International Journal of Industrial Engineering Computations 12 (2021) 63–78

Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

A multi-item batch fabrication problem featuring delayed product differentiation, outsourcing, and quality assurance

Yuan-Shyi Peter Chiu[a], Huei-Hsin Chang[b], Tiffany Chiu[c] and Singa Wang Chiu[d*]

[a]Department of Industrial Engineering & Management, Chaoyang University of Technology, Taichung 413, Taiwan
[b]Department of Financial, Chaoyang University of Technology, Taichung 413, Taiwan
[c]Anisfield School of Business, Ramapo College of New Jersey, Mahwah, NJ 07430, USA
[d]Department of Business Administration, Chaoyang University of Technology, Taichung 413, Taiwan

C H R O N I C L E

Article history:
Received June 16 2020
Received in Revised Format
August 12 2020
Accepted September 7 2020
Available online
September, 7 2020

Keywords:
Multi-item manufacturing
Outsourcing
Delayed differentiation
Rework
Scrap
Batch fabrication

A B S T R A C T

Variety, quality, and rapid response are becoming a trend in customer requirements in the contemporary competitive markets. Thus, an increasing number of manufacturers are frequently seeking alternatives such as redesigning their fabrication scheme and outsourcing strategy to meet the client's expectations effectively with minimum operating costs and limited in-house capacity. Inspired by the potential benefits of delay differentiation, outsourcing, and quality assurance policies in the multi-item production planning, this study explores a single-machine two-stage multi-item batch fabrication problem considering the abovementioned features. Stage one is the fabrication of all the required common parts, and stage two is manufacturing the end products. A predetermined portion of common parts is supplied by an external contractor to reduce the uptime of stage one. Both stages have imperfect in-house production processes. The defective items produced are identified, and they are either reworked or removed to ensure the quality of the finished batch. We develop a model to depict the problem explicitly. Modeling, formulation, derivation, and optimization methods assist us in deriving a cost-minimized cycle time solution. Moreover, the proposed model can analyze and expose the diverse features of the problem to help managerial decision-making. An example of this is the individual/ collective influence of postponement, outsourcing, and quality reassurance policies on the optimal cycle time solution, utilization, uptime of each stage, total system cost, and individual cost contributors.

1. Introduction

This study explores a single-machine two-stage multi-item batch fabrication problem featuring delayed product differentiation, outsourcing, and quality assurance. To rapid response to the customer's variety requirements, a growing number of manufacturers are frequently seeking effective alternatives such as redesigning the fabrication scheme. An example of this is to incorporate a delayed differentiation strategy in the multi-item fabrication planning with the existence of product commonality. Davis and Sasser (1995) discussed the idea of manufacturing process incorporating postponement. They specified that it is a fabrication scheme to keep the product generic until it is necessary to finish it up. They stated that it could lead to a reduction in overall supply-chain expenses by balancing stock savings and client service with raw material, design, and production costs. Labro (2004) conducted a literature review focusing on the component commonality in particularly from a point of view of cost effects. The author investigated the significant relationship between the postponement implementing models and their consequent benefits and concluded that there were insufficient empirical evidence to make a

* Corresponding author
E-mail: swang@cyut.edu.tw (S. W. Chiu)

2021 Growing Science Ltd.
doi: 10.5267/j.ijiec.2020.9.003

general statement concerning the influence of growing commonality on overall fabrication expenses. Ngniatedema et al. (2012) built a model to analyze global supply chains featuring postponement and variable international transfers and costs by service levels and shipping schedules. The influences of differences in postponement timing and decoupling points on the benefits of supply chains were explicitly explored. A real case concerning the fabrication of HP-printers was used to demonstrate their model's applicability. Berrade et al. (2017) built a model to analyze the necessary states of delay maintenance for a single component system. The authors specifically studied the cost-effective conditions for the postponement in maintenance relating to inspection frequency and system reliability issues. They found that significant cost-effectiveness of maintenance from delay time in maintenance and defect arrival time. Recent works (Chiu et al., 2018, 2019a, 2020; Sheikh et al., 2019; Vaid & Arora, 2019; Babu & Anuradha, 2020; Le Pape & Wang, 2020) also studied the impact of various characteristics of delay differentiation on production planning and intangible/tangible benefits in manufacturing firms.

In the manufacturing sector, the outsourcing strategy is an effective way to level the machine loadings in production planning. Hines and Rich (1998) explored the competitive advantage of implementing outsourcing. The authors stated how world-leading company - Toyota embedded skill and knowledge of supplier integration into its supply chain, especially an effective outsourcing strategy, to gain benefits and competitive advantage. The authors also discussed and illustrated using a UK context the influence of a four-stage model for supplier association, intending to provide a clear picture of the significant relationships among outsourcing, supplier association/networks, and competitive advantage. Alvarez and Stenbacka (2007) applied a real options method to explore a company's optimal organizational style. The authors found that the implementation of (partial) outsourcing becomes a growing function of a company facing the uncertain market. They also showed the correlation of market uncertainty and outsourcing proportion. Kopel et al. (2016) examined the sourcing alternatives for a multiproduct manufacturing firm wherein multiple inputs are required for the multiproduct fabrication. The authors specifically considered three distinct situations: (1) the optimal in-house production plan might have higher unit cost as compared to the least cost calculation, (2) aggregately outsourcing all required inputs may be profitable even though each item/input outsourced individually is not profitable, (3) the profit of the multiproduct fabrication declines as the supplier market's competition rises. Additional studies (Van Mieghem, 1999; Lee & Sung, 2008; Westphal & Sohal, 2016; Chiu et al., 2019b,c) explored the effect of diverse features of outsourcing strategies on operations management, production planning, and overall operating cost.

In most fabrication processes, it is inevitable to have nonconforming items produced due to diverse uncontrollable factors. These items must be either reworked/repaired or removed to ensure the desired quality of the finished batch. Calabria and Pulcini (1999) proposed a series of discontinuous-point processes for analyzing repairable items. The authors used maximum likelihood estimation/testing and focused on studying the repairable items' failure patterns other than those actions known as minimal repairs. Sarker et al. (2008) explored the optimal lot-size for a multiple stages fabrication system featuring rework alternatives. Two distinct rework policies were studied. The first policy is performing the rework in the same fabrication cycle and no shortage is allowed. The second policy is conducting the rework after N fabrication cycles and shortages in a cycle are permitted. The authors demonstrated that their problem has the nonlinear convex nature and provided solution procedure with sensitivity analyses of key system variables. The authors found that the second policy is in favor in terms of cost-saving under the conditions that the percentage of defectives and their holding costs are low. Moshtagh and Taleizadeh (2017) explored a combined manufacturing and remanufacturing closed-loop supply-chain system featuring rework, permitted shortage, and quality-relevant return rate. They assumed that both the manufacturing and remanufacturing processes are imperfect and defectives are reworked/repaired each cycle, the return rate is a variable, and the quality of returned goods is a random variable. Other than ordinary production-inventory related costs, for these returned goods salvage value, remanufacturing cost, and buyback cost are also considered. The authors examined three distinct mathematical models and each has its probability density functions. The solution procedures are presented accompanied by numerical illustrations to how their models' applicability. Recent works (Aringhieri et al., 2018; Abashar, 2018; Pearce et al., 2018; Sonntag & Kiesmüller, 2018; Rao & Singh, 2018; Istotskiy & Protasev, 2019; Iqbal et al., 2019; Larkin & Privalov, 2019; Noman et al., 2019; Ortiz-Servin et al., 2019; Tannady et al., 2019; Hammado et al., 2020; Lim, 2020; Zammori et al., 2020) examined the effect of various characteristics of imperfection in fabrication processes and rework/scrap of nonconforming items on operations management, fabrication planning, and overall operating cost. Since not many works can be found in the literature that focused on the exploration of the collective impact of commonality, outsourcing, and quality reassurance on the multi-item batch fabrication planning, we aim to bridge the gap.

2. Problem description and modeling

2.1. Nomenclature

λ_i = annual demand rate for end product i (where $i = 1, 2, \ldots, L$),
Q_i = the batch size for end product i,
$t_{1,i}$ = the uptime for fabricating the end product i in stage 2,
$t_{2,i}$ = the time required to rework the nonconforming end product i,
$t_{3,i}$ = the time required to deplete all of the end product i,
T = the common manufacturing cycle time - the decision variable,
t_i^* = the sum of optimal uptimes of the end products in stage two,

C_i = unit manufacturing cost for end product i in stage 2,
K_i = setup cost for end product i,
$h_{1,i}$ = holding cost for end product i,
S_i = setup time for end product i,
$P_{1,i}$ = annual fabrication rate for end product i,
x_i = random nonconforming portion in stage 2 when fabricating end product i,
$d_{1,i}$ = fabrication rate of random nonconforming end product i (i.e., $d_{1,i} = x_iP_{1,i}$),
$\theta_{1,i}$ = the scrap portion of the nonconforming end product i,
$P_{2,i}$ = annual reworking rate for end product i,
$C_{R,i}$ = unit rework cost for end product i,
$C_{S,i}$ = unit disposal cost for scrap product i,
$\theta_{2,i}$ = the scrap portion of the reworked nonconforming end product i,
$d_{2,i}$ = fabrication rate of scrap product i in $t_{2,i}$ (i.e., $d_{2,i} = \theta_{2,i}P_{2,i}$),
$h_{2,i}$ = holding cost for the reworked end product i in $t_{2,i}$,
$h_{4,i}$ = unit safety stock's holding cost for the end product i,
φ_i = the total scrap rate in fabricating end product i in stage 2,
$H_{1,i}$ = the inventory level of end product i at the end of the production process,
$H_{2,i}$ = the inventory level of end product i at the end of the rework process,
Q_0 = the in-house batch size for common parts in stage 1,
π_0 = the outsourcing portion of the required batch of common parts in stage 1,
$K_{\pi 0}$ = fixed outsourcing cost,
$C_{\pi 0}$ = unit outsourcing cost,
K_0 = in-house common part's setup cost,
C_0 = in-house common part's unit cost,
$\beta_{1,0}$ = the linking parameter between $K_{\pi 0}$ and K_0,
$\beta_{2,0}$ = the linking parameter between $C_{\pi 0}$ and C_0,
$t_{1,0}$ = the uptime for fabricating common parts when the outsourcing is implemented,
$t_{2,0}$ = the time required to rework the defective common parts,
$t_{3,0}$ = the time required to deplete the common parts in a fabrication cycle,
t_0^* = the sum of the optimal uptime and rework time ($t^*_{1,0} + t^*_{2,0}$) in stage one,
$h_{1,0}$ = holding cost for common parts,
λ_0 = the common parts' annual demand rate,
$P_{1,0}$ = annual fabrication rate for common parts,
x_0 = random nonconforming portion in stage 1 when producing the common parts,
$d_{1,0}$ = production rate of random nonconforming common parts (i.e., $d_{1,0} = x_0P_{1,0}$),
$\theta_{1,0}$ = the scrap portion of the nonconforming common parts,
$P_{2,0}$ = annual reworking rate for common parts,
$C_{R,0}$ = unit rework cost for common part,
$C_{S,0}$ = unit disposal cost for scrap common part,
$\theta_{2,0}$ = the scrap portion of the reworked nonconforming common parts,
$d_{2,0}$ = fabrication rate of scrap common parts in the rework time $t_{2,0}$ (i.e., $d_{2,0} = \theta_{2,0}P_{2,0}$),
φ_0 = overall scrap rate of common parts in stage one,
$h_{2,0}$ = holding cost for the reworked common parts in $t_{2,0}$,
$h_{4,0}$ = unit safety stock's holding cost for the common parts,
i_0 = the holding cost relating ratio (i.e., $h_{1,i} = (i_0)C_i$),
γ = the common part's completion rate as compared with a finished product,
S_0 = the setup time for common part,
$H_{1,0}$ = the in-house inventory level of common parts at the end of the fabrication process,
$H_{2,0}$ = the in-house inventory level of common parts at the end of the rework process,
$H_{3,0}$ = the inventory level of common parts after the outsourced common parts are received (which schedule is predetermined - at the end of rework process),
H_i = the inventory level of common parts at the end of the production process of product i,
$I(t)_i$ = the inventory level at time t of product i (where $i = 0, 1, 2, \ldots, L$),
$E[T]$ = the expected common production cycle time,
$TC(T)$ = total system cost in a replenishment cycle,
$E[TC(T)]$= the expected total system in a cycle,
$E[TCU(T)]$ = the expected system cost per unit time.

2.2. Description and modeling

A multi-item batch fabrication problem featuring delayed product differentiation, outsourcing, and quality assurance is explored. The detailed problem description and assumption are as follows: (i) It is a multi-item batch fabrication problem with the existence of a common part in these end products and a two-stage delayed differentiation production design, where all of the common parts are prepared in stage one and the end products are fabricated in stage two; (ii) The demand rate λ_i of L end products is constant (where $i = 1, 2, \ldots, L$); (iii) The completion rate γ of the common part (as compared with the end product) is a known constant and the production rate $P_{1,0}$ of the common parts depends on γ; (iv) The fabrication rate $P_{1,i}$ for L end products also depends on γ, e.g., if $\gamma = 50\%$, then $P_{1,i}$ and $P_{1,0}$ both become double as much as the standard rate of end product i in a single-stage production system; (v) To reduce the production time in stage one (since the fabrication of common parts may consume a large amount of production uptime), a π_0 portion of the required batch of common parts is outsourced, thus a different fixed cost $K_{\pi 0}$ and unit cost $C_{\pi 0}$ (see Eqs. (1) and (2)) are associated with the outsourcing activity; (vi) The random nonconforming portion x_0 and x_i exist in both production stages, and in stage one, a $\theta_{1,0}$ portion and in stage two, a $\theta_{1,i}$ portion among the nonconforming items are identified as scrap, and the others are reworked at the annual rate of $P_{2,0}$ and $P_{2,i}$, respectively. Also, during the rework processes, in stage one, a $\theta_{2,0}$ portion and in stage two, a $\theta_{2,i}$ portion of the reworked nonconforming items fail and become scrap; (vii) The outsourcing parts arrive as the in-house rework process completes.

$$K_{\pi 0} = \left(1 + \beta_{1,0}\right) K_0, \tag{1}$$

$$C_{\pi 0} = \left(1 + \beta_{2,0}\right) C_0, \tag{2}$$

where K_0, C_0, $\beta_{1,0}$, and $\beta_{2,0}$ denote in-house setup cost, unit cost, and the linking parameters, respectively. For example, $\beta_{1,0} = -0.6$ means that $K_{\pi 0}$ is 60% less than the in-house setup cost, and $\beta_{2,0} = 0.25$ means that the unit outsourcing cost is 25% more than the in-house unit cost, etc. Fig. 1 illustrates the inventory level of the proposed multi-item batch fabrication model with delayed product differentiation, outsourcing, and quality assurance.

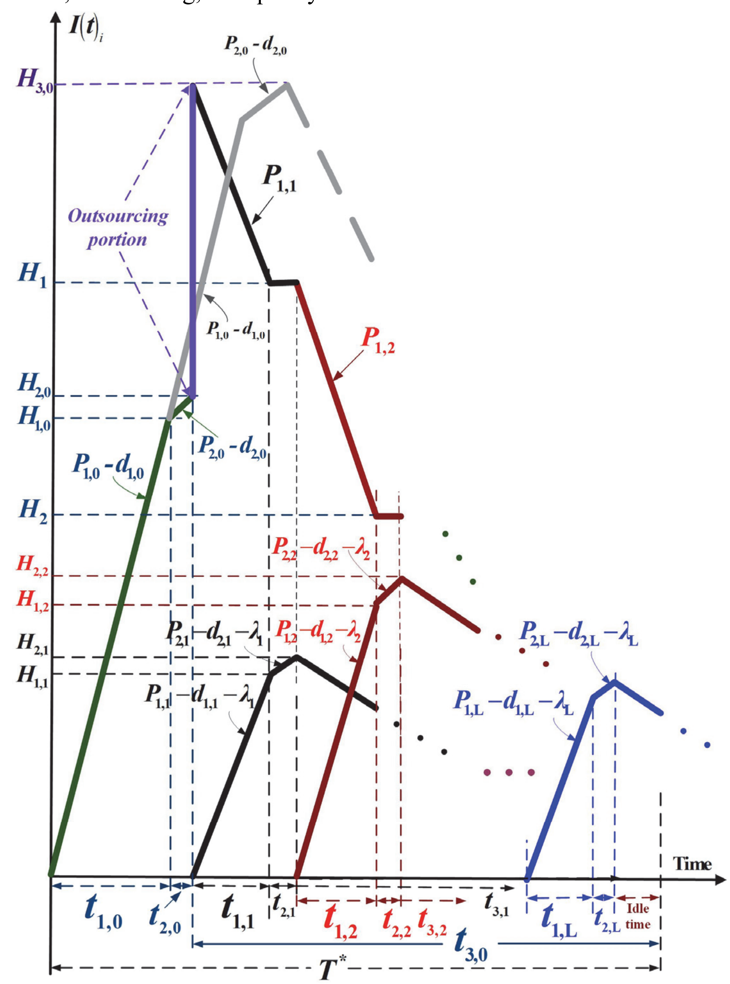

Fig. 1. The inventory level in the proposed multi-item batch fabrication model with delayed product differentiation, outsourcing, and quality assurance as compared with the same system without outsourcing policy (in grey)

Fig. 1 shows that in stage one, the level of inventory reaches $H_{1,0}$ at the end of uptime, and it arrives at $H_{2,0}$ when the rework process ends. Then, the outsourced items are received and the level of inventory reaches $H_{3,0}$ before the beginning of stage two, where the fabrication of L different end products starts. In stage two, for each product i (where $i = 1, 2, \ldots, L$), the level of inventory piles up to $H_{1,i}$ at the end of the uptime and it arrives at $H_{2,i}$ when the rework process ends. Because no stock-out situations are allowed, in stage one, $P_{1,0} - d_{1,0}$ must be greater than zero and in stage two, $P_{1,i} - d_{1,i} - \lambda_i$ must also be greater than zero.

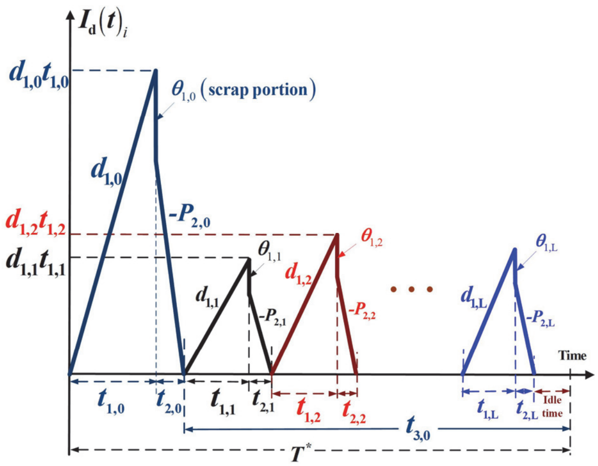

Fig. 2. The inventory status of nonconforming items

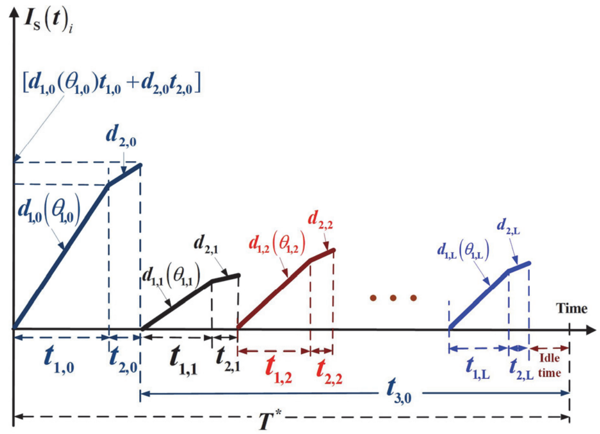

Fig. 3. The inventory status of scrap items in the proposed model

Figs. 2 and 3 illustrate the levels of nonconforming and scrap items in the proposed system, respectively. Fig. 2 shows that the maximal level of nonconforming common parts reaches ($d_{1,0}\, t_{1,0}$) at the end of uptime $t_{1,0}$, after removal of the scrap items, its level begins to decline during the rework process, and it depletes to zero at the end of rework. A similar inventory status occurs in stage two for the end products. Fig. 3 shows that the maximal level of scrap common parts reaches $[d_{1,0}(\theta_{1,0})t_{1,0} + d_{2,0}\, t_{2,0}]$ at the end of rework time $t_{2,0}$. Similarly, the level of scrap product *i* reaches $[d_{1,i}\,(\theta_{1,i})t_{1,i} + d_{2,i}\, t_{2,i}]$ in stage two.

2.3. Formulation of stage two

In stage two, the common parts are consumed to meet the fabrication requirements of the end products (see the inventory status of common parts in Fig. 4). The following formulas (for $i = 1, 2, \ldots, L$) can be observed according to the problem description and assumptions, and Figs. 1 through 4:

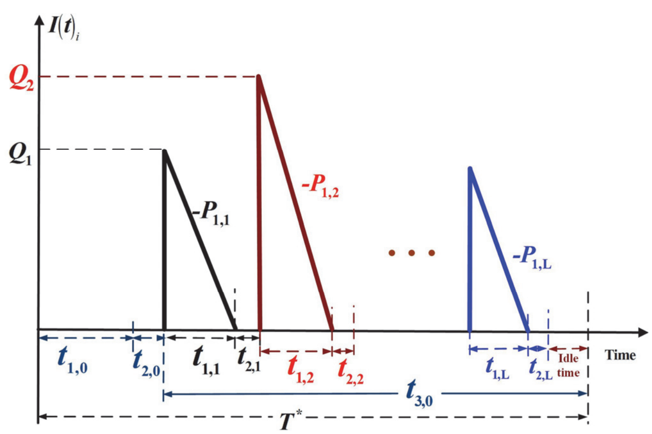

Fig. 4. The inventory status of common parts

$$T = t_{1,i} + t_{2,i} + t_{3,i} \tag{3}$$

$$Q_i = \frac{\lambda_i T}{1-\varphi_i x_i} \tag{4}$$

$$H_{1,i} = \left(P_{1,i} - d_{1,i} - \lambda_i\right) t_{1,i} \tag{5}$$

$$t_{1,i} = \frac{H_{1,i}}{P_{1,i} - d_{1,i} - \lambda_i} = \frac{Q_i}{P_{1,i}} \tag{6}$$

$$H_{2,i} = H_{1,i} + \left(P_{2,i} - d_{2,i} - \lambda_i\right) t_{2,i} \tag{7}$$

$$t_{2,i} = \frac{H_{2,i} - H_{1,i}}{P_{2,i} - d_{2,i} - \lambda_i} \tag{8}$$

$$t_{3,i} = \frac{H_{2,i}}{\lambda_i} \tag{9}$$

$$\varphi_i = \theta_{1,i} + \left(1 - \theta_{1,i}\right)\theta_{2,i} \tag{10}$$

Based on Eq. (4), we know that the requirement of the common parts at the end of stage one should be as follows:

$$H_{3,0} = \sum_{i=1}^{L} Q_i = \sum_{i=1}^{L} \frac{\lambda_i T}{1 - \varphi_i x_i} \tag{11}$$

2.4. Formulation of stage one

In stage 1, the following formulas can be observed according to the problem description and assumptions, and Figs. 1 and 2:

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T} \tag{12}$$

$$H_{3,0} = H_{2,0} + \pi_0 \left(\sum_{i=1}^{L} Q_i \right) \tag{13}$$

$$H_{2,0} = \left(1 - \pi_0\right) \left(\sum_{i=1}^{L} Q_i \right) \tag{14}$$

$$Q_0 = \frac{H_{2,0}}{1 - \varphi_0 x_0} \tag{15}$$

$$H_{2,0} = H_{1,0} + \left(P_{2,0} - d_{2,0}\right) t_{2,0} \tag{16}$$

$$t_{2,0} = \frac{H_{2,0} - H_{1,0}}{P_{2,0} - d_{2,0}} = \frac{Q_0 \left[x_0 \left(1 - \theta_{1,0}\right) \right]}{P_{2,0}} \tag{17}$$

$$H_{1,0} = \left(P_{1,0} - d_{1,0}\right) t_{1,0} \tag{18}$$

$$t_{1,0} = \frac{H_{1,0}}{P_{1,0} - d_{1,0}} = \frac{Q_0}{P_{1,0}} \tag{19}$$

$$\varphi_0 = \theta_{1,0} + \left(1 - \theta_{1,0}\right)\theta_{2,0} \tag{20}$$

$$T = t_{1,0} + t_{2,0} + t_{3,0} \tag{21}$$

$$H_1 = H_{3,0} - Q_1 \tag{22}$$

$$H_i = H_{(i-1)} - Q_i \text{ , } for\ i = 2,\ 3,\ \ldots, L \tag{23}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{24}$$

3. Cost analysis and solution

3.1. Cost analysis

TC(*T*), the total system cost in a replenishment cycle comprises the costs incurred in both stages as follows: (i) in stage one: the outsourcing and in-house production variable and setup costs, and in-house rework, disposal, and stock holding costs; (ii) in stage two: the sum of the fabrication variable, setup, rework, disposal, and stock holding costs for *L* different end products. Hence, *TC*(*T*) is

$$TC(T)=C_{\pi 0}\left[\pi_0\left(\sum_{i=1}^{L}Q_i\right)\right]+K_{\pi 0}+C_0Q_0+K_0+C_{R,0}\left[x_0\left(1-\theta_{1,0}\right)Q_0\right]+C_{S,0}\left(x_0\varphi_0Q_0\right)$$
$$+h_{2,0}\left(\frac{d_{1,0}t_{1,0}\left(1-\theta_{1,0}\right)}{2}\right)\left(t_{2,0}\right)+h_{1,0}\left[\frac{H_{1,0}t_{1,0}}{2}+\frac{H_{2,0}+H_{1,0}}{2}\left(t_{2,0}\right)+\frac{d_{1,0}t_{1,0}}{2}\left(t_{1,0}\right)+\sum_{i=1}^{L}\left[\frac{Q_i}{2}\left(t_{1,i}\right)+H_i\left(t_{1,i}+t_{2,i}\right)\right]\right]$$
$$+h_{4,0}\left(x_0\varphi_0Q_0\right)T+\sum_{i=1}^{L}\left\{\begin{array}{l}C_iQ_i+K_i+C_{R,i}\left[x_i\left(1-\theta_{1,i}\right)Q_i\right]+C_{S,i}\left(x_i\varphi_iQ_i\right)+h_{2,i}\left(\frac{d_{1,i}t_{1,i}\left(1-\theta_{1,i}\right)}{2}\right)\left(t_{2,i}\right)\\+h_{1,i}\left[\frac{H_{1,i}t_{1,i}}{2}+\frac{H_{2,i}+H_{1,i}}{2}\left(t_{2,i}\right)+\frac{H_{2,i}}{2}\left(t_{3,i}\right)+\frac{d_{1,i}t_{1,i}}{2}\left(t_{1,i}\right)\right]+h_{4,i}\left(x_i\varphi_iQ_i\right)T\end{array}\right\} \tag{25}$$

$E[TCU(T)]$ can be obtained after extra derivation efforts (see Appendix A for details) as follows:

$$E\left[TCU(T)\right]=\left\{\begin{array}{l}\frac{K_{\pi 0}}{T}+C_{\pi 0}\left[\pi_0\lambda_0\right]+C_0\left(1-\pi_0\right)\lambda_0E_{00}+\frac{K_0}{T}+C_{R,0}\left(1-\pi_0\right)\lambda_0\left(1-\theta_{1,0}\right)E_{10}+C_{S,0}\left(1-\pi_0\right)\lambda_0\varphi_0E_{10}\\+\frac{h_{2,0}\lambda_0{}^2T}{2}\left(1-\pi_0\right)^2\left(E_{10}\right)^2\left[\frac{\left(1-\theta_{1,0}\right)^2}{P_{2,0}}\right]+\frac{h_{1,0}\lambda_0{}^2T}{2}\left(1-\pi_0\right)^2\left(E_{00}\right)^2E_{0P}+h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i{}^2T}{2P_{1,i}}E_{0i}{}^2\right]\\+h_{1,0}\sum_{i=1}^{L}\left[\left(\frac{\lambda_iT}{P_{1,i}}E_{0i}+\frac{\lambda_iT\left(1-\theta_{1,i}\right)}{P_{2,i}}E_{1i}\right)\cdot\left(\sum_{i=1}^{L}\left(\lambda_iE_{0i}\right)-\sum_{j=1}^{i}\left(\lambda_jE_{0j}\right)\right)\right]+h_{4,0}\lambda_0\varphi_0T\left(1-\pi_0\right)E_{10}\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{array}{l}C_i\lambda_iE_{0i}+\frac{K_i}{T}+C_{R,i}\lambda_i\left(1-\theta_{1,i}\right)E_{1i}+C_{S,i}\lambda_i\varphi_iE_{1i}+\frac{h_{2,i}\lambda_i{}^2T}{2}\left(E_{1i}\right)^2\left[\frac{\left(1-\theta_{1,i}\right)^2}{P_{2,i}}\right]\\+\frac{h_{1,i}}{2}\lambda_i{}^2T\left[\frac{1}{\lambda_i}-\frac{E_{0i}{}^2\left(1-2\varphi_iE\left[x_i\right]\right)}{P_{1,i}}-\frac{E_{1i}{}^2\left(1-\theta_{1,i}\right)\left(1-\varphi_i\right)}{P_{2,i}}\right]+h_{4,i}T\lambda_i\varphi_iE_{1i}\end{array}\right\} \tag{26}$$

3.2. The common production cycle time solution

Apply the 1st and 2nd derivatives of $E[TCU(T)]$, one obtains:

$$\frac{dE\left[TCU(T)\right]}{dT}=\left\{\begin{array}{l}-\frac{K_{\pi 0}}{T^2}-\frac{K_0}{T^2}+\frac{h_{2,0}\lambda_0{}^2}{2}\left(1-\pi_0\right)^2\left(E_{10}\right)^2\left[\frac{\left(1-\theta_{1,0}\right)^2}{P_{2,0}}\right]+h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i{}^2}{2P_{1,i}}E_{0i}{}^2\right]\\+\frac{h_{1,0}\lambda_0{}^2}{2}\left(1-\pi_0\right)^2\left(E_{00}\right)^2E_{0P}+h_{4,0}\lambda_0\varphi_0\left(1-\pi_0\right)E_{10}\\+h_{1,0}\sum_{i=1}^{L}\left[\left(\frac{\lambda_i}{P_{1,i}}E_{0i}+\frac{\lambda_i\left(1-\theta_{1,i}\right)}{P_{2,i}}E_{1i}\right)\cdot\left(\sum_{i=1}^{L}\left(\lambda_iE_{0i}\right)-\sum_{j=1}^{i}\left(\lambda_jE_{0j}\right)\right)\right]\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{array}{l}-\frac{K_i}{T^2}+\frac{h_{2,i}\lambda_i{}^2}{2}\left(E_{1i}\right)^2\left[\frac{\left(1-\theta_{1,i}\right)^2}{P_{2,i}}\right]+h_{4,i}\lambda_i\varphi_iE_{1i}\\+\frac{h_{1,i}}{2}\lambda_i{}^2\left[\frac{1}{\lambda_i}-\frac{E_{0i}{}^2\left(1-2\varphi_iE\left[x_i\right]\right)}{P_{1,i}}-\frac{E_{1i}{}^2\left(1-\theta_{1,i}\right)\left(1-\varphi_i\right)}{P_{2,i}}\right]\end{array}\right\} \tag{27}$$

$$\frac{d^2E\left[TCU(T)\right]}{dT^2}=\frac{2K_{\pi 0}}{T^3}+\frac{2K_0}{T^3}+\sum_{i=1}^{L}\left\{\frac{2K_i}{T^3}\right\}>0 \tag{28}$$

In Eq. (28), because the setup costs $K_{\pi 0}$, K_0, K_i, and T are all positive, thus, $E[TCU(T)]$ is convex. It follows that by letting Eq. (26) = 0, the following optimal T^* is gained:

$$T^* = \sqrt{\frac{K_0\left(2+\beta_{1,0}\right)+\sum_{i=1}^{L}K_i}{\frac{h_{2,0}\lambda_0^{\,2}}{2}\left(1-\pi_0\right)^2\left(E_{10}\right)^2\left[\frac{\left(1-\theta_{1,0}\right)^2}{P_{2,0}}\right]+\frac{h_{1,0}\lambda_0^{\,2}}{2}\left(1-\pi_0\right)^2\left(E_{00}\right)^2 E_{0P}+h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i^{\,2}}{2P_{1,i}}E_{0i}^{\,2}\right] +h_{1,0}\sum_{i=1}^{L}E_{0ij}+h_{4,0}\lambda_0\varphi_0\left(1-\pi_0\right)E_{10}+\sum_{i=1}^{L}\left\{\frac{h_{2,i}\lambda_i^{\,2}}{2}\left(E_{1i}\right)^2\left[\frac{\left(1-\theta_{1,i}\right)^2}{P_{2,i}}\right]+\frac{h_{1,i}}{2}\lambda_i^{\,2}E_{2i}+h_{4,i}\lambda_i\varphi_i E_{1i}\right\}}} \tag{29}$$

where E_{0ij} and E_{2i} denote the following:

$$E_{0ij}=\left[\left(\frac{\lambda_i}{P_{1,i}}E_{0i}+\frac{\lambda_i\left(1-\theta_{1,i}\right)}{P_{2,i}}E_{1i}\right)\cdot\left(\sum_{i=1}^{L}\left(\lambda_i E_{0i}\right)-\sum_{j=1}^{i}\left(\lambda_j E_{0j}\right)\right)\right]\ for\ \ i=1,...,L;\ for\ \ j=1,...,i$$

$$E_{2i}=\left[\frac{1}{\lambda_i}-\frac{E_{0i}^{\,2}\left(1-2\varphi_i E\left[x_i\right]\right)}{P_{1,i}}-\frac{E_{1i}^{\,2}\left(1-\theta_{1,i}\right)\left(1-\varphi_i\right)}{P_{2,i}}\right]\ for\ \ i=1,...,L.$$

Lastly, if the sum of the setup times S_i is larger than the idle time (refer to Fig. 1) in T^*, then, one should further compute the $T_{\min}$ (as presented in Nahmias (2009)) and choose the maximum of (T^*, $T_{\min}$) as “the final cycle length” solution for the proposed problem to confirm that there is sufficient time for the setup, fabrication, and rework of the common parts and end products in both stages..

3.3. Prerequisite condition of this study

The following prerequisite situation must be true to ensure the machine has adequate capacity to produce and rework the common parts and L different end products in this study (Nahmias, 2009).

$$\left(\left(t_{1,0}+t_{2,0}\right)+\sum_{i=1}^{L}\left(t_{1,i}+t_{2,i}\right)\right)<T\ \text{ or}$$
$$\left[Q_0\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]\left(1-\theta_{1,0}\right)}{P_{2,0}}\right)+\sum_{i=1}^{L}Q_i\left(\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]\left(1-\theta_{1,i}\right)}{P_{2,i}}\right)\right]<T \tag{30}$$

or

$$\left\{\left(\frac{\lambda_0\left(1-\pi_0\right)}{\left[1-\varphi_0 E\left[x_0\right]\right]}\right)\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]\left(1-\theta_{1,0}\right)}{P_{2,0}}\right)+\sum_{i=1}^{L}\left(\frac{\lambda_i}{\left[1-\varphi_i E\left[x_i\right]\right]}\right)\left[\frac{1}{P_{1,i}}+\frac{E\left[x_i\right]\left(1-\theta_{1,i}\right)}{P_{2,i}}\right]\right\}<\ 1 \tag{31}$$

4. Example

Suppose that five distinct products with different annual demand rates must be satisfied by a two-stage batch fabrication plan featuring delayed product differentiation, outsourcing, and quality assurance. In the first production stage, where common parts for these distinct products are produced, the following relevant parameters’ values (in Table 1) are used:

Table 1
Parameters’ values used in the first production stage

π_0	$P_{1,0}$	$P_{2,0}$	x_0	$C_{S,0}$	$h_{1,0}$	$h_{2,0}$	$\beta_{1,0}$	$\beta_{2,0}$	γ
0.4	120000	96000	2.5%	$10	$8	$8	-0.7	0.4	0.5
K_0	λ_0	C_0	$C_{R,0}$	i_0	$\theta_{1,0}$	$\theta_{2,0}$	φ_0	δ	$h_{4,0}$
$8500	17406	$40	$25	0.2	4.6%	4.6%	9.0%	0.5	$8

Tables 2 and 3 exhibits the parameters’ values used in the second production stage for the distinct end products. While the parameters’ values used of the same problem under a single-stage production scheme are shown in Tables B-1 & B-2 (refer to Appendix B).

Table 2
Parameters' values used in the second production stage (1 of 2)

Product i	K_i	x_i	$P_{1,i}$	i_i	$h_{1,i}$	$\theta_{1,i}$	$h_{4,i}$
1	\$8500	2.5%	112258	0.2	\$16	4.6%	\$16
2	\$9000	7.5%	116066	0.2	\$18	9.4%	\$18
3	\$9500	12.5%	120000	0.2	\$20	14.6%	\$20
4	\$10000	17.5%	124068	0.2	\$22	20.0%	\$22
5	\$10500	22.5%	128276	0.2	\$24	25.8%	\$24

Table 3
Parameters' values used in the second production stage (2 of 2)

Product i	λ_i	C_i	$P_{2,i}$	$C_{R,i}$	$h_{2,i}$	$C_{S,i}$	$\theta_{2,i}$	φ_i
1	3000	\$40	89806	\$25	\$16	\$10	4.6%	9.0%
2	3200	\$50	92852	\$30	\$18	\$15	9.4%	18.0%
3	3400	\$60	96000	\$35	\$20	\$20	14.6%	27.0%
4	3600	\$70	99254	\$40	\$22	\$25	20.0%	36.0%
5	3800	\$80	102621	\$45	\$24	\$30	25.8%	45.0%

Apply Eqs. (29) and (26), the optimal solutions $T^* = 0.5541$ and $E[TCU(T^*)] = \$2,138,414$ are gained. Fig. 5 explicitly depicts the convexity of $E[TCU(T)]$ with regard to changes in T^*. Obviously, it illustrates that as the cycle length T deviates from T_A^*, $E[TCU(T)]$ knowingly increases both ways.

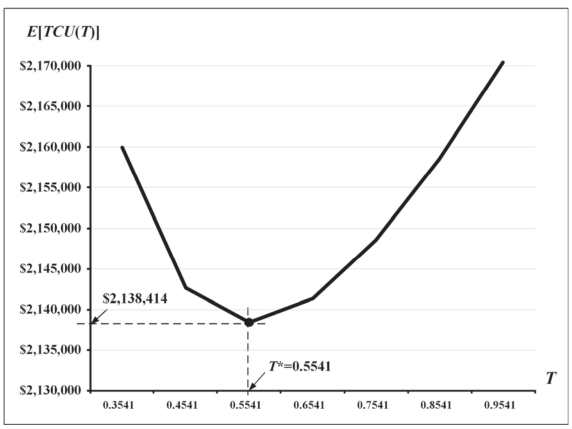

Fig. 5. The behavior of $E[TCU(T)]$ regarding T

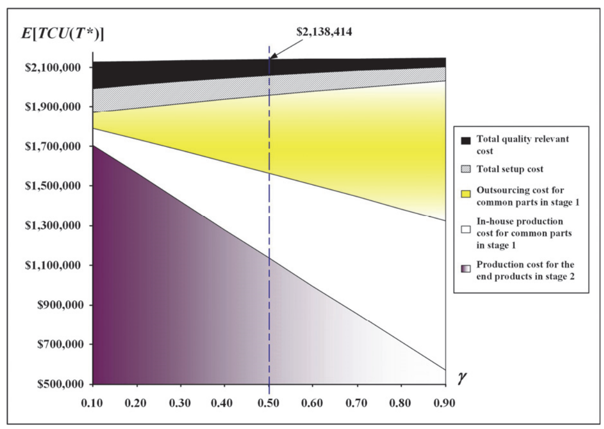

Fig. 6. The behavior of cost contributors to $E[TCU(T^*)]$ relating to γ

The behavior of cost contributors to $E[TCU(T^*)]$ relating to γ (the common part's completion rate) is depicted in Fig. 6. When $\gamma = 0.5$ as assumed in our example, $E[TCU(T^*)] = \$2,138,414$; as γ rises and since π_0 (the outsourcing portion of common parts) is set at 0.4, both the outsourcing cost and in-house production cost (i.e., the other $1 - \pi_0$ portion) for the common parts in stage 1 upsurges significantly. Conversely, as γ rises, the production cost for the end products in stage 2 declines severely; and the total quality relevant cost decreases reasonably because of the decreased number of items made in-house.

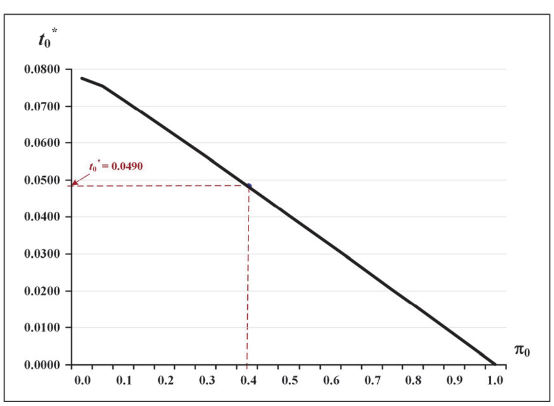

Fig. 7. The effect of differences in common-part outsourcing portions π_0 on t_0^*

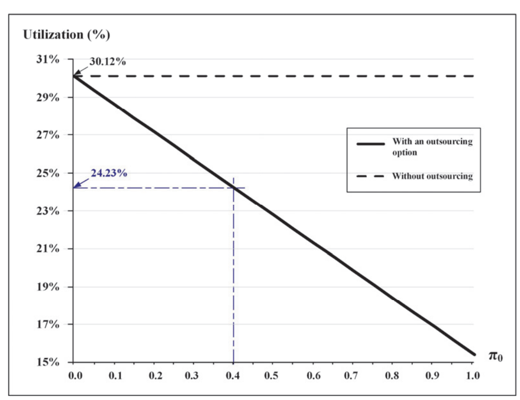

Fig. 8. The behavior of machine utilization (in percentage) vis-à-vis π_0

The impact of changes in outsourcing portion π_0 on diverse parameters of the system has been extensively explored and the results are depicted in Tables C-1 and C-2 (see Appendix C). From there, numerous in-depth characteristics of the system are exposed. For example, the effect of differences in π_0 on t_0^* is exhibited in Fig. 7. For $\pi_0 = 0.4$, the sum of the optimal in-house fabrication uptime and rework time t_0^* drops from 0.0785 to 0.0490 (year), which is a decrease of 37.58% (refer to Table C-1)); and as π_0 rises, t_0^* declines significantly. Fig. 8 shows the behavior of machine overall utilization (in percentage) vis-à-vis π_0 (the outsourcing portion of common parts). For $\gamma = 0.5$ and $\pi_0 = 0.4$ (as set in our example) the utilization drops from 30.12% to 24.23% (i.e., a 19.57% declines in utilization; please refer to Table C-1), and as π_0 rises, the utilization decreases knowingly. The behavior of cost contributors to $E[TCU(T^*)]$ concerning π_0 is portrayed in Fig. 9. For $\pi_0 = 0.4$ (as set in our example), $E[TCU(T^*)]$ = \$2,138,414, which is an 5.42% increase in the system cost compared to \$2,028,449 (when $\pi_0 = 0$; please refer to Table C-2), but with the benefit of a 19.57% declines in machine utilization as mentioned earlier (see Fig. 7). As π_0 rises, the outsourcing cost for common parts upsurges significantly; conversely, the in-house production cost for the other $(1 - \pi_0)$ common parts decreases extensively. All other cost contributors to $E[TCU(T^*)]$ have a trivial impact concerning π_0.

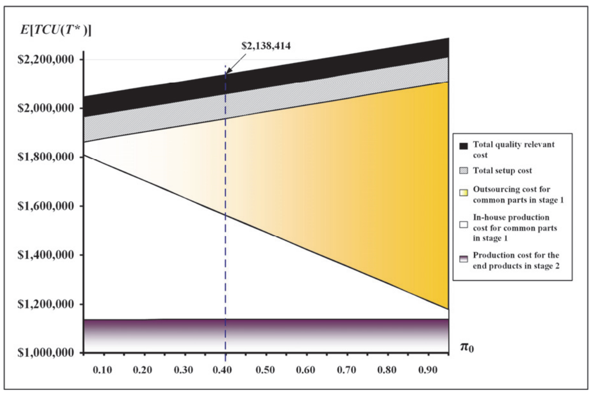

Fig. 9. The behavior of cost contributors to $E[TCU(T^*)]$ in relation to π_0

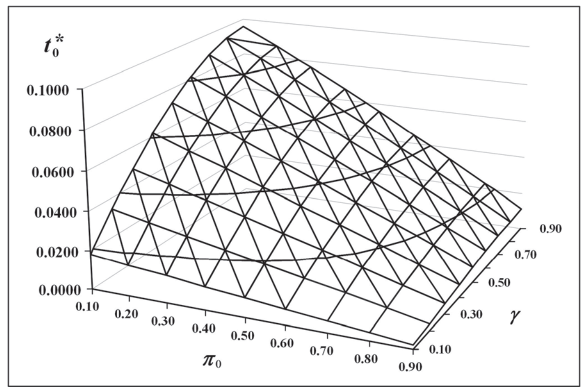

Fig. 10. The collective impact of π_0 and γ on the sum of optimal uptime and rework time t_0^*

The collective impact of outsourcing ratio π_0 and the common part's completion rate γ on t_0^* are analyzed and exhibited in Fig. 10. It reveals that as π_0 rises, t_0^* declines knowingly; especially when γ goes higher, t_0^* drops severely. Also, as γ increases, t_0^* rises accordingly; t_0^* surges drastically when π_0 is lower.

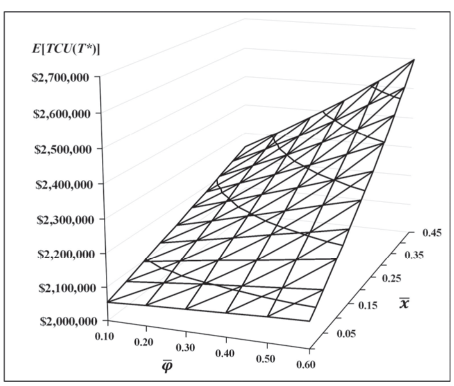

Fig. 11. The behavior of $E[TCU(T^*)]$ concerning the collective impact from and $\bar{x}$ and $\bar{\varphi}$

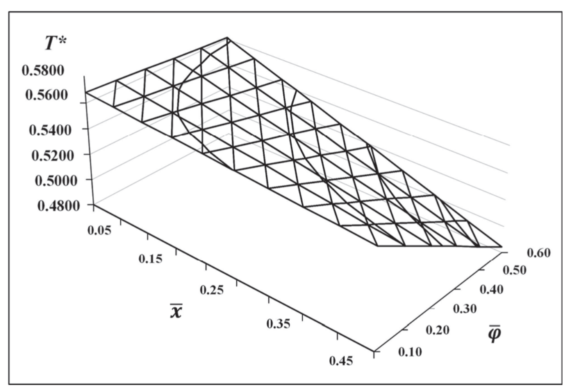

Fig. 12. The behavior of optimal T^* regarding the collective impact from and $\bar{x}$ and $\bar{\varphi}$

Figs. 11 and 12 illustrate the behavior of $E[TCU(T^*)]$ and T^* concerning the collective impact from $\bar{x}$ and $\bar{\varphi}$. Fig. 11 shows that as $\bar{x}$ rises, $E[TCU(T^*)]$ increases significantly and as $\bar{\varphi}$ increases, $E[TCU(T^*)]$ goes up accordingly. As both the $\bar{\varphi}$ and $\bar{x}$ rise, $E[TCU(T^*)]$ upsurges tremendously. Fig. 12 exposes that as $\bar{x}$ rises, T^* declines knowingly; and as $\bar{\varphi}$ surges, T^* decreases slightly. As both the quality-relevant factors increase, T^* drops hugely.

Fig. 13 exhibits the impact of differences in the common part's completion rate γ on the sum of optimal uptimes $(t_0^* + t_i^*)$. For $\gamma = 0.5$, the sum of the optimal uptimes $(t_0^* + t_i^*)$ declines from 0.2250 years to 0.1343, which is a decrease of 40.34%; and as γ rises, $(t_0^* + t_i^*)$ declines considerably due to the outsourcing strategy on common parts in stage one.

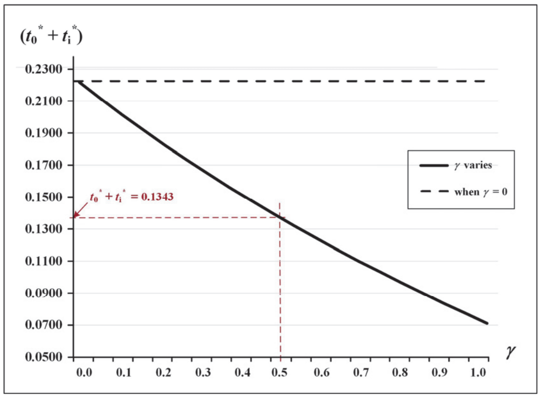

Fig. 13. Impact of changes in common part's completion rate γ on $(t_0^* + t_i^*)$

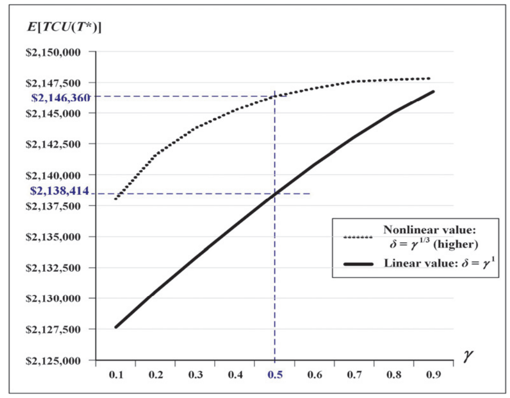

Fig. 14. The effect of linear and nonlinear relationships between δ and γ on $E[TCU(T^*)]$

Although in the example, we assume that the value of the common part is linearly related to its completion rate (i.e., $\delta = \gamma$), any other type of relationship between δ and γ can also be analyzed using our proposed model. Fig. 14 demonstrates the effect of the nonlinear relationship between δ and γ on $E[TCU(T^*)]$ as compared to the linear one. It specifies the values of $E[TCU(T^*)]$ at $\gamma = 0.5$ for both $\delta = \gamma^1$ (linear) and $\delta = \gamma^{1/3}$ (nonlinear) relationships. The sensitivity analysis of the influence of linear/nonlinear relationship between δ and γ on the decision variable T is explored and the outcome is illustrated in Fig. 15. It shows the values of T at $\gamma = 0.5$ for both $\delta = \gamma^1$ and $\delta = \gamma^{1/3}$.

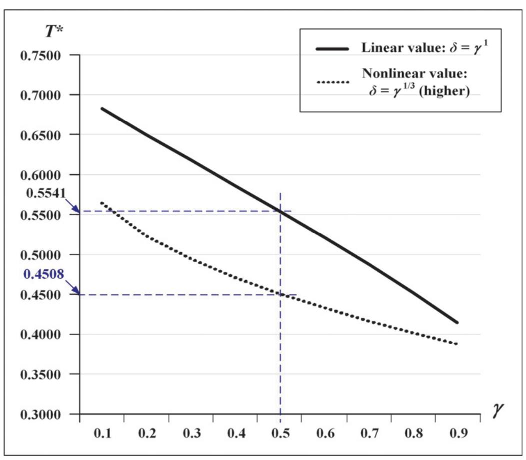

Fig. 15. Influence of linear/nonlinear relationship between δ and γ on T^*

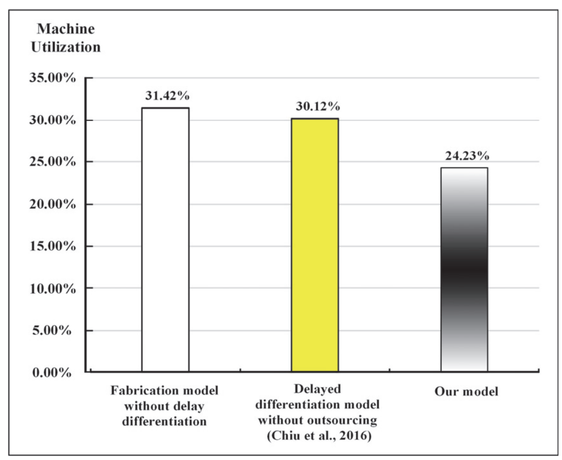

Fig. 16. Comparison of the utilization of our model with that in the similar models

Figs. 16 and 17 compare the utilization and $E[TCU(T^*)]$ of our model with that of similar models. From Fig. 16, the outsourcing option in the first stage allows our utilization (i.e., $(t_0^* + t_i^*) / T^*$) declines to 24.23% from 30.12%, which is a 19.57% decrease as compared to that of an existing delayed differentiation model without outsourcing (Chiu et al., 2016). Moreover, our utilization drops 22.88% as compared to a no delayed-differentiation fabrication model.

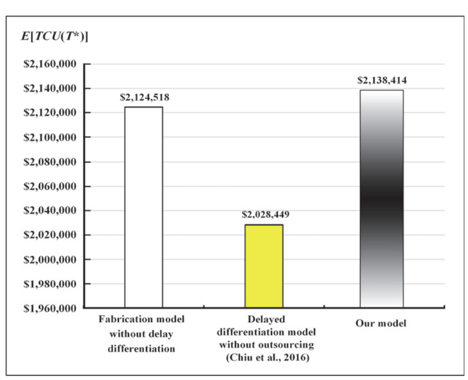

Fig. 17. Comparison of $E[TCU(T^*)]$ of our model with that in the similar models

Fig. 17 indicates that for a reduction of 19.57% in utilization, $E[TCU(T^*)]$ rises to $2,138,414 from $2,028,449, which is a 5.42% increase as compared to that of Chiu et al. (2016). Moreover, for a decline of 22.88% in utilization, our model pays the price of 0.65% in $E[TCU(T^*)]$ increase, as compared to a no delayed-differentiation fabrication model.

5. Conclusions

Inspired by the potential benefits of delay differentiation, outsourcing, and quality assurance policies in the multi-item production planning, this study explores a single-machine two-stage multi-item batch fabrication problem considering the abovementioned features. We develop a model to depict the problem explicitly (refer to subsection 2.2 and Fig. 1) and use modeling, formulation, derivation, and optimization methods to assist us in determining a cost-minimized cycle time solution (see from subsection 2.3 to 3.2). With the help of the proposed model, the diverse characteristics of the problem can now be disclosed to facilitate managerial decision-making. An example of this is the individual/collective influence of postponement, outsourcing, and quality reassurance policies on the optimal cycle time solution, utilization, uptime of each stage, total system cost, and individual cost contributors (refer to Figs. 5 to 15). Furthermore, we demonstrate how the proposed fabrication scheme with outsourcing and quality assurance policies outperforms other existing models (see Figs. 16 to 17). For future study, incorporating a discontinuous shipping policy for end products into the same context of this model will be an interesting subject.

Acknowledgments

The authors truly express appreciation to the Ministry of Science and Technology of Taiwan for supporting this project (grant#: MOST 107-2410-H-324-002).

References

Abashar, M.E.E. (2018). Ultra-clean hydrogen production by ammonia decomposition. *Journal of King Saud University - Engineering Sciences, 30*(1), 2-11.

Alvarez, L.H.R., & Stenbacka, R. (2007). Partial outsourcing: A real options perspective. *International Journal of Industrial Organization, 25*(1), 91-102.

Aringhieri, R., Duma, D., & Fragnelli, V. (2018). Modeling the rational behavior of individuals on an e-commerce system. *Operations Research Perspectives, 5,* 22-31.

Babu, K.V., Anuradha, B. (2020). Design of multi-band minkowski MIMO antenna to reduce the mutual coupling. *Journal of King Saud University - Engineering Sciences, 32*(1), 51-57.

Berrade, M.D., Scarf, P.A., & Cavalcante, C.A.V. (2017). A study of postponed replacement in a delay time model. *Reliability Engineering & System Safety, 168*, 70-79.

Calabria, R., & Pulcini, G. (1999). Discontinuous point processes for the analysis of repairable units. *International Journal of Reliability, Quality and Safety Engineering, 6*(4), 361-382.

Chiu, S.W., Kuo, J-S., Chiu, V., & Chiu, Y-S.P. (2016). Cost minimization for a multi-product fabrication-distribution problem with commonality, postponement, and quality assurance. *Mathematical and Computational Applications, 21* (3)*,* 1-17.

Chiu, Y-S.P., Lin, H-D., Wu, M-F., & Chiu, S.W. (2018). Alternative fabrication scheme to study effects of rework of nonconforming products and delayed differentiation on a multiproduct supply-chain system. *International Journal of Industrial Engineering Computations, 9*(2), 235-248.

Chiu, S.W., Kuo, J-S., Chiu, Y-S.P., & Chang, H-H. (2019a). Production and distribution decisions for a multi-product system with component commonality, postponement strategy and quality assurance using a two-machine scheme. *Jordan Journal of Mechanical & Industrial Engineering, 13*(2), 105-115.

Chiu, Y-S.P., Chiu, V., Lin, H-D., & Chang, H-H. (2019b). Meeting multiproduct demand with a hybrid inventory replenishment system featuring quality reassurance. *Operations Research Perspectives, 6, Art. No. 100112*, 1-8.

Chiu, Y-S.P., Chen, H-C., Chang, H-H., & Hwang, M-H. (2019c). Determining rotation cycle and distribution frequency for a vendor-buyer integrated multi-item system considering an external provider and rework. *International Journal of Industrial Engineering Computations, 10*(4), 505-520.

Chiu, Y-S.P., Chiu, V., Wang, Y., & Hwang, M-H. (2020). A postponement model for multi-item replenishment decision considering overtime, commonality, and quality reassurance. *International Journal of Industrial Engineering Computations, 11*(4), 509-524

Davis, T., & Sasser, M. (1995). Postponing product differentiation. *Mechanical Engineering*, *117*(11), 105-107.

Hammado, N., Utomo, S., & Budiyono (2020). Characteristic lignocellulose of sago solid waste for biogas production. *Journal of Applied Engineering Science, 18*(2), 157-164.

Hines, P., & Rich, N. (1998). Outsourcing competitive advantage: the use of supplier associations. *International Journal of Physical Distribution and Logistics Management, 28*(7), 524-546.

Iqbal, A., Ahmad, S.A., & Ojasvi (2019). Appropriate selections of distillation column control variable to improve integrating material recycle response. *Journal of King Saud University - Engineering Sciences, 31*(2)*,* 106-113.

Istotskiy, V., & Protasev, V. (2019). Design and manufacture of hob mills for the formation of straight slots using the principles of screw backing. *International Journal of Mathematical, Engineering and Management Sciences, 4*(4)*,* 936-945.

Kopel, M., Löffler, C., & Pfeiffer, T. (2016). Sourcing strategies of a multi-input-multi-product firm. *Journal of Economic Behavior & Organization, 127*, 30-45.

Labro, E. (2004). The cost effects of component commonality: A literature review through a management-accounting lens, *Manufacturing and Service Operations Management*, *6*(4), 358-367.

Larkin, E.V., & Privalov, A.N. (2019). Engineering method of fault -tolerant system simulations. *Journal of Applied Engineering Science, 17*(3), 295-303.

Le Pape, N., & Wang, Y. (2020). Can product differentiation mitigate conflicts of interest between principal stakeholders? A cournot-bertrand comparison. *Economic Modelling, 91*, 341-346.

Lee, I.S., & Sung, C.S. (2008). Minimizing due date related measures for a single machine scheduling problem with outsourcing allowed**.** *European Journal of Operational Research, 186*(3), 931-952.

Lim, S.-K. (2020). Sequential batching with minimum quantity commitment in N-level non-exclusive agglomerative hierarchical clustering structures. *International Journal of Industrial Engineering Computations, 11*(3), 317-340.

Moshtagh, M.S., & Taleizadeh, A.A. (2017). Stochastic integrated manufacturing and remanufacturing model with shortage, rework and quality based return rate in a closed loop supply chain. *Journal of Cleaner Production, 141*(10), 1548-1573.

Nahmias, S. (2009). *Production & Operations Analysis.* McGraw-Hill Inc. New York.

Ngniatedema, T., Kamga, T.R.D., Fono, L.A., & Mbondo, G.D., Li, S. (2012). Postponement and international transfer in global supply chains, *Journal of Operations Management, 30,* 167-179.

Noman, M.A., Nasr, E.S.A., Al-Shayea, A., & Kaid, H. (2019). Overview of predictive condition based maintenance research using bibliometric indicators. *Journal of King Saud University - Engineering Sciences, 31*(4), 355-367.

Ortiz-Servin, J.J., Castillo, A., & Talavera, F. (2019). A new methodology to optimize operation cycles in a bwr using heuristic techniques. *Journal of Applied Research and Technology, 17*(3), 155-162.

Pearce, A., Pons, D., & Neitzert, T. (2018). Implementing lean - outcomes from SME case studies. *Operations Research Perspectives, 5,* 94-104.

Rao, A.S., & Singh, A.K. (2018). Failure analysis of stainless steel lanyard wire rope. *Journal of Applied Research and Technology, 16*(1)*,* 35-40.

Sarker, B.R., Jamal, A.M.M., & Mondal, S. (2008). Optimal batch sizing in a multi-stage production system with rework consideration. *European Journal of Operational Research, 184*(3)*,* 915-929.

Sheikh, S., Komaki, G.M., Kayvanfar, V., & Teymourian, E. (2019). Multi-Stage assembly flow shop with setup time and release time. *Operations Research Perspectives, 6,* Art. No. 100111, 1-15.

Sonntag, D., & Kiesmüller, G.P. (2018). Disposal versus rework– Inventory control in a production system with random yield. *European Journal of Operational Research, 267*(1), 138-149.

Tannady, H., Gunawan, E., Nurprihatin, F., & Wilujeng, F.R. (2019). Process improvement to reduce waste in the biggest instant noodle manufacturing company in South East Asia. *Journal of Applied Engineering Science, 17*(2), 203-212.

Vaid, M.K., & Arora, G. (2019). Quintic B-spline technique for numerical treatment of third order singular perturbed delay differential equation. *International Journal of Mathematical, Engineering and Management Sciences, 4*(6), 1471-1482.

Van Mieghem, J.A. (1999). Coordinating investment, production and subcontracting. *Management Science, 45,* 954-971.

Westphal, P., & Sohal, A. (2016). Outsourcing decision-making: does the process matter? *Production Planning & Control, 27* (11), 894-908.

Zammori, F., Bertolini, M., & Mezzogori, D. (2020). A constructive algorithm to maximize the useful life of a mechanical system subjected to ageing, with non-resuppliable spares parts. *International Journal of Industrial Engineering Computations, 11*(1), 17-34.

Appendix - A

Details for the derivations of Eq. (26).
$E[TCU(T)]$ can be gained by the following steps: (i) applying the expected values $E[x_0]$ and $E[x_i]$ to cope with the randomness of the nonconforming common parts and the end product i, respectively; and (ii) substituting Eqs. (3) to (24) in Eq. (25) and calculating $E[TC(T)]/E[T]$. With extra derivation efforts, $E[TCU(T)]$ is obtained as shown in Eq. (A-1).

$$E\left[TCU(T))\right]=\left\{\begin{array}{l}\frac{K_{\pi 0}}{T}+C_{\pi 0}\left[\pi_0\lambda_0\right]+C_0\frac{(1-\pi_0)\lambda_0}{1-\varphi_0E[x_0]}+\frac{K_0}{T}+C_{R,0}\left[E[x_0](1-\theta_{1,0})\right]\left[\frac{(1-\pi_0)\lambda_0}{1-\varphi_0E[x_0]}\right]+h_{4,0}\varphi_0\lambda_0T\left[\frac{(1-\pi_0)E[x_0]}{1-\varphi_0E[x_0]}\right]\\ +\frac{h_{2,0}\lambda_0^{\,2}T}{2}\frac{(1-\pi_0)^2}{(1-\varphi_0E[x_0])^2}\left[\frac{E[x_0]^2(1-\theta_{1,0})^2}{P_{2,0}}\right]+h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i^{\,2}T}{2P_{1,i}}\frac{1}{(1-\varphi_iE[x_i])^2}\right]+C_{S,0}\left(E[x_0]\varphi_0\right)\left[\frac{(1-\pi_0)\lambda_0}{1-\varphi_0E[x_0]}\right]\\ +\frac{h_{1,0}\lambda_0^{\,2}T}{2}\frac{(1-\pi_0)^2}{(1-\varphi_0E[x_0])^2}\left[\frac{1}{P_{1,0}}+\frac{E[x_0](1-\theta_{1,0})(1-E[x_0]\varphi_0)}{P_{2,0}}+\frac{E[x_0](1-\theta_{1,0})(1-E[x_0])}{P_{2,0}}\right]\\ +h_{1,0}\sum_{i=1}^{L}\left[\left(\frac{\lambda_iT}{P_{1,i}}\frac{1}{1-\varphi_iE[x_i]}+\frac{\lambda_iTE[x_i](1-\theta_{1,i})}{P_{2,i}}\frac{1}{1-\varphi_iE[x_i]}\right)\left(\sum_{i=1}^{L}\frac{\lambda_i}{1-\varphi_iE[x_i]}-\sum_{j=1}^{i}\frac{\lambda_j}{1-\varphi_jE[x_j]}\right)\right]\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{array}{l}C_i\lambda_i\frac{1}{1-\varphi_iE[x_i]}+\frac{K_i}{T}+C_{R,i}\lambda_i\left[\frac{E[x_i](1-\theta_{1,i})}{1-\varphi_iE[x_i]}\right]+C_{S,i}\lambda_i\left[\frac{E[x_i]\varphi_i}{1-\varphi_iE[x_i]}\right]+\frac{h_{2,i}}{2}\frac{\lambda_i^{\,2}T}{(1-\varphi_iE[x_i])^2}\left[\frac{E[x_i]^2(1-\theta_{1,i})^2}{P_{2,i}}\right]\\ +\frac{h_{1,i}}{2}\frac{\lambda_i^{\,2}T}{(1-\varphi_iE[x_i])^2}\left[\frac{(1-E[x_i]\varphi_i)^2}{\lambda_i}-\frac{(1-2\varphi_iE[x_i])}{P_{1,i}}-\frac{E[x_i]^2(1-\theta_{1,i})(1-\varphi_i)}{P_{2,i}}\right]+h_{4,i}T\varphi_i\lambda_i\left(\frac{E[x_i]}{1-\varphi_iE[x_i]}\right)\end{array}\right\} \tag{A-1}$$

Let E_{00}, E_{10}, E_{0j}, E_{0P}, E_{0i}, and E_{1i} denote the following:

$$\begin{array}{l}E_{00}=\frac{1}{\left(1-\varphi_0E[x_0]\right)};\ E_{10}=\frac{E[x_0]}{\left(1-\varphi_0E[x_0]\right)};\ E_{0j}=\frac{1}{\left(1-\varphi_jE[x_j]\right)}\ \ for\ \ j=1,...,i\\ E_{0P}=\left[\frac{1}{P_{1,0}}+\frac{E[x_0](1-\theta_{1,0})(1-E[x_0]\varphi_0)}{P_{2,0}}+\frac{E[x_0](1-\theta_{1,0})(1-E[x_0])}{P_{2,0}}\right]\\ E_{0i}=\frac{1}{\left(1-\varphi_iE[x_i]\right)}\ \ for\ \ i=1,...,L;\ \ E_{1i}=\frac{E[x_i]}{\left(1-\varphi_iE[x_i]\right)}\ \ for\ \ i=1,...,L.\end{array} \tag{A-2}$$

Substitute Eq. (A-2) in Eq. (A-1), we find $E[TCU(T)]$ as follows:

$$E\left[TCU(T)\right]=\left\{\begin{array}{l}\frac{K_{\pi 0}}{T}+C_{\pi 0}\left[\pi_0\lambda_0\right]+C_0(1-\pi_0)\lambda_0E_{00}+\frac{K_0}{T}+C_{R,0}(1-\pi_0)\lambda_0(1-\theta_{1,0})E_{10}+C_{S,0}(1-\pi_0)\lambda_0\varphi_0E_{10}\\ +\frac{h_{2,0}\lambda_0^{\,2}T}{2}(1-\pi_0)^2(E_{10})^2\left[\frac{(1-\theta_{1,0})^2}{P_{2,0}}\right]+\frac{h_{1,0}\lambda_0^{\,2}T}{2}(1-\pi_0)^2(E_{00})^2E_{0P}+h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i^{\,2}T}{2P_{1,i}}E_{0i}^{\,2}\right]\\ +h_{1,0}\sum_{i=1}^{L}\left[\left(\frac{\lambda_iT}{P_{1,i}}E_{0i}+\frac{\lambda_iT(1-\theta_{1,i})}{P_{2,i}}E_{1i}\right)\cdot\left(\sum_{i=1}^{L}(\lambda_iE_{0i})-\sum_{j=1}^{i}(\lambda_jE_{0j})\right)\right]+h_{4,0}\lambda_0\varphi_0T(1-\pi_0)E_{10}\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{array}{l}C_i\lambda_iE_{0i}+\frac{K_i}{T}+C_{R,i}\lambda_i(1-\theta_{1,i})E_{1i}+C_{S,i}\lambda_i\varphi_iE_{1i}+\frac{h_{2,i}\lambda_i^{\,2}T}{2}(E_{1i})^2\left[\frac{(1-\theta_{1,i})^2}{P_{2,i}}\right]\\ +\frac{h_{1,i}}{2}\lambda_i^{\,2}T\left[\frac{1}{\lambda_i}-\frac{E_{0i}^{\,2}(1-2\varphi_iE[x_i])}{P_{1,i}}-\frac{E_{1i}^{\,2}(1-\theta_{1,i})(1-\varphi_i)}{P_{2,i}}\right]+h_{4,i}T\lambda_i\varphi_iE_{1i}\end{array}\right\} \tag{26}$$

Appendix - B

Table B-1
Parameters' values of the same problem under a single-stage production scheme -1

Product i	K_i	x_i	$P_{1,i}$	C_i	i	$h_{1,i}$	$C_{S,i}$	$h_{4,i}$
1	\$17000	5%	58000	\$80	0.2	\$16	\$20	\$16
2	\$17500	10%	59000	\$90	0.2	\$18	\$25	\$18
3	\$18000	15%	60000	\$100	0.2	\$20	\$30	\$20
4	\$18500	20%	61000	\$110	0.2	\$22	\$35	\$22
5	\$19000	25%	62000	\$120	0.2	\$24	\$40	\$24

Table B-2
Parameters' values of the same problem under a single stage production scheme -2

Product i	λ_i	$P_{2,i}$	$\theta_{1,i}$	$C_{R,i}$	$h_{2,i}$	$\theta_{2,i}$	φ_i
1	3000	46400	9.4%	$50	$16	9.4%	18.0%
2	3200	47200	14.6%	$55	$18	14.6%	27.0%
3	3400	48000	20.0%	$60	$20	20.0%	36.0%
4	3600	48800	25.8%	$65	$22	25.8%	45.0%
5	3800	49600	32.2%	$70	$24	32.2%	54.0%

Appendix - C

Table C-1
Impact of changes in π_0 on utilization & diverse system fabrication-related variables

π_0	t_0* (A)	(A)% decline	T*	Utilization (B)	(B) % decline	Total uptime (C)	(C) % decline	Total rework time (D)	(D) % decline
0.00	0.0785	-	0.5326	30.12%	-	0.1543	-	0.0061	-
0.05	0.0764	-2.61%	0.5460	29.39%	-2.45%	0.1542	-0.05%	0.0062	1.56%
0.10	0.0726	-7.50%	0.5474	28.65%	-4.89%	0.1506	-2.38%	0.0062	0.84%
0.15	0.0687	-12.44%	0.5486	27.91%	-7.34%	0.1470	-4.73%	0.0062	0.12%
0.20	0.0648	-17.40%	0.5499	27.18%	-9.78%	0.1433	-7.10%	0.0061	-0.63%
0.25	0.0609	-22.40%	0.5510	26.44%	-12.23%	0.1396	-9.50%	0.0061	-1.39%
0.30	0.0570	-27.43%	0.5521	25.70%	-14.68%	0.1359	-11.92%	0.0060	-2.16%
0.35	0.0530	-32.49%	0.5532	24.97%	-17.12%	0.1321	-14.36%	0.0060	-2.96%
0.40	**0.0490**	**-37.58%**	**0.5541**	**24.23%**	**-19.57%**	**0.1283**	**-16.82%**	**0.0059**	**-3.76%**
0.45	0.0450	-42.69%	0.5550	23.49%	-22.01%	0.1245	-19.30%	0.0059	-4.59%
0.50	0.0410	-47.82%	0.5558	22.76%	-24.46%	0.1207	-21.79%	0.0058	-5.43%
0.55	0.0369	-52.98%	0.5565	22.02%	-26.91%	0.1168	-24.31%	0.0058	-6.28%
0.60	0.0328	-58.15%	0.5572	21.28%	-29.35%	0.1129	-26.84%	0.0057	-7.15%
0.65	0.0288	-63.34%	0.5578	20.55%	-31.80%	0.1089	-29.39%	0.0057	-8.03%
0.70	0.0247	-68.55%	0.5583	19.81%	-34.25%	0.1050	-31.95%	0.0056	-8.93%
0.75	0.0206	-73.77%	0.5588	19.07%	-36.69%	0.1010	-34.53%	0.0055	-9.84%
0.80	0.0165	-79.00%	0.5591	18.33%	-39.14%	0.0970	-37.12%	0.0055	-10.77%
0.85	0.0124	-84.24%	0.5594	17.60%	-41.58%	0.0930	-39.72%	0.0054	-11.71%
0.90	0.0082	-89.49%	0.5596	16.86%	-44.03%	0.0890	-42.33%	0.0054	-12.66%
0.95	0.0041	-94.75%	0.5597	16.12%	-46.48%	0.0849	-44.95%	0.0053	-13.63%
1.00	0.0000	-	0.5176	15.39%	-48.92%	0.0748	-51.53%	0.0049	-21.05%

Table C-2
The impact of changes in π_0 on diverse system cost-relevant variables

π_0	E[$TCU(T^*)$] (E)	(E) % increase	Common part Outsourcing Cost (F)	(F) / (E) %	Common part Quality Relevant-cost	Common part Other production Relevant-cost	Finished items Quality Relevant-cost	Finished items Other production Relevant-ost
0.00	$2,028,449	-	0.00	0%	6175.83	723522.59	75754.46	1222996.51
0.05	$2,046,242	0.88%	$53,407	2.61%	5867.01	688044.65	75754.80	1223168.04
0.10	$2,059,326	1.52%	$102,132	4.96%	5558.18	652689.59	75754.84	1223191.36
0.15	$2,072,438	2.17%	$150,858	7.28%	5249.35	617361.31	75754.87	1223214.58
0.20	$2,085,578	2.82%	$199,585	9.57%	4940.53	582060.16	75754.90	1223237.53
0.25	$2,098,745	3.47%	$248,312	11.83%	4631.71	546786.51	75754.93	1223260.04
0.30	$2,111,940	4.12%	$297,039	14.06%	4322.89	511540.67	75754.96	1223281.94
0.35	$2,125,163	4.77%	$345,768	16.27%	4014.08	476322.99	75754.98	1223303.08
0.40	**$2,138,414**	**5.42%**	**$394,496**	**18.45%**	**3705.28**	**441133.74**	**75755.01**	**1223323.30**
0.45	$2,151,693	6.08%	$443,226	20.60%	3396.48	405973.22	75755.03	1223342.47
0.50	$2,165,001	6.73%	$491,956	22.72%	3087.68	370841.70	75755.05	1223360.46
0.55	$2,178,337	7.39%	$540,687	24.82%	2778.89	335739.41	75755.07	1223377.15
0.60	$2,191,702	8.05%	$589,418	26.89%	2470.11	300666.57	75755.09	1223392.42
0.65	$2,205,096	8.71%	$638,150	28.94%	2161.33	265623.39	75755.10	1223406.18
0.70	$2,218,519	9.37%	$686,883	30.96%	1852.55	230610.06	75755.11	1223418.33
0.75	$2,231,970	10.03%	$735,616	32.96%	1543.78	195626.72	75755.13	1223428.79
0.80	$2,245,451	10.70%	$784,350	34.93%	1235.01	160673.51	75755.13	1223437.50
0.85	$2,258,960	11.36%	$833,084	36.88%	926.25	125750.54	75755.14	1223444.39
0.90	$2,272,499	12.03%	$881,819	38.80%	617.50	90857.90	75755.15	1223449.43
0.95	$2,286,067	12.70%	$930,555	40.71%	308.75	55995.66	75755.15	1223452.59
1.00	$2,283,885	12.59%	$979,663	42.89%	0.00	5528.42	75754.08	1222939.15