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Optimizing an FPR-based supplier-retailer integrated problem with an outsourcer, rework, expedited rate, and probabilistic breakdown

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A B S T R A C T

Internal supply chains exist in many global enterprises, where manufacturing tasks and sales jobs operate separately, but the management needs to integrate their financial performance reports. In addition, the fabrication planning must meet specific operational goals, such as meeting external clients' requirements on quality and short order due dates, avoiding internal fabricating interruptions due to inevitable equipment breakdowns, and minimizing overall manufacturing and stock holding costs. Motivated by helping multinational corporations deal with the issues mentioned earlier, this study aims to optimize a finite production rate (FPR)-based supplier-retailer cooperative problem with multi-shipment, rework, subcontracting, probabilistic failure, and expedited rate. Wherein using an outsourcer and expedited-rate help shorten the needed batch producing time significantly; the rework of defects and corrective action on unanticipated breakdown assist in up-keeping the quality and avoiding fabricating delay. We develop an FPR-based model to cautiously represent the considered manufacturing features and activities involved in transporting end products and retailers' stock holding. Model's formulating and investigating assists us in gaining the function of operating costs. In addition, optimization procedures with a proposed algorithm help us verify its convexity and decide the model's best fabricating runtime solution. Finally, we validate how this study works and what important information our model can disclose using a numerical example to facilitate management's decision-making to end our work.

Nomenclature

T'_Z = cycle time in situation one,
λ = annual demands,
Q = production lot size,
β = mean annual Poisson-distributed breakdowns,
t = time to breakdown occurring,
t_r = equipment repair time,
M = equipment repair cost,
π = the outsourced portion of a batch (where $0 < \pi < 1$),
K_π = fixed subcontracting cost,
β_1 = the linking factor between K_π and K,
C_π = unit subcontracting cost,
β_2 = the linking factor between C_π and C,

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P_{1A} = annual adjustable production rate,
P_1 = standard production rate (i.e., without using the adjustable-rate),
α_1 = the linking factor between P_{1A} and P_1,
C_A = unit cost when adjustable-rate is used,
C = standard unit cost,
K_A = setup cost with adjustable-rate implmented,
K = standard setup cost,
α_2 = the linking factor between K_A and K,
x = annual uniform-distributed defective rate,
d_{1A} = defective item's production rate in t_{1Z}, so, $d_{1A} = xP_{1A}$,
P_{2A} = annual adjustable reworking rate,
P_2 = standard reworking rate,
C_{RA} = unit rework cost when adjustable reworking rate is used,
C_R = rework cost with P_2 implemented,
α_3 = the linking factor between C_A and C, and C_{RA} and C_R,
t_{1Z} = production uptime,
t'_{2Z} = situation 1's rework time,
t'_{3Z} = stock distributing time in situation one,
t'_{nZ} = the time between two consecutive deliveries in situation one,
h = unit holding cost,
h_1 = unit holding cost per reworked item,
h_3 = unit safety stock's holding cost,
C_1 = unit safety stock cost,
h_2 = unit holding cost at the buyer side,
C_T = unit distributing cost,
K_1 = fixed distributing cost,
n = the number of shipments in a cycle,
g = t_r, machine repair time,
H = stock level after receipt of the outsourced goods,
H_2 = stock level when rework time ends,
H_1 = stock level when uptime ends,
H_0 = situation 1's stock level,
T_Z = situation 2's cycle time,
t_{2Z} = rework time in situation two,
t_{3Z} = stock distributing time in situation two,
t_{nZ} = the time between two consecutive deliveries in situation two,
T = cycle time for a model without outsourcing, breakdown, nor adjustable-rate,
t_1 = uptime for a model without outsourcing, breakdown, nor adjustable-rate,
t_2 = the rework time for a system without outsourcing, breakdown, nor adjustable-rate,
t_3 = stock distributing time for a system without outsourcing, breakdown, nor adjustable- rate,
d_1 = defective items' producing rate for a model without outsourcing, breakdown, nor adjustable- rate,
D = the amount of goods per delivery,
I = the leftover goods when distributing interval ends,
$I(t)$ = stocks level at time t,
$I_c(t)$ = buyer's stock level at time t,
$I_d(t)$ = defective stock level at time t,
$I_F(t)$ = safety stock level at time t,
$TC(t_{1Z})_1$ = situation 1's total cost per cycle,
$E[TC(t_{1Z})_1]$ = situation 1's expected total cost per cycle,
$TC(t_{1Z})_2$ = situation 2's total cost per cycle,
$E[TC(t_{1Z})_2]$ = situation 2's expected total cost per cycle,
$E[T'_Z]$ = situation 1's expected cycle time,
$E[T_Z]$ = the expected cycle time in situation two,
$\boldsymbol{T}_Z$ = the proposed problem's cycle time,
$E[TCU(t_{1Z})]$ = our proposed model's expected annual operating cost.

1. Introduction

This study examined a FPR-based supplier-retailer cooperative problem with multi-shipment, subcontracting, probabilistic failure, expedited rate, and rework. Optimizing an FPR-based supplier-retailer integrated type of system helps global firms' intra-supply chains minimize their overall operating expenditures. Nachiappan and Jawahar (2007) explored the operational issues of a vendor-managed inventory (VMI) supply chain involving one vendor and the multi-buyer. The researchers focused on the supply chain's profit relating to the selling quantity and the contractual agreement price. To reach the agreement price, partners share their revenues. In addition, the study built a math model and formulation to explore the influence of different acceptable contractual prices to derive the optimal sales amount for each buyer. A heuristic using the genetic algorithm helped the study to resolve this integer programming model with nonlinear nature. Lastly, the researchers evaluated the quality of their obtained solution with sensitivity analyses. Hajej et al. (2015) explored the optimal production plan considering stochastic demand, system deterioration, service level requirement, maintenance, product returns, and shipment tasks. The customer's return items are assumed to be still new and re-saleable. The researchers formulated the studied problem as a quadratic model representing the relevant stock, shipment, fabrication, and maintenance. Lastly, they provided simulation outcomes to expose the shipping time impact on the planning of fabrication, maintenance, and shipment tasks. Sazvar et al. (2021) used a multi-objective capacity planning technique to explore a supply chain featuring sustainable-resilient for making strategic and tactical decisions. Wherein researchers focused on planning optimally for meeting clients' demands with weak operations and high disruption risks. The researchers developed a mathematical model with a resilient demand-side framework. They applied an actual case study of the influenza vaccine supply chain to validate their model and investigate the tradeoff between sustainability and resilience. In addition, the researchers used a vigorous fuzzy optimization method to cope with uncertainties, and they solved their multi-objective model by using goal programming with utility functions. Finally, after exploring the influences of the quantitative results of the problem's structural variables, the study suggested specific managerial insights into the situation. Other works (Cetinkaya & Lee, 2000; Sucky, 2005; García & de las Morenas, 2012; Sahebi et al., 2019; Brahmi et al., 2020; Pham & Doan, 2020; Pourmohammadi et al., 2020; Toncovich et al., 2020; Farmand et al., 2021; Nguyen et al., 2021) studied the impact of different shipping strategies on controlling and managing various types of supply chains.

Additionally, the fabrication planning must meet various operational goals, e.g., satisfying external clients' requirements on product quality and short order due dates, avoiding in-house fabricating interruptions due to inevitable equipment breakdowns, and minimizing total manufacturing and inventory holding expenses. Hence, implementing the screening and rework processes for random defects and instantly correcting the inevitable machine failures are critical for production managers. Kenne and Gharbi (2001) explored the optimal fabricating rate for a multiproduct parallel-machine manufacturing system subject to stochastic failure and repair rates. The researchers used an experimental design, Markov chain, discrete event simulation, and response surface techniques to handle their cost-minimization-oriented problem. Both exponential and non-exponential failure/repair times under constant demand rates are investigated using a simulation approach to estimate the optimal control and fabricating rate policy. Maggio et al. (2009) used a decomposition analytical approach to evaluate mean throughputs and buffers for an unreliable three-machine fabrication system. Their model considered deterministic machine processing times and geometric-distribution breakdown and repair rates. The researchers used simulation techniques to verify their model's results and discussed potential extensions to their model. Singh et al. (2014) studied a time-dependent demand economic production model considering multiple setups and rework. Their proposed green supply chain features a reverse logistic, i.e., considering one rework setup dealing with defects from multiple regular batch processes. Lastly, they offered a numerical illustration for their model with sensitivity analysis. Sharma and Rai (2021) explored failure modes for repairable systems using censored data analysis. In addition to conventional criteria in predicting failure modes (e.g., the time between machine failures), the researchers considered the virtual age models treating preventive and corrective maintenance as imperfect in their future reliability censored data analysis. Wherein their proposed models also offered a likelihood function for estimating system parameters. To help demonstrate the applicability of their methods and models, the researchers provided an aviation case study. Other works (Flapper et al., 2002; Öztürk et al., 2015; Chiu et al., 2018; Mallick et al., 2019; Pham & Doan, 2020; Gera, 2021; Hani, 2021; Shekhar et al., 2021) examined the effects of dissimilar issues of product defectives, rework, equipment failures, and various corrected actions on managing, planning, and controlling different manufacturing systems.

This study incorporates an outsourcer and expediting in-house fabricating rate to reduce the batch production time and satisfy external customers' needs of short order due dates. Antelo and Bru (2010) explored the influence of subcontracting strategy and restructuring the firm on operating in an uncertain environment. The researchers examined the option and role of outsourcing versus production cost and the potential timing and need for organizational restructuring to determine the optimal managerial decision in the firm's future. Rivera-Gómez et al. (2018) intended to simultaneously implement subcontracting, fabrication, and maintenance in a manufacturing system featuring various progressive deterioration processes. They explored such deterioration effects on the quality and machine reliability. The study assumed minimal repair and preventive maintenance to restore a failed machine to an operating condition and planned a proper subcontracting amount to meet client demand. Their study aimed to minimize overall operating expenses comprising fabrication, outsourcing, stock holding, preventive maintenance, product defects, machine repair, and backlog costs. By building a stochastic control model and using numerical methodologies, the study defined the control policies' structure and applied statistical analyses and optimization methods to decide their control policies' optimality. Lastly, through numerical illustrations, the study highlighted the relationships among fabrication, maintenance, subcontracting, deterioration, quality, and reliability to justify their results. Kershaw et al. (2021) incorporated machine learning into the decisions of predicting welding quality and adjusting welding speed. The researchers used two cameras to collect required real-time data for this preliminary study to develop appropriate training for machine learning. In addition, the study used a convolutional neural network to analyze the correlations from the collected top-side and back-side welding data sets to predict the quality and control speed of the welding processes.

Furthermore, the researchers used varying-speed experimental trials to explore the feasibility of applying bead-width welding speed in multi-layer perceptrons and provided an efficient algorithm with extra computational efforts to achieve an optimal/ideal bead-width rate for the welding process. Other works (Khouja, 2000; Bardhan et al., 2007; Jauhari and Pujawan, 2014; El-khalek et al., 2019; Mallick et al., 2019; Chiu et al., 2020; Gupta and Ivanov, 2020; Chiu et al., 2021; Hermawan, 2021) examined the influence of different controllable manufacturing rates and outsourcing options on planning, operating, and optimizing various global supply chains and production systems. Little works have explored optimizing an FPR-based supplier-retailer integrated problem with an outsourcer, rework, expedited rate, and probabilistic breakdown; we aim to fill this gap.

2. Model description

First, we provide a Nomenclature to ease the reading before describing the proposed FPR-based manufacturer-retailer collaboration system. Consider a batch manufacturing plan established to satisfy a λ demand rate of a specific product. The studied system is not reliable. It randomly fabricates x proportion of defective goods. In each cycle, right after the standard manufacturing process, a rework process can repair all faulty items made in this cycle. Further, the equipment is subject to Poisson-distributed failures with β as its annual mean. Hence, time t before a breakdown occurs adheres to an Exponential distribution with $\beta e^{-\beta t}$ as its density function. Upon occurrence of a failure, our study adopts an abort/resume policy, which puts the machine under repair at once and assuming a fixed repair time t_r. The production of the unfinished/interrupted lot resumes as soon as the machine is restored. This study uses dual uptime-reduced strategies. One is to subcontract a π portion of lot-size Q to the external suppliers, and the other is to accelerate the in-house manufacturing rate. A fixed setup cost K_π and unit cost C_π accompany this subcontracting policy, and another fixed setup cost K_A and unit cost C_A link to the adjusted-rate plan. Eq. (1) to Eq. (7) denote the relationships between each strategically-relevant parameter and its corresponding standard variable.

$$K_\pi = K\left(1+\beta_1\right) \tag{1}$$

$$C_\pi = C\left(1+\beta_2\right) \tag{2}$$

$$P_{1A} = P_1\left(1+\alpha_1\right) \tag{3}$$

$$K_A = K\left(1+\alpha_2\right) \tag{4}$$

$$C_A = C\left(1+\alpha_3\right) \tag{5}$$

$$P_{2A} = \left(1+\alpha_1\right)P_2 \tag{6}$$

$$C_{RA} = \left(1+\alpha_3\right)C_R \tag{7}$$

where α_1 denotes the proportion of speed P_{1A} is faster than the standard manufacturing rate P_1; β_i (for i = 1, 2) and α_i (for i = 2, 3) are the linking factors between cost-relevant parameters; P_{2A} stands for the accelerate reworking rate. The study allows no stock-out condition, i.e., $(P_{1A} - d_{1A} - \lambda) > 0$ must hold. The external supplier guarantees the outsourced products' quality. The scheduled receipt time for the outsourced goods is at the end of the in-house rework time (i.e., at the beginning of distributing time). Upon gathering the entire lot, n fixed-quantity shipments are distributed at t'_{nZ} (i.e., a constant time interval) during t'_{3Z} to the customer side. We build the following models to study different situations of stochastic breakdown occurrences explicitly:

2.1. *Situation 1: A breakdown happens in uptime* t_{1Z}

In situation one, the time to a breakdown occurs $t < t_{1Z}$. We adopt an A/R controlling policy to correct the failure and continue the production when the correction action is done. Figure 1 depicts the stock level of the proposed FPR-based problem (in thicker lines) compared to a problem with only rework (in thinner lines). Fig. 1 indicates stock level reaches H_0 when a failure occurs. It remains at H_0 during correction/repair time t_r. Once the device is restored, its level rises to H_1 when t_{1Z} ends. It arrives at H_2 when the rework time t'_{2Z} ends. By receiving the outsourced goods, the stock level increases to H, right before the beginning of stock distribution time t'_{3Z}. Finally, during t'_{3Z}, n equal-quantity shipments deplete the inventory level to zero (see Fig. 1 for details).

Fig. 2 illustrates safety inventory's level in situation 1. Since the cycle length T'_Z includes an extra repair time t_r, so the ready-to-delivery finished lot must consist of the safety inventories to meet the additional buyer's demand during t_r. Figure 3 exhibits the inventory level of defective products in situation 1 during T'_Z. According to Figures 1 to 3 and the model's assumptions, one directly observes the following equations:

$$H_0 = \left(P_{1A} - d_{1A}\right)t \tag{8}$$

$$H_1 = \left(P_{1A} - d_{1A}\right) t_{1Z} \tag{9}$$

$$H_2 = H_1 + P_{2A} t'_{2Z} \tag{10}$$

$$H = H_2 + \pi Q + \lambda t_r \tag{11}$$

$$T'_Z = t_{1Z} + t_r + t'_{2Z} + t'_{3Z} \tag{12}$$

$$t_{1Z} = \frac{Q(1-\pi)}{P_{1A}} = \frac{H_1}{P_{1A} - d_{1A}} \tag{13}$$

$$t'_{2Z} = \frac{x(1-\pi)Q}{P_{2A}} \tag{14}$$

$$t'_{3Z} = T'_Z - \left(t_{1Z} + t_r + t'_{2Z}\right) \tag{15}$$

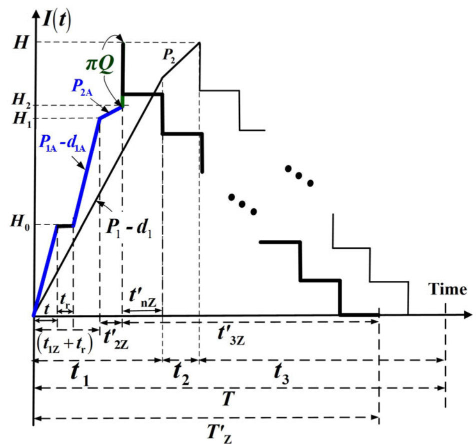

Fig. 1. Situation one's stock level of our proposed system (in thicker lines) compared to a model with rework only (in thinner lines)

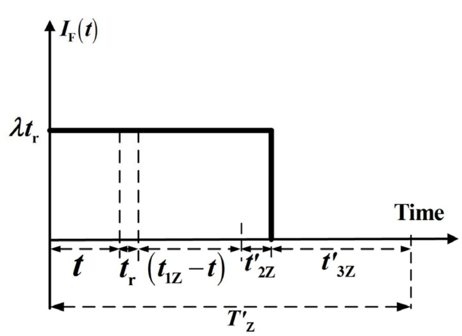

Fig. 2. The safety inventory's level in situation 1

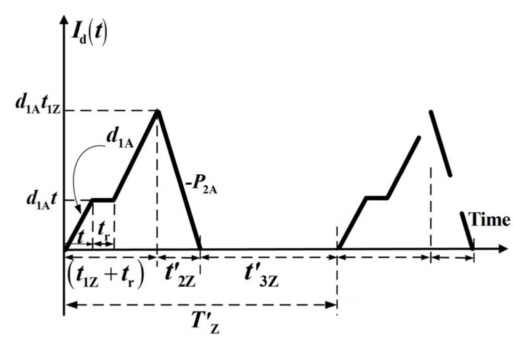

Fig. 3. The inventory level of defective products in situation 1 during T'_Z

The inventory level of defective products made is as follows:

$$d_{1A} t_{1Z} = Qx(1-\pi) = P_{1A} t_{1Z} x \tag{16}$$

Total stocks of finished items during t'_{3Z} are as follows (Chiu et al., 2020):

$$\left(\sum_{i=1}^{n-1} i\right) H\left(t'_{3Z}\right)\left(\frac{1}{n^2}\right) = \left[\frac{n(n-1)}{2}\right] H\left(t'_{3Z}\right)\left(\frac{1}{n^2}\right) = H\left(t'_{3Z}\right)\left(\frac{n-1}{2n}\right) \tag{17}$$

Fig. 4 illustrates situation 1's buyer inventory level in T'_Z (Chiu et al., 2020):

$$n\left(t'_{nZ}\right)\left(D - \frac{\lambda\left(t'_{nZ}\right)}{2}\right) + \left(t'_{nZ}\right)\frac{n(n-1)}{2} I + \frac{nI}{2}\left(t_{1Z} + t'_{2Z}\right) = \frac{1}{2}\left[\frac{Ht'_{3Z}}{n} + T'_Z\left(H - \lambda t'_{3Z}\right)\right] \tag{18}$$

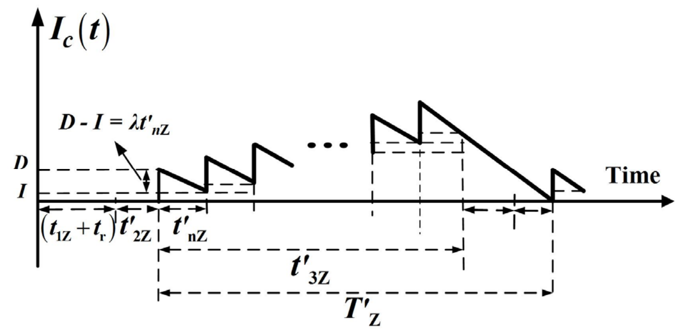

Fig. 4. The buyer side's stock level in T'_Z

Situation one's total operating expenses per cycle $TC(t_{1Z})_1$ consists of below: the variable and fixed subcontracting costs and in-house production cost, the variable and fixed distributing costs, machine repair cost, safety stock relating cost, rework related costs, and stock holding costs (comprising the defective, finished, and reworked products, and buyer side's stocks) during T'_Z as displayed in Eq. (19).

$$\begin{aligned} TC(t_{1Z})_1 &= (\pi Q)C_\pi + K_\pi + Q(1-\pi)C_A + K_A + (Q+\lambda t_r)C_T + nK_1 + (t_{1Z}+t_r+t'_{2Z})h_3(\lambda t_r) \\ &+M + \lambda t_r C_1 + \frac{P_{2A}t'_{2Z}}{2}(t'_{2Z})h_1 + Qx(1-\pi)C_{RA} + \left[\frac{Ht'_{3Z}}{n} + T'_Z(H-\lambda t'_{3Z})\right]\frac{h_2}{2} \\ &+h\left[(t_{1Z})\frac{H_1+d_{1A}t_{1Z}}{2} + (d_{1A}t)t_r + (t'_{2Z})\frac{H_1+H_2}{2} + (H_0 t_r) + (t'_{3Z})\left(\frac{n-1}{2n}\right)H\right] \end{aligned} \tag{19}$$

By substituting Eqs. (1) to (7), and (16) in Eq. (19), $TC(t_{1Z})_1$ becomes as follows:

$$\begin{aligned} TC(t_{1Z})_1 &= (1+\beta_2)(\pi Q)C + (1+\beta_1)K + (1+\alpha_2)K + Q(1-\pi)(1+\alpha_3)C + (Q+\lambda t_r)C_T \\ &+(t_{1Z}+t_r+t'_{2Z})(\lambda t_r)h_3 + nK_1 + \lambda t_r C_1 + M + (t'_{2Z})\frac{(1+\alpha_1)P_2 t'_{2Z}}{2}h_1 \\ &+(1+\alpha_3)Qx(1-\pi)C_R + \left[\frac{Ht'_{3Z}}{n} + T'_Z(H-\lambda t'_{3Z})\right]\frac{h_2}{2} \\ &+h\left[(t_{1Z})\frac{H_1 + x(1+\alpha_1)P_1 t_{1Z}}{2} + \frac{H_1+H_2}{2}(t'_{2Z}) + x(1+\alpha_1)P_1(t)t_r + H_0 t_r + Ht'_{3Z}\left(\frac{n-1}{2n}\right)\right] \end{aligned} \tag{20}$$

2.2. Situation 2: No breakdown happens in uptime

In situation two, the time before a breakdown occurs $t > t_{1Z}$. Fig. 5 displays situation two's stock level (in thicker lines) compared to a system with rework only (in thinner lines). It specifies that when the uptime ends, the level rises to H_1, and it further reaches H_2.when the rework finishes. By receiving the outsourced items, the inventory level arrives at H, before the beginning of product distributing time t_{3Z}. Fig. 6 exhibits the safety stock's level in situation two. Safety stock stays the same at all time. The inventory level of defective products in situation 2 is the same as Fig. 3, excluding t_r. Similar to subsection 2.1., we observe straightforward formulas as follows:

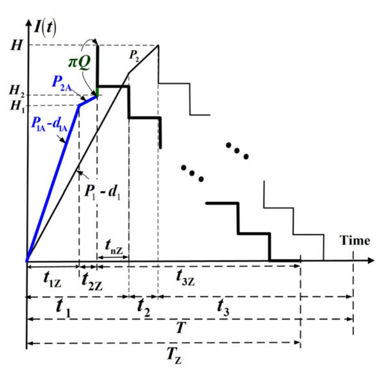

Fig. 5. Situation two's stock level (in thicker lines) compared to a system with rework only (in thinner lines)

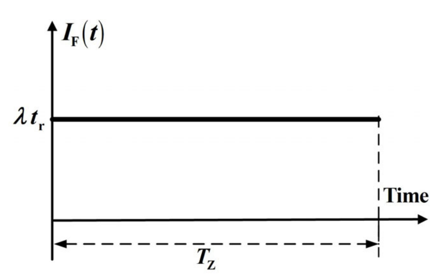

Fig. 6. Situation two's safety stock level

$$t_{1Z} = \frac{Q(1-\pi)}{P_{1A}} = \frac{H_1}{P_{1A} - d_{1A}} \tag{21}$$

$$t_{2Z} = \frac{x(1-\pi)Q}{P_{2A}} \tag{22}$$

$$T_Z = t_{1Z} + t_{2Z} + t_{3Z} \tag{23}$$

$$t_{3Z} = T_Z - (t_{1Z} + t_{2Z}) \tag{24}$$

$$H_1 = t_{1Z}(P_{1A} - d_{1A}) \tag{25}$$

$$H_2 = H_1 + P_{2A} t_{2Z} \tag{26}$$

$$H = H_2 + \pi Q \tag{27}$$

Total finished stocks in t_{3Z} for situation 2 are (Chiu et al., 2020):

$$\left(\sum_{i=1}^{n-1} i\right) H(t_{3Z}) \left(\frac{1}{n^2}\right) = \left[\frac{n(n-1)}{2}\right] H(t_{3Z}) \left(\frac{1}{n^2}\right) = H(t_{3Z}) \left(\frac{n-1}{2n}\right) \tag{28}$$

Total buyer side's stocks in T_Z for situation 2 are (Chiu et al., 2020):

$$(t_{nZ})\left(D - \frac{\lambda t_{nZ}}{2}\right) n + (t_{nZ}) I \frac{n(n-1)}{2} + (t_{1Z} + t_{2Z}) \frac{nI}{2} = \frac{1}{2}\left[\frac{H t_{3Z}}{n} + (H - \lambda t_{3Z}) T_Z\right] \tag{29}$$

Total operating expenses per cycle $TC(t_{1Z})_2$ in situation two consists of below: the variable and fixed subcontracting expense s and in-house production cost, the variable and fixed distributing costs (see Fig. 5), safety stock holding cost (see Fig. 6), re work related expenses, and total stock holding expenses (comprising the defective, finished, and reworked items, and buyer' s inventories) in T_Z as exhibited in Eq. (30).

$$\begin{aligned} TC(t_{1Z})_2 &= \pi Q C_\pi + K_\pi + [Q(1-\pi)] C_A + K_A + Q C_T + n K_1 + Q x (1-\pi) C_{RA} \\ &+ (t_{2Z}) \frac{P_{2A} t_{2Z}}{2} h_1 + T_Z(\lambda t_r) h_3 + \left[\frac{H t_{3Z}}{n} + T_Z (H - \lambda t_{3Z})\right] \frac{h_2}{2} \\ &+ \left[(t_{1Z}) \frac{H_1 + d_{1A} t_{1Z}}{2} + (t_{2Z}) \frac{H_1 + H_2}{2} + (t_{3Z}) H \left(\frac{n-1}{2n}\right)\right] h \end{aligned} \tag{30}$$

By substituting formulas (1) to (7), and (16) in formula (30), $TC(t_{1Z})_2$ becomes:

$$\begin{aligned} TC(t_{1Z})_2 &= (1+\beta_2)(Q\pi) C + (1+\beta_1) K + Q(1-\pi)(1+\alpha_3) C + (1+\alpha_2) K + n K_1 + (\lambda t_r) T_Z h_3 \\ &+ Q C_T + x Q (1-\pi)(1+\alpha_3) C_R + (t_{2Z}) \frac{(1+\alpha_1) P_2 t_{2Z}}{2} h_1 + \left[\frac{H t_{3Z}}{n} + T_Z (H - \lambda t_{3Z})\right] \frac{h_2}{2} \\ &+ \left[(t_{1Z}) \frac{H_1 + x(1+\alpha_1) P_1 t_{1Z}}{2} + (t_{2Z}) \frac{H_1 + H_2}{2} + t_{3Z} H \left(\frac{n-1}{2n}\right)\right] h \end{aligned} \tag{31}$$

2.3. Integrating of situations 1 and 2

The time to equipment failures adheres to the Exponential distribution with $\beta e^{-\beta t}$ and $F(t) = (1 - e^{-\beta t})$ as its density and cumulative density functions since the assumption of Poisson- distributed failure rate β. In addition, we apply x's expected values for coping with its randomness and use the renewal reward theorem to solve the $E[TCU(t_{1Z})]$ as follows:

$$E[TCU(t_{1Z})] = \frac{1}{E[\boldsymbol{T}_Z]} \left\{ \int_0^{t_{1Z}} E[TC(t_{1Z})_1] \cdot f(t) dt + \int_{t_{1Z}}^{\infty} E[TC(t_{1Z})_2] \cdot f(t) dt \right\} \tag{32}$$

where $E[\boldsymbol{T}_Z]$, $E[T'_Z]$, and $E[T_Z]$ denote the following:

$$E[\boldsymbol{T_Z}] = \int_0^{t_{1Z}} E[T'_Z] \cdot f(t) dt + \int_{t_{1Z}}^{\infty} E[T_Z] \cdot f(t) dt \tag{33}$$

$$E[T'_Z] = \frac{Q + \lambda t_r}{\lambda} = \frac{t_{1Z} P_{1A} \left[\frac{1}{(1-\pi)}\right] + \lambda t_r}{\lambda} \tag{34}$$

$$E[T_Z]=\frac{Q}{\lambda}=\frac{t_{1Z}P_{1A}\left[\frac{1}{(1-\pi)}\right]}{\lambda} \tag{35}$$

By substituting Eq. (20), Eq. (31), and Eq. (33) in Eq. (32) and with extra efforts in derivations, we gain the following $E[TCU(t_{1Z})]$ (see Appendix A):

$$E\left[TCU\left(t_{1Z}\right)\right]=\left[\frac{\lambda}{\delta_1+\frac{\left(1-e^{-\beta t_{1Z}}\right)\lambda g}{\left(1+\alpha_1\right)\left(t_{1Z}\right)P_1}}\right]\left[\begin{array}{l}\frac{\delta_2}{t_{1Z}}+\delta_3-\frac{y_1}{\left(t_{1Z}\right)}\left(e^{-\beta t_{1Z}}\right)+\frac{y_1}{t_{1Z}}+\delta_4\left(t_{1Z}\right)\\+y_2\left(e^{-\beta t_{1Z}}\right)+G_3\left(1-e^{-\beta t_{1Z}}\right)\end{array}\right] \tag{36}$$

2.4. *Solving the optimal t_{1Z}**

By applying the first- and second-derivative of $E[TCU(t_{1Z})]$, and we obtain Eqs. (A-6) and (A-7) (see Appendix A). It indicates $E[TCU(t_{1Z})]$ is convex if the 2nd term on the right-hand side (RHS) of Eq. (A-7) is positive (for the 1st term on the RHS of Eq. (A-7) is positive). Meaning, if $\omega(t_{1Z}) > t_{1Z} > 0$ is true (refer to Eq. (A-8) in Appendix A). Once we verify Eq. (A-8) is true, one resolves the optimal t_{1Z}* by letting the first-derivative of $E[TCU(t_{1Z})] = 0$ (see Eq. (A-6)). Since the 1st term on the RHS of Eq. (A-6) is positive, one has:

$$\left\{\begin{array}{l}\left\{\left[-\beta\delta_1 e^{-\beta t_{1Z}}\left(1+\alpha_1\right)P_1\right]\left(y_2-G_3\right)+\delta_4\left[\delta_1\left(1+\alpha_1\right)P_1-\lambda g\beta e^{-\beta t_{1Z}}\right]\right\}\left(t_{1Z}\right)^2\\+\left[-\left(\beta\lambda g e^{-\beta t_{1Z}}\right)\left(\delta_3+y_2\right)+2\lambda g\delta_4\left(1-e^{-\beta t_{1Z}}\right)+e^{-\beta t_{1Z}}y_1\beta\delta_1\left(1+\alpha_1\right)P_1\right]t_{1Z}\\-\left[\delta_1\left(1+\alpha_1\right)P_1+\lambda\beta e^{-\beta t_{1Z}}g\right]\left(\delta_2+y_1\right)-\lambda g\left(e^{-\beta t_{1Z}}-1\right)\left(\delta_3+G_3\right)\\+\lambda g\left(-e^{-2\beta t_{1Z}}+e^{-\beta t_{1Z}}\right)\left(y_2-G_3\right)+e^{-\beta t_{1Z}}y_1\left[\delta_1\left(1+\alpha_1\right)P_1+\beta\lambda g\right]\end{array}\right\}=0 \tag{37}$$

Let m_0, m_1, and m_2 denote:

$$m_0=\left[\begin{array}{l}-\left[\delta_1\left(1+\alpha_1\right)P_1+\lambda\beta e^{-\beta t_{1Z}}g\right]\left(\delta_2+y_1\right)-\lambda g\left(e^{-\beta t_{1Z}}-1\right)\left(\delta_3+G_3\right)\\+\lambda g\left(-e^{-2\beta t_{1Z}}+e^{-\beta t_{1Z}}\right)\left(y_2-G_3\right)-e^{-\beta t_{1Z}}y_3\left[\delta_1\left(1+\alpha_1\right)P_1+\beta\lambda g\right]\end{array}\right]$$

$$m_1=\left[-\left(\beta\lambda g e^{-\beta t_{1Z}}\right)\left(\delta_3+y_2\right)+2\lambda g\delta_4\left(1-e^{-\beta t_{1Z}}\right)-e^{-\beta t_{1Z}}y_3\beta\delta_1\left(1+\alpha_1\right)P_1\right]$$

$$m_2=\left[-\beta\delta_1 e^{-\beta t_{1Z}}\left(1+\alpha_1\right)P_1\right]\left(y_2-G_3\right)+\delta_4\left[\delta_1\left(1+\alpha_1\right)P_1-\lambda g\beta e^{-\beta t_{1Z}}\right]$$

Eq. (37) becomes:

$$m_2\left(t_{1Z}\right)^2+m_1\left(t_{1Z}\right)+m_0=0 \tag{38}$$

Apply the square roots solution, we derive t_{1Z}* below:

$$t_{1Z}{}^{*}=\frac{-m_1\pm\sqrt{m_1^2-4m_2m_0}}{2m_2} \tag{39}$$

Because $F(t_{1Z}) = (1 - e^{-\beta t1Z})$ falls in the interval [0, 1], so does $e^{-\beta t1Z}$ (i.e., its complement). Rearranging Eq. (37), we obtain $e^{-\beta t1Z}$ as follows:

$$e^{-\beta t_{1Z}}=\frac{\left[\left(1+\alpha_1\right)P_1\left(t_{1Z}\right)^2\delta_4\delta_1+2\lambda g\delta_4\left(t_{1Z}\right)-\delta_1\left(1+\alpha_1\right)P_1\left(\delta_2+y_1\right)+\lambda g\left(\delta_3+G_3\right)\right]}{\left\{\begin{array}{l}\left[\beta\delta_1\left(1+\alpha_1\right)P_1\right]\left(t_{1Z}\right)^2\left(y_2-G_3\right)+\delta_4\lambda g\beta\left(t_{1Z}\right)^2+2\lambda g\delta_4\left(t_{1Z}\right)\\+\left(\lambda g\beta\right)\left(t_{1Z}\right)\left(y_2+\delta_3\right)-y_1\beta\delta_1\left(1+\alpha_1\right)P_1\left(t_{1Z}\right)+\lambda\beta g\left(\delta_2+y_1\right)\\+\lambda g\left(\delta_3+G_3\right)+\lambda g\left(1-e^{-\beta t_{1Z}}\right)\left(y_2-G_3\right)-y_1\left[\delta_1\left(1+\alpha_1\right)P_1+\beta\lambda g\right]\end{array}\right\}} \tag{40}$$

First, let $e^{-\beta t1\pi} = 0$ and $e^{-\beta t1\pi} = 1$, calculate Eq. (39) to find the initial t_{1ZU} and t_{1ZL} (i.e., t_{1Z}'s upper and lower bounds). Then, use the current t_{1ZU} and t_{1ZL} values to calculate $e^{-\beta t1ZU}$ and $e^{-\beta t1ZL}$ values. Re-calculate Eq. (39) with the current $e^{-\beta t1ZU}$ and $e^{-\beta t1ZL}$ to update the new bounds t_{1ZU} and t_{1ZL}. t_{1Z}* is derived if $t_{1ZU} = t_{1ZL}$ (i.e., $t_{1Z}^* = t_{1ZU} = t_{1ZL}$); otherwise, repeat the steps above, until ($t_{1ZU} = t_{1ZL}$) holds.

3. Numerical illustration

To demonstrate the proposed model's capability and applicability, this section provides the following numerical example with the assumption of system variables, as shown in Table 1.

Table 1
The assumption of system variables

λ	C	h_2	C_R	P_1	C_1	K	α_1	K_1	C_T	β	β_1
4000	\$2	\$1.6	\$1	10000	\$2	\$200	0.5	\$90	\$0.01	1	-0.70
M	h	α_3	π	P_2	h_3	g	α_2	n	h_1	x	β_2
\$2500	\$0.4	0.1	0.4	5000	\$0.4	0.018	0.1	3	\$0.4	20%	0.5

First, verification of convexity of $E[TCU(t_{1Z})]$ is conducted by testing if the following condition holds: $\omega(t_{1Z}) > t_{1Z} > 0$ (as exhibited in Eq. (A-8) (Appendix A)). Since $e^{-\beta t1Z}$ falls in [0, 1], by assuming initially $e^{-\beta t1Z} = 0$ and $e^{-\beta t1Z} = 1$, and applying Eq. (39) to obtain $t_{1ZL} = 0.0687$ and $t_{1ZU} = 0.1961$. Next, using t_{1ZU} and t_{1ZL} to recalculate $e^{-\beta t1ZU}$ and $e^{-\beta t1ZL}$. Then, apply Eq. (B-8) with the current values of t_{1ZL}, t_{1ZU}, $e^{-\beta t1ZL}$, and $e^{-\beta t1ZU}$, to confirm that $\omega(t_{1ZL}) = 0.2911 > t_{1ZL} = 0.0687 > 0$ and $\omega(t_{1ZU}) = 0.4334 > t_{1ZU} = 0.1961 > 0$, respectively. Once $E[TCU(t_{1Z})]$ is convex, one knows that the optimal t_{1Z}* exists. A broader β-value range is applied to test for convexity of $E[TCU(t_{1Z})]$, and these results prove our proposed model's more general applicability (see Table 2).

Table 2
Verifying convexity of $E[TCU(t_{1Z})]$ against different βs

β	$\gamma(t_{1ZL})$	t_{1ZL}	$\gamma(t_{1ZU})$	t_{1ZU}
10	0.0471	0.0213	0.3675	0.1930
8	0.0574	0.0256	0.3258	0.1931
6	0.0737	0.0320	0.2999	0.1932
4	0.1028	0.0417	0.2933	0.1935
3	0.1286	0.0486	0.3025	0.1938
2	0.1742	0.0575	0.3319	0.1944
1	**0.2911**	**0.0687**	**0.4334**	**0.1961**
0.5	0.4950	0.0754	0.6317	0.1995
0.01	4.2325	0.0828	4.5725	0.4160

For $E[TCU(t_{1Z})]$ is convexity, we apply the solution procedure mentioned earlier for finding the optimal solution. First, by assume initially $e^{-\beta t1Z} = 0$ and $e^{-\beta t1Z} = 1$, and apply Eq. (39) to obtain $t_{1ZL} = 0.0687$ and $t_{1ZU} = 0.1961$. Then, utilizing the present t_{1ZU} and t_{1ZL} values to recalculate $e^{-\beta t1ZU}$ and $e^{-\beta t1ZL}$. Then, by re-applying Eq. (39) with the current $e^{-\beta t1ZU}$ and $e^{-\beta t1ZL}$ to update the t_{1ZU} and t_{1ZL} values. If ($t_{1ZL} = t_{1ZU}$) is true, then t_{1Z}* arrives (i.e., $t_{1Z}* = t_{1ZL} = t_{1ZU}$); otherwise, repeat the steps mentioned above until ($t_{1ZL} = t_{1ZU}$) is true. Table 3 shows the step-by-step iterative outcomes for locating t_{1Z}*. It discloses the initial t_{1Z}'s upper and lower bounds, t_{1Z}*, and the convexity of $E[TCU(t_{1Z})]$ concerning to t_{1Z}. Fig. 7 illustrates the $E[TCU(t_{1Z})]$'s behavior relating to t_{1Z}, and it indicates that $t_{1Z}* = 0.0838$ and $E[TCU(t_{1Z}*)] = \$12,870.75$.

Table 3
The step-by-step iterative outcomes for locating t_{1Z}**

Step	t_{1ZU}	$e^{-\beta t1ZU}$	t_{1ZL}	$e^{-\beta t1ZL}$	t_{1ZU} - t_{1ZL}	$E[TCU(t_{1ZU})]$	$E[TCU(t_{1ZL})]$
-	-	0	-	1	-	-	-
1	**0.1961**	0.8219	**0.0687**	0.9336	0.1274	\$13,674.65	\$12,911.94
2	0.0998	0.9050	0.0813	0.9219	0.0185	\$12,902.92	\$12,871.69
3	0.0863	0.9173	0.0834	0.9200	0.0029	\$12,871.68	\$12,870.78
4	0.0842	0.9192	0.0837	0.9197	0.0005	\$12,870.78	\$12,870.76
5	0.0839	0.9196	0.0838	0.9196	0.0001	\$12,870.76	\$12,870.75
6	**0.0838**	0.9196	**0.0838**	0.9196	**0.0000**	**\$12,870.75**	**\$12,870.75**

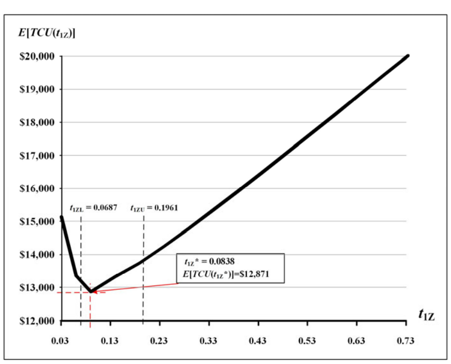

Fig. 7. The $E[TCU(t_{1Z})]$'s behavior relating to t_{1Z}

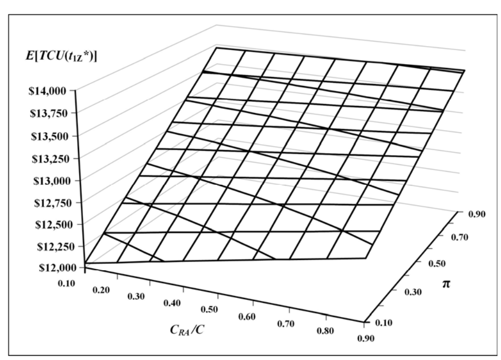

Fig. 8. Collective effect of differences in (C_{RA} / C) and π on $E[TCU(t_{1Z}*)]$

3.1. Joint effect of key features on the proposed study

The joint effect of differences in (C_{RA} / C) ratio and π on $E[TCU(t_{1Z}*)]$ are demonstrated in Fig. 8. It reveals that $E[TCU(t_{1Z}*)]$ significantly upsurges when both (C_{RA} / C) and π increase.

Fig. 9 displays the joint influence of changes in α_1 and n on $E[TCU(t_{1Z})]$. It shows $E[TCU(t_{1Z})]$ surges considerably as both n and α_1 rise. Fig. 10 illustrates the joint influence of differences in α_1 of the production rate and n on the t_{1Z}*. It indicates that t_{1Z}* considerably declines as α_1 rises, and t_{1Z}* increases significantly as n increases.

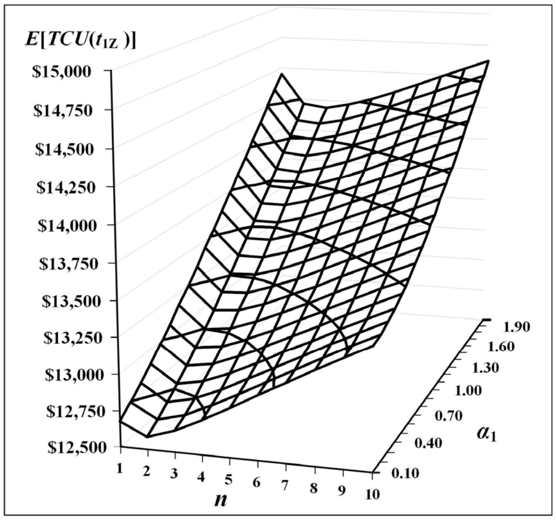

Fig. 9. Joint influence of variations in α_1 and n on $E[TCU(t_{1Z})]$

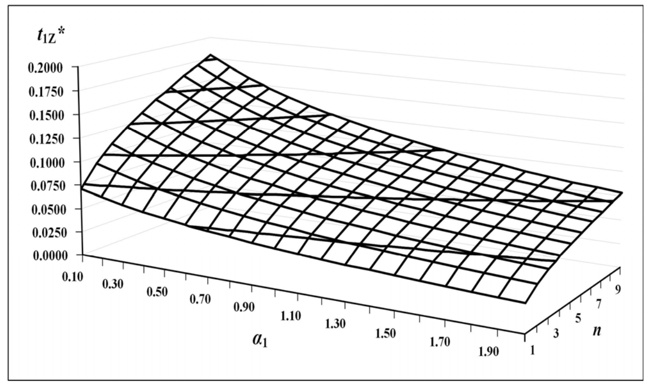

Fig. 10. Joint influence of changes in α_1 and n on t_{1Z}*

Fig. 11 shows the collective effect of differences in $1/\beta$ and π on runtime t_{1Z}*. It discloses that t_{1Z}* exceedingly drops as π rises. When π ≤ 0.4, the optimal t_{1Z}* declines noticeably as $1/\beta$ surges; and when π > 0.4, t_{1Z}* slightly drops as $1/\beta$ rises.

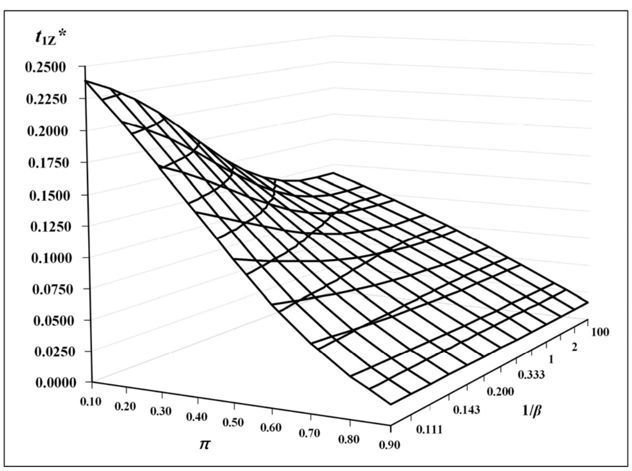

Fig. 11. Collective effect of differences in $1/\beta$ and π on t_{1Z}*

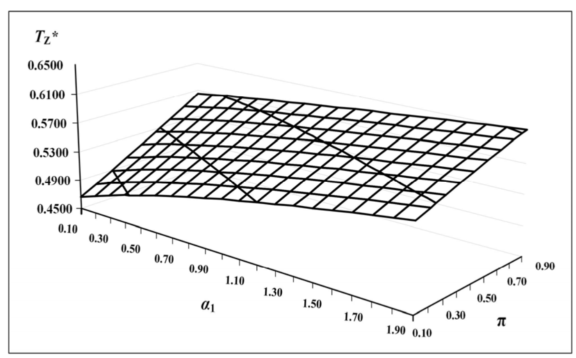

Fig. 12. Combined effect of changes in α_1 and π on t_{1Z}*

The combined effect of changes in α_1 and π on T_Z* is depicted in Fig. 12. It reveals that T_Z* noticeably increases as α_1 rises; T_Z* surges considerably as π upsurges. Fig. 13 exhibits the joint influence of difference in α_1 and π on utilization. It specifies that utilization exceedingly declines as π rises. When π ≤ 0.4, the machine utilization considerably drops as α_1 increases; and when π > 0.4, utilization reduces marginally as α_1 increases.

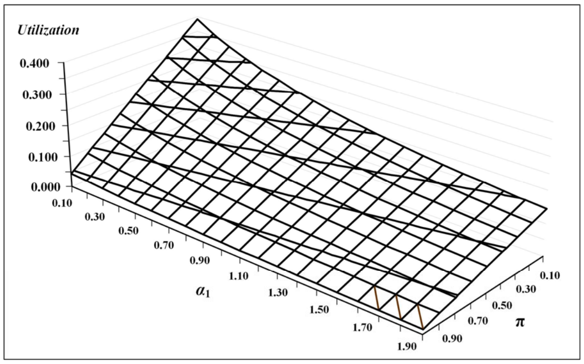

Fig. 13. Collective influence of difference in α_1 and π on utilization

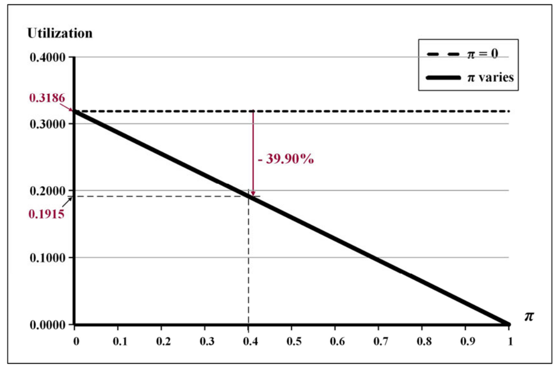

Fig. 14. Impact of differences in π on utilization

3.2. The impact of individual system feature on the proposed model

Fig. 14 displays the impact of differences in π on utilization. It exposes that utilization greatly declines as π rises. In our example, for π = 0.4, it shows that utilization drops from 0.3186 to 0.1915, a 39.90% decline due to our outsourcing strategy. Analytical results regarding the critical π value is depicted in Fig. 15. It discloses that the critical π = 0.7621. In other words, once the outsourcing portion rises to 0.7621 and beyond, it is more beneficial to utilize a pure 'buy' strategy.

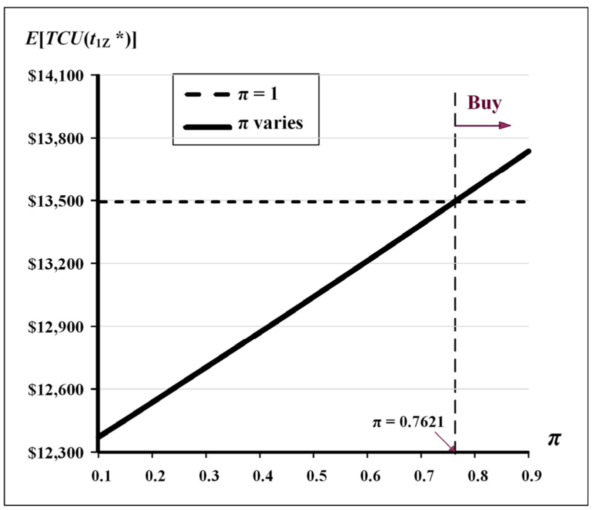

Fig. 15. A further investigative result concerning our example's critical π value

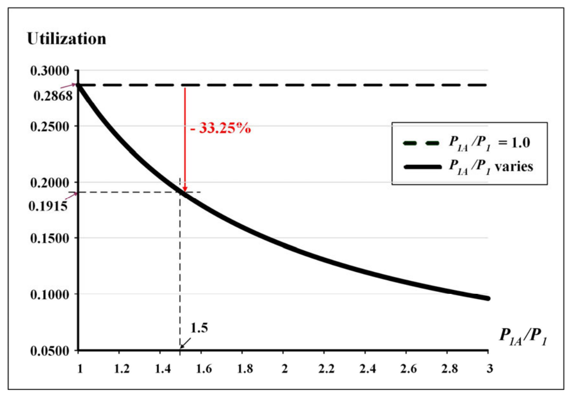

Fig. 16. The impact of variations in (P_{1A} / P_1) ratio on utilization

Fig. 16 depicts variations in the ratio of adjustable portion of production rate (P_{1A} / P_1) on machine utilization. It indicates that machine utilization radically drops as (P_{1A} / P_1) increases. In our example, for (P_{1A} / P_1) = 1.5, utilization reduces from 0.2868 to 0.1915, a 33.25% decrease due to our adjustable-rate strategy. Fig. 17 compares our utilization with other similar systems. Since our model considers dual utilization-reduction strategies (i.e., both outsourcing and adjustable-rate options), our utilization declines to 0.1915, or 33.2% lower than that in a similar system with outsourcing strategy only (see Fig. 17). Moreover, our utilization is 39.9% lower than that in a similar study with an adjustable-rate approach only (Chiu et al., 2020), or 59.8% less machine utilization than a similar system without implementing neither outsourcing nor adjustable-rate strategies (Chiu et al., 2018). For utilization reduction, as mentioned above, the prices we pay are 2.04%, 6.54%, and 10.35% increase in $E[TCU(t_{1Z}^*)]$, respectively.

A further explorative outcome reveals the impact of individual utilization-reduction strategy on utilization and $E[TCU(t_{1Z}^*)]$, as demonstrated in Fig. 18. It reveals that in our example, to reduce the utilization, a more cost-effective approach is to implement α_1 = 0.5 along with increasing π. It also confirms that in our example, π = 0.4 and α_1 = 0.5, utilization = 0.1915 and $E[TCU(t_{1Z}^*)]$ = $12,871. Moreover, this study can offer managerial decision-support information concerning how to efficiently implement the utilization-reduction strategy, as demonstrated in Fig. 19. For our example, it exposes that the most beneficial approach to reducing utilization starts with the following steps. (1) Initially, use the adjustable-rate strategy only, then continue to increase α_1 to 1.114. At this point, utilization declines to 0.2263, and $E[TCU(t_{1Z}^*)]$ rises to $12,877 (see the dash red-line in Fig. 19); (2) then, to further reduce utilization, the explorative result suggests to switch to the combined strategies; and starting with π = 0.527 and α_1 = 0. By keeping π at 0.527 and increasing α_1 (see the dash blue-line in Fig. 19). The suggestive steps mentioned above offer the most cost-effective approach to reduce utilization.

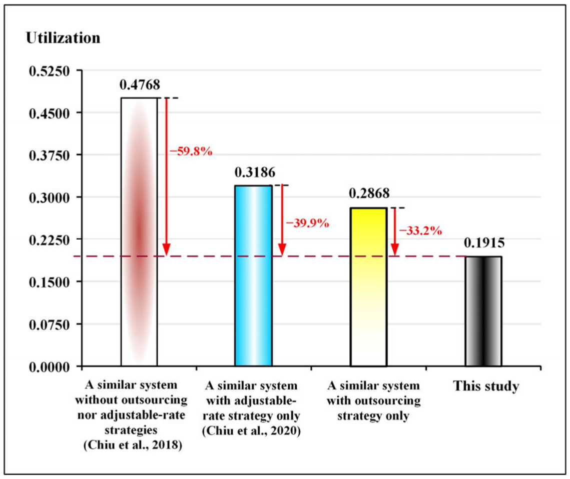

Fig. 17. Utilization comparison

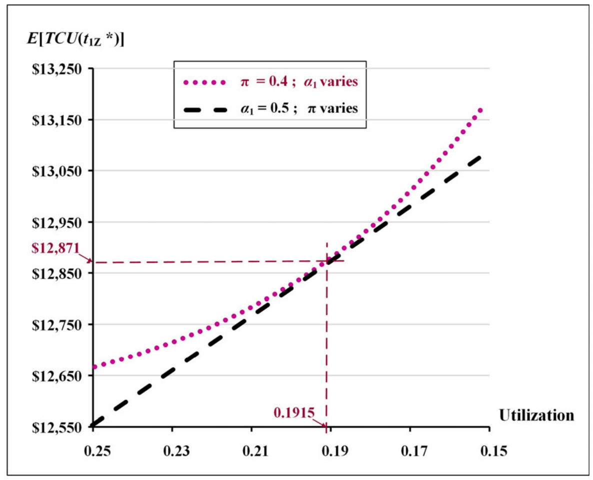

Fig. 18. The impact of individual utilization-reduction strategy on utilization and $E[TCU(t_{1Z}^*)]$

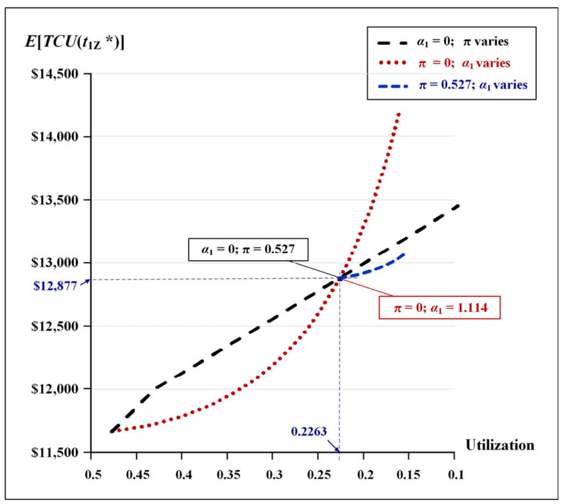

Fig. 19. Decision-support information concerning how to efficiently reduce utilization

4. Conclusions

This work derives an optimal batch fabricating time for an FPR-based supplier-retailer integrated problem with an outsourcer, rework, expedited-rate, and breakdown to assist the management of transnational enterprises in minimizing operating costs of their internal supply chains. The specific operational goals of the studied problem include satisfying external clients' requirements on quality and short order due dates, avoiding internal fabricating interruptions due to inevitable breakdowns, and minimizing overall manufacturing and stock holding costs. First, we build an FPR-based model featuring:
(1) The rework of defects and corrective action on the unanticipated breakdown to upkeep the quality and avoid fabricating delay;
(2) An outsourcer and expedited rate to significantly shorten the producing time; and
(3) Activities involved transporting end products and retailers' stock holding.

Through the model's formulating and investigating, we gain the function of operating costs. Then, we utilize optimization procedures with a proposed algorithm to verify its convexity and derive the model's best fabricating time solution. Finally, we validate how this study works and what important information our model can disclose using a numerical example (see the following) to facilitate management's decision-making:

(1) Table 2 verifies the convexity of operating costs against different βs, Table 3 exhibits the step-by-step iterating results for finding the optimal batch fabricating time, and Figure 7 depicts the operating cost's convexity;
(2) Fig. 8 to Fig. 13 demonstrates the joint impact of main system features (including the ratio of (C_{RA} / C), n, α_1, 1/β, and π) on the system operating costs, optimal runtime and batch cycle length, and utilization;
(3) Fig. 14 to Fig. 16 illustrates the impact of individual system feature (including the ratio of (P_{1A} / P_1) and π) on utilization,

optimal operating costs, and make-or-buy choices;
(4) Fig. 17 shows how our utilization outperforms that of existing studies;
(5) Fig. 18 and Fig.19 reveal the impact of individual utilization-reduction strategy on optimal operating costs and utilization and disclose crucial decision-support insights concerning the efficient and economical ways to reduce utilization.

Considering an uncertain annual product demand in the present problem and investigating its impact on the research outcomes is worth exploring for future work.

References

Antelo, M., & Bru, L. (2010). Outsourcing or restructuring: The dynamic choice. *International Journal of Production Economics. 123*(1), 1-7.

Bardhan, I., Mithas, S., & Lin, S. (2007). Performance impacts of strategy, information technology applications, and business process outsourcing in U.S. manufacturing plants. *Production and Operations Management, 16*(6), 747-762.

Brahmi, A., Hadj-Alouane, A.B., & Sboui, S. (2020). Dynamic and reactive optimization of physical and financial flows in the supply chain. *International Journal of Industrial Engineering Computations, 11*(1), 83-106.

Cetinkaya, S., & Lee, C.Y. (2000). Stock replenishment and shipment scheduling for vendor managed inventory systems. *Management Science, 46*(2), 217-232.

Chiu, S.W., Chen, H-M., Lin, H-D., & Chiu, Y-S.P. (2018) Optimization of an intra-supply chain system with unreliable production facility. *Journal of Applied Engineering Science, 16*(2), 192-201.

Chiu, S.W., You, L-W., Sung, P-C., & Wang, Y. (2020) Determining the fabrication runtime for a buyer-vendor system with stochastic breakdown, accelerated rate, repairable items, and multi-delivery strategy. *International Journal of Industrial Engineering Computations, 11*(4), 491-508.Chiu, Y.-S.P.,

Chiu, T., Pai, F.-Y., & Wu, H.Y. (2021). A producer-retailer incorporated multi-item EPQ problem with delayed differentiation, the expedited rate for common parts, multi-delivery and scrap. *International Journal of Industrial Engineering Computations, 12*(4), 427-440.

El-khalek, H.A., Aziz, R.F., & Morgan, E.S. (2019). Identification of construction subcontractor prequalification evaluation criteria and their impact on project success. *Alexandria Engineering Journal, 58*(1), 217-223.

Farmand, N., Zarei, H., & Rasti-Barzoki, M. (2021). Two meta-heuristic algorithms for optimizing a multi-objective supply chain scheduling problem in an identical parallel machines environment. *International Journal of Industrial Engineering Computations, 12*(3), 249-272.

Flapper, S.D.P., Fransoo, J.C., Broekmeulen, R.A., & Inderfurth, K. (2002). Planning and control of rework in the process industries: A review. *Production Planning & Control, 13*(1), 26-34.

Garcĺa, A., & de las Morenas, J. (2012). Integrated production attending to supply chain, materials handling and storage requirements. *International Journal of Advanced Logistics, 1*(1), 21-32.

Gera, A.E. (2021). The reliability of a system involving change points. *International Journal of Mathematical, Engineering and Management Sciences, 6*(1), 296-308.

Gupta, V., & Ivanov, D. (2020). Dual sourcing under supply disruption with risk-averse suppliers in the sharing economy. *International Journal of Production Research, 58*(1), 291-307.

Hajej, Z., Turki, S., & Rezg, N. (2015). Modelling and analysis for sequentially optimising production, maintenance and delivery activities taking into account product returns. *International Journal of Production Research, 53*(15), 4694-4719.

Hani, J.S.B. (2021). The moderating role of lean operations between supply chain integration and operational performance in Saudi manufacturing organizations. *Uncertain Supply Chain Management, 9*(1), 169-178.

Hermawan, D. (2021). The effect of competitive strategies on company performance with supply chain management as moderating variables in indonesian manufacturing corporations. *Uncertain Supply Chain Management, 9*(2), 237-246.

Jauhari, W.A., & Pujawan, I.N. (2014). Joint economic lot size (JELS) model for single-vendor single-buyer with variable production rate and partial backorder. *International Journal of Operational Research, 20*(1), 91-108.

Kenne, J.P., & Gharbi, A. (2001). A simulation optimization approach in production planning of failure prone manufacturing systems. *Journal of Intelligent Manufacturing, 12*, 421–431.

Kershaw, J., Yu, R., Zhang, Y., & Wang, P. (2021). Hybrid machine learning-enabled adaptive welding speed control. *Journal of Manufacturing Processes, 71,* 374-383.

Khouja, M. (2000). The economic lot and delivery scheduling problem: Common cycle, rework, and variable production rate. *IIE Transactions, 32*(8), 715-725.

Maggio, N., Matta, A., Gershwin, S.B., & Tolio, T. (2009). A decomposition approximation for three-machine closed-loop production systems with unreliable machines, finite buffers and a fixed population. *IIE Transactions, 41*(6), 562-574.

Mallick, B., Das, S., Sarkar, B., & Das, S. (2019). Application of the modified similarity-based method for multi-criteria inventory classification. *Decision Science Letters, 8*(4), 455-470.

Nachiappan, S.P., & Jawahar, N.A. (2007). A genetic algorithm for optimal operating parameters of VMI system in a two-echelon supply chain. *European Journal of Operational Research, 182*(3), 1433-1452.

Nguyen, T.T.C., Tran, Q.B., Ho, D.A., Duong, D.A., & Nguyen, T.B.T. (2021). The effect of supply chain linkages on the business performance: evidence from Vietnam. *Uncertain Supply Chain Management, 9*(3), 529-538.

Öztürk, H., Eroglu, A., & Lee, G.M. (2015). An economic order quantity model for lots containing defective items with rework option. *International Journal of Industrial Engineering: Theory Applications and Practice, 22*(6), 683-704.

Pham, T.H., & Doan, T.D.U. (2020). Supply chain relationship quality, environmental uncertainty, supply chain performance and financial performance of high-tech agribusinesses in Vietnam. *Uncertain Supply Chain Management, 8*(4), 663-674.

Pourmohammadi, F., Teimoury, E., & Gholamian, M.R. (2020). A scenario-based stochastic programming approach for designing and planning wheat supply chain (A case study). *Decision Science Letters, 9*(4), 537-546.

Rivera-Gómez, H., Gharbi, A., Kenné, J.-P., Montaño-Arango, O., & Hernández-Gress, E.S. (2018). Subcontracting strategies with production and maintenance policies for a manufacturing system subject to progressive deterioration. *International Journal of Production Economics, 200*, 103-118.

Sahebi, I.G., Masoomi, B., Ghorbani, S., & Uslu, T. (2019). Scenario-based designing of closed-loop supply chain with uncertainty in returned products. *Decision Science Letters, 8*(4), 505-518.

Sazvar, Z., Tafakkori, K., Oladzad, N., & Nayeri, S. (2021). A capacity planning approach for sustainable-resilient supply chain network design under uncertainty: A case study of vaccine supply chain. *Computers and Industrial Engineering, 159*, Art. No. 107406.

Sharma, G., & Rai, R.N. (2021). Failure modes based censored data analysis for repairable systems and its industrial perspective. *Computers and Industrial Engineering, 158*, Art. No. 107439.

Shekhar, C., Deora, P., Varshney, S., Singh, K.P., & Sharma, D.C. (2021). Optimal profit analysis of machine repair problem with repair in phases and organizational delay. *International Journal of Mathematical, Engineering and Management Sciences, 6*(1), 442-468.

Singh, S.R., Jain, S., & Pareek, S. (2014). An economic production model for time dependent demand with rework and multiple production setups. *International Journal of Industrial Engineering Computations, 5*(2), 305-314.

Sucky, E. (2005). Inventory management in supply chains: A bargaining problem. *International Journal of Production Economics, 93-94*(Spec. Issue), 253-262.

Toncovich, A.A., Rossit, D.A., Frutos, M., & Rossit, D.G. (2019). Solving a multi-objective manufacturing cell scheduling problem with the consideration of warehouses using a simulated annealing based procedure. *International Journal of Industrial Engineering Computations, 10*(1), 1-16.

Appendix – A

$E[TCU(t_{1Z})]$'s detailed derivations (i.e., Eq. (36)) and its convexity.

Apply x's expected values for dealing with its randomness, substitute Eq. (20), Eq. (31), and Eq. (33) in Eq. (32). With extra derivation efforts, $E[TCU(t_{1Z})]$ is found:

$$\begin{aligned}
E\left[TCU\left(t_{1Z}\right)\right] &= \left\{\int_0^{t_{1Z}} E\left[TC\left(t_{1Z}\right)_1\right]\cdot f(t)\,dt + \int_{t_{1Z}}^{\infty} E\left[TC\left(t_{1Z}\right)_2\right]\cdot f(t)\,dt\right\}\frac{1}{E[\boldsymbol{T}_Z]} \\
&= \left[\frac{1}{t_{1Z}}\delta_2 + \delta_3 + \left(h_1 - h\right)\left(t_{1Z}\right)\frac{E[x]^2 P_{1A}}{2P_{2A}} + \left(t_{1Z}\right)v_1\right]\left[\frac{\lambda}{\delta_1 + \dfrac{\left(1-e^{-\beta t_{1Z}}\right)\lambda g}{P_1\left(t_{1Z}\right)\left(1+\alpha_1\right)}}\right] \\
&+ \left[\begin{array}{l}
\left[\left(\lambda g\right)C_T + M + \left(\lambda g\right)C_1 + \left(\dfrac{\lambda g^2}{2}\right)h_2 + \left(\lambda g^2\right)h_3\right]\left[\dfrac{1-e^{-\beta t_{1Z}}}{P_1\left(1+\alpha_1\right)\left(t_{1Z}\right)}\right] \\
+\left[\left(-e^{-\beta t_{1Z}}\right)\left(t_{1Z}\right) - \dfrac{1}{\beta}e^{-\beta t_{1Z}} + \dfrac{1}{\beta}\right]h\dfrac{g}{\left(t_{1Z}\right)} + \left(\dfrac{g}{2n}\right)v_2\left(1-e^{-\beta t_{1Z}}\right)\left(h_2 - h\right) \\
+\left(\dfrac{g}{2}\right)v_3\left(1-e^{-\beta t_{1Z}}\right)\left(h_2 + 2h_3\right) + \left(1-e^{-\beta t_{1Z}}\right)h\cdot\left(\dfrac{g}{2}\right)v_2
\end{array}\right]\left[\frac{\lambda}{\delta_1 + \dfrac{\lambda g\left(1-e^{-\beta t_{1Z}}\right)}{P_1\left(t_{1Z}\right)\left(1+\alpha_1\right)}}\right]
\end{aligned} \tag{A-1}$$

where δ_1, δ_2, δ_3, v_1, v_2, and v_3 denote:

$$\begin{aligned}
\delta_1 &= \frac{1}{(1-\pi)} \\
\delta_2 &= \frac{\left(1+\beta_1\right)K}{\left(1+\alpha_1\right)P_1} + \frac{\left(1+\alpha_2\right)K}{\left(1+\alpha_1\right)P_1} + \frac{nK_1}{\left(1+\alpha_1\right)P_1}
\end{aligned} \tag{A-2}$$

$$\begin{aligned}
\delta_3 &= \left(1+\beta_2\right)C\left(\frac{\pi}{1-\pi}\right) + \left(1+\alpha_3\right)C + C_T\left(\frac{1}{1-\pi}\right) + \left(1+\alpha_3\right)C_R E[x] \\
v_1 &= \frac{\left(1+\alpha_1\right)P_1}{\left(1-\pi\right)^2}\left[\begin{array}{l}
\dfrac{h}{2\lambda}\left[1 + \dfrac{\lambda(1-\pi)(-\pi)}{\left(1+\alpha_1\right)P_1} + \dfrac{\lambda E[x](1-\pi)(1-2\pi)}{\left(1+\alpha_1\right)P_2}\right] + h_2\left(\dfrac{1}{2}\right)\left[\dfrac{1-\pi}{\left(1+\alpha_1\right)P_1} + \dfrac{E[x](1-\pi)}{\left(1+\alpha_1\right)P_2}\right] \\
+\left(h_2 - h\right)\left(\dfrac{1}{2\lambda n}\right)\left[1 - \dfrac{\lambda(1-\pi)}{\left(1+\alpha_1\right)P_1} - \dfrac{\lambda E[x](1-\pi)}{\left(1+\alpha_1\right)P_2}\right]
\end{array}\right] \\
v_2 &= \left(\frac{1}{1-\pi}\right)\left\{1 - \frac{\lambda(1-\pi)}{\left(1+\alpha_1\right)P_1} - \frac{\lambda E[x](1-\pi)}{\left(1+\alpha_1\right)P_2}\right\} \\
v_3 &= \left(\frac{1}{1-\pi}\right)\left\{1 + \frac{\lambda(1-\pi)}{\left(1+\alpha_1\right)P_1} + \frac{\lambda E[x](1-\pi)}{\left(1+\alpha_1\right)P_2}\right\}
\end{aligned} \tag{A-3}$$

Furthermore, suppose we let δ_4, y_1, y_2, G_0, G_1, G_2 and G_3 denote the following:

$$
\begin{aligned}
&\delta_4 = \frac{E[x]^2 P_{1A}}{2P_{2A}}\left(h_1 - h\right) + v_1 \\
&y_1 = \left[h_2\left(\frac{\lambda g^2}{2}\right) + C_T\left(\lambda g\right) + M + C_1\left(\lambda g\right) + h_3\left(\lambda g^2\right)\right]\frac{1}{(1+\alpha_1)P_1} + \frac{h \cdot g}{\beta} \\
&y_2 = -h \cdot g \\
&G_0 = \left(\frac{g}{2n}\right)v_2\left(h_2 - h\right);\ G_1 = \left(\frac{g}{2}\right)v_3\left(h_2 + 2h_3\right);\ G_2 = \left(\frac{g}{2}\right)v_2 h \cdot \\
&G_3 = G_2 + G_1 + G_0
\end{aligned}
\tag{A-4}
$$

$E[TCU(t_{1Z})]$ (i.e. Eq. (A-1),) becomes as follows (or as shown in Eq. (36)):

$$
E\left[TCU\left(t_{1Z}\right)\right] = \left[\frac{\lambda}{\delta_1 + \dfrac{\left(1-e^{-\beta t_{1Z}}\right)\lambda g}{\left(1+\alpha_1\right)\left(t_{1Z}\right)P_1}}\right]\left[\begin{array}{l} \dfrac{\delta_2}{t_{1Z}} + \delta_3 - \dfrac{y_1}{\left(t_{1Z}\right)}\left(e^{-\beta t_{1Z}}\right) + \dfrac{y_1}{t_{1Z}} + \delta_4\left(t_{1Z}\right) \\ +y_2\left(e^{-\beta t_{1Z}}\right) + G_3\left(1-e^{-\beta t_{1Z}}\right) \end{array}\right]
\tag{A-5}
$$

Apply $E[TCU(t_{1Z})]$'s first- and second-derivative, we obtain Eq. (A-6) and Eq. (A-7) as follows:

$$
\begin{aligned}
\frac{dE\left[TCU\left(t_{1Z}\right)\right]}{d\left(t_{1Z}\right)} = &\frac{P_1\left(1+\alpha_1\right)\lambda}{\left(\left(1-e^{-\beta t_{1Z}}\right)\lambda g + \left(1+\alpha_1\right)P_1\delta_1\left(t_{1Z}\right)\right)^2} \cdot \\
&\left[\begin{array}{l}
-\left(g\lambda\beta e^{-\beta t_{1Z}} + \delta_1\left(1+\alpha_1\right)P_1\right)\left(\delta_2 + y_1\right) \\
-\left(e^{-\beta t_{1Z}} - 1 + \beta e^{-\beta t_{1Z}} t_{1Z}\right)\lambda g\left(\delta_3 + G_3\right) \\
+\left(-\beta\delta_1 e^{-\beta t_{1Z}}\left(t_{1Z}\right)^2\left(1+\alpha_1\right)P_1 - \beta\lambda g e^{-\beta t_{1Z}}\left(t_{1Z}\right) + \lambda g e^{-\beta t_{1Z}} - \lambda g e^{-2\beta t_{1Z}}\right)\left(y_2 - G_3\right) \\
+\left(-\lambda g\beta e^{-\beta t_{1Z}}\left(t_{1Z}\right)^2 + \delta_1\left(t_{1Z}\right)^2\left(1+\alpha_1\right)P_1 + 2\lambda g\left(t_{1Z}\right)(1-e^{-\beta t_{1Z}}\right)\delta_4 \\
+\left(\beta v_0\left(t_{1Z}\right)(1+\alpha_1)P_1 + \delta_1\left(1+\alpha_1\right)P_1 + \beta\lambda g\right)e^{-\beta t_{1Z}} y_1
\end{array}\right]
\end{aligned}
\tag{A-6}
$$

$$
\begin{aligned}
\frac{d^2E\left[TCU\left(t_{1Z}\right)\right]}{d\left(t_{1Z}\right)^2} = &\left(\frac{P_1\left(1+\alpha_1\right)}{\left(\left(1+\alpha_1\right)P_1\delta_1\left(t_{1Z}\right) + \left(1-e^{-\beta t_{1Z}}\right)\lambda g\right)^3}\right)\lambda \cdot \\
&\left[\begin{array}{l}
\left(\begin{array}{l} \lambda^2 g^2\beta^2 e^{-2\beta t_{1Z}} + 4\delta_1\lambda g\beta e^{-\beta t_{1Z}}\left(1+\alpha_1\right)P_1 + \lambda^2 g^2\beta^2 e^{-\beta t_{1Z}} \\ +2\delta_1^2\left[\left(1+\alpha_1\right)P_1\right]^2 + \delta_1\lambda g\beta^2 e^{-\beta t_{1Z}}\left(1+\alpha_1\right)P_1\left(t_{1Z}\right) \end{array}\right)\left(\delta_2 + y_1\right) \\
+\left(\begin{array}{l} 2\lambda g\beta e^{-2\beta t_{1Z}} + \lambda g\beta^2 e^{-\beta t_{1Z}}\left(t_{1Z}\right) + \lambda g\beta^2 e^{-2\beta t_{1Z}}\left(t_{1Z}\right) - 2\lambda g\beta e^{-\beta t_{1Z}} - 2\delta_1\left(1+\alpha_1\right)P_1 \\ +2\beta\delta_1 e^{-\beta t_{1Z}}\left(1+\alpha_1\right)P_1\left(t_{1Z}\right) + \beta^2\delta_1 e^{-\beta t_{1Z}}\left(1+\alpha_1\right)P_1\left(t_{1Z}\right)^2 + 2\delta_1 e^{-\beta t_{1Z}}\left(1+\alpha_1\right)P_1 \end{array}\right)\lambda g\left(\delta_3 + G_3\right) \\
+\left(\begin{array}{l} \beta^2\lambda^2 g^2\left(t_{1Z}\right) + 2\beta\lambda^2 g^2 + \beta^2\lambda^2 g^2 e^{\beta t_{1Z}}\left(t_{1Z}\right) - 2\beta\lambda^2 g^2 e^{\beta t_{1Z}} \\ +\beta^2\delta_1\lambda g\left(t_{1Z}\right)^2\left(1+\alpha_1\right)P_1 + 4\beta\delta_1\lambda g\left(t_{1Z}\right)\left(1+\alpha_1\right)P_1 + 2\delta_1\lambda g\left(1+\alpha_1\right)P_1 \\ -2\beta\delta_1\lambda g e^{\beta t_{1Z}}\left(t_{1Z}\right)\left(1+\alpha_1\right)P_1 + 2\beta^2\delta_1\lambda g e^{\beta t_{1Z}}\left(t_{1Z}\right)^2\left(1+\alpha_1\right)P_1 \\ +\beta^2\delta_1^2 e^{\beta t_{1Z}}\left(t_{1Z}\right)^3\left[\left(1+\alpha_1\right)P_1\right]^2 - 2\delta_1\lambda g e^{\beta t_{1Z}}\left(1+\alpha_1\right)P_1 \end{array}\right)\left(y_2 - G_3\right)e^{-2\beta t_{1Z}} \\
+\left(\begin{array}{l} \lambda g\beta^2 e^{-2\beta t_{1Z}}\left(t_{1Z}\right)^2 + 2\lambda g + 2\lambda g e^{-2\beta t_{1Z}} + \lambda g\beta^2 e^{-\beta t_{1Z}}\left(t_{1Z}\right)^2 + 4\lambda g\beta e^{-2\beta t_{1Z}}\left(t_{1Z}\right) \\ -4\lambda g\beta e^{-\beta t_{1Z}}\left(t_{1Z}\right) - 4\lambda g e^{-\beta t_{1Z}} + \delta_1\beta^2 e^{-\beta t_{1Z}}\left(t_{1Z}\right)^3\left(1+\alpha_1\right)P_1 \end{array}\right)\lambda g\delta_4 \\
-\left(\begin{array}{l} 2\delta_1^2\left[\left(1+\alpha_1\right)P_1\right]^2 + \beta^2\delta_1^2\left(t_{1Z}\right)^2\left[\left(1+\alpha_1\right)P_1\right]^2 + 2\beta\delta_1^2\left(t_{1Z}\right)\left[\left(1+\alpha_1\right)P_1\right]^2 \\ +\beta^2\delta_1\lambda g e^{-\beta t_{1Z}}\left(t_{1Z}\right)\left(1+\alpha_1\right)P_1 + 2\beta^2\delta_1\lambda g\left(t_{1Z}\right)\left(1+\alpha_1\right)P_1 + 2\beta\delta_1\lambda g\left(1+\alpha_1\right)P_1 \\ +\beta^2\lambda^2 g^2 + 2\beta\delta_1\lambda g e^{-\beta t_{1Z}}\left(1+\alpha_1\right)P_1 + \beta^2\lambda^2 g^2 e^{-\beta t_{1Z}} \end{array}\right)e^{-\beta t_{1Z}} y_1
\end{array}\right]
\end{aligned}
\tag{A-7}
$$

Due to the 1st term on the RHS of Eq. (A-7) is positive, so $E[TCU(t_{1Z})]$ is convex if the 2nd term on the RHS of Eq. (A-7) is also positive. Meaning if the following $\omega(t_{1Z}) > t_{1Z} > 0$.

$$\omega(t_{1Z}) = \frac{\begin{bmatrix} \left(\lambda^2 g^2 \beta^2 e^{-2\beta t_{1Z}} + \lambda^2 g^2 \beta^2 e^{-\beta t_{1Z}} + 4\delta_1 \lambda g \beta e^{-\beta t_{1Z}} (1+\alpha_1) P_1 + 2\delta_1^2 \left[(1+\alpha_1) P_1\right]^2\right)(\delta_2 + y_1) \\ +\left(2\lambda g \beta e^{-2\beta t_{1Z}} - 2\lambda g \beta e^{-\beta t_{1Z}} + 2\delta_1 e^{-\beta t_{1Z}} (1+\alpha_1) P_1 - 2\delta_1 (1+\alpha_1) P_1\right)(\delta_3 + G_3)\lambda g \\ +\left(2\beta\lambda^2 g^2 - 2\beta\lambda^2 g^2 e^{\beta t_{1Z}} + 2\delta_1 \lambda g (1+\alpha_1) P_1 - 2\delta_1 \lambda g e^{\beta t_{1Z}} (1+\alpha_1) P_1\right)(y_2 - G_3) e^{-2\beta t_{1Z}} \\ +\left(2\lambda g + 2\lambda g e^{-2\beta t_{1Z}} - 4\lambda g e^{-\beta t_{1Z}}\right)\lambda g \delta_4 \\ -e^{-\beta t_{1Z}} y_1 \left(2\delta_1^2 [(1+\alpha_1) P_1]^2 + 2\beta\delta_1 \lambda g (1+\alpha_1) P_1 + 2\beta\delta_1 \lambda g e^{-\beta t_{1Z}} (1+\alpha_1) P_1 + \beta^2 \lambda^2 g^2 + \beta^2 \lambda^2 g^2 e^{-\beta t_{1Z}}\right) \end{bmatrix}}{-\begin{bmatrix} \left(\delta_1 \lambda g \beta^2 e^{-\beta t_{1Z}} (1+\alpha_1) P_1\right)(\delta_2 + y_1) \\ +\left(\lambda g \beta^2 e^{-2\beta t_{1Z}} + \lambda g \beta^2 e^{-\beta t_{1Z}} + \beta^2 \delta_1 e^{-\beta t_{1Z}} (1+\alpha_1) P_1 (t_{1Z}) + 2\beta\delta_1 e^{-\beta t_{1Z}} (1+\alpha_1) P_1\right)(\delta_3 + G_3)\lambda g \\ +\begin{pmatrix} \beta^2 \lambda^2 g^2 + \beta^2 \lambda^2 g^2 e^{\beta t_{1Z}} + \beta^2 \delta_1 \lambda g (t_{1Z})(1+\alpha_1) P_1 + 2\beta^2 \delta_1 \lambda g e^{\beta t_{1Z}} (t_{1Z})(1+\alpha_1) P_1 \\ +4\beta\delta_1 \lambda g (1+\alpha_1) P_1 - 2\beta\delta_1 \lambda g e^{\beta t_{1Z}} (1+\alpha_1) P_1 + \beta^2 \delta_1^2 e^{\beta t_{1Z}} (t_{1Z})^2 \left[(1+\alpha_1) P_1\right]^2 \end{pmatrix}(y_2 - G_3) e^{-2\beta t_{1Z}} \\ +\left(\lambda g \beta^2 e^{-2\beta t_{1Z}} (t_{1Z}) + \lambda g \beta^2 e^{-\beta t_{1Z}} (t_{1Z}) + 4\lambda g \beta e^{-2\beta t_{1Z}} - 4\lambda g \beta e^{-\beta t_{1Z}} + \delta_1 \beta^2 e^{-\beta t_{1Z}} (t_{1Z})^2 (1+\alpha_1) P_1\right)\lambda g \delta_4 \\ -\left(\beta^2 \delta_1^2 (t_{1Z}) [(1+\alpha_1) P_1]^2 + 2\beta\delta_1^2 [(1+\alpha_1) P_1]^2 + 2\beta^2 \delta_1 \lambda g (1+\alpha_1) P_1 + \beta^2 \delta_1 \lambda g e^{-\beta t_{1Z}} (1+\alpha_1) P_1\right) e^{-\beta t_{1Z}} y_1 \end{bmatrix}} > t_{1Z} > 0 \qquad \text{(A-8)}$$