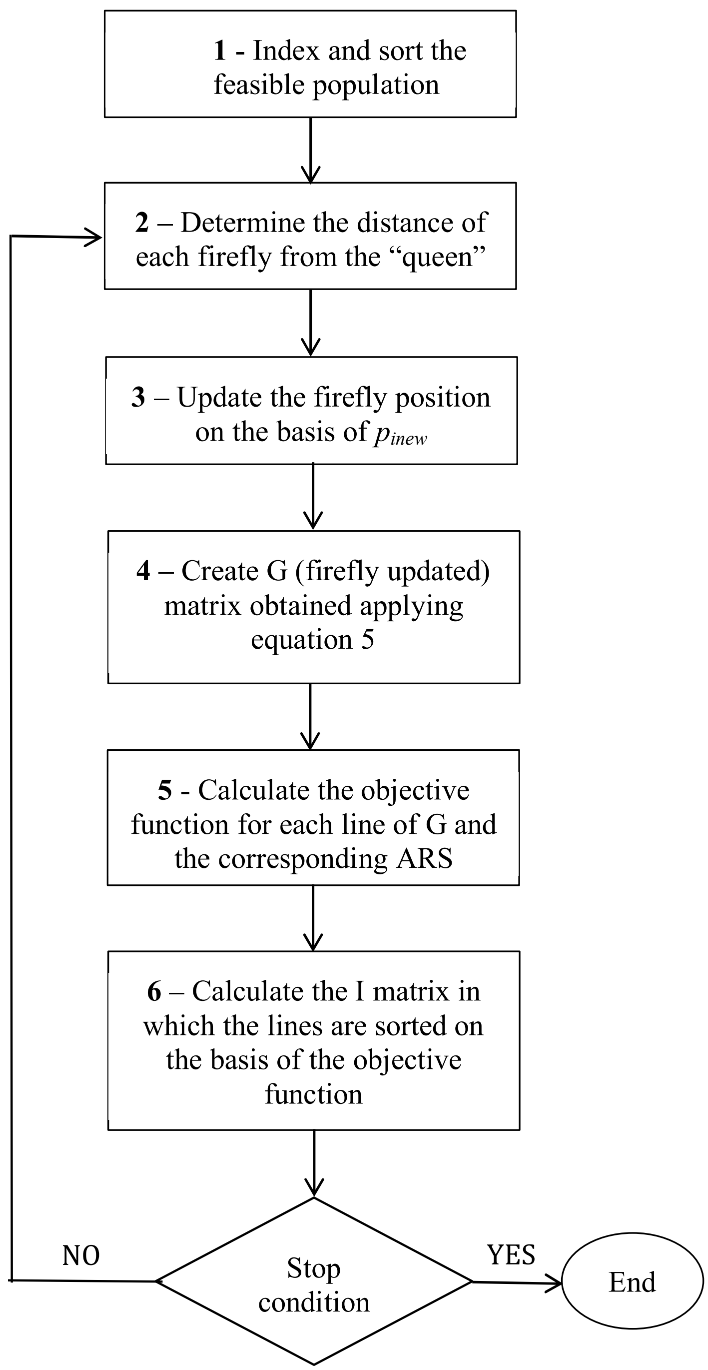

Fig. 2. Flow chart of FA optimization procedure

The fifth step computes the objective function for each line of the *G* matrix and the corresponding *ARS*.

Finally, in the **sixth step** a matrix $I=\begin{pmatrix} i_{11} & \cdots & i_{1j} \\ \vdots & \vdots & \vdots \\ i_{2z1} & \dots & i_{2zj} \end{pmatrix}$ sorted on the basis of the objective function, is created. The first line represents the new FQ that can or cannot coincide with the old one.
The software memorizes the sequence of the cuts and the procedure is repeated from step 2 until the stop condition (maximum number of iteration) is reached.

2.3 Objective function

In the proposed approach, the objective function (FO) is the minimization of the material handling cost:

$$Min\ (FO) = \sum_i \sum_j (f_{ij} c_{ij}) d_{ij}, \tag{6}$$

where f_{ij} is the material flow between the departments *i* and *j*, c_{ij} is the unit cost (the cost to move one unit load one distance from department *i* to department *j*) and d_{ij} is the distance between the centres of departments using the Manhattan distance. For each department, a specific aspect ratio is required, for instance in order to optimize the location of the machines inside. Let *h* and *w* be the two dimensions of the rectangle, the aspect ratio of the department *j* is defined as:

$$\gamma_j = \frac{max\{h_j, w_j\}}{min\{h_j, w_j\}}. \tag{7}$$

The degree of the aspect ratio satisfaction linearly decreases from an optimal to a minimum value if it is included in a given range, otherwise it drastically drops to zero (unfeasible layout). The simplest shape of such score function is given in (Aiello et al., 2006) where the upper limit was modified according to the instance of Armur and Buffa (1963) in which the maximum value of the range is 4. Moreover, in our approach a specific aspect ratio function could be associated to each department in order to consider the real industrial cases in which the aspect ratio requested could not be the same for all the departments, due to their different uses.

3. Numerical example

In this section the solution of unequal area FLPs is given following the layout solution representation and the layout arrangement procedure described in section 2. In order to validate the proposed algorithm, we consider the instance from Armour and Buffa (1963) to undertake experiments and comparisons. The authors give a specification of the problem set, departmental areas, product flows between departments and material handling costs respectively in tables 1, 2 ,3 and 4.

Table 1
Specification of the problem set

Problem name	Floor space dimension	Department requirements	Data reference
AB20	30.0 x 20.0	ARS $_{max}$=4	Armour and Buffa, 1963

Table 2
Department areas

Department	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Areas	27	18	27	18	18	18	9	9	9	24	60	42	18	24	27	75	64	41	27	45

Table 3
Product flows between departments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	120	80	0	0	0	0	0	0	40	80	0	0	80	0	0	0	0	0	0
2	120	0	80	1,630	30	0	930	0	80	90	0	0	0	0	0	0	0	0	460	0
3	80	80	0	0	0	130	0	0	210	260	0	0	0	870	0	0	0	0	910	0
4	0	1,630	0	0	60	380	500	0	130	0	0	70	0	0	0	0	0	100	1,050	0
5	0	30	0	60	0	0	150	90	0	60	0	0	0	0	90	0	0	0	0	0
6	0	0	130	380	0	0	410	0	0	0	0	30	0	0	0	0	0	70	0	0
7	0	930	0	500	150	410	0	1,600	0	110	0	0	0	60	0	0	0	110	0	250
8	0	0	0	0	90	0	1,600	0	0	0	0	0	40	0	0	0	0	0	500	2,230
9	0	80	210	130	0	0	0	0	0	0	0	0	0	500	0	0	500	0	0	0
10	40	90	260	0	60	0	110	0	0	0	30	800	0	1,240	160	0	0	0	350	0
11	80	0	0	0	0	0	0	0	0	30	0	150	0	200	80	1,500	350	90	0	0
12	0	0	0	70	0	30	0	0	0	800	150	0	0	0	110	0	1,000	0	560	0
13	0	0	0	0	0	0	0	40	0	0	0	0	0	500	40	500	0	40	0	0
14	80	0	870	0	0	0	60	0	500	1,240	200	0	500	0	650	0	0	60	0	0
15	0	0	0	0	90	0	0	0	0	160	80	110	40	650	0	0	350	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1,500	0	500	0	0	0	1,000	0	0	0
17	0	0	0	0	0	0	0	0	500	0	350	1,000	0	0	350	1,000	0	0	500	0
18	0	0	0	100	0	70	110	0	0	0	90	0	0	60	0	0	0	0	320	0
19	0	460	910	1,050	0	0	0	500	0	350	0	560	0	0	0	0	500	320	0	0
20	0	0	0	0	0	0	250	2,230	0	0	0	0	0	0	0	0	0	0	0	0

Table 4
Material handling costs

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0.015	0.015	0	0	0	0	0	0	0.026	0.014	0	0	0.015	0	0	0	0	0	0
2	0.015	0	0.012	0.015	0.026	0	0.015	0	0.015	0.015	0	0	0	0	0	0	0	0	0.015	0
3	0.015	0.012	0	0	0	0.017	0	0	0.015	0.015	0	0	0	0.015	0	0	0	0	0.015	0
4	0	0.015	0	0	0.018	0.015	0.015	0	0.018	0	0	0.020	0	0	0	0	0	0.015	0.015	0
5	0	0.026	0	0.018	0	0	0.015	0.015	0	0.026	0	0	0	0	0.015	0	0	0	0	0
6	0	0	0.017	0.015	0	0	0.015	0	0	0	0	0.015	0	0	0	0	0	0.015	0	0
7	0	0.015	0	0.015	0.015	0.015	0	0.015	0	0.017	0	0	0	0.016	0	0	0	0.015	0	0.015
8	0	0	0	0	0.015	0	0.015	0	0	0	0	0	0.015	0	0	0	0	0	0.015	0.015
9	0	0.015	0.015	0.018	0	0	0	0	0	0	0	0	0	0.015	0	0	0.015	0	0	0
10	0.026	0.015	0.015	0	0.026	0	0.017	0	0	0	0.012	0.015	0	0.015	0.012	0	0	0	0.015	0
11	0.014	0	0	0	0	0	0	0	0	0.012	0	0.015	0	0.015	0.012	0.015	0	0	0.015	0
12	0	0	0	0.020	0	0.015	0	0	0	0.015	0.015	0	0	0	0.015	0	0.015	0	0.015	0
13	0	0	0	0	0	0	0	0.015	0	0	0	0	0	0.016	0.026	0.012	0	0	0	0
14	0.015	0	0.015	0	0	0	0.016	0	0.015	0.015	0.015	0	0.016	0	0.015	0	0	0.015	0	0
15	0	0	0	0	0.015	0	0	0	0	0.012	0.012	0.015	0.026	0.015	0	0	0.015	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0.012	0	0	0	0.012	0	0	0
17	0	0	0	0	0	0	0	0	0.015	0	0	0.015	0	0	0.015	0.012	0	0	0.015	0
18	0	0	0	0.015	0	0.015	0.015	0	0	0	0	0	0	0.015	0	0	0	0	0.015	0
19	0	0.015	0.015	0.015	0	0	0	0.015	0	0.015	0.015	0.015	0	0	0	0	0.015	0.015	0	0
20	0	0	0	0	0	0	0.015	0.015	0	0	0	0	0	0	0	0	0	0	0	0

The algorithm is coded using Matlab software and simulations were conducted on an INTEL Core i7 (3.2 GHz) workstation with 16 Gb RAM. Table 5 shows the obtained non- dominated (Pareto) solutions in terms of objective function and *ARS* calculated as the mean value of the *ARS* of each department.

Table 5
Summary of the best solutions obtained

B vector	Material Handling cost	ARS
20-1-6-5-8-7-18-11-4-2-3-19-10-14-9-17-12-16-15-13	3228.89	0.7662
11-20-6-18-16-17-9-5-15-8-7-1-2-4-13-12-19-14-3-10	3391.90	0.8821
11-16-17-20-6-18-9-5-15-8-7-1-2-12-4-13-19-3-14-10	3410.62	0.8861
11-16-17-20-18-6-9-8-7-5-14-15-13-3-1-4-10-2-12-19	3525.49	0.9294

The block layouts of the optimal solutions generated by means of the proposed algorithm are reported in the figures below (Fig. 3). The layout reported to the left shows the position of the departments, whereas the one on the right highlights the *ASR* of each department. A scale of grey was used to differentiate the value of the aspect ratio. In particular, the lighter is the grey scale the nearest is the aspect ratio to the optimal value (*ARS*=1.5).

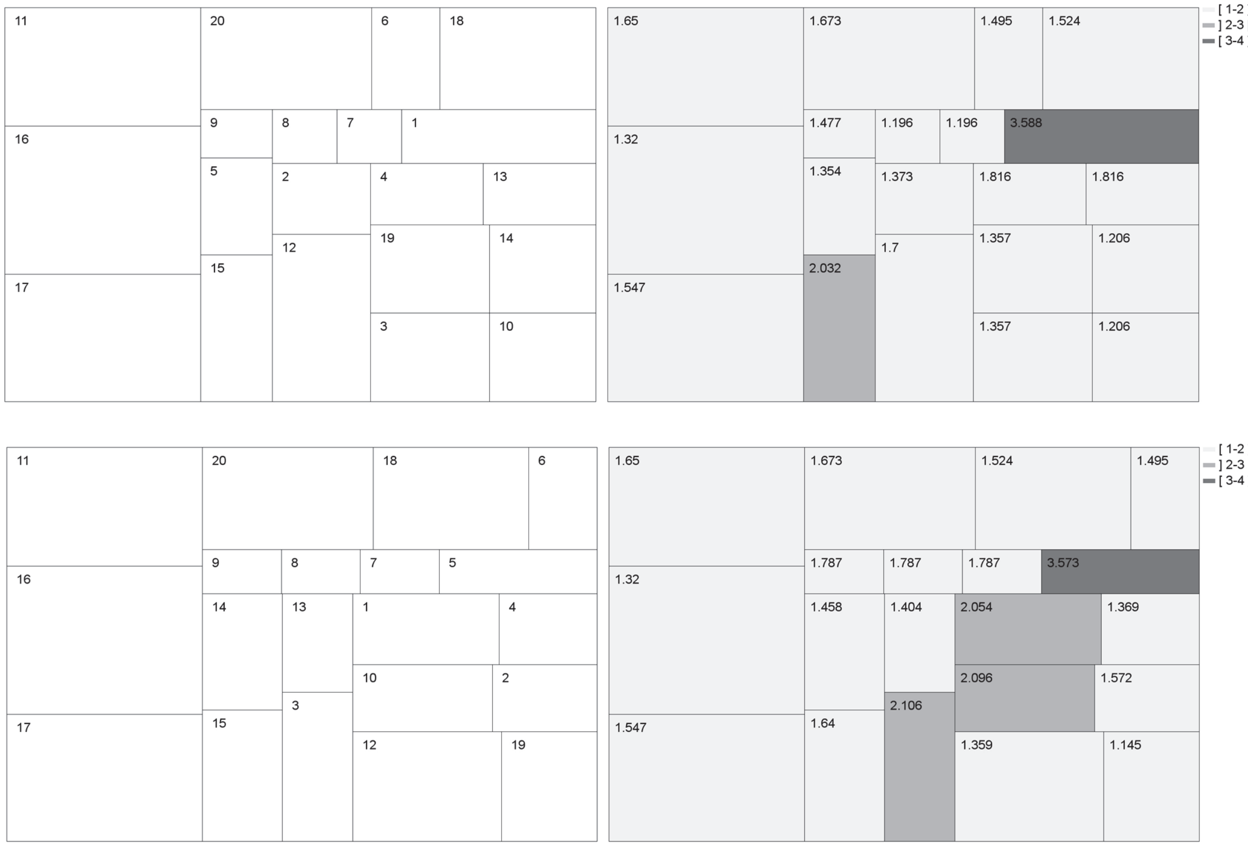

Fig. 3. Block layouts corresponding to the optimal solutions obtained

4. Benchmarking procedure and performance analysis

To evaluate the performance of the proposed approach, the results obtained were compared to the results achieved in the previous studies. In particular, three different approaches are used for the comparison: metaheuristic, mathematical programming and hybrid approaches (Kang & Chae, 2017). The overall comparison between the best solution obtained in this research and the best-known best solution provided by the previous studies are reported in table 6. It shows that the present approach is quite robust in terms of solution quality: it determined the best-known solution in terms of reduction of material handling cost maintaining a good degree of aspect ratio satisfaction. In particular, the comparison has been made in terms of a percentage reduction of the material handling cost obtained in Armour and Buffa (1963).

Table 6
Comparison of results in terms of reduction of Material handling Cost

Problem name	Sholz et al., 2009	Komarundin &Wong 2010	Kulturen Konak &Konak 2011	Gongalves & Resende 2015	Chang& Ku 2013	Kang& Chae 2017	FA approach
AB20	-33.54%	-36.75%	-32.12%	-36.13%	-34.47%	-36.92%	-58.93%

The proposed algorithm outperforms the previous best solution reported in figure 4 (Kang& Chae, 2017). Moreover, calculating the *ARS* for the layout obtained by Kang & Chae (2017) it is possible to notice that the proposed approach allows to obtain a layout with the best aspect ratio (0.76). Using the figure reported by the authors, the *ARS* of the layout has been calculated with the function adopted in this approach and its value is approximately 0.70.

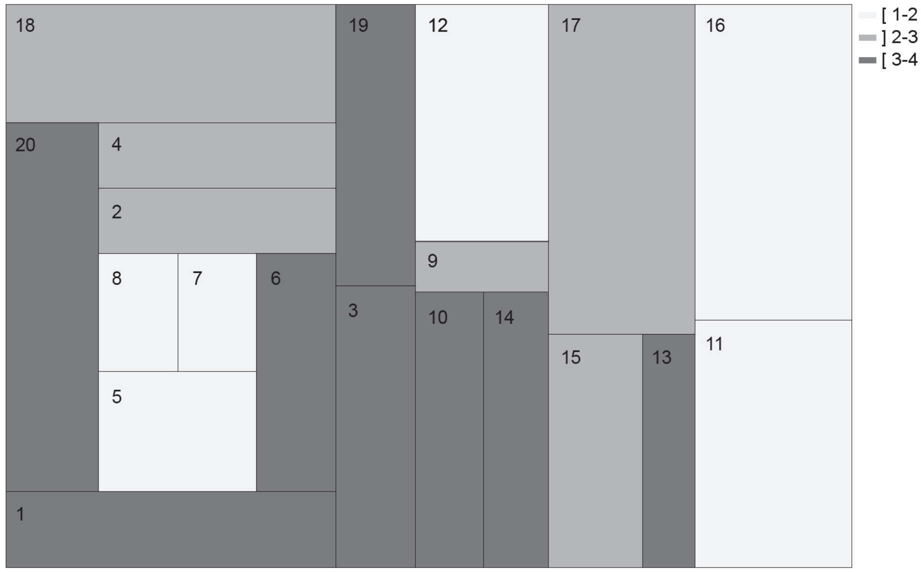

Fig. 4. Best layout obtained by Kang and Chae (2017)

Additionally, the evolution of Pareto-front obtained by taking into account the objective function and the aspect ratio has been determined. In particular, the non-dominated layout (i.e. the Pareto front) has been extracted in five different phases of the evolution procedure (namely 1000, 1500, 2000, 2500, 3000 iterations) as reported in figure 5. The results show that in the initial steps of the evolution, the Pareto fronts are very close and they could even overlap with each other. As the population evolves, however, better solutions are generated and the frontier moves to the upper right corner, in a more evident manner.

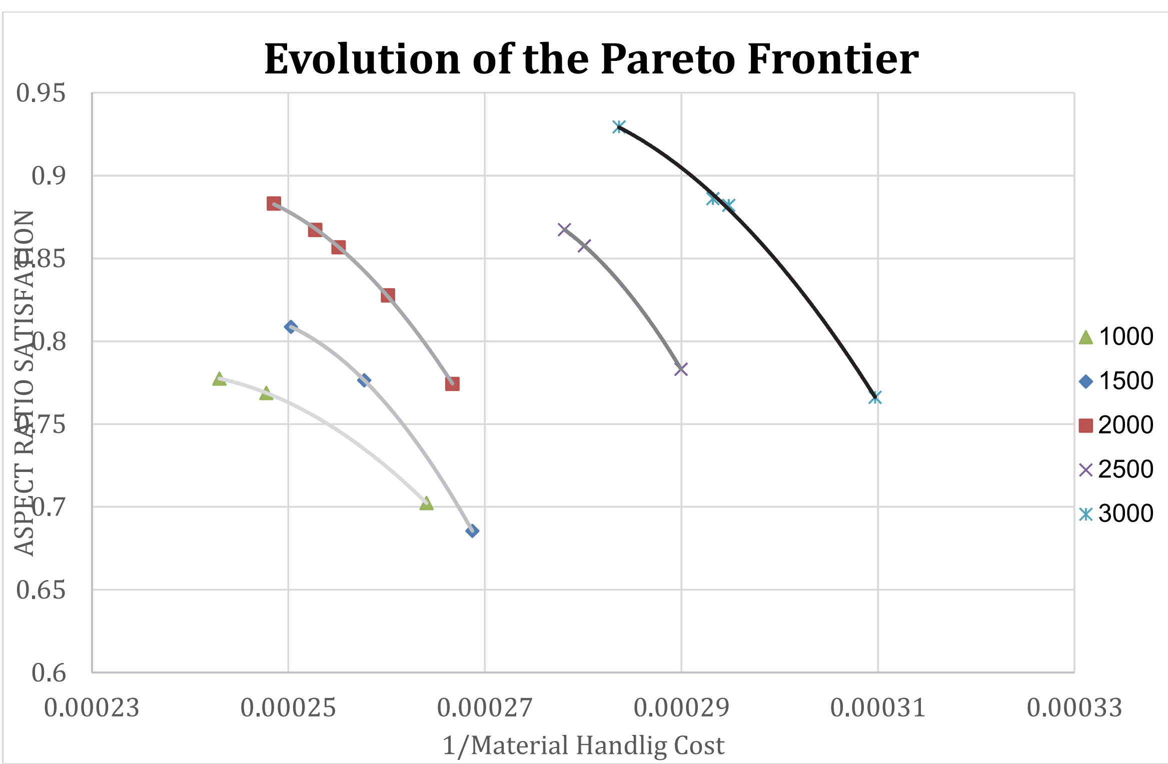

Fig. 5. Evolution progress of the Pareto front

5. Conclusions

The UA-FLP is a NP-hard optimization problem, which still involves designers and researchers in finding efficient and feasible solutions as the recent literature confirms. The development of innovative solution procedure is nowadays frequently considered in order to improve the effectiveness of the traditional approaches. This study has been conducted to propose an advanced method based on the firefly algorithm to solve the plant layout problems encountered in the industrial context. The problem is perceived in association with its multi-dimensional aspects, taking into account both material handling costs and department shapes. The benefits of the proposed method have emerged in the comparison with referenced results. Computational results show, in fact, that the proposed algorithm is robust because it determines the best-known solution to the problem set presented. Further improvements of the proposed methodology will include the comparison of the results obtained using more instances and the analysis of the effects of different parameters of the FA on the final solution for measuring the performance of the proposed approach on UA-FLPs. Moreover, the development of a methodology to treat the decision process considering the uncertainty related to the material flows between departments could be investigated using the fuzzy theory.

References

Aiello, G., Enea, M., & Galante, G. (2006). A multi-objective approach to facility layout problem by genetic search algorithm and Electre method. *Robotics and Computer-Integrated Manufacturing*, *22*(5-6), 447-455.

Aiello, G., La Scalia, G., & Enea, M. (2012). A multi-objective genetic algorithm for the facility layout problem based upon slicing structure encoding. *Expert Systems with Applications*, *39*(12), 10352–10358.

Aiello, G., La Scalia, G., & Enea, M. (2013). A non dominated ranking multi objective genetic algorithm and electre method for unequal area facility layout problems. *Expert Systems with Applications*, *40* (12), 4812–4819.

Armour, G.C., & Buffa, E.S., (1963). A heuristic algorithm and simulation approach to relative location of facilities. *Management Science*, *9*(2), 294–309.

Chang, M.S., & Ku, T.C. (2013). A slicing tree representation and QCP-model- based heuristic algorithm for the unequal-area block facility layout problem. *Mathematical Problems in Engineering*, 1–19.

Garey, M.R., & Johnson, D.S. (1979). *Computers and intractability: A guide to the theory of NP-completeness*. W.H. Freeman, New York.

Gonçalves, J.F., & Resende, M.G.C. (2015). A biased random-key genetic algorithm for the unequal area facility layout problem. *European Journal of Operational Research*, *246*(1), 86–107.

Kang, S., & Chae, J. (2017). Harmony search for the layout design of unequal area facility. *Experts systems with applications,* 79, 269-281.

Komarudin, Wong, K.Y. (2010). Applying ant system for solving unequal area facility layout problems. *European Journal of Operational Research*, *202*(3), 730–746.

Konak, A., Kulturel-Konak, S., Norman, B.A., & Smith, A.E. (2006). A new mixed integer programming formulation for facility layout design using flexible bays. *Operations Research Letters*, *34*(6), 660 – 672.

Kulturel-Konak, S., & Konak, A. (2011). A new relaxed flexible bay structure representation and particle swarm optimization for the unequal area facility layout problem. *Engineering Optimization*, *43*(12), 1263–1287.

McKendall Jr, A.R., & Jaramillo, J.R., (2006). A tabu search heuristic for the dynamic space allocation problem. *Computers and Operations Research*, *33*(3), 768–789.

Palomo-Romero, J.M., Salas-Morera, L., & Garcıa-Hernandez, L., (2017). An island model genetic algorithm for unequal area facility layout problems. *Experts Systems with Applications*, *68*, 151–162.

Şahin, R., Ertoğral, K., & Türkbey, O. (2010). A simulated annealing heuristic for the dynamic layout problem with budget constraint. *Computers and Industrial Engineering*, *59*(2), 308–313.

Scholz, D., Petrick, A., & Domschke, W. (2009). Stats: A slicing tree and tabu search based heuristic for the unequal area facility layout problem. *European Journal of Operational Research*, *197*(1), 166–178.

Scholz, D., Jaehn, F., & Junker, A. (2010). Extensions to STaTS for practical applications of the facility layout problem. *European Journal of Operational Research*, 204(3), 463–472.

Sönmez, Ö. E., & Baray, Ş. A. (2013). Application of firefly metaheuristic algorithm for the single row facility layout problem. In *Advances in Sustainable and Competitive Manufacturing Systems* (pp. 1165-1172). Springer, Heidelberg.

Tam, K.Y. (1992). Genetic algorithms, function optimization, and facility layout design. *European Journal of Operational Research*, 63(2), 322 – 346.

Tate, D.M. & Smith, A.E. (1995). Unequal area facility layout by genetic search. *IIE Transactions*, *27*(4), 465– 472.

Tavakkoli-Moghaddam, R., Torabi, N., & Ghaseminejad, A. (2015). A quaternion firefly algorithm to solve a multi-row facility layout problem. *International Journal of Engineering, Transactions B: Applications*, *28*(11), 1605-1613.

Ulutas, B.H., & Kulturel-Konak, S. (2012). An artificial immune system based algorithm to solve unequal area facility layout problem. *Expert Systems with Applications*, *39*(5), 5384–5395.

Yang, X.S. (2010). Firefly algorithm, stochastic test functions and design optimisation. *International Journal of Bio-Inspired Computation*, *2*(2), 78–84.