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International Journal of Industrial Engineering Computations

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Solving a hybrid batch production problem with unreliable equipment and quality reassurance

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C H R O N I C L E

Article history:
Received February 18 2021
Received in Revised Format
March 31 2020
Accepted April 1 2021
Available online
April, 3 2021

Keywords:
Hybrid economic production quantity
Poisson-distributed breakdown
Random scrap
Rework
Outsourcing
Production planning

A B S T R A C T

A hybrid batch fabrication plan involving an outsourcing option is often established to deal with the in-house capacity constraint and/or meet timely demand with a reduced cycle time. Besides, the occurrences of unpredictable equipment malfunction and imperfect product quality should also be effectively managed during in-house fabrication to meet the production schedule and the required quality level. To address these concerns, we examine a hybrid economic production quantity (EPQ) problem with an unreliable machine and quality reassurance; wherein, an outside provider helps supply a portion of the batch at a requested timing and desirable quality, but at the price of a higher than in-house unit cost. Corrective action is performed immediately when a Poisson-distributed breakdown occurs. Once the equipment repairing task completes, the interrupted lot's fabrication resumes. Random nonconforming products are identified, and the re-workable items among them are separated from the scraps. A rework task follows the regular process in each cycle at an extra cost. A portion of reworked items fails and are scrapped. A model portraying the problem's characteristics is built, and an optimization methodology is utilized to find the optimal runtime solution to the problem. A numerical example reveals our result's applicability, and specific managerial implications are suggested.

1. Introduction

This study investigates a hybrid batch fabrication problem with Poisson-distributed equipment failures and rework/scrap of random nonconforming products. Taft (1918) determined the most economical manufacturing batch size under the assumption of a perfect fabrication process; neither machine failures nor defective items were considered, and all items are produced in-house. But in real fabrication environments, the occurrence of random equipment malfunction is inevitable. It must be effectively handled to avoid delay in the production schedule, and consequently, increase costs due to the idling of production. Khandelwal et al. (1979) utilized the optimal control theory to determine the best preventive maintenance (PM) plans for a production facility featuring deterioration and random breakdown. The authors then extended their solution procedure to a group of independently working machines subject to the same situations with related repairing tasks and successfully derived the optimal PM policies. Hagstrom and Mak (1987) studied the impact of dependent component failures on the accuracy of system reliability estimation. The authors proposed general strategies to overcome the difficulty of reliability calculation due to dependent failures and compared the difference in computational efforts to that of independent component failures. Lee (1992) examined an imperfect fabrication process featuring defective products, rework, and machine failure correction (out-of-control) occurrence. The objective was to derive the lot-size decision that shortens the processing cycle time, and consequently, smoothing the fabrication schedule for a critical resource. Moini and Murthy (2000) examined several repair strategies to deal with an unreliable manufacturing system and find the optimal batch size that keeps the overall system cost

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2021 Growing Science Ltd.
doi: 10.5267/j.ijiec.2021.4.001

minimum. Nahas et al. (2009) studied the optimal design problem for unreliable series-parallel fabrication lines. They formulated it as a combinatorial optimization problem and considered types of buffers and number of different types of parallel machines as decision variables, wherein the cost, breakdown/repair rate, and processing time for different brands of machines, and size for different buffers were taken into account to maximize the fabrication rate under a cost constraint. The authors proposed an approximation using the analytical decomposition approach to measure the performance of series-parallel fabrication lines. Through simulation results of numerous test problems, the authors demonstrated their approach's accuracy compared to the results from approaches of simulated annealing and colony optimization. Srinivasa Rao and Naikan (2014) proposed a hybrid method combining Markov modeling and system dynamics/simulation approach to analyze the repairable systems' reliability and behavior. The authors demonstrated their proposed framework for a standby system with repair using simulation. They compared the results with that of the conventional Markov method to validate that it could serve as a substitute method for reliability analysis. Also, random imperfect quality items produced during in-house fabrication must be identified, and the re-workable ones should be separated from the scraps to ensure the entire finished lot meets the required quality level.

Shanthikumar and Tien (1983) examined a two-stage transfer line with a geometry-distributed failure and repair times along with scraps upon the machine failure condition. An algorithm was developed to calculate the fabrication rate of the transfer lines for various system parameters and reveal the behavior of the transfer lines to facilitate the decision on system design. Wein (1992) explored the rework and scrap decisions for a multistage fabrication system with deterministic demand and random yield. The author built a Markov model for the system, applied the linear cost related to the lot-size, and employed the dynamic programming method to solve the problem. The effect of each stage's random yield on the solution and diverse system parameters are explicitly investigated. Sha et al. (2001) examined and discussed three rework strategies in the literature and proposed two extra strategies. Through the simulation approach, the authors compared previous ones with newly proposed rework strategies and highlighted their strengths and weaknesses in terms of various performance measurements, such as machine utilization, cycle time, the length of queues, fabrication cycle time, and quantity of work-in-process. Giri and Chakraborty (2011) considered a single-vendor, single-buyer supply chain system with the deteriorating product, multiple shipments, stock-dependent demand rate, and imperfect fabrication process. The objective was to decide the optimal coordination order policy to minimize the supply chain's total cost. Through numerical illustrations, the authors showed their results are more cost-effective than the non-coordinated policy. Other recent studies (Mustajib et al., 2019; Ortiz-Servin et al., 2019; Bukljaš et al., 2020; Jat and Tiwari, 2020; Pinto et al., 2020; Poursoltan et al., 2020; Gera, 2021) explored the impact of stochastic breakdown, rework, and scrap issues on diverse aspects of unreliable manufacturing-inventory systems.

Furthermore, to deal with the in-house capacity constraint or meet timely demand with a reduced cycle time, outsourcing policy is an effective alternative. Bettis et al. (1992) observed that improper outsourcing usage contributes to many Western corporations' ongoing competitive decline in Europe, North America, and Asia regions. The impact of improper outsourcing practices on possible damage to future business was studied. In contrast, the proper outsourcing practices enable a firm to gain a significant competitive advantage. Brandes et al. (1997) studied the success or failure of the outsourcing strategy, with the focus on the following outsourcing characteristics: (1) decision, (2) process, (3) unit, and (4) result. Five different case studies were conducted, and the results for a success or failure outsourcing were fully discussed according to the following relevant outsourcing factors: (i) cost efficiency, (ii) core competence, (iii) strategic decision, (iv) emergency action, (v) high pace, (vi) high commitment, (vii) a vital resource, and (viii) a balanced customer case. Lee-Mortimer (2006) studied the relationship between outsourcing production and assembly overseas and an improved product design methodology. Many outsourcing firms/cases were reviewed and analyzed. The results suggested that a firm should conduct a detailed cost-benefit analysis of overseas manufacture and look into its current product design before initiating or reviewing an outsourcing plan. Assid et al. (2015) examined the joint fabrication and outsourcing planning for unreliable multiple facilities and multiproduct systems. Both regular demands and urgent orders were satisfied by the multiple fabrication facilities in the system. Each facility has a different capacity, and it may breakdown, and the system may also use the subcontracting options. The authors employed experimental design, the response surface method, analysis of variance, and simulation modeling to solve such a complex problem. A cost-minimization controlling policy was decided. A numerical example with sensitivity analysis was offered to show the robustness of their framework and solution process. The result indicated a 20% cost savings as compared to an existing method in the literature. Additional studies (Bui et al., 2019; Haoues et al., 2019; Chiu et al., 2020a-c; Barak and Javanmard, 2020; Iqbal et al., 2020; Nissas and Gasmi, 2021) addressed several outsourcing features and their impact on fabrication systems, business firms, and supply-chain systems. For very few prior studies have explored the collective influence of Poisson-distributed equipment failures, quality reassurance, and outsourcing on the batch decision, we aim to fill the research gap.

2. Description and modeling

A hybrid economic production quantity (EPQ) problem with an unreliable machine and quality reassurance is investigated in this study. Suppose a quality-ensured hybrid batch fabrication system with an annual production rate P_1 is used to satisfy the annual requirement λ of a certain product. To expedite the batch production's uptime and smoothen machine workloads, an external provider helps offer a fraction π of the batch quantity Q (where $0 < \pi < 1$). Accordingly, a particular outsourcing unit cost C_π and setup cost K_π are linked to outsourcing alternative, where $K_\pi = [(1 + \beta_1)K]$ and $C_\pi = [(1 + \beta_2)C]$. C and K are in-house unit and setup costs, β_1 denotes the connecting factor between K and K_π, where $-1 < \beta_1 < 0$; and β_2 stands for the connecting factor between C and C_π, where $\beta_2 > 0$. Due to diverse and unexpected factors in the in-house fabrication of the rest $[(1-\pi)Q]$ items, a random x portion of manufactured products has defects, and the production rate for defective items is d_1 (where $d_1 = P_1x$). No shortages are permitted in the proposed system, so $(P_1 - d_1 - \lambda) > 0$ must holds. Defective products are thoroughly inspected to separate the scrap (assuming a θ_1 portion among them) from the rework-able (the rest $(1-\theta_1)$ portion). The rework starts right after regular fabrication in each cycle, at a rate P_2, and extra unit rework cost C_R. A rework failure rate θ_2 is associated with the rework process. Items that fail rework become scrap, and unit disposal cost is C_S. So, the total scrap rate $\varphi = \theta_1 + (1-\theta_1)\theta_2$. The quality of all outsourced products is assumed to be quality-ensured by the external provider. Additionally, the unreliable equipment is subject to a Poisson-distributed failure with mean = β per year. The abort/resume controlling discipline is adopted when an equipment failure occurs. Under this discipline, the equipment repair task begins immediately, and the unfinished lot resumes when machine repair job completes. We assume a constant repairing time t_r. Appendix A offers a complete notation list. Due to the Poisson breakdown rate, the following separate conditions must be studied:

2.1. Condition 1: A random breakdown occurs during $T_{1\pi}$

That is $t < T_{1\pi}$. Fig. 1 illustrates the level of perfect on-hand stocks in this condition. It specifies that the inventory level is at H_0 when an equipment failure happens. It keeps growing after the repair time ends and reaches H_1 and H_2 when regular fabrication and rework processes stop, respectively. By the time we receive the outsourced items, the inventory level surges to H. Finally, the stock depletion time begins to bring the stock level down to zero before the next cycle initiates.

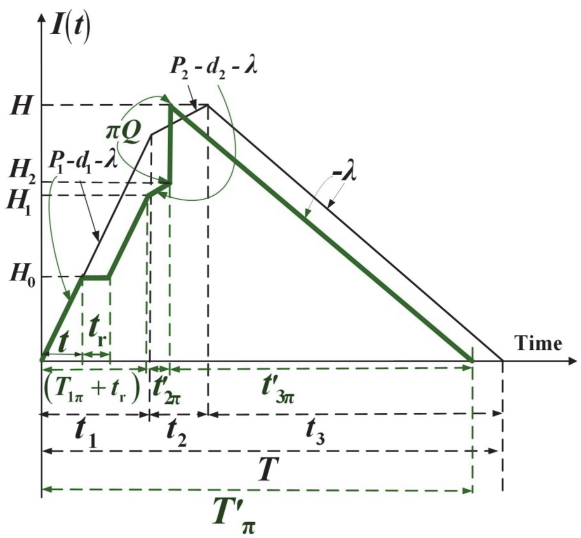

Fig. 1. Level of perfect on-hand stocks in the proposed problem (in green) compared to the same model with only quality reassurance (in black)

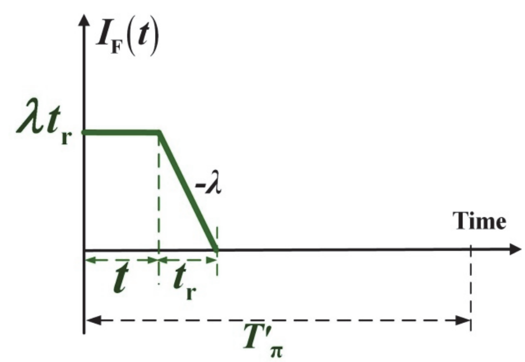

Fig. 2. Level of on-hand safety inventories in the proposed problem

Fig. 2 exhibits on-hand safety inventory level in the proposed problem. It explains how the safety stock is used to meet demand during t_r. Figs. 3 and 4 depict on-hand levels of defective and scrap items in the proposed problem.

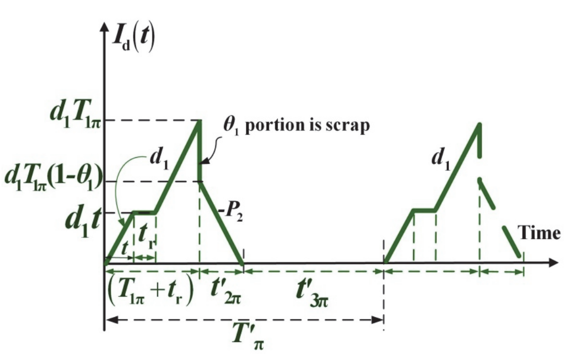

Fig. 3. Level of on-hand defective items in the proposed problem

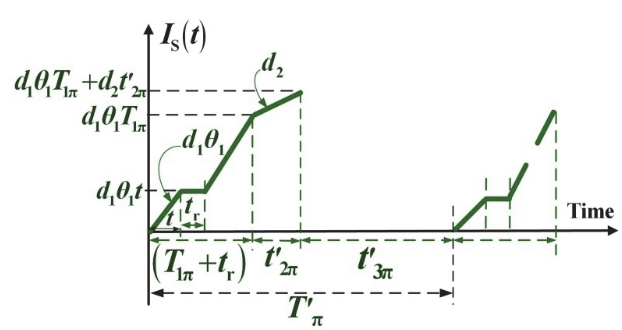

Fig. 4. Level of on-hand scrap items in the proposed Problem

The following formulas are observed from the problem description and Figs. 1 to 4:

$$T'_{\pi} = T_{1\pi} + t_{\mathrm{r}} + t'_{2\pi} + t'_{3\pi} \tag{1}$$

$$T_{1\pi} = \frac{(1-\pi)Q}{P_1} = \frac{H_1}{P_1 - d_1 - \lambda} \tag{2}$$

$$t'_{2\pi} = \frac{x\left[(1-\pi)Q\right](1-\theta_1)}{P_2} \tag{3}$$

$$t'_{3\pi} = \frac{H}{\lambda} = \frac{H_2 + \pi Q}{\lambda} \tag{4}$$

$$H_0 = (P_1 - d_1 - \lambda)t \tag{5}$$

$$H_1 = (P_1 - d_1 - \lambda)T_{1\pi} \tag{6}$$

$$H_2 = H_1 + (P_2 - d_2 - \lambda)t'_{2\pi} \tag{7}$$

$$H = H_2 + \pi Q = \lambda \cdot t'_{3\pi} \tag{8}$$

$$d_1 T_{1\pi} = x P_1 T_{1\pi} = x\left[(1-\pi)Q\right] \tag{9}$$

$$\varphi x\left[(1-\pi)Q\right] = \left[\theta_1 + (1-\theta_1)\theta_2\right] x\left[(1-\pi)Q\right]. \tag{10}$$

For the condition that a random equipment failure occurs during $T_{1\pi}$, the total cost in a cycle includes variable and fixed outsourcing costs, in-house variable and setup costs, equipment repairing cost, safety stock's holding, variable, and distribution costs, variable rework. and disposal costs, holding cost for reworked items, perfect stocks, and nonconforming items in $T_{1\pi}$, t_{r}, $t'_{2\pi}$, and $t'_{3\pi}$. Thus, $TC(T_{1\pi})_1$ is as follows:

$$\begin{aligned} TC(T_{1\pi})_1 &= C_\pi(\pi Q) + K_\pi + C\left[(1-\pi)Q\right] + K + M + h_3(\lambda t_r)\left(t + \frac{t_r}{2}\right) + C_1(\lambda t_r) \\ &+ C_T(\lambda t_r) + C_R x\left[(1-\pi)Q\right](1-\theta_1) + C_{\mathrm{S}}\varphi x\left[(1-\pi)Q\right] + h_1 \frac{P_2 t'_{2\pi}}{2}(t'_{2\pi}) \\ &+ h\left[\frac{H_1 + d_1 T_{1\pi}}{2}(T_{1\pi}) + (H_0 t_r) + (d_1 t)t_r + \frac{H_1 + H_2}{2}(t'_{2\pi}) + \frac{H}{2}(t'_{3\pi})\right] \end{aligned} \tag{11}$$

The following $E[TC(T_{1\pi})]_1$ can be obtained by substituting relevant parameters' relationships, including Eqs. (1) to (10) in Eq. (11), and applying $E[x]$ to cope with random nonconforming rate:

$$\begin{aligned} E\left[TC(T_{1\pi})\right]_1 &= \left[(1+\beta_2)C\right]\pi\left(\frac{T_{1\pi}P_1}{(1-\pi)}\right) + \left[(1+\beta_1)K\right] + K + C(T_{1\pi}P_1) + M \\ &+ h_3 t_r\left[\lambda t + \frac{\lambda t_r}{2}\right] + C_1(\lambda t_r) + C_T(\lambda t_r) + C_{\mathrm{R}} E[x](T_{1\pi}P_1)(1-\theta_1) \\ &+ C_{\mathrm{S}}\varphi E[x](T_{1\pi}P_1) + \frac{(T_{1\pi}P_1)^2 E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1) - h\right] + h(P_1 t t_r - t_r \lambda t) \\ &+ h\left(\frac{T_{1\pi}P_1}{(1-\pi)}\right)^2 \left[\begin{aligned} &\frac{1}{2\lambda}\left[1 - \varphi E[x](1-\pi)\right]^2 - \frac{(1-\pi)}{2P_1}\left[(1+\pi) - 2\varphi E[x](1-\pi)\right] \\ &+ \frac{E[x](1-\pi)(1-\theta_1)}{2P_2}\left[\varphi E[x](1-\pi) - 2\pi\right] \end{aligned}\right] \end{aligned} \tag{12}$$

The following $E[T'_{\pi}]$ can also be determined by applying $E[x]$:

$$E[T'_{\pi}] = \frac{Q\left[1 - \varphi \cdot E[x](1-\pi)\right]}{\lambda} = \frac{T_{1\pi}P_1\left[\frac{1}{(1-\pi)} - \varphi \cdot E[x]\right]}{\lambda} \tag{13}$$

2.2. *Condition 2: No breakdowns occur during $T_{1\pi}$*

That is $t > T_{1\pi}$. Fig. 5 exhibits the perfect on-hand stocks' level in this condition. It exposes the inventory level grows to H_1 and H_2 when regular fabrication and rework processes stop, respectively. Then, receipt of outsourced items brings the level of stock to H. Follows by depletion time, and stock level declines to zero before the next cycle starts. Fig. 6 displays on-hand safety stock level in the proposed problem with no random breakdown occurrence. It shows that safety stock level remains the same through-out the cycle length for no random breakdown occurs. The following straightforward relationships are observed from problem description and Figs. 5 to 6:

$$T_{\pi} = T_{1\pi} + t_{2\pi} + t_{3\pi} \tag{14}$$

$$T_{1\pi} = \frac{(1-\pi)Q}{P_1} = \frac{H_1}{P_1 - d_1 - \lambda} \tag{15}$$

$$t_{2\pi} = \frac{x\left[(1-\pi)Q\right](1-\theta_1)}{P_2} \tag{16}$$

$$t_{3\pi} = \frac{H}{\lambda} = \frac{H_2 + \pi Q}{\lambda} \tag{17}$$

$$H_1 = (P_1 - d_1 - \lambda)T_{1\pi} \tag{18}$$

$$H_2 = H_1 + (P_2 - d_2 - \lambda)t_{2\pi} \tag{19}$$

$$H = H_2 + \pi Q = \lambda \cdot t_{3\pi} \tag{20}$$

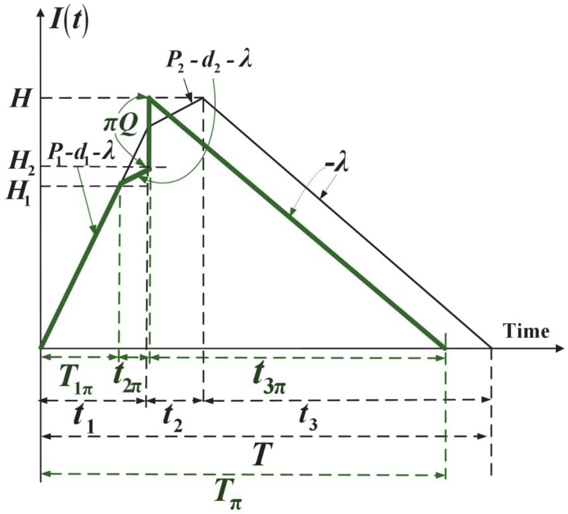

Fig. 5. Level of perfect on-hand stocks in the proposed problem with no breakdown occurrence (in green) compared to the same problem with only quality reassurance (in black)

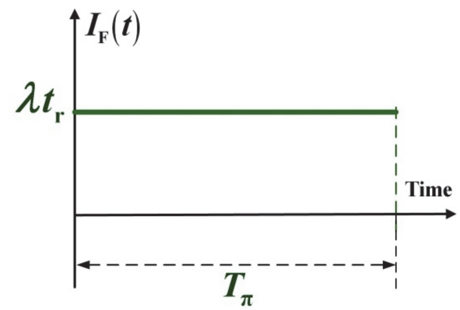

Fig. 6. Level of on-hand safety stocks in the proposed problem with no random breakdown occurrence

For condition two that no random breakdown occurs during $T_{1\pi}$, the total cost per cycle includes variable and fixed outsourcing costs, in-house variable and setup costs, safety stock's holding cost, rework and disposal costs, holding cost for reworked items, perfect stocks, and defective items in $T_{1\pi}$, $t_{2\pi}$, and $t_{3\pi}$. Thus, $TC(T_{1\pi})_2$ is as follows:

$$\begin{aligned} TC(T_{1\pi})_2 &= C_\pi(\pi Q) + K_\pi + C\left[(1-\pi)Q\right] + K + h_3(\lambda t_r)T_\pi + C_R x\left[(1-\pi)Q\right](1-\theta_1) + C_S\varphi x\left[(1-\pi)Q\right] + h_1\frac{P_2 t_{2\pi}}{2}(t_{2\pi}) \\ &+ h\left[\frac{H_1 + d_1 T_{1\pi}}{2}(T_{1\pi}) + \frac{H_1 + H_2}{2}(t_{2\pi}) + \frac{H}{2}(t_{3\pi})\right] \end{aligned} \tag{21}$$

The following $E[TC(T_{1\pi})]_2$ – the expected total cost per cycle for no breakdown occurrence case, can be derived by substituting all relevant relationships of variables, including Eqs. (9), (10), and (14) to (20) in Eq. (21), and applying $E[x]$ to cope with the random nonconforming rate:

$$\begin{aligned} E\left[TC(T_{1\pi})\right]_2 &= \left[(1+\beta_2)C\right]\pi\left(\frac{T_{1\pi}P_1}{(1-\pi)}\right) + \left[(1+\beta_1)K\right] + K + C(T_{1\pi}P_1) + h_3(\lambda t_r)T_\pi + C_R E[x](T_{1\pi}P_1)(1-\theta_1) + C_S\varphi E[x](T_{1\pi}P_1) \\ &+ \frac{(T_{1\pi}P_1)^2 E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1) - h\right] + h\left(\frac{T_{1\pi}P_1}{(1-\pi)}\right)^2 \left[\begin{array}{l} \frac{1}{2\lambda}\left[1-\varphi E[x](1-\pi)\right]^2 - \frac{(1-\pi)}{2P_1}\left[(1+\pi) - 2\varphi E[x](1-\pi)\right] \\ + \frac{E[x](1-\pi)(1-\theta_1)}{2P_2}\left[\varphi E[x](1-\pi) - 2\pi\right] \end{array}\right] \end{aligned} \tag{22}$$

The following $E[T_\pi]$ – the expected cycle length when no breakdown taking place, can be derived by applying $E[x]$ to cope with random nonconforming rate:

$$E[T_\pi] = \frac{Q\left[1-\varphi\cdot E[x](1-\pi)\right]}{\lambda} = \frac{T_{1\pi}P_1\left[\frac{1}{(1-\pi)} - \varphi\cdot E[x]\right]}{\lambda} \tag{23}$$

3. Solution to the proposed problem

Due to the Poisson-distributed failure-rate β, the time to failure follows Exponential distribution with density and cumulative density functions as $f(t) = \beta e^{-\beta t}$ and $F(t) = (1 - e^{-\beta t})$, respectively. Because of the random scrap rate φ, our cycle length is variable. Therefore, we apply the renewal reward theorem to handle variable cycle length. Hence, $E[TCU(T_{1\pi})]$ is as follows:

$$E\left[TCU\left(T_{1\pi}\right)\right]=\frac{\left\{\int_{0}^{T_{1\pi}}E\left[TC\left(T_{1\pi}\right)\right]_{1}\cdot f\left(t\right)dt+\int_{T_{1\pi}}^{\infty}E\left[TC\left(T_{1\pi}\right)\right]_{2}\cdot f\left(t\right)dt\right\}}{E[\boldsymbol{T}_{\pi}]} \tag{24}$$

where $E[\boldsymbol{T}_{\pi}]$ is

$$E[\boldsymbol{T}_{\boldsymbol{\pi}}]=\int_{0}^{T_{1\pi}}E[T'_{\pi}]\cdot f\left(t\right)dt+\int_{T_{1\pi}}^{\infty}E[T_{\pi}]\cdot f\left(t\right)dt \tag{25}$$

Substitute Eqs. (12), (22), and (25) in Eq. (24), and with extra derivations, $E[TCU(T_{1\pi})]$ is gained as follows (the details are exhibited in Appendix B):

$$E\left[TCU\left(T_{1\pi}\right)\right]=\frac{\lambda\left(1-\pi\right)}{\left[1-\varphi E\left[x\right]\left(1-\pi\right)\right]}\left\{\begin{array}{l}\dfrac{Z_{1}}{T_{1\pi}}+\dfrac{W_{1}}{T_{1\pi}}+W_{2}e^{-\beta T_{1\pi}}+\dfrac{W_{3}e^{-\beta T_{1\pi}}}{T_{1\pi}}\\+T_{1\pi}\left[\dfrac{P_{1}E\left[x\right]^{2}\left(1-\theta_{1}\right)}{2P_{2}}\left[h_{1}\left(1-\theta_{1}\right)-h\right]+h(W_{4})\right]\\+\left[\dfrac{\pi\left(1+\beta_{2}\right)C}{\left(1-\pi\right)}+C+C_{R}E\left[x\right]\left(1-\theta_{1}\right)+C_{S}\varphi E\left[x\right]\right]\\+\dfrac{h_{3}g\left[1-\varphi E\left[x\right]\left(1-\pi\right)\right]}{\left(1-\pi\right)}\left(e^{-\beta T_{1\pi}}\right)\end{array}\right\} \tag{26}$$

3.1. Convexity of the system cost function

First, we apply the first- and second-derivatives of $E[TCU(T_{1\pi})]$ and gain the following:

$$\frac{dE\left[TCU\left(T_{1\pi}\right)\right]}{d\left(T_{1\pi}\right)}=\frac{\lambda\left(1-\pi\right)}{\left[1-\varphi E\left[x\right]\left(1-\pi\right)\right]}\left\{\begin{array}{l}-\dfrac{Z_{1}}{\left(T_{1\pi}\right)^{2}}-\dfrac{W_{1}}{\left(T_{1\pi}\right)^{2}}-\beta W_{2}e^{-\beta T_{1\pi}}+W_{3}\left[-\dfrac{e^{-\beta T_{1\pi}}}{\left(T_{1\pi}\right)^{2}}-\dfrac{\beta e^{-\beta T_{1\pi}}}{T_{1\pi}}\right]\\+\left[\dfrac{P_{1}E\left[x\right]^{2}\left(1-\theta_{1}\right)}{2P_{2}}\left[h_{1}\left(1-\theta_{1}\right)-h\right]+h(W_{4})\right]\\-\dfrac{\beta h_{3}g\left[1-\varphi E\left[x\right]\left(1-\pi\right)\right]e^{-\beta T_{1\pi}}}{\left(1-\pi\right)}\end{array}\right\} \tag{27}$$

and

$$\frac{d^{2}E\left[TCU\left(T_{1\pi}\right)\right]_{1}}{d\left(T_{1\pi}\right)^{2}}=\frac{\lambda\left(1-\pi\right)}{\left[1-\varphi E\left[x\right]\left(1-\pi\right)\right]}\left\{\begin{array}{l}\dfrac{2Z_{1}}{\left(T_{1\pi}\right)^{3}}+\dfrac{2W_{1}}{\left(T_{1\pi}\right)^{3}}+\beta^{2}W_{2}e^{-\beta T_{1\pi}}\\+W_{3}\left[\dfrac{2e^{-\beta T_{1\pi}}}{\left(T_{1\pi}\right)^{3}}+\dfrac{2\beta e^{-\beta T_{1\pi}}}{\left(T_{1\pi}\right)^{2}}+\dfrac{\beta^{2}e^{-\beta T_{1\pi}}}{T_{1\pi}}\right]\\+\dfrac{\beta^{2}h_{3}g\left[1-\varphi E\left[x\right]\left(1-\pi\right)\right]e^{-\beta T_{1\pi}}}{\left(1-\pi\right)}\end{array}\right\} \tag{28}$$

Because the first term $\lambda(1-\pi)/[1-\varphi E[x](1-\pi)]$ on RHS (right-hand side) of Eq. (28) is positive, it follows that $E[TCU(T_{1\pi})]$ is convex if the second term on RHS of Eq. (28) is also positive. That is if Eq. (29) holds.

$$\delta\left(T_{1\pi}\right)=\frac{-2\left(Z_{1}+W_{1}+W_{3}e^{-\beta T_{1\pi}}\right)}{T_{1\pi}{}^{2}\beta^{2}e^{-\beta T_{1\pi}}\left[W_{2}+h_{3}g\left[\dfrac{1-\varphi E\left[x\right]\left(1-\pi\right)}{\left(1-\pi\right)}\right]\right]+W_{3}\left(2\beta e^{-\beta T_{1\pi}}+T_{1\pi}\beta^{2}e^{-\beta T_{1\pi}}\right)}>T_{1\pi}>0 \tag{29}$$

*3.2. Determining the optimal $T_{1\pi}$**

Under the condition that Eq. (29) is confirmed, we can set the first-derivative of $E[TCU(T_{1\pi})]=0$ and derive $T_{1\pi}$*.

$$\frac{\lambda\left(1-\pi\right)}{\left[1-\varphi E\left[x\right]\left(1-\pi\right)\right]}\left\{\begin{array}{l}-\dfrac{Z_{1}}{\left(T_{1\pi}\right)^{2}}-\dfrac{W_{1}}{\left(T_{1\pi}\right)^{2}}-\beta W_{2}e^{-\beta T_{1\pi}}+W_{3}\left[-\dfrac{e^{-\beta T_{1\pi}}}{\left(T_{1\pi}\right)^{2}}-\dfrac{\beta e^{-\beta T_{1\pi}}}{T_{1\pi}}\right]\\+\left[\dfrac{P_{1}E\left[x\right]^{2}\left(1-\theta_{1}\right)}{2P_{2}}\left[h_{1}\left(1-\theta_{1}\right)-h\right]+h(W_{4})\right]\\-\dfrac{\beta h_{3}g\left[1-\varphi E\left[x\right]\left(1-\pi\right)\right]e^{-\beta T_{1\pi}}}{\left(1-\pi\right)}\end{array}\right\}=0 \tag{30}$$

or

$$\left\{\begin{array}{l}(T_{1\pi})^2\left\{\frac{P_1E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1)-h\right]+h(W_4)-\beta e^{-\beta T_{1\pi}}\left[\frac{h_3 g\left[1-\varphi E[x](1-\pi)\right]}{(1-\pi)}+W_2\right]\right\}\\+(T_{1\pi})\left(-W_3\beta e^{-\beta T_{1\pi}}\right)+\left(-Z_1-W_1-W_3e^{-\beta T_{1\pi}}\right)\end{array}\right\}=0 \tag{31}$$

Let y_2, y_1, and y_0 represent the following:

$$\begin{aligned}&y_2=\left\{\frac{P_1E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1)-h\right]+h(W_4)-\beta e^{-\beta T_{1\pi}}\left[\frac{h_3 g\left[1-\varphi E[x](1-\pi)\right]}{(1-\pi)}+W_2\right]\right\};\\&y_1=\left(-W_3\beta e^{-\beta T_{1\pi}}\right);\\&y_0=\left(-Z_1-W_1-W_3e^{-\beta T_{1\pi}}\right).\end{aligned} \tag{32}$$

Eq. (31) becomes as follows:

$$y_2\left(T_{1\pi}\right)^2+y_1\left(T_{1\pi}\right)+y_0=0 \tag{33}$$

By applying the square roots solution, we find the following $T_{1\pi}$*:

$$T_{1\pi}{}^*=\frac{-y_1\pm\sqrt{y_1^2-4y_2y_0}}{2y_2} \tag{34}$$

$$T_{1\pi}{}^*=\frac{W_3\beta e^{-\beta T_{1\pi}}\pm\sqrt{\left(W_3\beta e^{-\beta T_{1\pi}}\right)^2-4\left\{\begin{array}{l}\frac{P_1E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1)-h\right]+h(W_4)\\-\beta e^{-\beta T_{1\pi}}\left[\frac{h_3g\left[1-\varphi E[x](1-\pi)\right]}{(1-\pi)}+W_2\right]\end{array}\right\}\left(-Z_1-W_1-W_3e^{-\beta T_{1\pi}}\right)}}{2\left\{\frac{P_1E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1)-h\right]+h(W_4)-\beta e^{-\beta T_{1\pi}}\left[\frac{h_3g\left[1-\varphi E[x](1-\pi)\right]}{(1-\pi)}+W_2\right]\right\}} \tag{35}$$

*3.2.1. Algorithm for finding the $T_{1\pi}$**

As $F(T_{1\pi}) = (1 - e^{-\beta T1\pi})$ is the cumulative density function of Exponential distribution, so it is throughout [0, 1], so does its complement $e^{-\beta T1\pi}$. Also, we can rearrange Eq. (31) as follows:

$$e^{-\beta T_{1\pi}}=\frac{\left\{\frac{P_1E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1)-h\right]+h(W_4)\right\}\left(T_{1\pi}\right)^2-Z_1-W_1}{\beta\left[\frac{h_3g\left[1-\varphi E[x](1-\pi)\right]}{(1-\pi)}+W_2\right]\left(T_{1\pi}\right)^2+W_3\beta\left(T_{1\pi}\right)+W_3} \tag{36}$$

Initially, let $e^{-\beta T1\pi} = 0$ and $e^{-\beta T1\pi} = 1$, apply Eq. (35) to obtain the upper and lower bounds for uptime, i.e., $T_{1\pi U}$ and $T_{1\pi L}$. Next, we use the current $T_{1\pi U}$ and $T_{1\pi L}$ to recalculate the update values of $e^{-\beta T1\pi U}$ and $e^{-\beta T1\pi L}$. Repeat the aforementioned steps, that are to apply Eq. (35) with the current $e^{-\beta T1\pi U}$ and $e^{-\beta T1\pi L}$ to compute a new set of $T_{1\pi U}$ and $T_{1\pi L}$, until $T_{1\pi U} = T_{1\pi L}$. Then, the optimal $T_{1\pi}$* is found, that is $T_{1\pi}* = T_{1\pi U} = T_{1\pi L}$.

4. Numerical example

A numerical example is offered in this section to show the proposed hybrid EPQ problem's applicability with unreliable machine and product quality issues. The following assumptions of values for system parameters are made (see **Table 1**):

Table 1
Assumptions of values for system parameters

β	C_π	C	β_2	K_π	K	C_R	C_S	C_T	C_1	λ	P_1
1	2.8	2.0	0.4	135	450	1.0	0.3	0.01	2.0	4000	10000
π	θ_1	θ_2	β_1	x	g	φ	h	h_1	h_3	M	P_2
0.4	0.3	0.3	-0.70	20%	0.018	0.51	0.8	0.8	0.8	2500	5000

Convexity of $E[TCU(T_{1\pi})]$ is first examined by applying Eq. (29) for β value at 1.0. Let $e^{-\beta T1\pi} = 0$ and $e^{-\beta T1\pi} = 1$, then use Eq. (35) to gain $T_{1\pi U} = 0.4530$ and $T_{1\pi L} = 0.1221$, respectively. Next, use the current $T_{1\pi U}$ and $T_{1\pi L}$ to recalculate the update values of $e^{-\beta T1\pi U}$ and $e^{-\beta T1\pi L}$. Finally, we apply Eq. (29) with current values of $T_{1\pi U}$, $T_{1\pi L}$, $e^{-\beta T1\pi U}$, and $e^{-\beta T1\pi L}$ to obtain $\delta(T_{1\pi U}) = 0.7381 > T_{1\pi U} = 0.4530 > 0$ and $\delta(T_{1\pi L}) = 0.3459 > T_{1\pi L} = 0.1221 > 0$, respectively. So, for $\beta = 1.0$ the convexity of $E[TCU(T_{1\pi})]$ is verified, and the optimal replenishment uptime $T_{1\pi}$* exists. To demonstrate our model's border applicability, different breakdown rates other than one are computed to test the system cost function's convexity further. An algorithm along with equations (35) and (36) (refer to subsection 3.2.) is employed to derive the optimal solution of $T_{1\pi}* = 0.1965$ and $E[TCU(T_{1\pi}*)]$ = \$11,966 (detailed derivations of $T_{1\pi}*$ are exhibited in **Table C-2** (Appendix C)).

4.1. Influence of the main system factor on the problem

The behavior of $E[TCU(T_{1\pi})]$ relating to the process of seeking $T_{1\pi}*$ and to different values of $T_{1\pi}$ is depicted in Fig. 7.

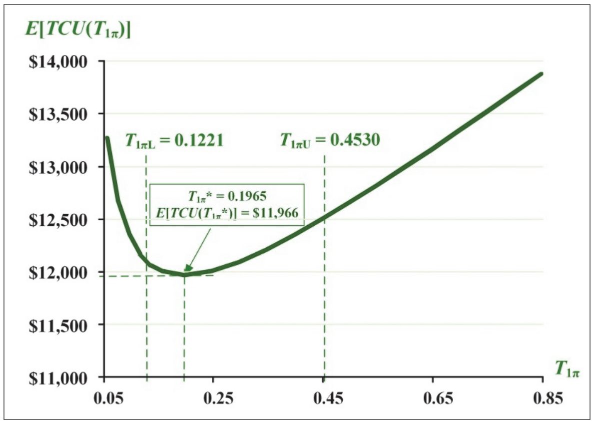

Fig. 7. Behavior of $E[TCU(T_{1\pi})]$ relating to the process of seeking $T_{1\pi}*$ and different values of $T_{1\pi}$

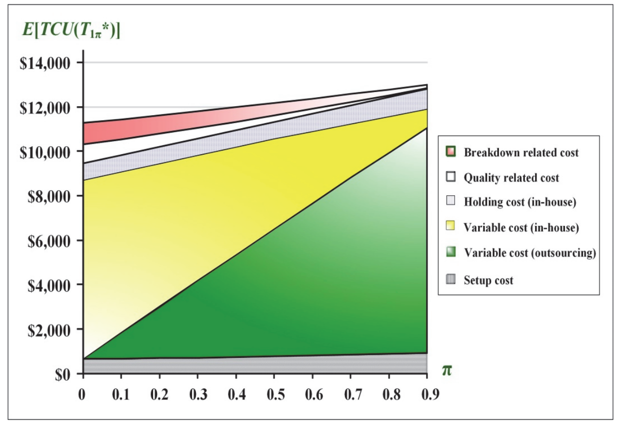

Fig. 8. Variations in outsourcing ratio π effect on different contributors of $E[TCU(T_{1\pi}*)]$

Variations in outsourcing ratio π effect on different contributors of $E[TCU(T_{1\pi}*)]$ are analyzed and displayed in Fig. 8. It specifies that as π increases, variable outsourcing cost upsurges significantly; but, quite the reverse, the in-house variable fabrication cost decreases considerably, so do the breakdown and quality-related costs.

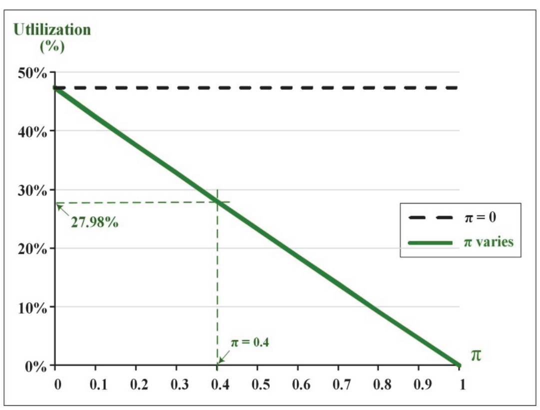

Fig. 9. The impact of outsourcing proportion π on utilization

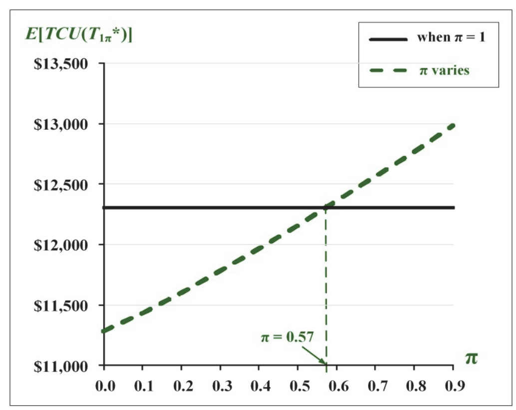

Fig. 10. The effect of outsourcing proportion π on $E[TCU(T_{1\pi}*)]$

Fig. 9 depicts the impact of outsourcing ratio π on utilization. It indicates that utilization declines considerably as π increases, and it drops to 27.98% at $\pi = 0.4$. The effect of outsourcing proportion π on $E[TCU(T_{1\pi}*)]$ is demonstrated in Fig. 10. It discloses that $E[TCU(T_{1\pi}*)]$ increases noticeably as π rises, and it further exposes that when π increases to 0.57, the "buy" decision is more economical. The breakup of $E[TCU(T_{1\pi}*)]$ (at $\pi = 0.4$ and $\beta = 1$) is demonstrated in Fig. 11. It discloses the setup and variable costs for outsourcing (i.e., 1.42% and 38.62%) and diverse in-house relevant costs. The contributors of product quality-related cost are further analyzed and displayed in Fig. 12. It reveals that the variable rework cost is the most significant contributor (i.e., 46.03%) of quality-related price. The second one is making extra items to compensate for the scrap items, which takes 40.24% of the quality-related price. Figure 13 exhibits the impact of different mean-time-to-failure values $1/\beta$ with different overall scrap rates φ on $E[TCU(T_{1\pi}*)]$. From Fig. 13, one finds that as $1/\beta$ surges to 0.2 and up, $E[TCU(T_{1\pi}*)]$ starts to decline noticeably; and as $1/\beta$ approaches 100 (or ∞), the studied problem comprises no breakdown instance, and $E[TCU(T_{1\pi}*)]$ drops to \$11,342. Also, it shows that as φ rises, $E[TCU(T_{1\pi}*)]$ increases accordingly. Fig. 14 depicts the influence of changes in φ with different x values on $E[TCU(T_{1\pi}*)]$. It exposes that as both x and φ rise, $E[TCU(T_{1\pi}*)]$ increases accordingly.

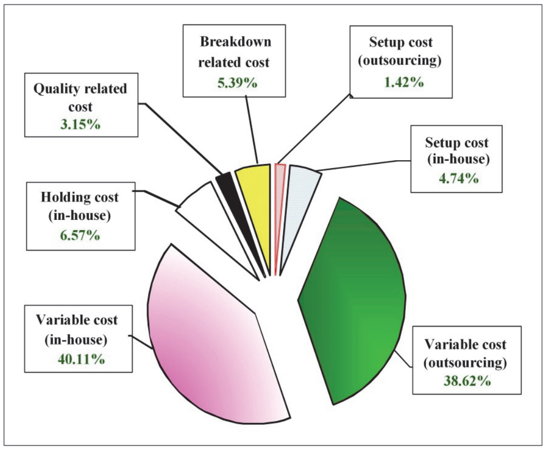

Fig. 11. The breakup of $E[TCU(T_{1\pi}^*)]$ (at $\pi = 0.4$ and $\beta = 1$)

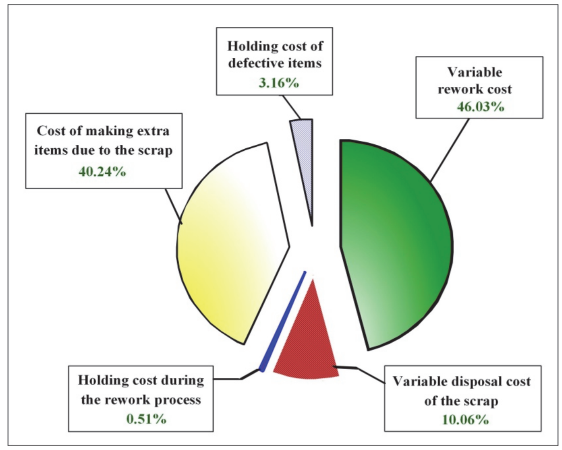

Fig. 12. The contributors of machine failure and product quality relevant costs

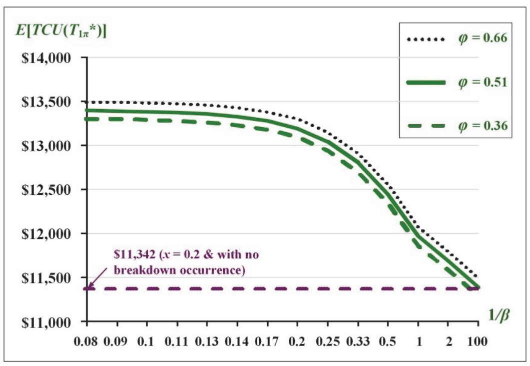

Fig. 13. The impact of different $1/\beta$ values along with various φ values on $E[TCU(T_{1\pi}^*)]$

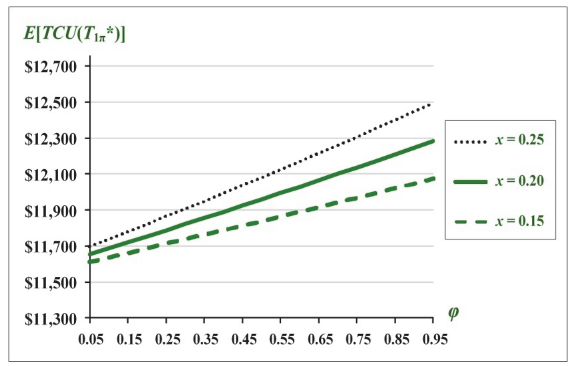

Fig. 14. The influence of φ along with different x values on $E[TCU(T_{1\pi}^*)]$

4.2. Joint influence of the core system factors on the problem

Fig. 15 analyzes and illustrates the joint influence of variations in outsourcing ratio π and overall scrap rate φ on $E[TCU(T_{1\pi}^*)]$. It indicates that $E[TCU(T_{1\pi}^*)]$ increases as both π and φ go up, and the impact from π is significantly higher than that from φ.

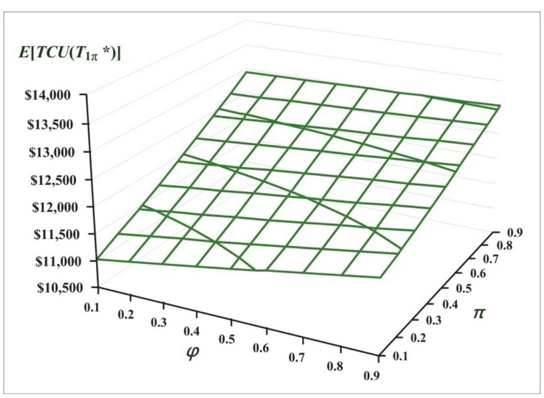

Fig. 15. Joint influence of π and φ on $E[TCU(T_{1\pi}^*)]$

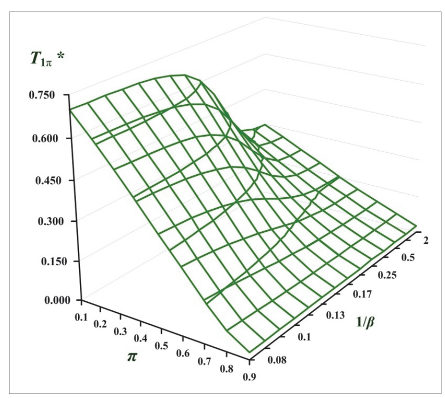

Fig. 16. Combined effect of $1/\beta$ and π on $T_{1\pi}^*$

The combined effect of mean-time-to-failure $1/\beta$ and outsourcing ratio π on $T_{1\pi}^*$ is studied, and the outcome is displayed in Fig. 16. It discloses that $T_{1\pi}^*$ declines enormously as π rises; and as $1/\beta$ surges and when $\pi < 0.5$, $T_{1\pi}^*$ decreases considerably; but, when $\pi > 0.5$, $T_{1\pi}^*$ declines slightly as $1/\beta$ rises. The joint impact of outsourcing unit cost added fraction β_2 and overall scrap rate φ on $E[TCU(T_{1\pi}^*)]$ is investigated and depicted in Fig. 17. It exposes that $E[TCU(T_{1\pi}^*)]$ rises as both β_2 and φ increase, and the influence from β_2 is much higher than that from φ. The combined impact of x and φ on $T_{1\pi}^*$ is studied and illustrated in Fig. 18. It exposes that $T_{1\pi}^*$ increases as both x and φ go up; and the influence of x is noticeably more significant than that from φ.

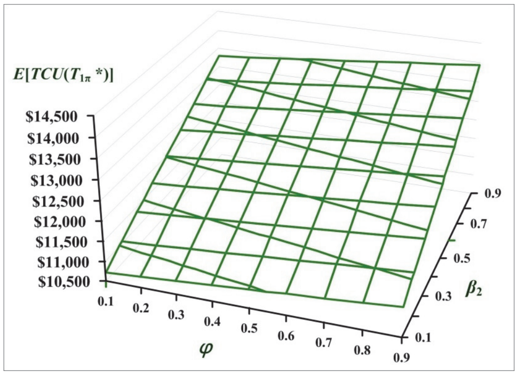

Fig. 17. Joint impact of β_2 and φ on $E[TCU(T_{1\pi}*)]$

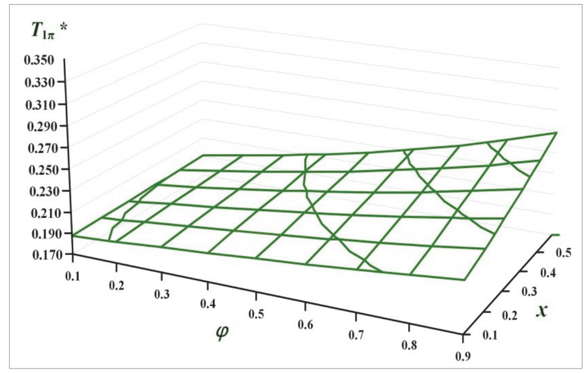

Fig. 18. Combined influence of x and φ on $T_{1\pi}*$

5. Conclusions

A hybrid EPQ problem considering an unreliable machine and quality reassurance is explored to deal with the in-house capacity constraint and meet the fabrication schedule and the required quality standard. The outsourced items are received at a requested time and a desirable quality, and the occurrences of Poisson-distributed breakdown and imperfect product quality in the in-house fabrication are both effectively managed. A precise model portraying the problem's characteristics is built explicitly. The optimal replenishment runtime is derived through mathematical analyses and an optimization approach. A numerical example reveals our result's applicability, and specific managerial implications are suggested (refer to Figs. 7-18). For future research, an interesting subject is to study the influence of multi-shipment policy on the optimal replenishing decision of the problem.

Acknowledgment

The authors thank the Ministry of Science and Technology of Taiwan for supporting this project under fund-ID#: MOST 107-2221-E-324-015.

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Appendix – A

Nomenclature

t = fabrication time before a random breakdown occurs (in years),
β = mean breakdowns per year, a Poisson-distributed variable,
t_r = machine repair time,
Q = batch size,
M = machine repair cost,
x = random defective rate during fabrication,
θ_1 = the scrap potion among defective items,
θ_2 = the scrap portion among reworked items,
φ = overall scrap rate (sum of scrap rates in $T_{1\pi}$ and $t'_{2\pi}$),
$T_{1\pi}$ = replenishment uptime in the proposed batch fabrication problem with an unreliable equipment and quality reassurance – the decision variable,
$t'_{2\pi}$ = rework time in the proposed problem with breakdown occurrence,
$t'_{3\pi}$ = stock depletion time in the proposed problem with breakdown occurrence,
T'_π = cycle length in the proposed problem with breakdown occurrence,
d_2 = production rate of scrap items during the reworking time $t'_{2\pi}$,
C = unit production cost,
K = in-house setup cost,
h = unit holding cost,
h_1 = holding cost per reworked item,
h_3 = holding cost per safety item,
C_R = unit reworking cost,
C_S = unit disposal cost,
K_π = outsourcing setup (order) cost,
C_π = unit outsourcing cost,
β_1 = the connecting factor between K_π and K, where $K_\pi = (1 + \beta_1)K$ and $-1 < \beta_1 < 0$,
β_2 = the connecting factor between C_π and C, where $C_\pi = (1 + \beta_2)C$ and $\beta_2 > 0$,

C_1 = unit cost of safety item,
C_T = unit delivery cost of safety item,
g = t_r, fixed machine repair time,
H_0 = level of perfect on-hand stocks when a breakdown occurs,
H_1 = level of perfect on-hand stocks when regular fabrication process finishes,
H_2 = level of perfect on-hand stocks when the rework process finishes,
H = level of on-hand stocks after receipt of outsourced products,
$I(t)$ = perfect on-hand stocks at time t,
$I_F(t)$ = on-hand safety stocks at time t,
$I_d(t)$ = on-hand defective stocks at time t,
$I_s(t)$ = level of on-hand scrap at time t,
$TC(T_{1\pi})_1$ = total system costs per cycle in the case of breakdown occurrence,
$E[TC(T_{1\pi})]_1$ = the expected total system costs per cycle for the proposed problem with breakdown occurrence,
$t_{2\pi}$ = rework time in the proposed problem without breakdown occurrence,
$t_{3\pi}$ = stock depletion time in the proposed problem without breakdown occurrence,
T_π = cycle length for the proposed problem without breakdown occurrence,
$TC(T_{1\pi})_2$ = total system costs per cycle for the proposed problem without breakdown occurrence,
$E[TC(T_{1\pi})]_2$ = the expected total system costs per cycle for the proposed problem without breakdown occurrence,
$E[TCU(T_{1\pi})]$ = the expected system costs per unit time for the proposed problem with an unreliable machine and quality reassurance (with/without a breakdown occurrence),
t_1 = replenishment uptime in a system with quality reassurance, without breakdown nor outsourcing,
t_2 = rework time in a system with quality reassurance,
t_3 = stock depletion time in a system with quality reassurance,
T = cycle length in a system with quality reassurance,
$\boldsymbol{T_\pi}$ = replenishment cycle length for the proposed problem with an unreliable machine and quality reassurance (with/without a breakdown occurrence).

Appendix – B

Detailed derivations of Eq. (26) are as follows:
Upon obtaining $E[TC(T_{1\pi})]_1$ (i.e., Eq. (12)) and $E[TC(T_{1\pi})]_2$ (Eq. (22)) suppose we let

$$z_1 = \left[(1+\beta_2)C\right]\frac{\pi P_1}{(1-\pi)} + CP_1 + C_R E[x]P_1(1-\theta_1) + C_S\varphi E[x]P_1 \tag{B-1}$$

$$z_2 = \frac{P_1^2 E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1)-h\right] + h\left[\begin{array}{l}\dfrac{P_1^2}{2\lambda(1-\pi)^2}\left[1-\varphi E[x](1-\pi)\right]^2 - \dfrac{P_1}{2(1-\pi)}\left[(1+\pi)-2\varphi E[x](1-\pi)\right] \\ +\dfrac{P_1^2 E[x](1-\theta_1)}{2P_2(1-\pi)}\left[\varphi E[x](1-\pi)-2\pi\right]\end{array}\right] \tag{B-2}$$

then $E[TC(T_{1\pi})]_1$ and $E[TC(T_{1\pi})]_2$ become as follows:

$$E\left[TC(T_{1\pi})\right]_1 = z_2(T_{1\pi})^2 + z_1(T_{1\pi}) + \left\{(1+\beta_1)K + K + M + \lambda g\left[h_3\left(t+\frac{g}{2}\right)+C_1+C_T\right] + h\left(P_1 tg - \lambda gt\right)\right\} \tag{B-3}$$

$$E\left[TC(T_{1\pi})\right]_2 = z_2(T_{1\pi})^2 + z_1(T_{1\pi}) + \left[(1+\beta_1)K\right] + K + h_3(\lambda g)T_\pi \tag{B-4}$$

Apply the following Eq. (24) and substitute equations (B-3), (B-4), and (25) in Eq. (24), and with extra derivations, Eq. (B-5) can be gained.

$$E\left[TCU(T_{1\pi})\right] = \frac{\left\{\int_0^{T_{1\pi}} E\left[TC(T_{1\pi})\right]_1 \cdot f(t)\,dt + \int_{T_{1\pi}}^{\infty} E\left[TC(T_{1\pi})\right]_2 \cdot f(t)\,dt\right\}}{E[\boldsymbol{T_\pi}]} \tag{24}$$

$$E\left[TCU(T_{1\pi})\right] = \frac{\lambda(1-\pi)}{\left[1-\varphi E[x](1-\pi)\right]}\left\{\begin{array}{l}\dfrac{(2+\beta_1)K}{T_{1\pi}P_1} + \dfrac{\pi(1+\beta_2)C}{(1-\pi)} + C + C_R E[x](1-\theta_1) + C_S\varphi E[x] \\ +\dfrac{T_{1\pi}P_1E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1)-h\right] + \dfrac{\left[1-\varphi E[x](1-\pi)\right]^2}{\lambda(1-\pi)}\left[\dfrac{T_{1\pi}hP_1}{2(1-\pi)}\right] \\ -\dfrac{T_{1\pi}h}{2(1-\pi)}\left[(1+\pi)-2\varphi E[x](1-\pi)\right] + \dfrac{T_{1\pi}hP_1E[x](1-\theta_1)}{2P_2(1-\pi)}\left[\varphi E[x](1-\pi)-2\pi\right] \\ +\dfrac{1}{T_{1\pi}}\left[\dfrac{M}{P_1} + \dfrac{h_3\lambda g^2}{2P_1} + \dfrac{hg}{\beta} + \dfrac{C_1\lambda g}{P_1} + \dfrac{C_T\lambda g}{P_1} - \dfrac{h\lambda g}{P_1\beta} + \dfrac{h_3\lambda g}{P_1\beta}\right] \\ +e^{-\beta T_{1\pi}}\left(-hg - \dfrac{h_3\lambda g}{P_1} + \dfrac{h\lambda g}{P_1}\right) + \dfrac{h_3 g\left[1-\varphi E[x](1-\pi)\right]}{(1-\pi)}\left(e^{-\beta T_{1\pi}}\right) \\ +\dfrac{e^{-\beta T_{1\pi}}}{T_{1\pi}}\left[-\dfrac{M}{P_1} - \dfrac{h_3\lambda g^2}{2P_1} - \dfrac{hg}{\beta} - \dfrac{C_1\lambda g}{P_1} - \dfrac{C_T\lambda g}{P_1} + \dfrac{h\lambda g}{P_1\beta} - \dfrac{h_3\lambda g}{P_1\beta}\right]\end{array}\right\} \tag{B-5}$$

Suppose we let Z_1, W_1, W_2, and W_3 stand for the following:

$$Z_1 = \frac{(2+\beta_1)K}{P_1} \tag{B-6}$$

$$W_1 = \left[\frac{M}{P_1} + \frac{h_3\lambda g^2}{2P_1} + \frac{hg}{\beta} + \frac{C_1\lambda g}{P_1} + \frac{C_T\lambda g}{P_1} - \frac{h\lambda g}{P_1\beta} + \frac{h_3\lambda g}{P_1\beta}\right] \tag{B-7}$$

$$W_2 = \left[-hg - \frac{h_3\lambda g}{P_1} + \frac{h\lambda g}{P_1}\right] \tag{B-8}$$

$$W_3 = \left[-\frac{M}{P_1} - \frac{h_3\lambda g^2}{2P_1} - \frac{hg}{\beta} - \frac{C_1\lambda g}{P_1} - \frac{C_T\lambda g}{P_1} + \frac{h\lambda g}{P_1\beta} - \frac{h_3\lambda g}{P_1\beta}\right] \tag{B-9}$$

$$W_4 = \left[\begin{array}{l} \dfrac{\left[1-\varphi E[x](1-\pi)\right]^2}{\lambda(1-\pi)}\left[\dfrac{P_1}{2(1-\pi)}\right] - \dfrac{1}{2(1-\pi)}\left[(1+\pi) - 2\varphi E[x](1-\pi)\right] \\ + \dfrac{P_1 E[x](1-\theta_1)}{2P_2(1-\pi)}\left[\varphi E[x](1-\pi) - 2\pi\right] \end{array}\right] \tag{B-10}$$

then

$$E\left[TCU(T_{1\pi})\right] = \frac{\lambda(1-\pi)}{\left[1-\varphi E[x](1-\pi)\right]}\left\{\begin{array}{l} \dfrac{Z_1}{T_{1\pi}} + \dfrac{W_1}{T_{1\pi}} + W_2 e^{-\beta T_{1\pi}} + \dfrac{W_3 e^{-\beta T_{1\pi}}}{T_{1\pi}} \\ +T_{1\pi}\left[\dfrac{P_1 E[x]^2(1-\theta_1)}{2P_2}\left[h_1(1-\theta_1) - h\right] + h(W_4)\right] \\ +\left[\dfrac{\pi(1+\beta_2)C}{(1-\pi)} + C + C_R E[x](1-\theta_1) + C_S\varphi E[x]\right] \\ +\dfrac{h_3 g\left[1-\varphi E[x](1-\pi)\right]}{(1-\pi)}\left(e^{-\beta T_{1\pi}}\right) \end{array}\right\} \tag{26}$$

Appendix – C

Table C-1
Results of convexity examinations on $E[TCU(T_{1\pi})]$ for diverse β values

β	$T_{1\pi U}$	$\delta(T_{1\pi U})$	$T_{1\pi L}$	$\delta(T_{1\pi L})$
12	0.4440	5.7592	0.0182	0.0389
10	0.4442	3.2193	0.0217	0.0465
8	0.4445	1.8944	0.0269	0.0579
6	0.4449	1.1945	0.0353	0.0764
4	0.4457	0.8310	0.0507	0.1117
3	0.4465	0.7322	0.0642	0.1446
2	0.4481	0.6861	0.0857	0.2038
1	**0.4530**	**0.7381**	**0.1221**	**0.3459**
0.5	0.4626	0.9184	0.1494	0.5639
0.01	1.0355	4.4855	0.1835	3.6187

Table C-2
Detailed derivations of $T_{1\pi}$* from a proposed algorithm

Step #	$T_{1\pi U}$	$e^{-\beta T1\pi U}$	$T_{1\pi L}$	$e^{-\beta T1\pi L}$	$T_{1\pi U}$ - $T_{1\pi L}$	$E[TCU(T_{1\pi U})]$	$E[TCU(T_{1\pi L})]$
-	-	0	-	1	-	-	-
1	**0.4530**	0.6357	**0.1221**	0.8850	0.3308	$12,517.24	$12,136.31
2	0.2631	0.7687	0.1716	0.8423	0.0915	$12,029.76	$11,979.70
3	0.2163	0.8055	0.1886	0.8282	0.0277	$11,972.94	$11,967.37
4	0.2026	0.8166	0.1940	0.8237	0.0086	$11,966.79	$11,966.22
5	0.1984	0.8200	0.1957	0.8222	0.0027	$11,966.17	$11,966.11
6	0.1971	0.8211	0.1963	0.8218	0.0008	$11,966.10	$11,966.10
7	0.1967	0.8214	0.1965	0.8216	0.0002	$11,966.10	$11,966.10
8	0.1966	0.8214	0.1965	0.8215	0.0001	$11,966.10	$11,966.10
9	**0.1965**	0.8215	**0.1965**	0.8215	0.0000	**$11,966.10**	**$11,966.10**