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Manufacturing runtime problem with an expedited fabrication rate, random failures, and scrap

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A B S T R A C T

When operating in highly competitive business environments, contemporary manufacturing firms must persistently find ways to fulfill timely orders with quality ensured merchandise, manage the unanticipated fabrication disruptions, and minimize total operating expenses. To address the aforementioned concerns, this study explores the optimal runtime decision for a manufacturing system featuring an expedited fabrication rate, random equipment failures, and scrap. Specifically, the proposed study considers an expedited rate that is linked to higher setup and unit costs. The fabrication process is subject to random failure and scrap rates. The failure instance follows a Poisson distribution, is repaired right away, and the fabrication of interrupted batch resumes when the equipment is restored. The defective goods are identified and scrapped. Mathematical modeling and optimization method are used to find the total system cost and the optimal runtime of the problem. The applicability and sensitivity analyses of research outcome are illustrated through a numerical example. Diverse critical information regarding the individual/joint impacts of variations in stochastic time-to-failure, expedited rate, and random scrap on the optimal runtime decision, total system expenses, different cost components, and machine utilization, can now be revealed to assist in in-depth problem analyses and decision makings.

1. Introduction

A manufacturing runtime problem with an expedited fabrication rate, random failures, and scrap is investigated in this study. Taft (1918) is believed to be the first who proposed a mathematical approach to calculate the most economical production lot (or called the economic production quantity (EPQ) model), by balancing costs of setup and holding to decide the lot size per cycle which minimizes total relevant expenses. A perfect manufacturing process with finite fabrication rate is assumed in the original EPQ model. But, in real production environments, due to varied unexpected situations, fabrication of defective/scrap goods and machine failure instances are inevitable. Literatures on production inventory systems with defective/scrap items are surveyed as follows: Schwaller (1988) examined three separate inventory models; namely, the basic economic order quantity (EOQ) model, EOQ with non-instantaneous replenishment, and EOQ with backlogging, by incorporating the product screening cost into total inventory relevant costs. Effects of defective proportion and inspection cost on these models were

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investigated to obtain in-depth information on inventory policy making. Richter (1996) considered an EOQ model with the repairable and scrapped items using two preset variables n and m to control production and repair mode within a finite interval of time. The author started with the assumption of a constant scrap rate and cost minimum setup numbers to derive the minimum cost. Then, the author treated the optimal/ minimum cost as a function of scrap rate to explore the convex/concave relationships between the cost and the scrap rate, and further discussed the optimality on cost. Konstantaras et al. (2007) studied the imperfect EOQ/EPQ system with an in-house 100% product screening process. In the end of inspection, two options were proposed to handle the defective goods: (i) to sell them at a discount price to a secondary marketplace; and (ii) to repair them completely with an extra rework cost per item. For both cases, the perfect goods are transported to warehouse in equal-size shipments. Their objectives were to not only retain the product quality, but also to decide the optimal lot-size and frequency of shipments per order that kept the system cost at minimal. A numerical illustration with sensitivity analysis showed how their system works. Extra works (Porteus, 1986; So & Tang, 1995; Grosfeld-Nir & Gerchak, 2002; Giri & Chakraborty, 2011; Abubakar et al., 2017; Chiu et al., 2017; Shakoor et al., 2017; Sher et al., 2017; Chiu et al., 2018a,b; Gan et al., 2018; Imbachi et al., 2018; Pearce et al., 2018) focused on fabrication/inventory systems with different features of nonconforming goods and their consequent treatments.

Due to different unpredicted reasons in real manufacturing process, random machine failure is inevitable. Groenevelt et al. (1992) explored the optimal fabrication lot-size problem considering equipment failures along with two distinct controlling disciplines on breakdown corrections; namely, the no-resumption (NR) and the abort-resume (AR) disciplines. According to the NR-discipline, the fabrication of interrupted lot is not resumed after a failure occurs. While under the AR-discipline, if the current stock level falls below a preset threshold point, fabrication of the interrupted lot is resumed right away, when the equipment is fixed. Analyses, solution procedures, outcomes, and impacts of these disciplines on the optimal lot-size were separately carried out, illustrated, and discussed. Kuhn (1997) examined a dynamic batch size problem featuring exponential breakdowns. Two distinct scenarios were investigated. The first scenario assumed the setup is completely lost after a machine failed, and the second scenario assumed that the resuming setup cost is significantly lower than the standard setup expense, when the fabrication of interrupted batch is resumed. The author showed that under scenario one, if the production planner ignors the factor of machine failures the cost penalty will be remarkably higher than that of the classic EPQ model. Besides, the author recommended a conditional resumption idea for scenario two and suggested an approach by using dynamic programming for finding optimal lot-size solutions for both decisions for both scenarios. Dehayem Nodem et al. (2009) considered the production rate and repair/replacement decision makings for a fabrication system featuring random breakdowns. Consequent actions/ policies immediately after a breakdown instance include (i) machine is under repair, or (ii) an identical spare machine is used. Their objective was to choose the optimal manufacturing rate and the repair/replacement policy that keep the long-run system expenses at minimum. A semi-Markov decision model (SMDM) was employed to help first decide the best repair/replacement policy, then, based on this policy the production rate was determined. By the use of numerical methods, the authors showed the optimality conditions, and revealed that as the number of machine failures rises, the on-hand stock level must be adjusted higher to avoid shortage. Chakraborty et al. (2013) studied an EMQ system which is subject to stochastic failure, repair, and stock threshold level (STL). The fabrication rate was considered as a decision variable and it links to the machine failure rate. The authors allocate additional capacity to hedge against random breakdowns. According to general distributions of breakdown and repair time, the basic model was built and two computational algorithms were proposed to help solve the optimal fabrication and STL that keep the long-run system expenses at minimal um. Extra works (Berg et al., 1994; Giri & Dohi, 2005a; Dahane et al., 2012; Liu et al., 2017a; Muzamil et al., 2017; Vujosevic et al., 2017; Luong & Karim, 2017; Nobil et al., 2018; Mohammed et al., 2018; Zhang et al., 2018) were also carried out to study the effect of diverse aspects of failures and their corresponding actions on manufacturing systems.

To shorten manufacturing completion time to fulfill buyer's timely orders, production managers often expedite the fabrication rate. Arcelus and Srinivasan (1987) studied an EOQ- based system with several optimizing measures and different demands and prices. With the aim of making profit, the authors established decision rules to manage inventories of finished stocks, defined price using a markup rate of cost, and assumed demand as price-dependent function. Both order quantity and markup rate are decision variables, and optimization of their system was determined by three broadly used performance evaluators, which include profits, return on investment, and residual income. Balkhi and Benkherouf (1996) examined an inventory system featuring deteriorating products, time-varying demand and fabrication rates. A precise method was presented to derive the optimal stock refilling schedule for the proposed inventory system and the method was illustrated via a numerical example. Gharbi et al. (2006) investigated an unreliable multiproduct multi-machine fabrication system with adjustable fabrication rates and setup actions. Different setup times and costs are linked to each switching process, no matter it is a product- or machine-type of switch. Their objective was to minimize the overall operating costs by deciding the best fabrication rates and the optimal sequence of setups. The authors used the following approaches to solve this complex problem: (i) the stochastic optimal control theory, (ii) experimental design, (iii) discrete event simulation, and (iv) response surface method. Two different cases were studied: case one considered single machine unreliable system featuring exponential breakdown and repair time distribution, and case two considered five machines system featuring non-exponential breakdown and repair time distributions. The experimental results revealed that an extended Hedging Corridor policy gave better performance on these two cases. Numerical examples were provided to illustrate contribution of the paper. Zanoni et al. (2014) considered energy reduction in a two-stage fabrication system with controllable fabrication rates, wherein, a single product is first fabricated on an equipment, and then transported in batch shipments to the subsequent fabrication stage. In each stage, the finite fabrication rate is assumed to be adjustable, and this rate is linked to specific energy consumption based on the type of process involved. The purpose of their work was to propose a model in the production planning phase, to analyze the system and minimize its overall costs, including production, inventory, and energy costs. The research result showed that significantly savings were realized as compared to a production plan without considering energy consumption. Additional works (Khouja & Mehrez, 1994; Giri & Dohi, 2005b; Sana, 2010; Liu et al., 2017b; Bottani, et al., 2017; Chiu et al., 2018c,d; Ameen et al., 2018) were also conducted to address various issues and influences of variable fabrication rates on manufacturing systems. As little attention has been paid to study the joint influences of stochastic failures, random scrap, and expedited fabrication rate on the manufacturing runtime decision, this work aims to link the gap.

2. The proposed manufacturing runtime problem

A manufacturing runtime problem with an expedited fabrication rate, random failures, and scrap is explored. Consider a manufacturing system is used to satisfy annual demand rate λ of a particular product. The production equipment is subject to stochastic failures which follows Poisson distribution with mean equal to β breakdowns per year. An abort/resume (A/R) stock control policy is used when a failure encounters. According to the A/R policy, malfunction equipment is under repair right away, and fabrication of the interrupted/unfinished lot will be resumed when repair task is successfully done. A constant repair time t_r is assumed in this study. To reduce cycle length of the batch fabrication plan, this study adopts an expedited rate. Let α_1 represent extra percentage of production rate and consequently, the speedy rate related parameters are defined as follows:

$$P_{1A} = (1+\alpha_1)P_1 \tag{1}$$

$$K_A = (1+\alpha_2)K \tag{2}$$

$$C_A = (1+\alpha_3)C \tag{3}$$

where P_{1A} is the speedy rate, K_A and C_A are speedy rate relevant setup and unit costs; and P_1, K, C, α_2, and α_3 denote the standard rate, standard setup and unit costs, and connecting variables between K_A and K, and between C_A and C, respectively. For instance, for $\alpha_1 = 0.3$, it represents that speedy manufacturing rate is 30% higher than standard rate; and $\alpha_3 = 0.15$ means unit cost is 15% greater than standard unit cost due to the expedited rate. The manufacturing process randomly fabricate x portion of scrap goods at a rate d_{1A} due to diverse unanticipated reasons. Shortages are not permitted, so $(P_{1A} - d_{1A} - \lambda) > 0$ must hold, where d_{1A} is as follows:

$$d_{1A} = xP_{1A} \tag{4}$$

Appendix A includes other notation used in this study. The following two cases must be explored separately because of the random failures:

2.1. *Case 1: A random failure taking place*

In case 1, time to failure $t < t_{1A}$ and the on-hand stock status in a cycle is illustrated in Fig. 1. It shows that the stock level at H_0 when a random failure takes place. After the equipment is repaired and restored, the stock level keeps piling up and arrives at H when manufacturing uptime ends. Then, stock depletes to empty before next cycle starts.

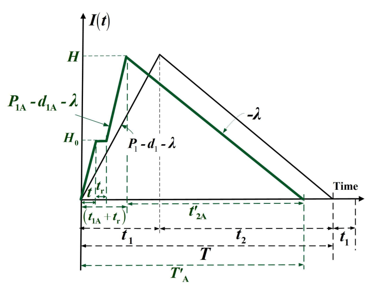

Fig. 1. On-hand stock status in the proposed system with expedited rate, random failures, and scrap (in green) as compared to that of a fabrication system with scrap (in black)

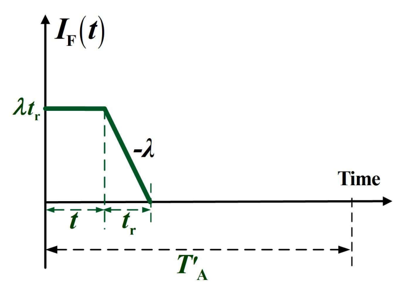

Fig. 2.The on-hand status of safety stocks in the proposed system with expedited rate, random failures, and scrap

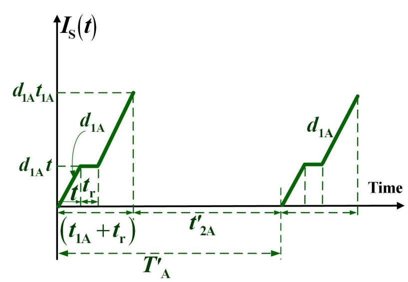

Fig. 3. The on-hand status of scraps in the proposed system with expedited rate, random failures, and scrap

The on-hand status for safety stock is exhibited in Fig. 2. It shows that safety stock being used to satisfy demand during t_r, right after a failure taking place. Fig. 3 depicts the on-hand status of scrap in the proposed system with expedited rate, random failures, and scrap. From problem statement and by observing Fig. 1 to Fig. 3, one can obtain the following basic relationships:

$$t_{1A} = \frac{Q}{P_{1A}} = \frac{H}{P_{1A} - d_{1A} - \lambda} \tag{5}$$

$$t'_{2A} = \frac{H}{\lambda} \tag{6}$$

$$T'_A = t_{1A} + t_r + t'_{2A} \tag{7}$$

$$H_0 = (P_{1A} - d_{1A} - \lambda)t \tag{8}$$

$$H = (P_{1A} - d_{1A} - \lambda)t_{1A} \tag{9}$$

$$d_{1A}t_{1A} = xP_{1A}t_{1A} = xQ \tag{10}$$

In the case of a random failure taking place, total cost per cycle $TC(t_{1A})_1$ comprises variable manufacturing, setup, and disposal costs, fixed equipment repair cost, safety stocks' variable, holding, and delivery costs, and holding costs for perfect and scrap items during manufacturing uptime and depletion time. Hence, $TC(t_{1A})_1$ is

$$\begin{aligned} TC(t_{1A})_1 &= C_A Q + K_A + C_S xQ + M + \left[C_1 \lambda t_r + h_3(\lambda t_r)(t + t_r/2) + C_T \lambda t_r\right] \\ &+ h\left[\frac{H + d_{1A} t_{1A}}{2}(t_{1A}) + (H_0 t_r) + (d_{1A} t) t_r + \frac{H}{2}(t'_{2A})\right] \end{aligned} \tag{11}$$

By applying $E[x]$ to cope with random scrap rate and substituting Eq. (1) to Eq. (10) in Eq. (11), the following $E[TC(t_{1A})]_1$ can be derived:

$$\begin{aligned} E\left[TC(t_{1A})\right]_1 &= \left[(1+\alpha_3)C\right](1+\alpha_1)P_1 t_{1A} + (1+\alpha_2)K + C_S E[x](1+\alpha_1)P_1 t_{1A} \\ &+ M + C_1 \lambda g + h_3 \lambda g\left(t + \frac{g}{2}\right) + C_T \lambda g + h\left[(1+\alpha_1)P_1 - \lambda\right] tg \\ &+ \frac{h\left[(1+\alpha_1)P_1 t_{1A}\right]^2}{2}\left\{\frac{\left[1 - E[x]\right]^2}{\lambda} + \frac{\left[2E[x] - 1\right]}{(1+\alpha_1)P_1}\right\} \end{aligned} \tag{12}$$

The expected cycle time $E[T'_A]$, in the case of a random failure taking place, can be derived as follows:

$$E[T'_A] = \frac{Q\left[1 - E[x]\right]}{\lambda} = \frac{t_{1A} P_{1A}\left[1 - E[x]\right]}{\lambda} \tag{13}$$

2.1. *Case 2: No random failures taking place*

In case 2, time to failure $t > t_{1A}$ and the on-hand stock status in a cycle is illustrated in Fig. 4. It shows that the stock level reaches H when manufacturing uptime finishes. Then, it depletes to empty before next cycle starts.

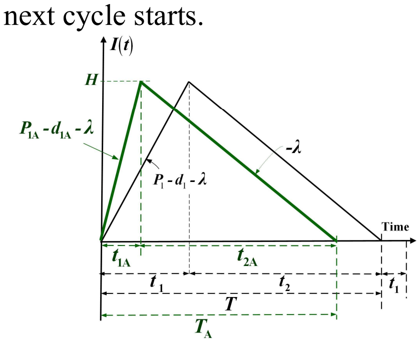

Fig. 4. On-hand stock status in the proposed system with expedited rate and scrap, but without failures taking place (in green) as compared to that of a manufacturing system with scrap (in black)

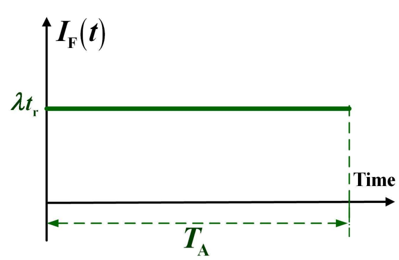

Fig. 5. The on-hand status of safety stocks in the proposed system with scrap and an expedited rate, but without failures taking place

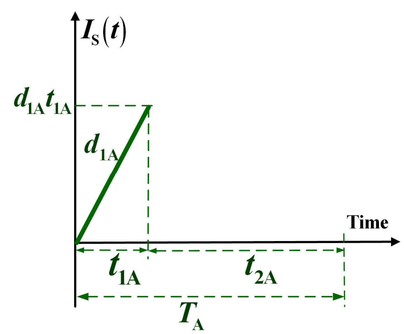

Fig. 6. The on-hand status of safety stocks in the proposed system with scrap and expedited rate, but without failures taking place

Fig. 5 displays the on-hand status for safety stock. It shows that safety stocks have not been used because no machine failures take place. Fig. 6 illustrates the on-hand status of scrap in the proposed system with expedited rate and scrap, but without failures taking place. From problem statement and also by observing Fig. 4 to Fig. 6, one can obtain the following basic relationships:

$$t_{1A} = \frac{Q}{P_{1A}} = \frac{H}{P_{1A} - d_{1A} - \lambda} \tag{14}$$

$$t_{2A} = \frac{H}{\lambda} \tag{15}$$

$$H = \left(P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda\right)t_{1\mathrm{A}} \tag{16}$$

$$T_{\mathrm{A}} = t_{1\mathrm{A}} + t_{2\mathrm{A}} \tag{17}$$

In the case of no machine failures, total cost per cycle $TC(t_{1\mathrm{A}})_2$ comprises variable manufacturing, setup, and disposal costs, safety stocks' holding cost, and holding costs for perfect and scrap items during manufacturing uptime and depletion time. Hence, $TC(t_{1\mathrm{A}})_2$ is

$$TC\left(t_{1\mathrm{A}}\right)_2 = C_{\mathrm{A}}Q + K_{\mathrm{A}} + C_{\mathrm{S}}xQ + h_3\left(\lambda t_r\right)T_{\mathrm{A}} + h\left[\frac{H + d_{1\mathrm{A}}t_{1\mathrm{A}}}{2}\left(t_{1\mathrm{A}}\right) + \frac{H}{2}\left(t_{2\mathrm{A}}\right)\right] \tag{18}$$

By applying $E[x]$ to handle with random scrap rate and substituting Eqs. (1-4) and Eqs. (14-17) in Eq. (18), the following $E[TC(t_{1\mathrm{A}})]_2$ can be derived:

$$\begin{aligned} E\left[TC\left(t_{1\mathrm{A}}\right)\right]_2 =& \left[\left(1+\alpha_3\right)C\right]\left(1+\alpha_1\right)P_1t_{1\mathrm{A}} + \left(1+\alpha_2\right)K + C_{\mathrm{S}}E\left[x\right]\left(1+\alpha_1\right)P_1t_{1\mathrm{A}} \\ &+ h_3\lambda gT_{\mathrm{A}} + \frac{h\left[\left(1+\alpha_1\right)P_1t_{1\mathrm{A}}\right]^2}{2}\left\{\frac{\left[1-E\left[x\right]\right]^2}{\lambda} + \frac{\left[2E\left[x\right]-1\right]}{\left(1+\alpha_1\right)P_1}\right\} \end{aligned} \tag{19}$$

3. Solving the proposed runtime problem

Because equipment failure is assumed to be a random variable which follows the Poisson distributed with mean equal to β per year, hence, time-to-failure t obeys an Exponential distribution with $f(t) = \beta e^{-\beta t}$ and $\mathrm{F}(t) = (1 - e^{-\beta t})$ as its density and cumulative density functions, respectively. Therefore, the expected annual system cost $E[TCU(t_{1\mathrm{A}})]$ can be determined as follows:

$$E\left[TCU\left(t_{1\mathrm{A}}\right)\right] = \frac{\left\{\int_0^{t_{1\mathrm{A}}} E\left[TC\left(t_{1\mathrm{A}}\right)\right]_1 \cdot f\left(t\right)dt + \int_{t_{1\mathrm{A}}}^{\infty} E\left[TC\left(t_{1\mathrm{A}}\right)\right]_2 \cdot f\left(t\right)dt\right\}}{E[\boldsymbol{T}_{\mathbf{A}}]} \tag{20}$$

where $E[\boldsymbol{T}_{\mathbf{A}}]$ is

$$E[\boldsymbol{T}_{\mathbf{A}}] = \int_0^{t_{1\mathrm{A}}} E[T'_{\mathrm{A}}]\cdot f\left(t\right)dt + \int_{t_{1\mathrm{A}}}^{\infty} E[T_{\mathrm{A}}]\cdot f\left(t\right)dt = \frac{t_{1\mathrm{A}}\left[\left(1+\alpha_1\right)P_1\right]\left[1-E\left[x\right]\right]}{\lambda} \tag{21}$$

The following $E[TCU(t_{1\mathrm{A}})]$ can be gained by substituting Eq. (12), Eq. (19), and Eq. (21) in Eq. (20) (for detailed derivations please see Appendix B):

$$E\left[TCU\left(t_{1\mathrm{A}}\right)\right] = \frac{\lambda}{\left[1-E\left[x\right]\right]}\left\{\begin{array}{l} \dfrac{Z_1}{t_{1\mathrm{A}}} + \dfrac{Z_2}{t_{1\mathrm{A}}} + Z_3e^{-\beta t_{1\mathrm{A}}} + \dfrac{Z_4e^{-\beta t_{1\mathrm{A}}}}{t_{1A}} + \left[\left(1+\alpha_3\right)C\right] + C_{\mathrm{S}}E\left[x\right] \\ + t_{1\mathrm{A}}\left\{+\dfrac{h(1+\alpha_1)P_1}{2}\left[\dfrac{\left[1-E\left[x\right]\right]^2}{\lambda} + \dfrac{\left[2E\left[x\right]-1\right]}{\left(1+\alpha_1\right)P_1}\right]\right\} \\ + h_3g\left[1-E\left[x\right]\right]\left(e^{-\beta t_{1\mathrm{A}}}\right) \end{array}\right\} \tag{22}$$

3.1 Convexity of $E[TCU(t_{1\mathrm{A}})]$

We first apply the first- and second-derivatives of $E[TCU(t_{1\mathrm{A}})]$ and gain the following:

$$\frac{dE\left[TCU\left(t_{1\mathrm{A}}\right)\right]}{dt_{1\mathrm{A}}} = \frac{\lambda}{\left[1-E\left[x\right]\right]}\left\{\begin{array}{l} \dfrac{-Z_1}{t_{1\mathrm{A}}^2} - \dfrac{Z_2}{t_{1\mathrm{A}}^2} - \beta Z_3e^{-\beta t_{1\mathrm{A}}} - \dfrac{Z_4e^{-\beta t_{1\mathrm{A}}}}{t_{1\mathrm{A}}^2} - \dfrac{Z_4\beta e^{-\beta t_{1\mathrm{A}}}}{t_{1\mathrm{A}}} \\ + \left\{\dfrac{h(1+\alpha_1)P_1}{2}\left[\dfrac{\left[1-E\left[x\right]\right]^2}{\lambda} + \dfrac{\left[2E\left[x\right]-1\right]}{\left(1+\alpha_1\right)P_1}\right]\right\} \\ -\beta h_3g\left(1-E\left[x\right]\right)\left(e^{-\beta t_{1\mathrm{A}}}\right) \end{array}\right\} \tag{23}$$

and

$$\frac{d^2E\left[TCU\left(t_{1A}\right)\right]}{dt_{1A}^2}=\frac{\lambda}{\left[1-E\left[x\right]\right]}\left\{\begin{array}{l}\frac{2Z_1}{t_{1A}^3}+\frac{2Z_2}{t_{1A}^3}+\beta^2Z_3e^{-\beta t_{1A}}+\frac{\beta^2Z_4e^{-\beta t_{1A}}}{t_{1A}}+\frac{2\beta Z_4e^{-\beta t_{1A}}}{t_{1A}^2}\\+\frac{2Z_4e^{-\beta t_{1A}}}{t_{1A}^3}+\beta^2h_3g\left(1-E\left[x\right]\right)\left(e^{-\beta t_{1A}}\right)\end{array}\right\} \tag{24}$$

From Eq. (24), because the first term λ /(1–$E[x]$) on RHS (right-hand side) is positive, it follows that $E[TCU(t_{1A})]$ is convex if the second term on RHS of Eq. (24) is also positive. That is if Eq. (25) holds.

$$\gamma\left(t_{1A}\right)=\frac{2\left(Z_1+Z_2+Z_4e^{-\beta t_{1A}}\right)}{\left(-t_{1A}^2\beta^2Z_3e^{-\beta t_{1A}}-t_{1A}^2\beta^2h_3g\left(1-E\left[x\right]\right)\left(e^{-\beta t_{1A}}\right)-t_{1A}\beta^2Z_4e^{-\beta t_{1A}}-2\beta Z_4e^{-\beta t_{1A}}\right)}>t_{1A}>0 \tag{25}$$

*3.2 Searching for t_{1A}**

We set the first-derivative of $E[TCU(t_{1A})]$ equal to zero to search for optimal runtime t_{1A}* under the condition that Eq. (25) holds.

$$\frac{\lambda}{\left[1-E\left[x\right]\right]}\left\{\begin{array}{l}\frac{-Z_1}{t_{1A}^2}-\frac{Z_2}{t_{1A}^2}-\beta Z_3e^{-\beta t_{1A}}-\frac{Z_4e^{-\beta t_{1A}}}{t_{1A}^2}-\frac{Z_4\beta e^{-\beta t_{1A}}}{t_{1A}}-\beta h_3g\left(1-E\left[x\right]\right)\left(e^{-\beta t_{1A}}\right)\\+\left\{\frac{h(1+\alpha_1)P_1}{2}\left[\frac{\left[1-E\left[x\right]\right]^2}{\lambda}+\frac{\left[2E\left[x\right]-1\right]}{\left(1+\alpha_1\right)P_1}\right]\right\}\end{array}\right\}=0 \tag{26}$$

or

$$\left\{t_{1A}^2\left\{\begin{array}{l}-\beta Z_3e^{-\beta t_{1A}}-\beta h_3g\left(1-E\left[x\right]\right)\left(e^{-\beta t_{1A}}\right)\\+\frac{h(1+\alpha_1)P_1}{2}\left[\frac{\left[1-E\left[x\right]\right]^2}{\lambda}+\frac{\left[2E\left[x\right]-1\right]}{\left(1+\alpha_1\right)P_1}\right]\end{array}\right\}+t_{1A}\left(-Z_4\beta e^{-\beta t_{1A}}\right)+\left(-Z_1-Z_2-Z_4e^{-\beta t_{1A}}\right)\right\}=0 \tag{27}$$

Let r_2, r_1, and r_0 stand for the following:

$$\begin{aligned}r_2&=\left\{-\beta Z_3e^{-\beta t_{1A}}-\beta h_3g\left(1-E\left[x\right]\right)\left(e^{-\beta t_{1A}}\right)+\frac{h(1+\alpha_1)P_1}{2}\left[\frac{\left[1-E\left[x\right]\right]^2}{\lambda}+\frac{\left[2E\left[x\right]-1\right]}{\left(1+\alpha_1\right)P_1}\right]\right\};\\r_1&=\left(-Z_4\beta e^{-\beta t_{1A}}\right);\\r_0&=\left(-Z_1-Z_2-Z_4e^{-\beta t_{1A}}\right)\end{aligned} \tag{28}$$

Eq. (28) becomes

$$r_2\left(t_{1A}\right)^2+r_1\left(t_{1A}\right)+r_0=0 \tag{29}$$

We can now apply Eq. (30) (i.e., the square roots solution procedure) to search for t_{1A}*:

$$t_{1A}{}^{*} = \frac{-r_1 \pm \sqrt{r_1^2 - 4r_2 r_0}}{2r_2} \tag{30}$$

$$t_{1A}^{*} = \frac{Z_4 \beta e^{-\beta t_{1A}} \pm \sqrt{\left(Z_4 \beta e^{-\beta t_{1A}}\right)^2 - 4\left\{\begin{array}{l} -\beta Z_3 e^{-\beta t_{1A}} - \beta h_3 g\left(1-E[x]\right)\left(e^{-\beta t_{1A}}\right) \\ + \dfrac{h(1+\alpha_1)P_1}{2}\left[\dfrac{\left[1-E[x]\right]^2}{\lambda} + \dfrac{\left[2E[x]-1\right]}{(1+\alpha_1)P_1}\right] \end{array}\right\}\left(-Z_1 - Z_2 - Z_4 e^{-\beta t_{1A}}\right)}}{2\left\{-\beta Z_3 e^{-\beta t_{1A}} - \beta h_3 g\left(1-E[x]\right)\left(e^{-\beta t_{1A}}\right) + \dfrac{h(1+\alpha_1)P_1}{2}\left[\dfrac{\left[1-E[x]\right]^2}{\lambda} + \dfrac{\left[2E[x]-1\right]}{(1+\alpha_1)P_1}\right]\right\}} \tag{31}$$

*3.2.1 Recursive algorithm for locating t_{1A}**

The cumulative density function of Exponential distribution $F(t_{1A}) = (1 - e^{-\beta t1A})$ and it is over the interval of [0, 1], so does its complement $e^{-\beta t1A}$. Furthermore, one can rearrange Eq. (27) as follows:

$$e^{-\beta t_{1A}} = \frac{t_{1A}^2\left\{\dfrac{h(1+\alpha_1)P_1}{2}\left[\dfrac{\left[1-E[x]\right]^2}{\lambda} + \dfrac{\left[2E[x]-1\right]}{(1+\alpha_1)P_1}\right]\right\} - Z_1 - Z_2}{t_{1A}^2\left[\beta Z_3 + \beta h_3 g\left(1-E[x]\right)\right] + t_{1A}\left(\beta Z_4\right) + Z_4} \tag{32}$$

The proposed recursive algorithm is as follows: Step (1): first set $e^{-\beta t1A} = 0$ and $e^{-\beta t1A} = 1$, use Eq. (32) to compute the upper and lower bounds for uptime t_{1A} (i.e., t_{1AU} and t_{1AL}). Step (2): use current t_{1AU} and t_{1AL} to calculate and update $e^{-\beta t1AU}$ and $e^{-\beta t1AL}$. Step (3): use current $e^{-\beta t1AU}$ and $e^{-\beta t1AL}$ to compute Eq. (29) again and update t_{1AU} and t_{1AL} for uptime t_{1A}. Step (4): verify whether or not $(t_{1AU} - t_{1AL}) = 0$, if it is true, then go to Step (5); otherwise, go to Step (2). Step (5): t_{1A}* is located (it is either t_{1AU} or t_{1AL}).

4. Numerical demonstration

To demonstrate applicability of the proposed manufacturing runtime problem, an example with the following assumption of system parameters (see Table 1) is considered:

Table 1
Assumptions of system parameters in the numerical demonstration

Parameters	λ	x	α_1	P_{1A}	P_1	α_2	K_A	K	α_3	β
	4000	20%	0.5	15000	10000	0.1	495	450	0.25	1
Parameters	C_A	C	C_S	M	g	h	h_3	C_T	C_1	
	2.5	2.0	0.3	2500	0.018	0.8	0.8	0.01	2.0	

Prior to solving t_{1A}* for the problem, the convexity of $E[TCU(t_{1A})]$ must be verified first. That is, to test whether $\gamma(t_{1A}) > t_{1A} > 0$ (i.e., Eq. (25)). We start with setting $e^{-\beta t1A} = 0$ and $e^{-\beta t1A} = 1$, then applying Eq. (31) to obtain $t_{1AU} = 0.5000$ and $t_{1AL} = 0.1130$. Finally, by computing Eq. (25) using these t_{1AU} and t_{1AL} values, we confirmed $\gamma(t_{1AU}) = 0.7466 > t_{1AU} > 0$ and $\gamma(t_{1AL}) = 0.2966 > t_{1AL} > 0$. So, the convexity of $E[TCU(t_{1A})]$ is proved. Furthermore, to demonstrate the proposed study is applicable for broader range of mean machine failure rates, extra convexity tests were carried out and results are exhibited in Table C-1 (Appendix C). A recursive algorithm (see subsection 3.2.1) is utilized to locate t_{1A}*. The detailed solution processes are shown in Table 2, where $t_{1A}^* = 0.2015$ and $E[TCU(t_{1A}^*)] = \$13,536$ are obtained.

The impact of differences in t_{1A} on diverse cost elements in $E[TCU(t_{1A})]$ are depicted in Fig. 7. It indicates that as uptime t_{1A} deviates from its optimal position (i.e., 0.2015), $E[TCU(t_{1A})]$ begins to boost; and as t_{1A} increases, holding cost goes up significantly and quality relevant cost raises accordingly; but the setup cost declines drastically. Fig. 8 exhibits the effect of variations in random scrap rate x on optimal manufacturing uptime t_{1A}*. It shows that optimal uptime t_{1A}* increases notably, as scrap rate x raises; and at x = 0.2 (as assumed in the example), t_{1A}* = 0.2015 years. The influence of variations in mean-time-to-failure $1/\beta$ together with various defective rates x on $E[TCU(t_{1A}^*)]$ is illustrated in Fig. 9.

Table 2
Solution processes of the proposed manufacturing runtime problem

Step no.	t_{1AU}	$e^{-\beta t_{1AU}}$	t_{1AL}	$e^{-\beta t_{1AL}}$	Difference between t_{1AU} and t_{1AL}	$E[TCU(t_{1AU})]$	$E[TCU(t_{1AL})]$
-	-	0	-	1	-	-	-
1	**0.5000**	0.6065	**0.1130**	0.8932	0.3870	\$14,189.93	\$13,787.81
2	0.2887	0.7493	0.1669	0.8463	0.1218	\$13,632.38	\$13,562.57
3	0.2310	0.7937	0.1886	0.8281	0.0424	\$13,550.13	\$13,539.65
4	0.2119	0.8090	0.1968	0.8214	0.0151	\$13,538.30	\$13,536.84
5	0.2053	0.8144	0.1998	0.8189	0.0055	\$13,536.68	\$13,536.48
6	0.2029	0.8164	0.2009	0.8180	0.0020	\$13,536.46	\$13,536.44
7	0.2020	0.8171	0.2013	0.8177	0.0007	\$13,536.43	\$13,536.43
8	0.2017	0.8173	0.2014	0.8176	0.0003	\$13,536.43	\$13,536.43
9	0.2016	0.8174	0.2015	0.8175	0.0001	\$13,536.43	\$13,536.43
10	**0.2015**	0.8175	**0.2015**	0.8175	**0.0000**	**\$13,536.43**	**\$13,536.43**

It specifies that as $1/\beta$ increases (which implies the occurrence of a machine failure is less likely), $E[TCU(t_{1A}^*)]$ declines significantly, especially when $1/\beta \geq 0.25$. It also shows that as scrap rate x goes up, $E[TCU(t_{1A}^*)]$ increases notably.

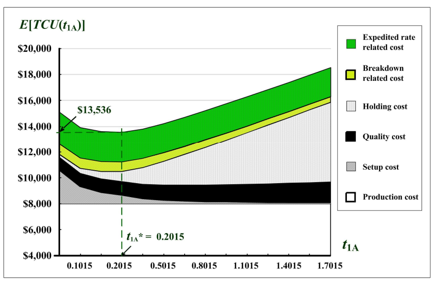

Fig. 7. The impact of differences in t_{1A} on diverse cost elements in $E[TCU(t_{1A})]$

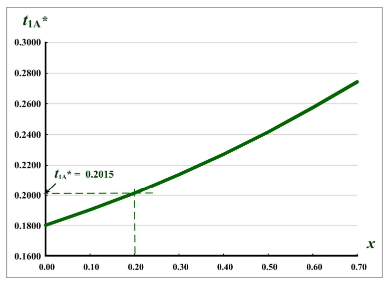

Fig. 8. The effect of variations in random scrap rate x on optimal manufacturing uptime t_{1A}*

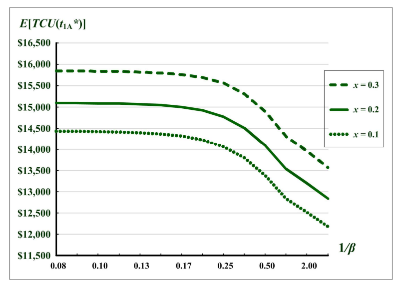

Fig. 9. The influence of variations in mean-time-to-failure $1/\beta$ together with various x on $E[TCU(t_{1A}^*)]$

Analytical results on cost elements in $E[TCU(t_{1A}^*)]$ is exhibited in Fig. 10. It reveals that 16.9% of $E[TCU(t_{1A}^*)]$ is related to the expedited rate option, 8.1% of system cost is concerned with product quality matters, and 5.6% is regarding the machine failure matter, etc. The impact of changes in expedited ratio P_{1A}/P_1 on $E[TCU(t_{1A}^*)]$ is shown in Fig. 11. It indicates that as P_{1A}/P_1 increases, $E[TCU(t_{1A}^*)]$ goes up significantly; and it reconfirms that when P_{1A}/P_1 = 1.5, $E[TCU(t_{1A}^*)]$ = \$13,536 (as assumed in our example).

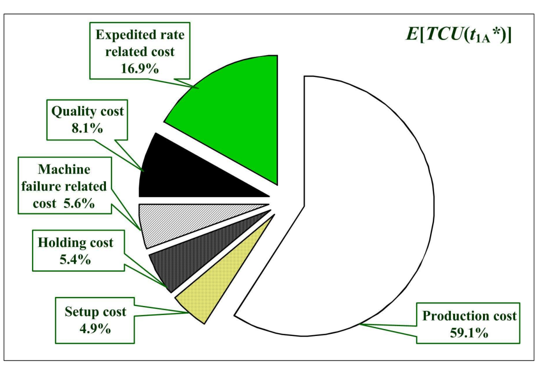

Fig. 10. Analytical results on cost elements in $E[TCU(t_{1A}^*)]$

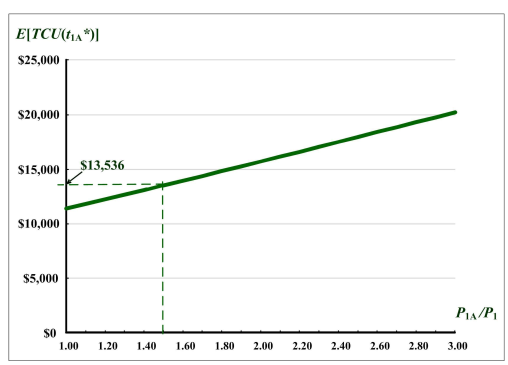

Fig. 11. The impact of changes in expedited ratio P_{1A}/P_1 on $E[TCU(t_{1A}^*)]$

Fig. 12 displays the combined effect of differences in factor of expedited rate α_1 and mean-time-to-failure $1/\beta$ on t_{1A}*. It points out that as $1/\beta$ raises, t_{1A}* decreases considerably; and as the factor of expedited rate α_1 increases, the optimal uptime t_{1A}* decreases significantly (this reconfirms that manufacturing uptime reduced extensively as expedited rate boosts). Fig. 13 illustrates the influence of variations in expedited rate P_{1A}/P_1 on utilization (i.e., t_{1A} / $E[T_A]$). It shows that as P_{1A}/P_1 increases, machine utilization decreases notably; and at P_{1A}/P_1 = 1.5 (as assumed in our example), utilization declines to 0.2963 from 0.4444 (see Table 3 for details).

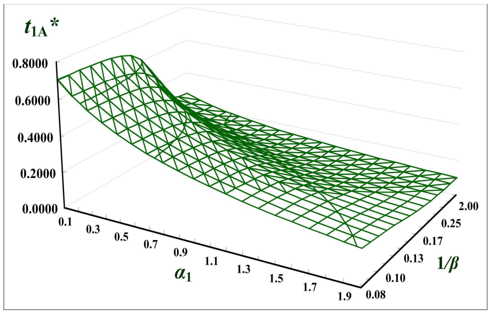

Fig. 12. Combined effect of differences in expedited rate factor α_1 and mean-time-to-failure $1/\beta$ on t_{1A}*

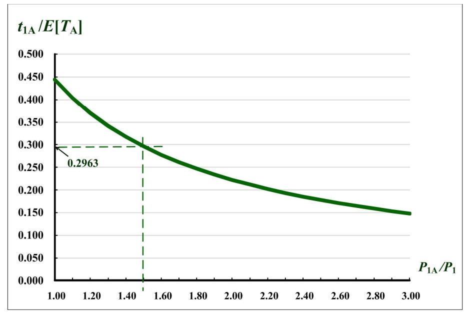

Fig. 13. The influence of variations in expedited ratio P_{1A}/P_1 on machine utilization

Fig. 14. exhibits the impact of changes in random scrap rate x on different cost elements in $E[TCU(t_{1A}*)]$. It specifies that as x increases, product quality cost boosts extensively.

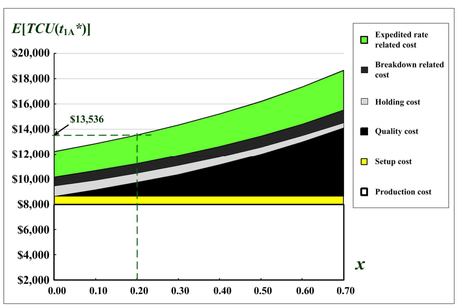

Fig. 14. The impact of variations in random scrap rate x on different cost contributors of $E[TCU(t_{1A}*)]$

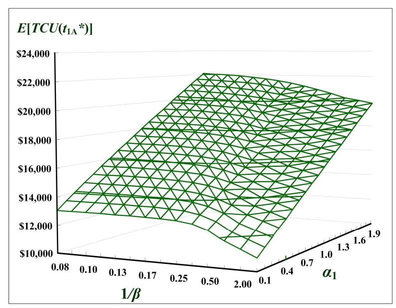

Fig. 15. Combined influence of variations in $1/\beta$ and factor of expedited rate α_1 on $E[TCU(t_{1A}*)]$

Table 3

Analytical results of influence of changes in (P_{1A}/P_1) on t_{1A}, utilization, and $E[TCU(T_A^*)]$

P_{1A}/P_1	t_{1A}	Increase %	Utilization (t_{1A}/T_A)	Decline %	$E[TCU(T_A^*)]$	Increase %
1.00	0.3442	-	0.4444	-	$11,397	-
1.10	0.2988	-13.20%	0.4040	-9.09%	$11,820	3.71%
1.20	0.2652	-22.96%	0.3704	-16.67%	$12,245	7.44%
1.30	0.2392	-30.50%	0.3419	-23.08%	$12,673	11.19%
1.40	0.2185	-36.52%	0.3175	-28.57%	$13,103	14.97%
1.50	**0.2015**	-41.46%	0.2963	-33.33%	**$13,536**	18.77%
1.60	0.1873	-45.58%	0.2778	-37.50%	$13,972	22.59%
1.70	0.1753	-49.09%	0.2614	-41.18%	$14,410	26.43%
1.80	0.1649	-52.10%	0.2469	-44.44%	$14,849	30.29%
1.90	0.1558	-54.73%	0.2339	-47.37%	$15,290	34.16%
2.00	0.1479	-57.04%	0.2222	-50.00%	$15,733	38.04%
2.10	0.1408	-59.09%	0.2116	-52.38%	$16,177	41.94%
2.20	0.1345	-60.92%	0.2020	-54.55%	$16,622	45.84%
2.30	0.1289	-62.57%	0.1932	-56.52%	$17,068	49.76%
2.40	0.1237	-64.06%	0.1852	-58.33%	$17,515	53.68%
2.50	0.1191	-65.41%	0.1778	-60.00%	$17,963	57.61%
2.60	0.1148	-66.65%	0.1709	**-61.54%**	$18,411	**61.54%**
2.70	0.1109	-67.78%	0.1646	-62.96%	$18,860	65.48%

2.80	0.1073	-68.83%	0.1587	-64.29%	$19,310	69.42%
2.90	0.1040	-69.80%	0.1533	-65.52%	$19,760	73.37%
3.00	0.1009	-70.69%	0.1481	-66.67%	$20,210	77.33%

Combined influence of variations in mean-time-to-failure $1/\beta$ and the factor of expedited rate α_1 on $E[TCU(t_{1A}*)]$ is depicted in Fig. 15. It indicates that as the mean-time-to-failure $1/\beta$ increases, $E[TCU(t_{1A}*)]$ declines noticeably; and as the factor of expedited rate α_1 increases, $E[TCU(t_{1A}*)]$ boosts drastically. Fig. 16 illustrates the joint effect of differences in manufacturing uptime t_{1A} and random scrap rate x on $E[TCU(t_{1A}*)]$. It points out the effect of t_{1A} and convexity on cost function, and as random scrap rate x goes up, $E[TCU(t_{1A}*)]$ increases radically.

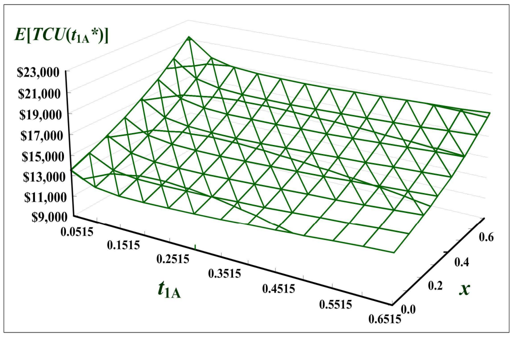

Fig. 16. The joint effect of differences in uptime t_{1A} and random scrap rate x on $E[TCU(t_{1A}*)]$

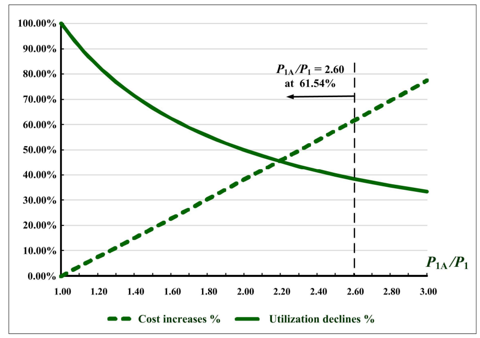

Fig. 17. Behavior of "system cost increase percentage" versus "machine utilization decline percentage"

Fig. 17 demonstrates the behavior of "system cost increase percentage" versus "machine utilization decline percentage." It reveals that the linear breakeven point of cost/benefit is at $P_{1A}/P_1 = 2.60$, where the percentage of cost increase is equal to the percentage of utilization decline (refer to Table 3 for details).

5. Concluding Remarks

This study explores the optimal runtime solution for a manufacturing system featuring an expedited fabrication rate, random equipment failures, and scrap. In order to explicitly represent realistic features of the studied problem, an accurate model comprising two separate situations is constructed (refer to the cases 1 and 2 in subsections 2.1 and 2.2, respectively.) Mathematical derivation and optimization approaches (including a proposed algorithm) are used to find the system cost and optimal runtime solution for the problem (see section 3). Finally, the applicability and sensitivity analyses of research outcome are illustrated via a numerical example (refer to section 4).

In addition to deriving the optimal runtime policy for the problem, the major contribution of the present work includes to reveal diverse critical information regarding the individual/joint influences of variations in the expedited rate, random scrap rate, and stochastic time-to-failure on the optimal runtime decision (see Figures 8 and 12), on total system expenses (refer to Figures 7, 9, 11, 15, and 16), on different cost components (see Figures 10 and 14), and on machine utilization (see Figures 13 and 17). In summary, the research outcomes not only enable the in-depth problem analyses, but also facilitate managerial decision makings. For future study, consideration of a random demand in the context of the same problem will be an interesting direction.

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References

Abubakar, M., Basheer, U., & Ahmad, N. (2017). Mesoporosity, thermochemical and probabilistic failure analysis of fired locally sourced kaolinitic clay. *Journal of the Association of Arab Universities for Basic and Applied Sciences, 24,* 81-88.

Ameen, W., AlKahtani, M., Mohammed, M.K., Abdulhameed, O., & El-Tamimi, A.M. (2018). Investigation of the effect of buffer storage capacity and repair rate on production line efficiency. *Journal of King Saud University - Engineering Sciences, 30(3),* 243-249

Arcelus, F.J., & Srinivasan, G. (1987). Inventory policies under various optimizing criteria and variable markup rates. *Management Science, 33(6)*, 756-762.

Balkhi, Z.T., & Benkherouf, L. (1996). On the optimal replenishment schedule for an inventory system with deteriorating items and time-varying demand and production rates. *Computers and Industrial Engineering, 30(4),* 823-829.

Berg, M., Posner, M.J.M., & Zhao, H., (1994). Production-inventory systems with unreliable machines. *Operations Research 42,* 111-118.

Bottani, E., Centobelli, P., Cerchione, R., Del Gaudio, L., & Murino, T. (2017). Solving machine loading problem of flexible manufacturing systems using a modified discrete firefly algorithm. *International Journal of Industrial Engineering Computations, 8(3),* 363-372.

Chakraborty, T., Chauhan, S.S., & Giri, B.C. (2013). Joint effect of stock threshold level and production policy on an unreliable production environment. *Applied Mathematical Modelling, 37(10-11),* 6593-6608.

Chiu, Y.-S.P., Liu, C.-J., & Hwang, M.-H. (2017). Optimal batch size considering partial outsourcing plan and rework. *Jordan Journal of Mechanical and Industrial Engineering, 11(3),* 195-200.

Chiu, Y.-S.P., Chen, H.-Y., Chiu, S.W., & Chiu, V. (2018a). Optimization of an economic production quantity-based system with random scrap and adjustable production rate. *Journal of Applied Engineering Science, 16(1),* 11-18.

Chiu, S.W., Lin, H.-D., Chou, C.-L., & Chiu, Y.-S.P. (2018b). Mathematical modeling for exploring the effects of overtime option, rework, and discontinuous inventory issuing policy on EMQ model. *International Journal of Industrial Engineering Computations, 9(4),* 479-490.

Chiu, S.W., Wu, H-Y., Chiu, Y-S.P., & Hwang, M-H. (2018c). Exploration of finite production rate model with overtime and rework of nonconforming products. *Journal of King Saud University-Engineering Sciences, 30(3),* 224-231.

Chiu, Y-S.P., Chen, H-Y., Chiu, T., & Chiu, S.W. (2018d). Incorporating flexible fabrication rate and random scrap into a FPR-based supply-Chain system. *Economic Computation and Economic Cybernetics Studies and Research, 52(2),* 157-174.

Dahane, M., Rezg, N., & Chelbi, A. (2012). Optimal production plan for a multi-products manufacturing system with production rate dependent failure rate. *International Journal of Production Research, 50(13),* 3517-3528.

Dehayem Nodem, F.I., Kenne, J.P., & Gharbi, A. (2009). Hierarchical decision making in production and repair/replacement planning with imperfect repairs under uncertainties. *European Journal of Operational Research*, *198(1),* 173-189.

Gan, S.-S., Pujawan, I.N., Suparno, & Widodo, B. (2018). Pricing decisions for short life-cycle product in a closed-loop supply chain with random yield and random demands. *Operations Research Perspectives, 5,* 174-190.

Gharbi, A., Kenné, J.-P., & Hajji, A. (2006). Operational level-based policies in production rate control of unreliable manufacturing systems with set-ups. *International Journal of Production Research, 44(3),* 545-567.

Giri, B.C., & Dohi, T. (2005a). Exact formulation of stochastic EMQ model for an unreliable production system. *Journal of the Operational Research Society, 56(5),* 563-575.

Giri, B.C., & Dohi, T. (2005b). Computational aspects of an extended EMQ model with variable production rate. *Computers and Operations Research, 32(12),* 3143-3161.

Giri, B.C., & Chakraborty, A. (2011). Supply chain coordination for a deteriorating product under stock-dependent consumption rate and unreliable production process. *International Journal of Industrial Engineering Computations, 2(2),* 263-272.

Groenevelt, H., Pintelon, L., & Seidmann, A. (1992). Production lot sizing with machine breakdowns.

Management Sciences, 38, 104-123.

Grosfeld-Nir, A., & Gerchak, Y. (2002). Multistage production to order with rework capability. *Management Science*, *48(5),* 652-664.

Imbachi, C.G.A., Larrahondo, A.F.C., Orozco, D.L.P., Cadavid, L.R., & Bastidas, J.J.B. (2018). Design of an inventory management system in an agricultural supply chain considering the deterioration of the product: The case of small citrus producers in a developing country. *Journal of Applied Engineering Science, 16(4),* 523-537.

Konstantaras, I., Goyal, S. K., & Papachristos, S. (2007). Economic ordering policy for an item with imperfect quality subject to the in-home inspection. *International Journal of Systems Science*, *38(6),* 473-482.

Khouja, M. & Mehrez, A. (1994). Economic production lot size model with variable production rate and imperfect quality. *Journal of the Operational Research Society, 45(12),* 1405-1417.

Kuhn, H. (1997). A dynamic lot sizing model with exponential machine breakdowns. *European Journal of Operational Research, 100(3),* 514-536.

Liu, Y., Wu, H., Hou, J., Wei, C., & Ren, W. (2017a). An injection/production rate allocation method applied for polymer-surfactant flooding. *Journal of Engineering Research, 5(2),* 250-267.

Liu, L., Liu, X., Wang, X., Wang, Y., & Li, C. (2017b). Reliability analysis and evaluation of a brake system based on competing risks. *Journal of Engineering Research, 5(3),* 150-161.

Luong, H.T., & Karim, R. (2017). An integrated production inventory model of deteriorating items subject to random machine breakdown with a stochastic repair time. *International Journal of Industrial Engineering Computations, 8(2),* 217-236.

Mohammed, B.K., Roman, M., Odah, M.H., & Bager, A.S.M. (2018). Testing reliability: Factorial design with data from a log-epsilon-skew-normal distribution. *Economic Computation and Economic Cybernetics Studies and Research, 51(3),* 143-160.

Muzamil, M., Siddiqui, M.A., & Samiuddin, M. (2017). Experimental investigation and optimization of process parameters for through induction hardening using factorial design of experiments. *Journal of Engineering Research, 5(3),* 174-185.

Nobil, A.H., Niaki, S.T.A., & Nobil, E. (2017). An effective and simple algorithm to solve the discrete multi-product economic production quantity model. *Economic Computation and Economic Cybernetics Studies and Research, 51(3),* 251-261.

Pearce, A., Pons, D., & Neitzert, T. (2018). Implementing lean - outcomes from SME case studies. *Operations Research Perspectives, 5,* 94-104.

Porteus, E. L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction, *Operations Research, 34,* 137-144.

Richter, K. (1996). The EOQ repair and waste disposal model with variable setup numbers. *European Journal of Operational Research, 95(2),* 313-324.

Sana, S.S. (2010). A production-inventory model in an imperfect production process. *European Journal of Operational Research, 200(2),* 451-464.

Schwaller, R. (1988). EOQ under inspection costs. *Production and Inventory Management, 29,* 22-24.

Shakoor, M., Abu Jadayil, W., Jabera, N., & Jaber, S. (2017). Efficiency assessment in emergency department using lean thinking approach. *Jordan Journal of Mechanical and Industrial Engineering, 11(2),* 97-103.

Sher, H.A., Addoweesh, K.E., Khalid, Z., & Khan, Y. (2017). Theoretical and experimental analysis of inverter fed induction motor system under DC link capacitor failure. *Journal of King Saud University - Engineering Sciences, 29(2),* 103-111.

So, K.C., & Tang, C.S. (1995). Optimal operating policy for a bottleneck with random rework. *Management Science, 41(4),* 620-636.

Taft, E.W. (1918). The most economical production lot. *Iron Age, 101,* 1410–1412.

Vujosevic, M., Makajic-Nikolic, D., & Pavlovic, P. (2017). A new approach to determination of the most critical multi-state components in multi-state systems. *Journal of Applied Engineering Science, 15(4),* 401-405.

Zhang, Y., Wu, W., & Tang, Y. (2017). Analysis of an k-out-of-n:G system with repairman's single vacation and shut off rule. *Operations Research Perspectives, 4,* 29-38.

Zanoni, S., Bettoni, L., & Glock, C.H. (2014). Energy implications in a two-stage production system with controllable production rates. *International Journal of Production Economics, 149,* 164-171.

Appendix – A

The following are other notation used by this study:

t = manufacturing time (in years) before a random failure taking place,
M = fixed cost to repair the machine failure,
Q = lot size,
T'_A = cycle time of the studied system,
$E[x]$= the expected random scrap rate,
H_0 = level of perfect stock when a random failure takes place,
H = level of perfect stock when manufacturing uptime ends,
h = unit holding cost,
C_1 = unit cost for safety stock,
h_3 = unit holding cost for safety stock,
t_{1A} = uptime – the decision variable of the proposed manufacturing runtime problem,
t'_{2A} = stock depletion time of the proposed system with a random failure taking place,
g = fixed machine repair time, thus, $g = t_r$,
$I(t)$ = on-hand stock status at time t,
$I_s(t)$= on-hand status of scraps at time t,
$I_F(t)$= on-hand status of safety stocks at time t,
$E[T'_A]$ = expected cycle length in the case of a random failure taking place,
$TC(t_{1A})_1$ = total cost per cycle in the case of a random failure taking place,
$E[TC(t_{1A})]_1$ = expected total cost per cycle in the case of a random failure taking place,
t_{2A} = stock depletion time in the case of no machine failure taking place,
T_A = cycle time in the case of no machine failure taking place,
$E[T_A]$= expected cycle time in the case of no machine failure taking place,
$TC(t_{1A})_2$ = total cost per cycle in the case of no machine failure taking place,
$E[TC(t_{1A})]_2$ = expected total cost per cycle in the case of no machine failure taking place,
t_1 = uptime for a manufacturing system with scrap only,
t_2 = stock depletion time for a manufacturing system with scrap,
T = cycle time for a manufacturing system with scrap,
$\boldsymbol{T_A}$ = cycle time for the proposed system with expedited rate, random failures, and scrap,
$E[\boldsymbol{T_A}]$ = expected cycle time for the proposed system with expedited rate, random failures, and scrap,
$E[TCU(t_{1A})]$ = expected annual system cost for the proposed system with expedited rate, random failures, and scrap.

Appendix – B

The following are detailed derivations for $E[TCU(t_{1A})]$ (i.e., Eq. (20)):
Let u_1 and u_2 be the following:

$$u_1 = \left[(1+\alpha_3)C\right](1+\alpha_1)P_1 + C_S E[x](1+\alpha_1)P_1 \tag{B-1}$$

$$u_2 = \frac{h\left[(1+\alpha_1)P_1\right]^2}{2}\left\{\frac{\left[1-E[x]\right]^2}{\lambda} + \frac{\left[2E[x]-1\right]}{(1+\alpha_1)P_1}\right\} \tag{B-2}$$

then $E[TC(t_{1A})]_1$ (Eq. (10)) and $E[TC(t_{1A})]_2$ (Eq. (17)) become the following:

$$E\left[TC\left(t_{1\mathrm{A}}\right)\right]_1 = u_1 t_{1\mathrm{A}} + \left(1+\alpha_2\right)K + M + \lambda g\left[C_1 + h_3\left(t+\frac{g}{2}\right) + C_T\right] + h\left[\left(1+\alpha_1\right)P_1 - \lambda\right]tg + u_2\left(t_{1\mathrm{A}}\right)^2 \tag{B-3}$$

$$E\left[TC\left(t_{1\mathrm{A}}\right)\right]_2 = u_1 t_{1\mathrm{A}} + \left(1+\alpha_2\right)K + h_3\lambda g T_{\mathrm{A}} + u_2\left(t_{1\mathrm{A}}\right)^2 \tag{B-4}$$

Applying the following Eq. (18) and substituting Eqs. (B-3), (B-4) and (19) in Eq. (18), Eq. (B-5) can be gained after extra derivation efforts.

$$E\left[TCU\left(t_{1\mathrm{A}}\right)\right] = \frac{\left\{\int_0^{t_{1\mathrm{A}}} E\left[TC\left(t_{1\mathrm{A}}\right)\right]_1 \cdot f(t)\,dt + \int_{t_{1\mathrm{A}}}^{\infty} E\left[TC\left(t_{1\mathrm{A}}\right)\right]_2 \cdot f(t)\,dt\right\}}{E[\boldsymbol{T}_{\mathbf{A}}]} \tag{18}$$

$$\begin{aligned}
E\left[TCU\left(t_{1\mathrm{A}}\right)_1\right] &= h_3\lambda g\left(e^{-\beta t_{1\mathrm{A}}}\right) \\
&+ \frac{\lambda}{\left[1-E[x]\right]}\left\{\begin{aligned}
&\frac{\left[\left(1+\alpha_2\right)K\right]}{(1+\alpha_1)P_1 t_{1\mathrm{A}}} + \left[\left(1+\alpha_3\right)C\right] + C_S E[x] + t_{1\mathrm{A}}\left\{\frac{h(1+\alpha_1)P_1}{2}\left\{\frac{\left[1-E[x]\right]^2}{\lambda} + \frac{\left[2E[x]-1\right]}{\left(1+\alpha_1\right)P_1}\right\}\right\} \\
&+\frac{1}{t_{1\mathrm{A}}}\left[\begin{aligned}&\frac{C_1\lambda g}{\left(1+\alpha_1\right)P_1} + \frac{C_T\lambda g}{\left(1+\alpha_1\right)P_1} + \frac{M}{(1+\alpha_1)P_1} + \frac{h_3\lambda g^2}{2(1+\alpha_1)P_1} + \frac{hg}{\beta} + \frac{h_3\lambda g}{\left(1+\alpha_1\right)P_1\beta} \\ &- \frac{h\lambda g}{\left(1+\alpha_1\right)P_1\beta}\end{aligned}\right] \\
&+ e^{-\beta t_{1\mathrm{A}}}\left[-hg - \frac{h_3\lambda g}{\left(1+\alpha_1\right)P_1} + \frac{h\lambda g}{\left(1+\alpha_1\right)P_1}\right] \\
&+ \frac{e^{-\beta t_{1\mathrm{A}}}}{t_{1\mathrm{A}}}\left[\begin{aligned}&-\frac{C_1\lambda g}{\left(1+\alpha_1\right)P_1} - \frac{C_T\lambda g}{\left(1+\alpha_1\right)P_1} - \frac{M}{(1+\alpha_1)P_1} - \frac{h_3\lambda g^2}{2(1+\alpha_1)P_1} - \frac{hg}{\beta} \\ &- \frac{h_3\lambda g}{\left(1+\alpha_1\right)P_1\beta} + \frac{h\lambda g}{\left(1+\alpha_1\right)P_1\beta}\end{aligned}\right]
\end{aligned}\right\}
\end{aligned} \tag{B-5}$$

Suppose we let Z_1, Z_2, Z_3, and Z_4 denote the following:

$$Z_1 = \left[\frac{\left(1+\alpha_2\right)K}{(1+\alpha_1)P_1}\right] \tag{B-6}$$

$$Z_2 = \left[\frac{C_1\lambda g}{\left(1+\alpha_1\right)P_1} + \frac{C_T\lambda g}{\left(1+\alpha_1\right)P_1} + \frac{M}{(1+\alpha_1)P_1} + \frac{h_3\lambda g^2}{2(1+\alpha_1)P_1} + \frac{hg}{\beta} + \frac{h_3\lambda g}{\left(1+\alpha_1\right)P_1\beta} - \frac{h\lambda g}{\left(1+\alpha_1\right)P_1\beta}\right] \tag{B-7}$$

$$Z_3 = \left[-hg - \frac{h_3\lambda g}{\left(1+\alpha_1\right)P_1} + \frac{h\lambda g}{\left(1+\alpha_1\right)P_1}\right] \tag{B-8}$$

$$Z_4 = \left[-\frac{C_1\lambda g}{\left(1+\alpha_1\right)P_1} - \frac{C_T\lambda g}{\left(1+\alpha_1\right)P_1} - \frac{M}{(1+\alpha_1)P_1} - \frac{h_3\lambda g^2}{2(1+\alpha_1)P_1} - \frac{hg}{\beta} - \frac{h_3\lambda g}{\left(1+\alpha_1\right)P_1\beta} + \frac{h\lambda g}{\left(1+\alpha_1\right)P_1\beta}\right] \tag{B-9}$$

then

$$E\left[TCU\left(t_{1A}\right)\right]=\frac{\lambda}{\left[1-E\left[x\right]\right]}\left\{\begin{array}{l}\frac{Z_1}{t_{1A}}+\frac{Z_2}{t_{1A}}+Z_3e^{-\beta t_{1A}}+\frac{Z_4e^{-\beta t_{1A}}}{t_{1A}}+\left[\left(1+\alpha_3\right)C\right]+C_SE\left[x\right]\\+t_{1A}\left\{\frac{h(1+\alpha_1)P_1}{2}\left[\frac{\left[1-E\left[x\right]\right]^2}{\lambda}+\frac{\left[2E\left[x\right]-1\right]}{\left(1+\alpha_1\right)P_1}\right]\right\}\\+h_3g\left[1-E\left[x\right]\right]\left(e^{-\beta t_{1A}}\right)\end{array}\right\} \tag{20}$$

Appendix – C

Table C-1

Results of extra convexity tests with broader range of β values

β	t_{1AU}	$\gamma(t_{1AU})$	t_{1AL}	$\gamma(t_{1AL})$
10	0.4853	4.3920	0.0184	0.0389
7	0.4860	1.8245	0.0259	0.0551
4	0.4878	0.9254	0.0435	0.0939
3	0.4892	0.7925	0.0556	0.1221
2	0.4919	0.7215	0.0758	0.1733
1	**0.5000**	**0.7466**	**0.1130**	**0.2966**
0.5	0.5158	0.8897	0.1430	0.4768
0.01	1.3575	3.4131	0.1832	2.2267