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Fabrication runtime decision for a hybrid system incorporating probabilistic breakdowns, scrap, and overtime

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ABSTRACT

Manufacturers today need to optimize their fabrication runtime decision by meeting short customer order due dates externally and managing the potentially unreliable machines and manufacturing processes internally. Outsourcing and overtime are commonly utilized strategies to expedite fabricating time. Additionally, detailed analyses and necessary actions on inevitable product defects (i.e., removal of scraps) and equipment breakdowns (such as machine repairing) are prerequisites to fabrication runtime planning. Motivated by assisting today's manufacturers decide the best batch runtime plan under the situations mentioned above, this study applies mathematical modeling to a hybrid fabrication problem that incorporates partial overtime and outsourcing, inevitable product defects, and a Poisson-distributed breakdown. We develop a model to accurately represent the problem's characteristics. Formulations and detailed model analyses allow us to find the cost function first. Differential equations and algorithms help us confirm the gain function's convexity and find the best runtime decision. Lastly, we use numerical illustrations to show our study's applicability by revealing in-depth crucial managerial information of the studied problem.

Nomenclature

P_{1A} = annual manufacturing rate with the overtime strategy,
C_A = unit manufacturing cost with the overtime strategy,
K_A = manufacturing setup cost with the overtime strategy,
P_1 = standard manufacturing rate (i.e., without overtime strategy),
C = standard unit production cost,
K = standard setup cost,
α_1 = the connecting factor between P_1 and P_{1A},
α_3 = the connecting factor between C and C_A,
α_2 = the connecting factor between K and K_A,
λ = product demand per year,
Q = manufacturing batch size,
t_{1Z} = manufacturing runtime (uptime) of the proposed problem – the decision variable,
π = the outsourced proportion of a batch (where $0 < \pi < 1$),
C_π = unit outsourcing cost,
K_π = setup cost of the outsourcing strategy,
β_2 = the connecting factor between C and C_π,
β_1 = the connecting factor between K and K_π,

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x = Uniform-distributed scrap rate,
d_{1A} = manufacturing rate of scrap items, where $d_{1A} = P_{1A}\,x$,
C_S = disposal cos per scrapped item,
β = mean Poisson-distributed breakdowns per year,
t = mean time to machine breakdowns,
M = fixed mending cost per breakdown occurrence,
t_r = needed/permitted time to fix a breakdown occurrence,
H_0 = product level when a stochastic breakdown occurs,
H_1 = product level when the manufacturing uptime finishes,
H = product level upon receipt of the outsourced products,
t'_{2Z} = finished items issuing time in the breakdown occurring case,
T'_Z = manufacturing cycle length,
g = t_r, needed/permitted time to fix a breakdown occurrence,
h = unit holding cost,
h_3 = holding cost per safety item,
C_1 = unit cost of safety item,
C_T = unit issuing cost,
T_Z = cycle length in the no breakdown occurring case,
t_{2Z} = finished items issuing time in the no breakdown occurring case,
t_1 = runtime for a model without overtime, outsourcing, nor breakdown,
t_2 = finished items issuing time for a model without overtime, outsourcing, nor breakdown,
d_1 = manufacturing rate of scrap items for a model without overtime, outsourcing, nor breakdown,
T = cycle length for a model without overtime, outsourcing, nor breakdown,
$I(t)$ = product level at time t,
$I_F(t)$ = safety product's level at time t,
$I_s(t)$ = scrap product's level at time t,
$TC(t_{1Z})_1$ = total cost per cycle in the breakdown occurring case,
$E[TC(t_{1Z})_1]$ = the expected total cost per cycle in the breakdown occurring case,
$E[T'_Z]$ = the expected cycle length in the breakdown occurring case,
$TC(t_{1Z})_2$ = total cost per cycle in the no breakdown occurring case,
$E[TC(t_{1Z})_2]$ = the expected total cost per cycle in the no breakdown occurring case,
$E[T_Z]$ = the expected cycle time in the no breakdown occurring case,
$\boldsymbol{T_Z}$ = replenishing cycle length of the problem,
$E[TCU(t_{1Z})]$ = the expected annual system cost of the problem.

1. Introduction

Rapidly responding to clients' orders externally and successfully dealing with internal unreliable equipment and processes become critical operational tasks to manufacturers to stay competitive in today's turbulent marketplaces. Outsourcing and overtime are commonly utilized strategies to expedite fabricating time to quick response to customers' short order lead time. Shy and Stenbacka (2003) showed how companies utilized their efficient production design mode as a strategic tool to gain a competitive advantage in a differentiated end products marketplace. The researchers demonstrated how the input supplier introduction could achieve a lower than average component-making cost without losing the economies of scale exploitability, for equilibrium companies would outsource production to a cooperative subcontractor. The authors think the equilibrium is robust whether supplies come from subcontractors or market rivals. Lagodimos and Mihiotis (2010) explored an efficient daily workforce shift and overtime planning for the identical manning lines in packing shops to minimize the operating cost for these packing lines. The researchers showed that the studied problem has an NP-hard nature and focused on solving the exceptional cases of identical-manning lines. Using a heuristic, the researchers found the optimal solution's properties and proved that excluding overtime options, the O(N log N) algorithm could arrive at its optimality. Finally, the researchers used a commercial optimizer to perform computations and comparisons for 2- and 3-shift problems and demonstrate how efficient their algorithm is for these types of issues. Westphal and Sohal (2016) studied the required process for outsourcing decision-making, focusing mainly on information technology (IT). The researchers started with a brief review of IT outsourcing literature to explore (i) its decision processes lacking empirical studies and (ii) how the IT outsourcing decision affects the outcomes. From the literature of strategic decision-making, the authors identified and applied the relevant constructs of decision processes to four Australian firms to investigate their impact on these firms. Two companies among them had already decided to outsource their application development offshore, and the influential stakeholders pushed their decisions. In comparison, the other two firms are at the stage of renewing their outsourcing contracts through a highly rigorous decision procedure. As a result, the researchers found that the context of the decision process affects the adoption type, leading to different outcomes. Therefore, the managers must follow a more formal and rational decision procedure to achieve a better result. Abdul Halim et al. (2021) used lingo software to optimize an overtime deteriorating goods fabrication-inventory model with linear inventory-dependent demand and nonlinear price. The researchers cautiously evaluated and optimized this nonlinear-nature problem with a numerical illustration, performed sensitivity analyses on system parameter changes, and drew conclusions to facilitate managerial decision-making. Other works (Van Mieghem, 1999; Choi, 2007; Conway and Sturges, 2014; Raut et al., 2018; Sumrit, 2020; Astuty et al., 2021; Chiu et al., 2021) also explored the influence of various features of overtime/outsourcing strategies on production planning and control in manufacturing firms.

Promptly and cautiously handling potentially unreliable machines and manufacturing processes are critical for production managers to avoid delay-in-production and retain high product quality. Yeh and Chen (2006) derived the optimal batch size and product screening strategy for deteriorating manufacturing systems, wherein free minimal warranty is associated with all products sold. In addition, the researchers implemented an inspection strategy for the last K products in a lot and reworked the resulting nonconforming items if found. Accordingly, they created a model to simultaneously find the optimal batch size and screening policy that kept the minimum annual expenses for the system. As a result, they found an approximate solution with an upper bound using a proposed algorithm through numerical examples since no closed-form solution concerning the optimal batch size could arrive. Dehayem Nodem et al. (2011) studied production planning considering repair and replacement switching strategies for unreliable fabrication systems subject to machine failures. The manufacturing rate and the corrected action upon the breakdown instance are the decision variables of their model, and they aimed to minimize the overall operating cost, including repair/replacement, stock holding, and backlogging expenses. Further, the researchers assumed a semi-Markovian process on failures and repair-history-dependent corrected activities. Accordingly, the authors developed the optimality conditions and demonstrated how their model and control policies work through stochastic dynamic programming and numerical illustration. Moussawi-Haidar et al. (2016) examined a fabrication batch size problem incorporating quality inspection and reworking of defects focusing on the impact of screening time on the studied problem. The researchers specifically explored two consequences of handling the random, imperfect quality items: (1) through a discount sale and (2) repairing them via rework, wherein only the perfect quality products are used to meet the customer demand. In addition, the renewal reward theorem was also applied to develop the anticipated profit functions and derive the optimal closed-form batch size solution. Finally, they offered numerical illustrations to demonstrate how their model worked. Larbi Rebaiaia and Ait-kadi (2021) evaluated various maintenance policies with minimal corrected actions at the breakdown and conditional replacements under different disciplines to determine the most cost-saving approach. Their proposed replacement policies include (1) only at the first breakdown occurrence and (2) at each breakdown instance. Accordingly, the researchers constructed mathematical models to incorporate the above corrective actions and preventive maintenance at component and system levels. In addition, they developed an algorithm to estimate the Weibull-distributed replacement functions and find the cost-minimization maintenance strategy in multiple component industrial cases. Finally, they performed a sensitivity analysis and compared opportunistic maintenance policies for further potential cost reduction. As a result, they found that the strategy of replacing a piece of new equipment at each breakdown instance turned out to be the most cost-reduced and efficient one. Other works (Martorell et al., 1999; Grosfeld-Nir and Gerchak, 2002; Ouyang et al., 2015; Khanna et al., 2020; Daryanto and Christata, 2021; Suroso et al., 2021; Terdpaopong et al., 2021; Yera et al., 2021) also examined the effect of various characteristics of deteriorating manufacturing systems with defeats and equipment failures on planning, controlling, and management of diverse fabrication systems. Few prior works have studied the fabrication runtime decision for a hybrid system incorporating probabilistic breakdowns, scrap, and overtime. This work intends to fill this gap.

2. The proposed problem

The present work explores the fabrication runtime decision for a hybrid system incorporating probabilistic breakdowns, scrap, and overtime. The proposed problem is described below. Consider a hybrid batch manufacturing system that needs to meet a product requirement of λ per year. A proportion π of its batch size Q is supplied by an external source to cut down its uptime. Besides, the proposed model utilizes an overtime strategy to shorten the cycle runtime further. The following are the relationships of overtime and outsourcing relevant parameters (refer to the Nomenclature for details) versus their corresponding standard parameters:

$$P_{1A} = (1+\alpha_1) P_1 \tag{1}$$

$$C_A = (1+\alpha_3) C \tag{2}$$

$$K_A = (1+\alpha_2) K \tag{3}$$

$$C_\pi = (1+\beta_2) C \tag{4}$$

$$K_\pi = (1+\beta_1) K \tag{5}$$

A random x portion of the batch produced is scrap. The quality assurance asks for a full screening and disposing of all faulty produced by the in-house manufacturing process. The proposed problem does not permit a stock-out condition; hence, $(P_{1A} - d_{1A} - \lambda) > 0$ must hold. Moreover, the manufacturing equipment is subject to a Poisson-distributed breakdown with β as the mean per year. The time to a machine breaks down t obeys an Exponential distribution with $f(t) = \beta e^{-\beta t}$ as its density function.

Upon a breakdown occurrence, we use an abort/resume stock control discipline. It resumes manufacturing the unfinished batch right away once the correction action on the breakdown instance is done. We assume a fixed/allowable machine repair time t_r; however, if actual machine repairing time exceeds t_r, a rental/spare equipment takes over the manufacturing task to prevent the undesirable production schedule delay. Due to the stochastic breakdowns, we explicitly investigate the following two distinct conditions:

2.1. Condition one: The manufacturing equipment breaks down during the uptime

This condition means that the time machine breaks down $t < t_{1Z}$. Fig. 1 exhibits the product level of the proposed hybrid manufacturing runtime problem incorporating stochastic breakdowns, overtime, and scrap (in thicker lines) compared to the same problem with scrap items only (in thinner lines). The product level in Fig. 1 reaches H_0 when a breakdown takes place, and during machine repair time t_r it remains the same. Upon restoration of equipment, the product level grows again and up to H_1 when uptime t_{1Z} ends. Meantime, the receipt of outsourced products is added and brings the product level to H. Lastly, it finishes at zero when the stock issuing time ends, and the next replenishing cycle initiates.

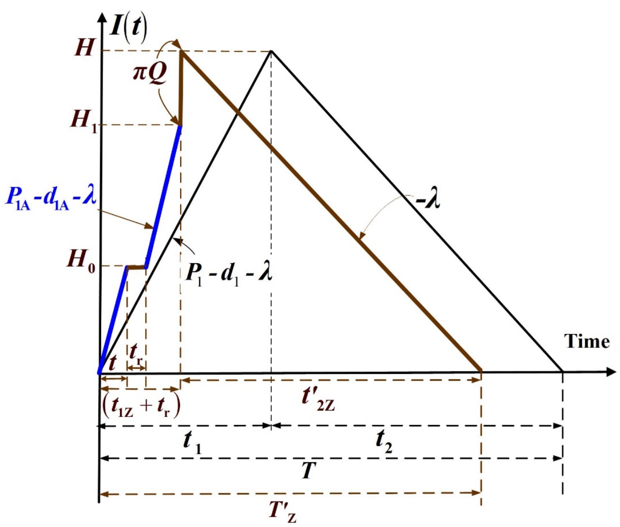

Fig. 1. The product level of the proposed hybrid fabrication runtime problem incorporating probabilistic breakdowns, overtime, and scrap (in thicker lines) compared to the same problem with scrap items only (in thinner lines)

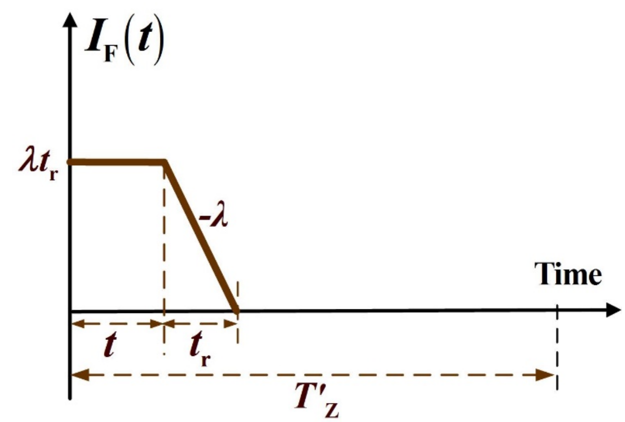

Fig. 2. The safety product level in condition one

Fig. 2 illustrates the safety product level in condition one. Because a breakdown takes place, the safety products are utilized to meet the demand in machine-repair time t_r. Fig. 3 shows the scrap product level in condition one. The maximum scrap level is as follows:

$$d_{1A}t_{1Z} = xP_{1A}t_{1Z} = x(1-\pi)Q \tag{6}$$

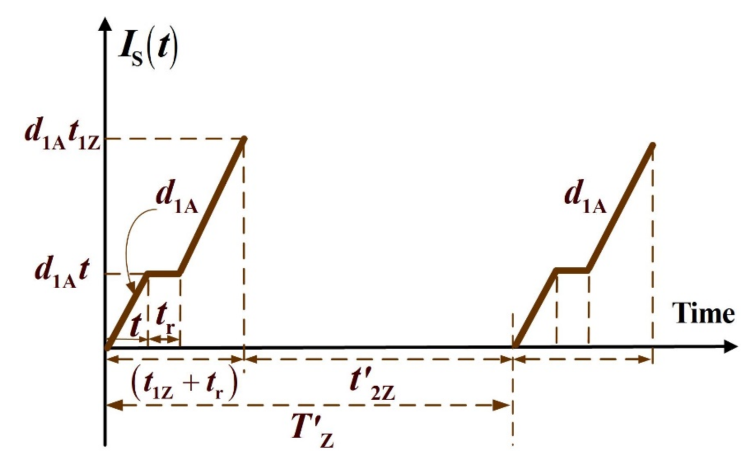

Fig. 3. The scrap product level in condition one

We can observe the following equations based on the problem description and from Fig. 1 to Fig. 3:

$$H_0 = \left(P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda\right)t \tag{7}$$

$$t_{1\mathrm{Z}} = \frac{H_1}{P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda} = \frac{(1-\pi)Q}{P_{1\mathrm{A}}} \tag{8}$$

$$H_1 = \left(P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda\right)t_{1\mathrm{Z}} \tag{9}$$

$$H = H_1 + \pi Q \tag{10}$$

$$t'_{2\mathrm{Z}} = \frac{H}{\lambda} \tag{11}$$

$$T'_{\mathrm{Z}} = t_{1\mathrm{Z}} + t_{\mathrm{r}} + t'_{2\mathrm{Z}} \tag{12}$$

$TC(t_{1\mathrm{Z}})_1$, the total cost per cycle, in condition one includes the following (as exhibited in Eq. (13)): both the variable and fixed in-house fabrication and outsourcing costs, safety product relevant expense (see Fig. 2), breakdown correction repair cost, and scrap disposal cost (see Figure 3), and overall holding costs during T'_{Z} (including the perfect and scrap products).

$$\begin{aligned} TC\left(t_{1\mathrm{Z}}\right)_1 &= C_A\left[(1-\pi)Q\right] + K_A + C_\pi\left(\pi Q\right) + K_\pi + C_1\left(\lambda t_r\right) + C_T\left(\lambda t_r\right) + h_3\left(\lambda t_r\right)\left(t + \frac{t_r}{2}\right) \\ &+ M + C_S x\left[(1-\pi)Q\right] + h\left[\frac{H_1 + d_{1A}t_{1\mathrm{Z}}}{2}\left(t_{1\mathrm{Z}}\right) + \left(H_0 t_r\right) + \left(d_{1A}t\right)t_r + \frac{H}{2}\left(t'_{2Z}\right)\right] \end{aligned} \tag{13}$$

Substitute Eqs. (1) to (6) in Eq. (13), $TC(t_{1\mathrm{Z}})_1$ becomes as shown in Eq. (14).

$$\begin{aligned} TC\left(t_{1\mathrm{Z}}\right)_1 &= \left(1+\alpha_3\right)C\left[(1-\pi)Q\right] + \left(1+\alpha_2\right)K + \left(1+\beta_2\right)C\left(\pi Q\right) + \left(1+\beta_1\right)K \\ &+ C_1\left(\lambda t_r\right) + C_T\left(\lambda t_r\right) + h_3\left(\lambda t_r\right)\left(t + \frac{t_r}{2}\right) + M + C_S x\left[(1-\pi)Q\right] \\ &+ h\left[\frac{H_1 + x\left(1+\alpha_1\right)P_1 t_{1\mathrm{Z}}}{2}\left(t_{1Z}\right) + \left(H_0 t_r\right) + x\left(1+\alpha_1\right)P_1\left(t\right)t_r + \frac{H}{2}\left(t'_{2Z}\right)\right] \end{aligned} \tag{14}$$

2.2. Condition two: No breakdown occurrence during the uptime

No breakdown occurrence during $t_{1\mathrm{Z}}$, meaning that time to a breakdown occurrence $t > t_{1\mathrm{Z}}$. Fig. 4 displays the product level of the proposed hybrid manufacturing runtime problem incorporating overtime, scrap, but with no stochastic breakdowns (in thicker lines) compared to the same problem with scrap items only (in thinner lines). Fig. 4 points out that the product level grows up to H_1 when uptime $t_{1\mathrm{Z}}$ ends. It arrives at H when the outsourced items are received. Then, the product level finishes at zero when the issuing time $t_{2\mathrm{Z}}$ ends, and the next replenishing cycle initiates.

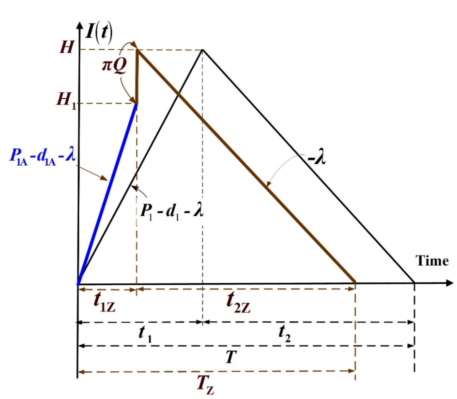

Fig. 4. The product level of the proposed hybrid manufacturing runtime problem incorporating overtime and scrap, but with no stochastic breakdowns, (in thicker lines) compared to the same problem with scrap items only (in thinner lines)

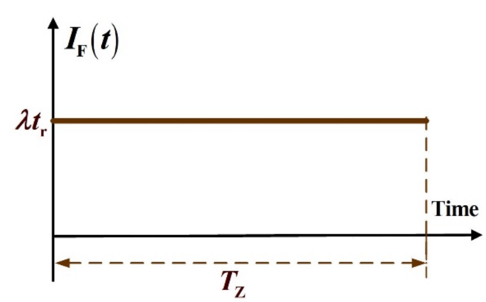

Fig. 5. The safety product level in condition two

Fig. 5 illustrates the safety product level in condition two. Since no breakdown takes place, the safety product level stays the same in the cycle.

Fig. 6 shows the scrap product level in condition two. It points out that the maximum scrap level is $d_{1A}t_{1Z}$. We can directly observe the following formulas based on the problem description and from Fig. 4 to Fig. 6:

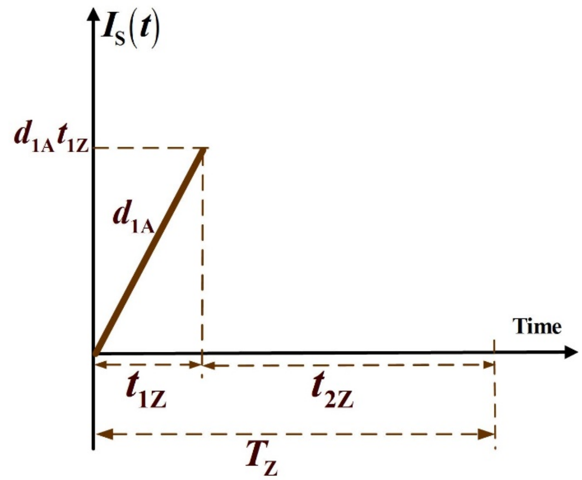

Fig. 6. The scrap product level in condition two

$$t_{1Z} = \frac{H_1}{P_{1A} - d_{1A} - \lambda} = \frac{Q(1-\pi)}{P_{1A}} \tag{15}$$

$$H_1 = (P_{1A} - d_{1A} - \lambda)t_{1Z} \tag{16}$$

$$H = H_1 + \pi Q \tag{17}$$

$$t_{2Z} = \frac{H}{\lambda} \tag{18}$$

$$T_Z = t_{1Z} + t_{2Z} \tag{19}$$

$TC(t_{1Z})_2$, the total cost per cycle, in condition two includes the following (as shown in Eq. (20)): both the variable and setup in-house fabrication and outsourcing costs, safety product holding relating cost (see Fig. 5), disposal and total holding costs during T_Z (including the finished and scrap products).

$$\begin{aligned} TC(t_{1Z})_2 &= C_A\left[(1-\pi)Q\right] + K_A + C_\pi(\pi Q) + K_\pi + h_3(\lambda t_r)T_Z \\ &+ C_S x\left[(1-\pi)Q\right] + h\left[\frac{H_1 + d_{1A}t_{1Z}}{2}(t_{1Z}) + \frac{H}{2}(t_{2Z})\right] \end{aligned} \tag{20}$$

Substitute Eqs. (1) to (6) in Eq. (20), $TC(t_{1Z})_2$ becomes as shown in Eq. (21).

$$\begin{aligned} TC(t_{1Z})_2 &= (1+\alpha_3)C\left[(1-\pi)Q\right] + (1+\alpha_2)K + (1+\beta_2)C(\pi Q) + (1+\beta_1)K \\ &+ h_3(\lambda t_r)T_Z + C_S x\left[(1-\pi)Q\right] + h\left[\frac{H_1 + x(1+\alpha_1)P_1 t_{1Z}}{2}(t_{1Z}) + \frac{H}{2}(t_{2Z})\right] \end{aligned} \tag{21}$$

2.3. Combining both conditions one and two

This study assumes the stochastic machine breakdowns that follow the Poisson distribution with β instances as mean per year. Therefore, time to a breakdown taking place obeys an Exponential distribution with density function $f(t) = \beta e^{-\beta t}$ and cumulative density function $F(t) = (1 - e^{-\beta t})$. Furthermore, we apply the expected scrap rate $E[x]$ to deal with its randomness in Eqs. (14) and (21), and employ the renewal reward theorem to cope with variable cycle length caused by random scrap rate assumption. Therefore, $[TCU(t_{1Z})]$ can be derived as follows:

$$E\left[TCU(t_{1Z})\right] = \frac{\left\{\int_0^{t_{1Z}} E\left[TC(t_{1Z})_1\right]\cdot f(t)\,dt + \int_{t_{1Z}}^{\infty} E\left[TC(t_{1Z})_2\right]\cdot f(t)\,dt\right\}}{E[\boldsymbol{T}_Z]} \tag{22}$$

where $E[\boldsymbol{T}_Z]$, $E[T'_Z]$, and $E[T_Z]$ represent the following:

$$E[\boldsymbol{T_Z}] = \int_0^{t_{1Z}} E[T'_Z] f(t)\,dt + \int_{t_{1Z}}^{\infty} E[T_Z] f(t)\,dt \tag{23}$$

$$E[T'_Z] = \frac{Q\left[1 - E[x](1-\pi)\right] + \lambda t_r}{\lambda} = \frac{t_{1Z} P_{1A}\left[\frac{1}{(1-\pi)} - E[x]\right] + \lambda t_r}{\lambda} \tag{24}$$

$$E[T_Z] = \frac{Q\left[1 - E[x](1-\pi)\right]}{\lambda} = \frac{t_{1Z} P_{1A}\left[\frac{1}{(1-\pi)} - E[x]\right]}{\lambda} \tag{25}$$

Substitute Eqs. (14), (21), and (23) in Eq. (22), with further derivations, we can derive $E[TCU(t_{1Z})]$ as exhibited in Eq. (26) (for details, please refer to Appendix A):

$$E\left[TCU\left(t_{1Z}\right)\right] = \frac{\lambda}{\delta_0 + \dfrac{\lambda g\left(1-e^{-\beta t_{1Z}}\right)}{t_{1Z}\left(1+\alpha_1\right)P_1}} \cdot \left\{ \begin{array}{l} \dfrac{\delta_1}{t_{1Z}} + \delta_2 + \left(t_{1Z}\right)\delta_3 + \dfrac{G_0}{t_{1Z}}\left(1-e^{-\beta t_{1Z}}\right) - G_1\left(e^{-\beta t_{1Z}}\right) \\ + \dfrac{G_2}{t_{1Z}}\left(1-e^{-\beta t_{1Z}}\right) + G_3\left(1-e^{-\beta t_{1Z}}\right) \end{array} \right\} \tag{26}$$

3. Solving the optimal runtime of the problem

Formulas (A-5) and (A-6) exhibit the first- and second-derivative of $E[TCU(t_{1Z})]$ (see Appendix A), and since the first term on the right-hand side (RHS) of Eq. (A-6) is positive, so $E[TCU(t_{1Z})]$ is convex, if the second term on the RHS of Eq. (A-6) is also positive. That is, if $\omega(t_{1Z}) > t_{1Z} > 0$ holds (see formulas (A-7) in Appendix A). As we confirm that formula (A-7) is true, the optimal t_{1Z}^* can be determined by setting the first-derivative of $E[TCU(t_{1Z})]$ equal to zero (refer to Eq. (A-5)). Because the first term on the RHS of Eq. (A-5) is positive, we have the following:

$$\left\{ \begin{array}{l} \left[\begin{array}{l} \delta_3\left(\delta_4 - \lambda g \beta e^{-\beta t_{1Z}}\right) \\ + \delta_4\left(G_1 + G_3\right)\left(\beta e^{-\beta t_{1Z}}\right) \end{array} \right] \left(t_{1Z}\right)^2 \\ + \left[-\delta_2 \lambda g\left(\beta e^{-\beta t_{1Z}}\right) + \delta_3\left[2\lambda g\left(1-e^{-\beta t_{1Z}}\right)\right] + \delta_4\left(G_0 + G_2\right)\left(\beta e^{-\beta t_{1Z}}\right) + G_1\left(\beta \lambda e^{-\beta t_{1Z}} g\right) \right]\left(t_{1Z}\right) \\ + \left[\begin{array}{l} -\delta_1\left(\lambda g \beta e^{-\beta t_{1Z}} + \delta_4\right) + \delta_4\left(G_0 + G_2\right)\left(e^{-\beta t_{1Z}} - 1\right) - \delta_2 \lambda g\left(e^{-\beta t_{1Z}} - 1\right) \\ + \left(G_1 + G_3\right)\lambda e^{-2\beta t_{1Z}} g - \left(G_1 + 2G_3\right)\left(\lambda e^{-\beta t_{1Z}} g\right) + G_3 \lambda g \end{array} \right] \end{array} \right\} = 0 \tag{27}$$

Let m_2, m_1, and m_0 represent the following:

$$\begin{aligned} m_2 &= \delta_3\left(\delta_4 - \lambda g \beta e^{-\beta t_{1Z}}\right) + \delta_4\left(G_1 + G_3\right)\left(\beta e^{-\beta t_{1Z}}\right) \\ m_1 &= -\delta_2 \lambda g\left(\beta e^{-\beta t_{1Z}}\right) + \delta_3\left[2\lambda g\left(1-e^{-\beta t_{1Z}}\right)\right] + \delta_4\left(G_0 + G_2\right)\left(\beta e^{-\beta t_{1Z}}\right) + G_1\left(\beta \lambda e^{-\beta t_{1Z}} g\right) \\ m_0 &= \left[\begin{array}{l} -\delta_1\left(\lambda g \beta e^{-\beta t_{1Z}} + \delta_4\right) + \delta_4\left(G_0 + G_2\right)\left(e^{-\beta t_{1Z}} - 1\right) - \delta_2 \lambda g\left(e^{-\beta t_{1Z}} - 1\right) \\ + \left(G_1 + G_3\right)\lambda e^{-2\beta t_{1Z}} g - \left(G_1 + 2G_3\right)\left(\lambda e^{-\beta t_{1Z}} g\right) + G_3 \lambda g \end{array} \right] \end{aligned}$$

Formula (27) becomes as follows:

$$m_2\left(t_{1Z}\right)^2 + m_1\left(t_{1Z}\right) + m_0 = 0 \tag{28}$$

The optimal runtime t_{1Z}^* can be determined by utilizing the square roots solution:

$$t_{1Z}^* = \frac{-m_1 \pm \sqrt{m_1^2 - 4m_2 m_0}}{2m_2} \tag{29}$$

Since $F(t_{1Z}) = (1 - e^{-\beta t1Z})$, the cumulative density function falls in [0, 1], so does $e^{-\beta t1Z}$ (i.e., its complement). By rearranging Eq. (27), we arrive at $e^{-\beta t1Z}$ as shown in Eq. (30).

$$e^{-\beta t_{1Z}} = \frac{-\left[\delta_4\delta_3(t_{1Z})^2 + 2\delta_3\lambda g(t_{1Z}) - \delta_4\delta_1 + \delta_2\lambda g - \delta_4(G_0+G_2) + (G_1+G_3)\lambda e^{-2\beta t_{1Z}} g + G_3\lambda g\right]}{\begin{Bmatrix}(G_1+G_3)\beta\delta_4(t_{1Z})^2 - \delta_3\lambda g\beta(t_{1Z})^2 - \delta_2\lambda g(\beta)(t_{1Z}) + \delta_4(G_0+G_2)(\beta)(t_{1Z}) \\ -2\delta_3\lambda g(t_{1Z}) + G_1(\beta\lambda g)(t_{1Z}) - \delta_1\lambda g\beta - \delta_2\lambda g + \delta_4(G_0+G_2) - (G_1+2G_3)(\lambda g)\end{Bmatrix}} \tag{30}$$

*3.1. A proposed recursive algorithm for locating t_{1Z}**

The following is a proposed recursive algorithm to locate the optimal runtime t_{1Z}*:

(1) Start with letting $e^{-\beta t1Z} = 0$ and $e^{-\beta t1Z} = 1$ to compute formula (29) and find the upper and lower bounds for t_{1Z} (i.e., t_{1ZU} and t_{1ZL}).
(2) Update the values of $e^{-\beta t1ZU}$ and $e^{-\beta t1ZL}$ using current values of t_{1ZU} and t_{1ZL}.
(3) Re-compute formula (29) with the updated $e^{-\beta t1ZU}$ and $e^{-\beta t1ZL}$ to gain a new/updated set of bounds t_{1ZU} and t_{1ZL}.
(4) Test if ($t_{1ZU} = t_{1ZL}$) holds. If yes, then t_{1Z}* is located. Meaning that the optimal runtime $t_{1Z}* = t_{1ZL} = t_{1ZU}$; otherwise, repeat on to procedure (2).

4. Result demonstration with an example

The proposed model and result's demonstration is depicted with an example in this section. Variables' assumptions are given as displayed in Table 1.

Table 1
Variables' assumptions in our numerical demonstration

P_1	β	π	K	α_1	α_3	h	C_S	C_1	P_2
10000	1	0.4	$200	0.5	0.1	$0.4	$0.1	$2	5000
λ	M	β_2	β_1	x	α_2	C	h_1	h_3	g
4000	$2500	0.5	-0.70	20%	0.1	$2	$0.4	$0.4	0.018

First of all, we must make certain $E[TCU(t_{1Z})]$ is convex by verifying that Eq. (A-7) (i.e., $\omega(t_{1Z}) > t_{1Z} > 0$) holds. Since the value of $e^{-\beta t1Z}$ is within [0, 1], by letting $e^{-\beta t1Z} = 0$ and $e^{-\beta t1Z} = 1$ first and applying Eq. (29), $t_{1ZL} = 0.0694$ and $t_{1ZU} = 0.3628$ can be gained. Secondly, computing Eq. (A-7) with $e^{-\beta t1ZL}$ and $e^{-\beta t1ZU}$, $\omega(t_{1ZL}) = 0.1831 > t_{1ZL} > 0$ and $\omega(t_{1ZU}) = 0.5013 > t_{1ZU} > 0$ are obtained. So, for $\beta = 1$, $E[TCU(t_{1Z})]$' convexity is confirmed and optimal t_{1Z}* exists. Boarder applicability of our model and results are demonstrated with more comprehensive β values.

Table 2
Our model's applicability is demonstrated with wider β values

β	$\omega(t_{1ZU})$	t_{1ZU}	$\omega(t_{1ZL})$	t_{1ZL}
11	1.6526	0.3564	0.0220	0.0106
8	0.9235	0.3566	0.0298	0.0143
5	0.5866	0.3572	0.0465	0.0221
4	0.5251	0.3575	0.0572	0.0270
3	0.4844	0.3581	0.0744	0.0344
2	0.4676	0.3593	0.1058	0.0468
1	**0.5013**	**0.3628**	**0.1831**	**0.0694**
0.5	0.6017	0.3697	0.3025	0.0876
0.01	2.9169	0.7956	2.2260	0.1118

To derive t_{1Z}*, we applying sub-section 3.1's algorithm. Table 3 gives the detailed iterative outcomes from executing the algorithm and arrives at $t_{1Z}* = 0.1175$ and $E[TCU(t_{1Z}*)]$ = $11,973. Fig.7 demonstrates the $E[TCU(t_{1Z})]$'s convexity and behavior relating to t_{1Z}. It points out t_{1Z}'s starting lower and upper bounds, t_{1Z}*, and $E[TCU(t_{1Z})]$'s convexity and behavior concerning t_{1Z}.

Table 3
The detailed iterative outcomes from the recursive algorithm for $t_{1Z}*$

Step	t_{1ZL}	$e^{-\beta t1ZL}$	$E[TCU(t_{1ZL})]$	t_{1ZU}	$e^{-\beta t1ZU}$	$E[TCU(t_{1ZU})]$	t_{1ZU} - t_{1ZL}
-	-	1	-	-	0	-	-
1	**0.0694**	0.9330	$12087.94	**0.3628**	0.6957	$12546.93	0.2934
2	0.0998	0.9050	$11983.96	0.1841	0.8318	$12056.39	0.0843
3	0.1112	0.8947	$11974.36	0.1389	0.8703	$11984.50	0.0277
4	0.1153	0.8911	$11973.29	0.1247	0.8828	$11974.58	0.0094
5	0.1167	0.8898	$11973.16	0.1200	0.8870	$11973.32	0.0033
6	0.1172	0.8894	$11973.15	0.1183	0.8884	$11973.17	0.0011
7	0.1174	0.8892	$11973.15	0.1178	0.8889	$11973.15	0.0004
8	0.1175	0.8892	$11973.15	0.1176	0.8891	$11973.15	0.0001
9	**0.1175**	0.8892	**$11973.15**	**0.1175**	0.8891	**$11973.15**	**0.0000**

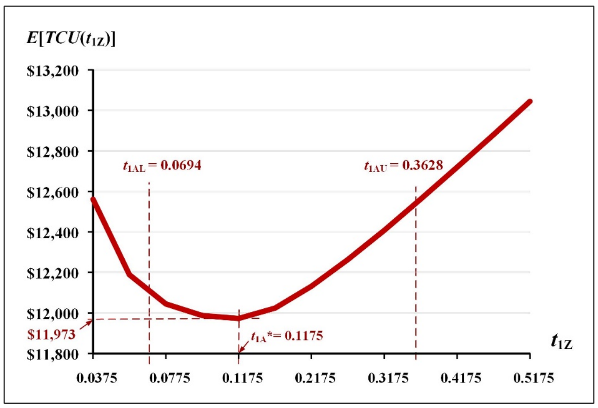

Fig. 7. The convexity and behavior of $E[TCU(t_{1Z})]$ relating to t_{1Z}

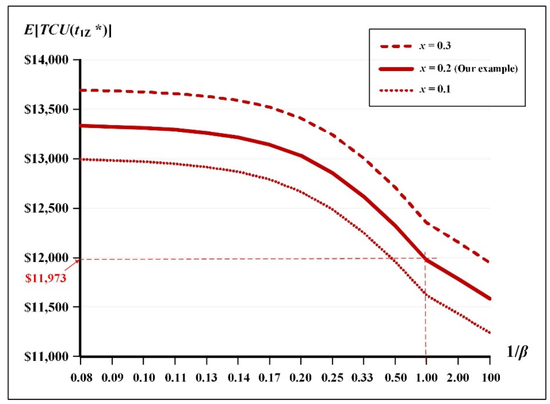

Fig. 8. The combined effect of changes in $1/\beta$ and x on $E[TCU(t_{1Z}*)]$

3.1. The impact of stochastic breakdowns and scrap

The impact of stochastic machine breakdowns and random scrap are explicitly investigated as follows: Fig. 8 exhibits the combined effect of changes in $1/\beta$ (i.e., mean-time-to-breakdown) values and a few defective rate x on $E[TCU(t_{1Z}*)]$. It discloses that $E[TCU(t_{1Z}*)]$ considerably decreases as $1/\beta$ rises beyond 0.20. As x increases, $E[TCU(t_{1Z}*)]$ noticeable upsurges. Fig. 9 illustrates the collective influence of variations $1/\beta$ and x on the optimal runtime $t_{1Z}*$. It exposes that $t_{1Z}*$ drastically declines as $1/\beta$ rises beyond 0.20. As x increases, $t_{1Z}*$ surges.

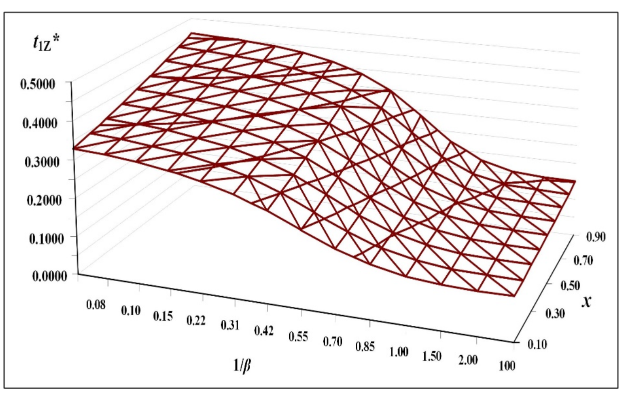

Fig. 9. The collective influence of variations in $1/\beta$ and x on $t_{1Z}*$

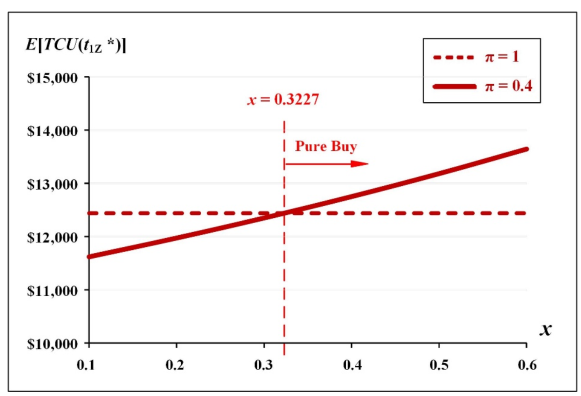

Fig. 10. The critical defective rate x on the 'make-or-buy' decision-making

The analytical outcome of the critical defective rate x (i.e., 0.3227) on the 'make-or-buy' decision-making is illustrated in Fig. 10. It reveals that as x rises up and over 0.3227, a 'pure buy' decision is more beneficial than a hybrid stock replenishing system. The detailed cost contributors of $E[TCU(t_{1Z}*)]$ are analyzed and illustrated in Fig. 11. It discloses that the outsourcing variable cost and in-house variable cost are the two major cost contributors to $E[TCU(t_{1Z}*)]$. They add up to 82.26% (i.e., 42.53% + 39.73%). The reliability cost adds up to 6.58%, which includes a 3.57% relating to random breakdowns and a 3.01% concerning disposal cost and extra production cost due to the random defective items.

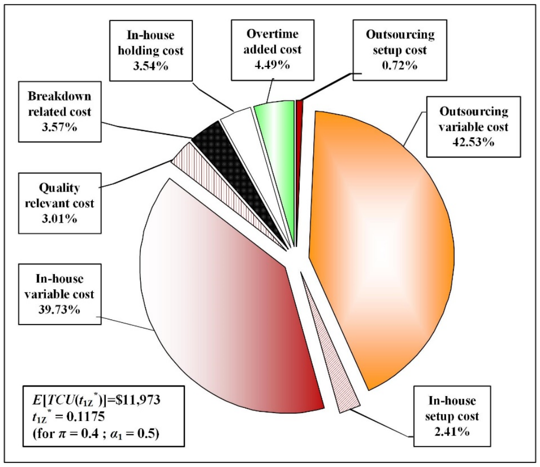

Fig. 11. The detailed cost contributors to $E[TCU(t_{1Z}*)]$

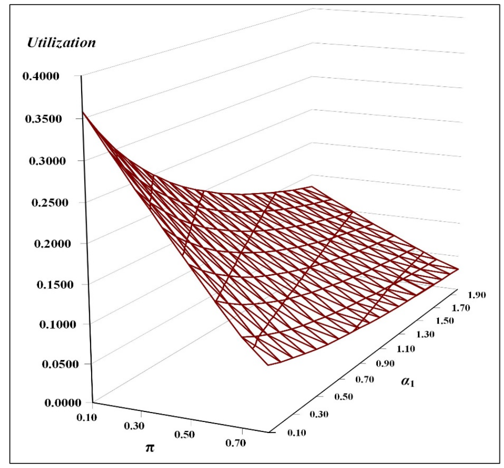

Fig. 12. The combined impact of π and α_1 on utilization

3.2. Influence of dual uptime/utilization reduction strategies

The influence of dual uptime/utilization reduction strategies are examined explicitly as follows: Fig. 12 portrays the combined impact of outsourcing proportion π and overtime factor α_1 on utilization. It exposes that utilization significantly drops as both π and α_1 rise. Also, it reveals π has more impact on utilization's decrease than that of α_1. A further explorative result shows the impact of overtime factor α_1 on utilization, as depicted in Fig. 13. As α_1 rises, the utilization radically decreases. For α_1 = 0.5, this example's utilization declines 33.25% to 0.1697 (from 0.2543, i.e., utilization for a model without overtime option).

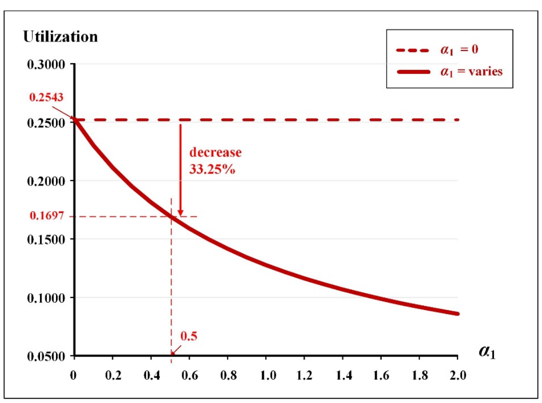

Fig. 13. The impact of α_1 on utilization

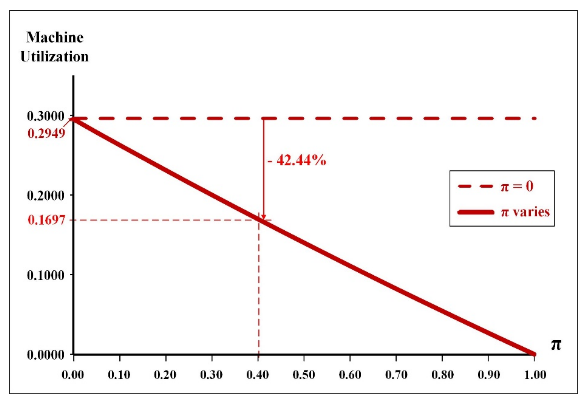

Fig. 14. The investigative outcome of the influence of π on utilization

Fig. 14 exhibits the investigative outcome of the influence of π on utilization. The machine utilization severely decreases as π increases. For π = 0.4, this example's utilization drops 42.44% to 0.1697 (from 0.2949, i.e., utilization for a model without outsourcing). This study implements dual uptime-reduction strategies (i.e., the outsourcing and overtime options) to significantly reduce machine utilization. Fig. 15 illustrates the explorative result from comparing this study's utilization with that of existing studies. In addition to what was described in Fig. 13 and Fig. 14, this study's utilization drops 61.56% compared to an existing study that did not adopt any uptime-reduction strategies (Chiu et al., 2013). For a 33.25%, 42.44%, and 61.56% utilization decline, this study pays the prices of a 3.67%, 6.81%, and 14.01% surge in $E[TCU(t_{1Z}*)]$, respectively. Specifically, $E[TCU(t_{1Z}*)]$ upsurges to $11,973 from $11,549, $11,210, and $10,502, respectively. Fig. 16 depicts the collective influence of overtime factor α_1 and outsourcing proportion π on $E[TCU(t_{1Z}*)]$. It discloses that $E[TCU(t_{1Z}*)]$ significantly upsurges as both α_1 and π rise. This example indicates π has more influence on $E[TCU(t_{1Z}*)]$'s increase than that of α_1.

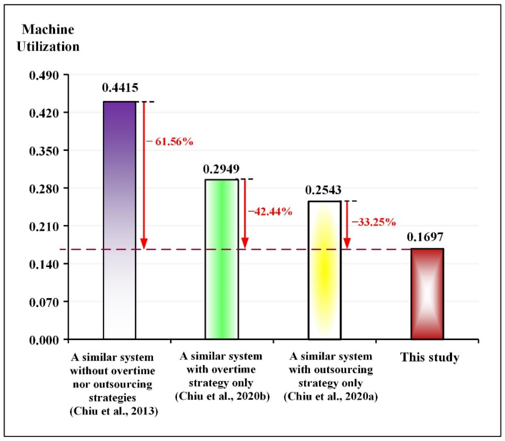

Fig. 15. The explorative result from comparing this study's utilization with existing studies

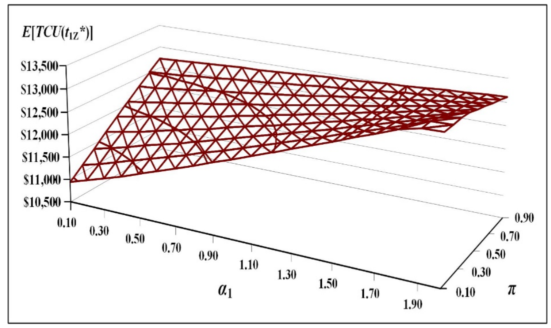

Fig. 16. The collect influence of α_1 and π on $E[TCU(t_{1Z}*)]$

Furthermore, Fig. 17 provides the managerial insights concerning an economic/effective utilization-reduction strategy. It suggests an economic/practical uptime-reduction approach, that is: (1) by setting π = 0 and increasing α_1 initially (see the bold brown dash line); (2) once the utilization decreases to 0.2612 (i.e., when α_1 reached 0.6942 and π = 0), then switches to the strategy of resetting α_1 = 0 and π = 0.3845; and (3) let π remain at 0.3845 and starts to increase α_1 (see the bold brown solid line).

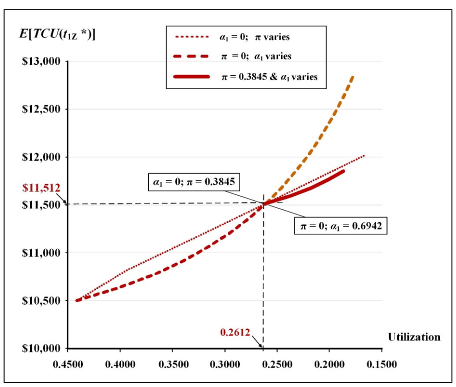

Fig. 17. Managerial insights concerning an economic/practical utilization-reduction strategy

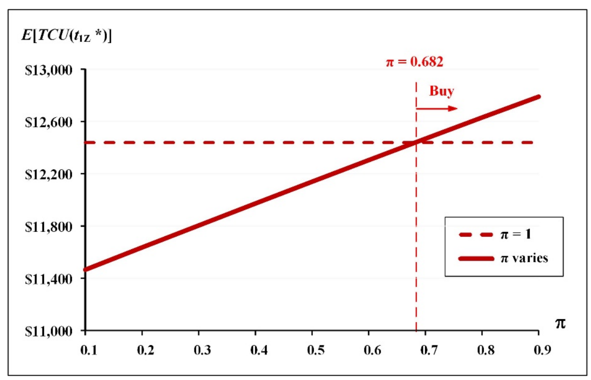

Fig. 18. The critical outsourcing proportion π on 'make-or-buy' decision

3.3. *The critical values for 'make-or-buy' decision-making*

Lastly, this study provides additional managerial decision-supporting information regarding critical parameter values on the 'make-or-buy' decision making. Fig. 18 discloses the critical outsourcing proportion π (i.e., 0.682) on the 'make-or-buy' decision. It specifies as π surges to and over 0.682; a 'pure buy' decision is more economical. Fig. 19 analyzes and exhibits critical value for outsourcing cost-added β_2 on the 'pure buy' decision making. It exposes the critical β_2 = 0.2751 for the 'pure make' decision-making. As β_2 increases to over 27.51%, a 'pure make' decision is a more economical choice.

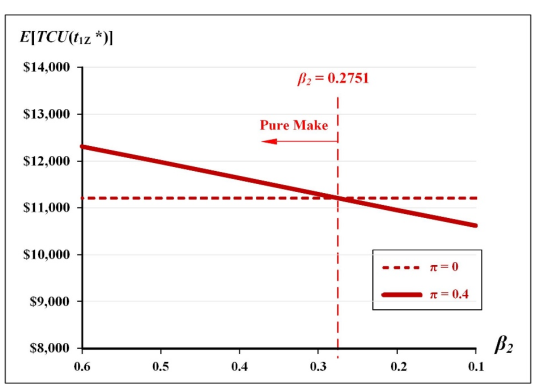

Fig. 19. The critical outsourcing cost-added factor β_2 on the 'pure make' decision-making

4. Conclusions

This study applies mathematical modeling to explore a hybrid fabrication runtime problem with partial overtime and outsourcing, inevitable product defects, and a Poisson-distributed failure to assist today's manufacturers in deciding the best runtime plan that meets order due dates externally and manages unreliable breakdowns and product defects internally. The formulations, detailed model analyses, differential equations, and algorithm allow us to obtain the cost function, confirm the gained function's convexity, and derive the best runtime decision. Lastly, numerical illustrations help us prove our study's applicability by exposing numerous crucial managerial information about the studied problem. For example:

(1) Reconfirmation of the convexity of $E[TCU(t_{1Z})]$ (see Fig. 7);
(2) The impact of stochastic breakdowns and scrap on $E[TCU(t_{1Z}^*)]$ and t_{1Z}^* (refer to Fig. 8 to Fig. 11);
(3) The influence of dual fabricating uptime and utilization reduction strategies on $E[TCU(t_{1Z}^*)]$ and utilization (see Fig. 12 to Fig. 17);
(4) The critical values for 'make-or-buy' decision-making (refer to Fig. 18 to Fig. 19).

One of the worth investigative subjects for future study is to include stochastic annual demand into the problem.

Acknowledgment

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Appendix – A

Derivations of $E[TCU(t_{1Z})]$ (Eq. (26)) and its convexity.

This study applies the expected scrap rate $E[x]$ to deal with its randomness in Eqs. (14) and (21), and employs the renewal reward theorem to cope with variable cycle length caused by random scrap rate assumption. Then, by substituting Eqs. (14), (21), and (23) in Eq. (22), with further derivations, one can derive $E[TCU(t_{1Z})]$ as exhibited below.

$$E\left[TCU\left(t_{1Z}\right)\right]=\frac{\left\{\int_0^{t_{1Z}} E\left[TC\left(t_{1Z}\right)_1\right]f\left(t\right)dt+\int_{t_{1Z}}^{\infty} E\left[TC\left(t_{1Z}\right)_2\right]f\left(t\right)dt\right\}}{E[\boldsymbol{T}_Z]}$$

$$=\left[\frac{\lambda}{\delta_0+\dfrac{\lambda g\left(1-e^{-\beta t_{1Z}}\right)}{\left(t_{1Z}\right)\left(1+\alpha_1\right)P_1}}\right]\left[\begin{array}{l}\dfrac{\left(1+\beta_1\right)K}{t_{1Z}\left(1+\alpha_1\right)P_1}+\dfrac{\left(1+\alpha_2\right)K}{t_{1Z}\left(1+\alpha_1\right)P_1}\\ +\left[\left(1+\beta_2\right)C\dfrac{\pi}{1-\pi}+\left(1+\alpha_3\right)C+C_S E\left[x\right]\right]\\ +\left(t_{1Z}\right)\left\{\begin{array}{l}\dfrac{h}{2\lambda}\dfrac{\left(1+\alpha_1\right)P_1}{\left(1-\pi\right)^2}\cdot\\ \left\{-\dfrac{\lambda\left(1-\pi\right)}{\left(1+\alpha_1\right)P_1}\left[\left(1+\pi\right)-2E\left[x\right]\left(1-\pi\right)\right]+\left[1-E\left[x\right]\left(1-\pi\right)\right]^2\right\}\end{array}\right\}\\ +\left[M+C_1\lambda g+C_T\lambda g+h_3\left(\dfrac{\lambda g^2}{2}\right)\right]\left(\dfrac{1-e^{-\beta t_{1Z}}}{t_{1Z}\left(1+\alpha_1\right)P_1}\right)\\ +\dfrac{\left[hg\left(\left(1+\alpha_1\right)P_1-\lambda\right)+h_3\lambda g\right]}{t_{1Z}\left(1+\alpha_1\right)P_1}\left(-t_{1Z}e^{-\beta t_{1Z}}-\dfrac{1}{\beta}e^{-\beta t_{1Z}}+\dfrac{1}{\beta}\right)\\ +\dfrac{h_3 g}{1-\pi}\left[1-E\left[x\right]\left(1-\pi\right)\right]\left(1-e^{-\beta t_{1Z}}\right)\end{array}\right] \quad \text{(A-1)}$$

where δ_0 represents the following:

$$\delta_0=\left[\frac{1}{\left(1-\pi\right)}-E\left[x\right]\right] \quad \text{(A-2)}$$

Let δ_1, δ_2, δ_3, G_0, G_1, G_2, and G_3 represent the following:

$$
\begin{aligned}
&\delta_1=\frac{(1+\beta_1)K}{(1+\alpha_1)P_1}+\frac{(1+\alpha_2)K}{(1+\alpha_1)P_1}\\
&\delta_2=\left[(1+\beta_2)C\frac{\pi}{1-\pi}+(1+\alpha_3)C+C_S E[x]\right]\\
&\delta_3=\frac{h}{2\lambda}\frac{(1+\alpha_1)P_1}{(1-\pi)^2}\left\{-\frac{\lambda(1-\pi)}{(1+\alpha_1)P_1}\left[(1+\pi)-2E[x](1-\pi)\right]+\left[1-E[x](1-\pi)\right]^2\right\}\\
&\delta_4=\delta_0(1+\alpha_1)P_1
\end{aligned}
\tag{A-3}
$$

$$
\begin{aligned}
&G_0=\frac{M+C_1\lambda g+C_T\lambda g+h_3\left(\dfrac{\lambda g^2}{2}\right)}{(1+\alpha_1)P_1};\quad G_1=\frac{\left[hg\left((1+\alpha_1)P_1-\lambda\right)+h_3\lambda g\right]}{(1+\alpha_1)P_1}\\
&G_2=\frac{\left[hg\left((1+\alpha_1)P_1-\lambda\right)+h_3\lambda g\right]}{(1+\alpha_1)P_1\beta};\quad G_3=\frac{h_3 g}{1-\pi}\left[1-E[x](1-\pi)\right].
\end{aligned}
\tag{A-4}
$$

Then, Eq. (B-1) (i.e., $E[TCU(t_{1Z})]$) becomes as follows:

$$
E\left[TCU(t_{1Z})\right]=\frac{\lambda}{\delta_0+\dfrac{\lambda g\left(1-e^{-\beta t_{1Z}}\right)}{t_{1Z}(1+\alpha_1)P_1}}\cdot\left\{\begin{aligned}&\frac{\delta_1}{t_{1Z}}+\delta_2+(t_{1Z})\delta_3+\frac{G_0}{t_{1Z}}\left(1-e^{-\beta t_{1Z}}\right)-G_1\left(e^{-\beta t_{1Z}}\right)\\&+\frac{G_2}{t_{1Z}}\left(1-e^{-\beta t_{1Z}}\right)+G_3\left(1-e^{-\beta t_{1Z}}\right)\end{aligned}\right\}
\tag{26}
$$

The first- and second-derivative of $E[TCU(t_{1Z})]$ can also be derived as follows:

$$
\frac{dE\left[TCU(t_{1Z})\right]}{d(t_{1Z})}=\frac{\lambda(1+\alpha_1)P_1}{\left[\delta_4 t_{1Z}+\lambda g\left(1-e^{-\beta t_{1Z}}\right)\right]^2}\left\{\begin{aligned}&-\delta_1\left[\lambda g\beta e^{-\beta t_{1Z}}+\delta_4\right]-\delta_2\lambda g\left(\beta e^{-\beta t_{1Z}}t_{1Z}+e^{-\beta t_{1Z}}-1\right)\\&+\delta_3\left[t_{1Z}{}^2\delta_4-\lambda g\beta e^{-\beta t_{1Z}}t_{1Z}{}^2-2\lambda g e^{-\beta t_{1Z}}t_{1Z}+2\lambda g t_{1Z}\right]\\&+(G_0+G_2)\delta_4\left(\beta e^{-\beta t_{1Z}}t_{1Z}+e^{-\beta t_{1Z}}-1\right)\\&-G_1\left(-\beta e^{-\beta t_{1Z}}t_{1Z}{}^2\delta_4-\beta\lambda e^{-\beta t_{1Z}}t_{1Z}g-\lambda e^{-2\beta t_{1Z}}g+\lambda e^{-\beta t_{1Z}}g\right)\\&+G_3\left(\beta e^{-\beta t_{1Z}}t_{1Z}{}^2\delta_4-2\lambda e^{-\beta t_{1Z}}g+\lambda g e^{-2\beta t_{1Z}}+\lambda g\right)\end{aligned}\right\}
\tag{A-5}
$$

$$
\begin{aligned}
\frac{d^2E\left[TCU(t_{1Z})\right]}{d(t_{1Z})^2}=&\frac{\lambda(1+\alpha_1)P_1}{\left[\delta_4 t_{1Z}+\lambda g\left(1-e^{-\beta t_{1Z}}\right)\right]^3}\cdot\\
&\left\{\begin{aligned}&\delta_1\begin{pmatrix}\beta^2e^{-2\beta t_{1Z}}\lambda^2g^2+\beta^2e^{-\beta t_{1Z}}\lambda^2g^2+4\beta e^{-\beta t_{1Z}}\lambda g\delta_4+2\delta_4{}^2\\+\beta^2e^{-\beta t_{1Z}}t_{1Z}\lambda g\delta_4\end{pmatrix}\\
&+\delta_2\lambda g\begin{pmatrix}2e^{-\beta t_{1Z}}\delta_4-2\delta_4+2\beta e^{-2\beta t_{1Z}}\lambda g-2\beta e^{-\beta t_{1Z}}\lambda g\\+\beta^2e^{-\beta t_{1Z}}t_{1Z}{}^2\delta_4+2\beta e^{-\beta t_{1Z}}t_{1Z}\delta_4+\beta^2e^{-2\beta t_{1Z}}\lambda g t_{1Z}+\beta^2e^{-\beta t_{1Z}}\lambda g t_{1Z}\end{pmatrix}\\
&+\delta_3\lambda g\begin{pmatrix}2\lambda g+2e^{-2\beta t_{1Z}}\lambda g-4e^{-\beta t_{1Z}}\lambda g+\beta^2e^{-2\beta t_{1Z}}t_{1Z}{}^2\lambda g+\beta^2e^{-\beta t_{1Z}}t_{1Z}{}^2\lambda g\\+4\beta e^{-2\beta t_{1Z}}t_{1Z}\lambda g-4\beta e^{-\beta t_{1Z}}t_{1Z}\lambda g+\beta^2e^{-\beta t_{1Z}}t_{1Z}{}^3\delta_4\end{pmatrix}\\
&-\delta_4(G_0+G_2)\begin{pmatrix}2\beta\lambda e^{-2\beta t_{1Z}}g+2e^{-\beta t_{1Z}}\delta_4-2\beta\lambda e^{-\beta t_{1Z}}g+\beta^2e^{-\beta t_{1Z}}t_{1Z}{}^2\delta_4\\-2\delta_4+\beta^2\lambda e^{-2\beta t_{1Z}}t_{1Z}g+\beta^2\lambda e^{-\beta t_{1Z}}t_{1Z}g+2\beta e^{-\beta t_{1Z}}t_{1Z}\delta_4\end{pmatrix}\\
&-G_1e^{-2\beta t_{1Z}}\begin{pmatrix}2\beta\lambda^2g^2-2\beta\lambda^2e^{\beta t_{1Z}}g^2+2\lambda g\delta_4-2\lambda e^{\beta t_{1Z}}g\delta_4+\beta^2\lambda^2g^2t_{1Z}\\+\beta^2\lambda^2e^{\beta t_{1Z}}g^2t_{1Z}+4\beta\lambda g t_{1Z}\delta_4+\beta^2\lambda g t_{1Z}{}^2\delta_4\\+2\beta^2\lambda e^{\beta t_{1Z}}g t_{1Z}{}^2\delta_4-2\beta\lambda e^{\beta t_{1Z}}g t_{1Z}\delta_4+\beta^2e^{\beta t_{1Z}}t_{1Z}{}^3\delta_4{}^2\end{pmatrix}\\
&-\delta_4G_3\begin{pmatrix}2\lambda g+2\lambda e^{-2\beta t_{1Z}}g-4\lambda e^{-\beta t_{1Z}}g+4\beta\lambda e^{-2\beta t_{1Z}}t_{1Z}g+\beta^2\lambda e^{-2\beta t_{1Z}}t_{1Z}{}^2g\\+\beta^2\lambda e^{-\beta t_{1Z}}t_{1Z}{}^2g-4\beta\lambda e^{-\beta t_{1Z}}t_{1Z}g+\beta^2e^{-\beta t_{1Z}}t_{1Z}{}^3\delta_4\end{pmatrix}\end{aligned}\right\}
\end{aligned}
\tag{A-6}
$$

The first term on the right-hand side (RHS) of Eq. (A-6) is positive, so if the second term on the RHS of Eq. (A-6) is also po sitive, then $E[TCU(t_{1Z})]$ is convex. That is, if $\omega(t_{1Z}) > t_{1Z} > 0$ (as shown in Eq. (A-7) holds.

$$\omega(t_{1Z}) = \frac{\left\{\begin{array}{l} \delta_1\left(\beta^2 e^{-2\beta t_{1Z}}\lambda^2 g^2 + \beta^2 e^{-\beta t_{1Z}}\lambda^2 g^2 + 4\beta e^{-\beta t_{1Z}}\lambda g\delta_4 + 2\delta_4^{\,2}\right) \\ +\delta_2\lambda g\left(2e^{-\beta t_{1Z}}\delta_4 - 2\delta_4 + 2\beta e^{-2\beta t_{1Z}}\lambda g - 2\beta e^{-\beta t_{1Z}}\lambda g\right) \\ +\delta_3\lambda g\left(2\lambda g + 2e^{-2\beta t_{1Z}}\lambda g - 4e^{-\beta t_{1Z}}\lambda g\right) \\ -\delta_4\left(G_0+G_2\right)\left(2\beta\lambda e^{-2\beta t_{1Z}} g + 2e^{-\beta t_{1Z}}\delta_4 - 2\beta\lambda e^{-\beta t_{1Z}} g - 2\delta_4\right) \\ -G_1 e^{-2\beta t_{1Z}}\left(2\beta\lambda^2 g^2 - 2\beta\lambda^2 e^{\beta t_{1Z}} g^2 + 2\lambda g\delta_4 - 2\lambda e^{\beta t_{1Z}} g\delta_4\right) \\ -\delta_4 G_3\left(2\lambda g + 2\lambda e^{-2\beta t_{1Z}} g - 4\lambda e^{-\beta t_{1Z}} g\right) \end{array}\right\}}{-\left\{\begin{array}{l} \delta_1\left(\beta^2 e^{-\beta t_{1Z}}\lambda g\delta_4\right) + \delta_2\lambda g\left(\beta^2 e^{-\beta t_{1Z}} t_{1Z}\delta_4 + 2\beta e^{-\beta t_{1Z}}\delta_4 + \beta^2 e^{-2\beta t_{1Z}}\lambda g + \beta^2 e^{-\beta t_{1Z}}\lambda g\right) \\ +\delta_3\lambda g\left(\beta^2 e^{-2\beta t_{1Z}} t_{1Z}\lambda g + \beta^2 e^{-\beta t_{1Z}} t_{1Z}\lambda g + 4\beta e^{-2\beta t_{1Z}}\lambda g - 4\beta e^{-\beta t_{1Z}}\lambda g + \beta^2 e^{-\beta t_{1Z}} t_{1Z}^{\,2}\delta_4\right) \\ -\delta_4\left(G_0+G_2\right)\left(\beta^2 e^{-\beta t_{1Z}} t_{1Z}\delta_4 + \beta^2\lambda e^{-2\beta t_{1Z}} g + \beta^2\lambda e^{-\beta t_{1Z}} g + 2\beta e^{-\beta t_{1Z}}\delta_4\right) \\ -G_1 e^{-2\beta t_{1Z}}\left(\begin{array}{l} \beta^2\lambda^2 g^2 + \beta^2\lambda^2 e^{\beta t_{1Z}} g^2 + 4\beta\lambda g\delta_4 + \beta^2\lambda g t_{1Z}\delta_4 \\ +2\beta^2\lambda e^{\beta t_{1Z}} g t_{1Z}\delta_4 - 2\beta\lambda e^{\beta t_{1Z}} g\delta_4 + \beta^2 e^{\beta t_{1Z}} t_{1Z}^{\,2}\delta_4^{\,2} \end{array}\right) \\ -\delta_4 G_3\left[4\beta\lambda e^{-2\beta t_{1Z}} g + \beta^2\lambda e^{-2\beta t_{1Z}} t_{1Z} g + \beta^2\lambda e^{-\beta t_{1Z}} t_{1Z} g - 4\beta\lambda e^{-\beta t_{1Z}} g + \beta^2 e^{-\beta t_{1Z}} t_{1Z}^{\,2}\delta_4\right] \end{array}\right\}} > t_{1Z} > 0 \tag{A-7}$$

Appendix B

Table B-1

Probabilities of different Poisson-distributed breakdown-rates

β	t_{1Z} *	$P(x=0)$	$P(x=1)$	$P(x\le 1)$	$P(x>1)$
8.0	0.3030	8.85%	21.46%	30.31%	69.69%
6.0	0.2507	22.22%	33.42%	55.64%	44.36%
5.0	0.2123	34.60%	36.72%	71.32%	28.68%
4.0	0.1735	49.96%	34.67%	84.63%	15.37%
3.0	0.1444	64.85%	28.09%	92.94%	7.06%
2.0	0.1267	77.62%	19.66%	97.28%	2.72%
1.0	0.1175	88.92%	10.45%	99.37%	0.63%
0.5	0.1154	94.39%	5.45%	99.84%	0.16%
0.01	0.1149	99.89%	0.11%	100.00%	0.00%

$$\frac{e^{-\beta t_{1Z}^*}\left(\beta t_{1Z}^*\right)^x}{x!} \tag{B-1}$$

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