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Decision analysis of individual supplier in a vendor-managed inventory program with revenue-sharing contract

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A B S T R A C T

As a useful strategy to improve the flexibility of the system to manage uncertainty in supply and demand and to improve the sustainability of the supply chain, vendor-managed inventory (VMI) programs have attracted widespread attention in the field of supply chain management. However, a growing body of empirical literature has shown that participants' decisions deviate significantly from the standard theoretical predictions. Under a VMI program, the supplier bears not only the production cost, but also the risk of leftover inventory. Moreover, the inequality among participants and different personalities of decision-makers in VMI programs may lead to the divergence of decision-making. To understand the supplier's replenishment decision in view of the behavioral pattern, we propose a new inventory model for the supplier with the focus theory of choice. The proposed model conceives that the retailer evaluates each replenishment quantity based on the most salient demand for him/her instead of calculating the expected utility. By employing this inventory model, we construct a two-tier supply chain model with revenue-sharing contract and theoretically derive the optimal sharing percentage of the revenue and replenishment quantity. Results analysis gains managerial insights into the strategic selection of the retailer who faces suppliers with different personalities. Comparisons between the classic revenue-sharing contract model and the proposed model are also carried out by illustrative examples. This research provides a new perspective to analyze individual supplier's behavior in a VMI program with revenue-sharing contracts.

1. Introduction

The vendor-managed inventory (VMI) program studies an inventory management problem faced by an upstream supplier that is in a collaborative agreement (Kadiyala et al., 2020). Under a VMI program, the supplier decides on the quantity and time of replenishment and the retailer, on the other hand, is relieved of keeping track of its inventory and placing orders with the supplier from time to time. The well-known pioneer that adopts such a VMI program is Wal-Mart partnering with P&G and many other suppliers. Many other companies have followed the general trend, e.g. Campbell Soup, Barilla, GE and Intel. As one centralized program, VMI has been proved to improve supply chain's efficiency and responsiveness to customer needs (Aviv, 2007; Bookbinder et al. 2010). VMI practices decrease supply chain greenhouse gas emissions because they improve the flexibility of the system to manage uncertainty in supply and demand (Ugarte et al. 2016). However, it is also reported that behaviors related to the inequality and decline of trust between two parties may lead to the poor performance of VMI programs (Zammori et al., 2009). Since the VMI supply chain cannot realize the expected benefits, analyzing the behavior of supply chain participants in VMI programs is becoming an important research topic of supply chain management. Till now, a considerable amount of studies have appeared in the VMI literature (see, e.g., Beheshti et al., 2020; Hariga et al., 2019; Hong et al., 2016; Zanoni et al., 2012).

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To better coordinate the channel members, designing coordinating contracts for VMI programs has received a great deal of attention. Coordinating contracts can motivate channel members to behave in ways that are best for the whole channel while maximizing their own profits (Phan et al., 2019). This situation results in a coordination of the VMI supply chain. In practice, many coordination mechanisms such as buyback, consignment, quantity discount, stockout-cost sharing, revenue-sharing contracts and others have been introduced and implemented in industry (see, e.g., Bai et al., 2019; Chakraborty et al., 2015; Hu et al., 2018; Lee et al., 2016; Sainathan & Groenevelt, 2019). As one payment scheme, the VMI program with revenue sharing has been widely employed in industries, especially in retail businesses (Gerchak & Wang, 2004). For instance, Amazon, Tmall, JD and other online platforms will charge a certain percentage of commission according to the transaction amount realized on the platforms. Due to its wide application, revenue-sharing contracts in VMI programs attract a lot of attention from many researchers. In traditional research, most of them predict the equilibrium outcomes in a game by maximizing the expected utility with the assumption of rational decision-makers. For example, Cai et al. (2017) established dynamic game relationships under a revenue-sharing contract and constructed three subsidy contracts to coordinate a VMI supply chain with service-level sensitive customers. Hu et al. (2017) considered a random demand and customers' uncertain return behavior and used a backward induction to solve equilibrium solutions. Other related areas have also been studied (see, e.g., Cachon & Lariviere, 2005; Chen & Wei, 2012; Gerchak & Wang, 2004; Li et al., 2009; Lim et al., 2015). However, Zhao et al. (2019) conducted experiments to reveal that decisions under revenue-sharing contracts deviate significantly from the standard theoretical predictions. Meanwhile, many empirical studies proved that the decision maker may be irrational through experiments. These observations motivate us to study the behaviors of supply chain participants. Accordingly, to better coordinate the VMI supply chain, it is significant to understand the behaviors of supply chain members and their impact on supply chain performance.

To understand the supplier's replenishment decision in view of the behavioral pattern, in this paper we propose a new inventory model for the supplier with the focus theory of choice. Different from the classic inventory model in which the supplier seeks an optimal replenishment quantity by maximizing the weighted average profit considering all possible demands, the proposed model conceives that the supplier evaluates each replenishment quantity by examining the most salient demand (called focus) for him/her instead of calculating the expected value. The focus theory of choice initially proposed by Guo (2019) is based on the bounded rationality of individuals (Simon, 1976) due to their limited attention (Masatlioglu et al., 2012). The focus theory of choice axiomatized a decision-making procedure and explained several well-known anomalies such as St. Petersburg paradoxes, Allais, Ellsberg paradoxes, violations of stochastic dominance and so on. The core idea of the focus theory of choice is that the most salient event corresponds to the most-preferred action, which plays a significant role in human decision-making. Decision-maker's specific frame of mind and behavioral patterns may affect the salience (attention-grabbing information), which is consistent with the results of the psychological experiments conducted by Stewart et al. (2016). Moreover, a number of literature has also shown the importance of salience information in human decision making under uncertainty (Lacetera et al., 2012; Busse et al., 2013; Brandstätter & Körner, 2014). As a special case of focus theory of choice, the one-shot decision theory (Guo, 2011) has been applied in a variety of fields and solved several decision problems, such as duopoly markets of innovative products (Guo, 2010; Guo et al., 2010), newsvendor problems for innovative products (Guo & Ma, 2014; Zhu & Guo, 2017), auction problems (Wang & Guo, 2017), production planning problems (Zhu & Guo, 2020) and supply chain management (Fang et al., 2021; Ma, 2019).

In this paper, we assume the supplier is an optimistic decision maker so that we employ the positive evaluation system of the focus theory of choice to analyze the supplier's replenishment decision in a VMI program with revenue-sharing contracts. We consider a two-tier VMI supply chain consisting of a single supplier and a single retailer for an innovative product. Due to the short life cycle of such a product (Fisher, 1997), the supplier usually has only one chance to replenish the product. With the assumption of personality information sharing, the operational procedure for revenue-sharing contract is that a rational retailer first sets a percentage of revenue and then the supplier determines a replenishment quantity. Within the framework of the focus theory of choice, the supplier makes the decision through a two-step process shown below. First, for each potential replenishment quantity, the supplier chooses his/her most salient demand while considering the payoff caused by the occurrence of the demand and the probability of the demand occurring. Each selected demand is called the focus of the replenishment quantity. Second, the supplier selects the most-preferred replenishment quantity by considering the focus of all possible replenishment quantities.

Under the proposed inventory model, we construct a Stackelberg supply chain game under the revenue-sharing contract mode. We theoretically derive the optimal sharing percentage of the revenue for the retailer by maximizing his/her expected profits. Theoretical analysis is consistent with several situations in reality and can help us to gain managerial insights into the strategic selection of the supplier and retailer under a revenue-sharing contract. Further, comparisons between the classic revenue-sharing contract model and the proposed model are also carried out by illustrative examples. This research provides a new perspective to analyze individual supplier's behavior in a VMI program with revenue-sharing contracts.

The remainder of this paper is organized as follows. In Section 2, we propose a new inventory model in a VMI program by employing the focus theory of choice under the positive evaluation system. In Section 3, we theoretically derive the optimal replenishment quantity for the supplier with the focus theory of choice under the positive evaluation system, and discuss the existence of equilibrium solutions to the new revenue-sharing contract model. In Section 4, we conduct a numerical example to specifically solve the proposed model and gain the managerial insights through results analysis. Finally, conclusions and future research are given in Section 5.

2. Revenue-sharing contract model with the focus theory of choice under the positive evaluation system

The VMI supply chain considered in this paper consists of a single supplier and a single retailer, they make their own decisions of a short life cycle product according to a revenue-sharing contract in this two-tier supply chain. First, the retailer determines the percentage s $(0 < s \leq 1)$ of revenue shared with the supplier. Second, after receiving the value of s, the supplier determines the replenishment quantity $q > 0$ before the selling season. The supplier's unit production cost is $c > 0$. We assume that the customer demand for the product is a random variable, denoted by X, and it has a density function $f(\cdot)$ and follows a cumulative distribution function $F(\cdot)$. We assume that the demand lies on an interval $[l, h]$ where $0 \leq l \leq h$. Clearly, the replenishment quantity should also lie within the demand interval $[l, h]$. When the demand x is observed, the retailer will sell units (limited by the supply q and the demand x) at the exogenous unit retail price r $(r > c)$. Based on the above description, the supplier's payoff function can be given as follows:

$$v(s,q,x) = \begin{cases} srx - cq, & \text{if } x < q, \\ srq - cq, & \text{if } x \geq q, \end{cases} \tag{1}$$

where s represents the sharing percentage provided by the retailer. The supplier needs to determine a replenishment quantity referring to the sharing percentage and demand. In this paper, we do not consider the unit opportunity cost and assume that the residual value of unsold products is zero. According to (1), it can be conceived that the highest profit of the supplier is

$$v_u = srh - ch,$$

that is, the supplier replenishes the most and the demand is the largest. To ensure $v_u \geq 0$, we assume $s \geq c/r$. The lowest profit is the case that the supplier replenishes the most but demand is lowest, that is

$$v_l = srl - ch.$$

As we know, many practical VMI supply chains consider the following operational procedure, which can be simplified into three stages. In the first stage, the retailer proposes the revenue-sharing contract and shares the percentage s of revenue with the supplier. In the second stage, after receiving the percentage s from the retailer, the supplier decides the replenishment quantity q with a total cost cq. In the last stage, after the demand is realized, they distribute the resultant revenue according to the contract terms of the revenue-sharing contracts. In general, over the above procedure, the retailer and the supplier make respective decisions at different stages. With the above operational procedure, we will establish a Stackelberg game model in the VMI supply chain where the participants (a retailer and a supplier) follow a revenue-sharing contract. The retailer acts as the leader and decides its revenue-sharing percentage at first. Then, the supplier as the follower will refer to the percentage s provided by the retailer to determine his/her optimal replenishment quantity $q(s)$. Based on standard backward induction, we can analyze the equilibrium of the game under the assumption of complete information. The main analysis process includes: Firstly, since the supplier decides the replenishment quantity after observing the sharing percentage, we use the focus theory of choice to derive his optimal replenishment quantity under the positive evaluation system; Then, with optimal replenishment quantity of the supplier, we proceed to solve the optimal revenue-sharing percentage of the retailer. Therefore, a perfectly rational retailer may infer the supplier's personality traits based on the past transactions and determine the revenue-sharing percentage in the contract by conjecturing the supplier's decision in this VMI supply chain. By maximizing his/her expected profit, the optimal sharing percentage can be solved. Thus, the equilibrium strategy of this Stackelberg game is obtained.

2.1. The inventory model with the focus theory of choice

With the increasing popularity of the VMI program in practice, many problems of its implementations have also been exposed. Among these problems, behaviors such as the irrational factors (social preferences and/or decision bias) of partners may lead to the poor performance of VMI supply chains (Zhao et al., 2019). In recent years, a growing number of literatures have shown salience rather than the expected value plays an important role in decision making (Lacetera et al., 2012; Busse et al., 2013; Brandstätter & Körner, 2014). In light of these observations, we apply the focus theory of choice framework in the revenue-sharing contract model and suppose that a supplier evaluates each replenishment quantity by examining the most salient demand (called focus) for him/her rather than the expected profit. To better construct a new revenue-sharing contract model with the focus theory of choice, we next convert the payoff function into a satisfaction function and the supplier's probability density function to a relative likelihood function.

Definition 1. *Let V be the range of the supplier's payoff. The function $u: V \to [0,1]$ is called a satisfaction function if $u(v_1) > u(v_2) \Leftrightarrow v_1 > v_2$, $\forall\, v_1, v_1 \in V$, and $\exists\, v_c \in V$ such that $u(v_c) = \max_{v \in V} u(v) = 1$.*

For simplicity of writing, we denote the composite function $u\big(v(s,q,x)\big)$ as $u(s,q,x)$. For any given sharing percentage $s \in$

$[c/r, 1]$ and replenishment quantity $q \in [l, h]$, $u(s, q, x)$ represents the supplier's satisfaction level about the resulting payoff if the demand arises as x. The satisfaction function represents the relative position of different payoffs exogenously determined by the decision maker (supplier). In this paper, we take the following satisfaction function:

$$u(s,q,x) := \frac{v(s,q,x) - v_l}{v_u - v_l} = \begin{cases} \dfrac{x-l}{h-l} + \dfrac{c(h-q)}{sr(h-l)}, & \text{if } x \le q, \\ \dfrac{q-l}{h-l} + \dfrac{c(h-q)}{sr(h-l)}, & \text{if } x > q. \end{cases} \tag{2}$$

Definition 2. *Let* $f: [l,h] \to \mathbb{R}_+$ *be the density function of stochastic demand. The function* $\pi: [l,h] \to [0,1]$ *is called the relative likelihood function if it satisfies that*

$\pi(x_1) > \pi(x_2) \Leftrightarrow f(x_1) > f(x_2)$, $\forall\, x_1, x_2 \in [l,h]$, *and* $\exists\, x_c \in [l,h]$, *such that* $\pi(x_c) = \max_{x \in [l,h]} \pi(x) = 1$.

For any $x \in [l,h]$, we call $\pi(x)$ as the relative likelihood degree of x. The relative likelihood function represents the relative position of the probability of different demands. In this paper, we take the following relative likelihood function:

$$\pi(x) := \frac{f(x)}{\max_{x \in [l,h]} f(x)}. \tag{3}$$

The focus theory of choice takes the relative likelihood and satisfaction functions as basic decision inputs, which are more convenient to obtain, instead of adopting original probability and profit functions. Compared to absolute values in human decision making, the relative values are suggested by a large amount of evidence that they are more perceptible and accessible (Frank, 1985; Solnick & Hemenway, 1998).

To analyze the revenue-sharing contract model, we make the following basic assumptions on the uncertain demand:

Assumption 1. *The probability density function* f *is continuous and strictly quasi-concave on the interval* $[l,h]$, *and* $\exists\, m \in [l,h]$ *such that* $f(m) = \max_{x \in [l,h]} f(x)$.

Since l and h are the lower and upper bounds of the demand respectively, $f(x)$ is increasing in the interval $[l,m]$ and decreasing in the interval $[m,h]$. Thus, we know that $\pi(x)$ attaining its unique maximum at $x = m$, $\pi(x)$ is strictly increasing on $[l,m]$ and decreasing on $[m,h]$. As we know, many common distributions satisfy Assumption 1, such as triangular distribution, truncated (logarithmic) normal distribution and truncated gamma distribution. The focus theory of choice owns two different evaluation systems: positive and negative. In the positive evaluation system, an event that has a relatively high satisfaction level and a relatively high likelihood possesses a relatively high salience. Contrary to it, in the negative evaluation system, an event that has a relatively low satisfaction level and a relatively high likelihood stands out as more salient. In line with what is described above, the focus theory of choice can characterize decision-makers' behavioral patterns in face of decision under risk. As for which one is at work, it depends on the frames of mind and personal traits of the decision maker. For example, when a decision maker is optimistic, the positive evaluation system is usually active. On the contrary, the negative system corresponds to a pessimistic decision maker. In this paper, we analyze the vendor-managed inventory problem with the focus theory of choice under the positive evaluation system. Since the life cycle of innovative products is generally shorter than the procurement lead-time, the supplier needs to take into account that which demand should be considered because there is only one demand occurring and he/she has only one chance to determine the replenishment quantity. According to the focus theory of choice, the procedure for decision-making is separated into two steps. In the first step, the supplier selects most salient outcomes (referred to as foci) from all possible demands of each action, which is the most appropriate scenario for him/her. In the second step, the supplier chooses the optimal action (replenishment quantity) by assessing all associated foci. Next, we introduce the specific model of the supplier's strategy selection. Under the positive evaluation system, we know that the supplier identifies the most salient demand with a relatively high satisfaction level and a relatively high likelihood. Hence, for any given sharing percentage s and replenishment quantity $q \in [l,h]$, we denote $P(s,q)$ as the set of optimal solutions to the following optimization problem:

$$\max_{x \in [l,h]} \ \min\{\varphi * \pi(x), u(s,q,x)\} \tag{4}$$

where parameter φ is a positive real number and denoted as a scaling factor that directly decides whether the focus with a higher satisfaction level or a higher likelihood from the inner minimization operation in Eq. (4). The optimization problem (4) is derived from the positive focus representation theorem in Guo (2019). Given q, for $x_1, x_2 \in [l,h]$, if $\pi(x_1) \ge \pi(x_2)$ and $u(s,q,x_1) \ge u(s,q,x_2)$, then we have $\min\{\varphi * \pi(x_1), u(s,q,x_1)\} \ge \min\{\varphi * \pi(x_2), u(s,q,x_2)\}$. Clearly, for any replenishment quantity q in a VMI program, Eq, (4) seeks the demand that processes a relatively high satisfaction level and a relatively high likelihood degree. In the above optimization problem, parameter φ can be interpreted as a weight that measures the supplier's emphasis on the satisfaction level and likelihood. Therefore, φ can represent the optimistic level of the supplier:

the higher the value of φ, the more optimistic the supplier. Next, we define the positive focus of the replenishment quantity q.

Definition 3. *If there is only one element in $P(s,q)$, then it is the positive focus of the replenishment quantity q, denoted by $x(s,q)$. If there exists more than one element in $P(s,q)$ and $\nexists\, x \in P(s,q)$ such that $\pi(x) > \pi\big(x(s,q)\big)$, $u(s,x,q) \geq u(s,x(s,q),q)$ or $\pi(x) \geq \pi\big(x(s,q)\big)$, $u(s,x,q) > u(s,x(s,q),q)$ for $x(s,q) \in P(s,q)$, then $x(s,q)$ is called the positive focus of q.*

From Definition 3, we know that the dominated outcomes are excluded and $x(s,q)$ is the most favorable demand for replenishment quantity q. If multiple positive foci exist, we denote the set of positive foci $x(s,q)$ as $P_+(s,q)$. Based on the above procedure, the optimal replenishment quantity is determined in the next step. Among all the possible positive foci for different replenishment quantities, we derive the optimal replenishment quantity by the following optimization problem:

$$\max_{q \in [l,h]} \max_{x(s,q) \in P_+(s,q)} \min\{\kappa * \pi(x(s,q)), u(s,x(s,q),q)\} \tag{5}$$

where parameter κ is a positive real number. Eq. (5) is derived from the optimal action representation theorem under the positive evaluation system in Guo (2019). We define the set of optimal solutions of Eq. (5) as $Q(s)$. For any $q_1, q_2 \in [l,h]$, if $\pi\big(x(s,q_1)\big) \geq \pi\big(x(s,q_2)\big)$ and $u(s,x(s,q_1),q_1) \geq u(s,x(s,q_2),q_2)$, then we have $\min\{\kappa * \pi\big(x(s,q_1)\big), u(s,x(s,q_1),q_1)\} \geq \min\{\kappa * \pi\big(x(s,q_2)\big), u(s,x(s,q_2),q_2)\}$. It implies that Eq. (5) seeks an optimal replenishment quantity whose focus has a relatively high likelihood and generates a relatively high satisfaction level. Similar to the interpretation of parameter φ, we know that increasing κ will result in a replenishment quantity whose positive focus has a relatively high satisfaction level but a relatively low likelihood. Hence, the parameter κ can measure the confidence index of the supplier to his/her decision: the higher the value of κ, the more confident the supplier.

Definition 4. *If there is only one element in $Q(s)$, then it is the optimal replenishment quantity under the positive evaluation system, denoted by $q(s)$. If there exists more than one element in $Q(s)$ and $\nexists\, q \in Q(s)$ such that $\pi(x(s,q)) > \pi\Big(x\big(s,q(s)\big)\Big), u(s,x(s,q),q) \geq u(s,x(s,q(s)),q(s))$ or $\pi(x(s,q)) \geq \pi\Big(x\big(s,q(s)\big)\Big), u(s,x(s,q),q) > u(s,x(s,q(s)),q(s))$ for $q(s) \in Q(s)$, then $q(s)$ is the optimal replenishment quantity under the positive evaluation system.*

From Definition 4, we know that the optimal replenishment quantity $q(s)$ weakly dominates all other elements in $Q(s)$ if it contains multiple optimal replenishment quantities under the positive evaluation system. We denote the set of optimal replenishment quantities under the positive evaluation system as $Q_+(s)$.

2.2. The Stackelberg game model with revenue-sharing contract

In this subsection, we will establish a Stackelberg game model in the VMI supply chain where the retailer and supplier follow a revenue-sharing contract. First, the retailer acts as the leader and decides its revenue-sharing percentage first. Then, the supplier as the follower will refer to the percentage provided by the retailer to determine his/her optimal replenishment quantity. With conjecturing the supplier's replenishment quantity $q(s)$, the retailer decides an optimal sharing percentage s^* by maximizing his/her expected profits. That is, solving the following optimization problem:

$$\max_{c/r \leq s \leq 1} \max_{q(s) \in Q_+(s)} r(1-s)\mathbb{E}\{\min(X, q(s))\} \tag{6}$$

After observing the sharing percentage s^* provided by the retailer, the supplier replenishes an optimal quantity $q(s^*)$ as the Stackelberg follower. In the next section, we will derive the optimal replenishment quantity for the supplier with the focus theory of choice under the positive evaluation system, and discuss the existence of equilibrium solutions to the Stackelberg game model with the revenue-sharing contract.

3. Theoretical analysis of revenue-sharing contract model under a VMI program

This section is separated into two parts: in the first part, we will derive the focus and optimal replenishment quantity of the supplier with the focus theory of choice under the positive evaluation system; in the second part, we will specifically discuss the existence of the retailer's optimal sharing percentage by solving the problem (6). Note that the proofs of all lemmas and theorems in this section are presented in Appendix A.

3.1 Optimal replenishment quantity for the supplier

We first propose some lemmas and theorems in this part to derive the optimal solution and characterize the properties of the optimal solution with the focus theory of choice under the positive evaluation system. As described below, Lemma 1 can be proposed to characterize the focus demand for any given replenishment quantity under the positive evaluation system, which is the solution to the lower level program (4).

Lemma 1. *For any given sharing percentage* $s \in [c/r, 1]$ *and replenishment quantity* $q \in [l, h]$, *its positive focus* $x(s, q)$ *is characterized as follows:*

(i) *If* $\varphi > \frac{u(s,q,q)}{\pi(q)}$, *for* $q \in [l, m]$, $x(s, q) = m$; *for* $q \in (m, h]$, $x(s, q) = q$;

(ii) *If* $\frac{u(s,m,q)}{\pi(m)} \le \varphi \le \frac{u(s,q,q)}{\pi(q)}$, *for* $q \in [l, m]$, $x(s, q) = m$; *for* $q \in (m, h]$, *there is a unique solution* $x(s, q)$ *to the equation* $u(s, x, q) = \varphi * \pi(x)$ *for* x *on* $[m, q]$;

(iii) *If* $0 < \varphi < \frac{u(s,m,q)}{\pi(m)}$, $x(s, q) = m$.

According to Lemma 1, we know that for any replenishment quantity $q \in [l, m]$, no matter what the value of φ, the focus demand $x(s, q)$ always equals to m. It illustrates that the choice of positive focus is unconcerned with an optimistic index φ for any $q \in [l, m]$ and the replenishment quantity q may affect the choice of the positive focus. In this case, the relative likelihood function stands out in determining the focus. Lemma 1 also characterizes the importance of parameter φ in determining the focus for any given replenishment quantity $q \in (m, h]$ under the positive evaluation system. When parameter φ is sufficiently high in case (i), the satisfaction function plays an important role in determining the focus $x(s, q) = q$, which has the highest satisfaction level at an even lower likelihood. After that, when φ decreases into the middle range in case (ii), the focus possesses a lower satisfaction level, but a higher likelihood. When parameter φ decreases to sufficiently small in case (iii), the relative likelihood function stands out in identifying the focus $x(s, q) = m$, which possesses the highest likelihood at a lower satisfaction level. Based on lemma 1, we confirm the following result.

Theorem 1. *For any given sharing percentage* $s \in [c/r, 1]$, $x(s, q)$ *equals to* m *for any* $q \in [l, m]$ *whenever* $\varphi > 0$, *and increases as* q *increases in the interval* $[m, h]$.

Theorem 1 demonstrates the relationship between the replenishment quantity and positive focus. It represents that for any given replenishment quantity $q_i \in [l, h]$, $i = 1,2$, if $q_1 < q_2$, then the positive focus of q_1 is smaller than or equal to the positive focus of q_2. Meanwhile, it implies that for any $q \in [m, h]$, the positive decision-maker tends to focus on a larger demand when he/she is given a larger replenishment quantity. Since $\pi(x)$ increases on $[l, m]$ and decreases on $[m, h]$, the following result is natural.

Theorem 2. *For any given sharing percentage* $s \in [c/r, 1]$, $\pi\big(x(s, q)\big)$ *equals to* $\pi(m)$ *for any* $q \in [l, m]$ *whenever* $\varphi > 0$, *and decreases as* q *increases on* $[m, h]$.

Theorem 2 illustrates that the relative likelihood function of the positive focus of q is a quasi-concave function. When φ is sufficient small as per Lemma 1 (iii), it also holds when $0 < \varphi < \frac{u(s,m,q)}{\pi(m)}$ for any $q \in [l, h]$. Meanwhile, it still holds for any replenishment quantity $q \in [l, m]$ *whenever* $\varphi > 0$, in the case that we have $\pi\big(x(s, q)\big) = \pi(m)$. To study the monotonicity of the function $u(s, x(s, q), q)$, we give the following lemma.

Lemma 2. *For any given sharing percentage* $s \in [c/r, 1]$ *and replenishment quantity* $q \in [m, h]$, *its positive focus* $x(s, q)$ *is determined as follows:*

(i) *If* $\varphi > \frac{u(s,h,h)}{\pi(h)}$, *then* $x(s, q) = q$;

(ii) *If* $\frac{u(s,m,m)}{\pi(m)} \le \varphi \le \frac{u(s,h,h)}{\pi(h)}$, *then there is a unique solution* $x_\varphi(s)$ *for* x *on the interval* $[m, h]$ *to the equation* $\varphi * \pi(x) = u(s, x, x)$, *such that*

$$x(s, q) = \begin{cases} q, & \text{for } q \in [m, x_\varphi], \\ x_r(s), & \text{for } q \in (x_\varphi, h], \end{cases}$$

where $x_r(s)$ *is the unique solution to the equation* $u(s, x, q) = \varphi * \pi(x)$ *for* x *on the interval* $[m, q]$;

(iii) $\frac{u(s,m,h)}{\pi(m)} \le \varphi < \frac{u(s,m,m)}{\pi(m)}$, *then there is a unique solution* $q_\varphi(s)$ *for* q *on the interval* $[m, h]$ *to the equation* $\varphi * \pi(m) = u(s, m, q)$, *such that*

$$x(s, q) = \begin{cases} m, & for\ q \in [m, q_\varphi], \\ x_r(s), & for\ q \in (q_\varphi, h], \end{cases}$$

where $x_r(s)$ *is the unique solution to the equation* $u(s, x, q) = \varphi * \pi(x)$ *for* x *on the interval* $[m, q]$;

(iv) $0 < \varphi < \frac{u(s,m,h)}{\pi(m)}$, *then* $x(s,q) = m$.

Lemma 2 further derives the positive focus of the replenishment quantity q on $[m,h]$. With the support from Lemma 1, we know that different positive focus arises within different ranges of parameter φ. Moreover, the thresholds of φ that we can now pinpoint are independent of replenishment quantity. Lemma 2 can help the decision-maker (supplier) get managerial insights on how a supplier chooses the most salient demand given different replenishment quantities and how his/her personality traits that are reflected by the parameter φ can contribute to his/her behavioral decisions. With the assistance of Lemma 1, Theorem 1 and Lemma 2, we can further obtain the following result.

Theorem 3. *For any given sharing percentage* $s \in [c/r, 1]$*, the function* $u(s,x(s,q),q)$ *is continuous and strictly quasi-concave of* q *on the interval* $[l,h]$.

(i) *If* $\varphi > \frac{u(s,h,h)}{\pi(h)}$, *then* $u(s,x(s,q),q)$ *is strictly increasing on* $[l,h]$.

(ii) *If* $\frac{u(s,m,m)}{\pi(m)} \le \varphi \le \frac{u(s,h,h)}{\pi(h)}$, *then* $u(s,x(s,q),q)$ *is strictly increasing on* $[l,x_\varphi]$ *and decreasing on* $[x_\varphi(s),h]$ *where* $x_\varphi(s)$ *is the unique solution to the equation* $u(s,x,x) = \varphi * \pi(x)$ *for* x *on* $[m,h]$.

(iii) *If* $\frac{u(s,m,h)}{\pi(m)} \le \varphi < \frac{u(s,m,m)}{\pi(m)}$, *then* $u(s,x(s,q),q)$ *is strictly increasing on* $[l,m]$ *and strictly decreasing on* $[m,h]$.

(iv) *If* $0 < \varphi < \frac{u(s,m,h)}{\pi(m)}$, *then* $u(s,x(s,q),q)$ *is strictly increasing on* $[l,m]$ *and strictly decreasing on* $[m,h]$.

We are ready to derive the optimal replenishment quantity of the inventory model, which is the solution to the upper level program (5).

Theorem 4. *For any given sharing percentage* $s \in [c/r, 1]$*, the optimal replenishment quantity* $q(s)$ *under the positive evaluation system and its corresponding optimal positive focus* $x(s,q(s))$ *are defined as follows:*

(i) *When* $\varphi > \frac{u(s,h,h)}{\pi(h)}$,

$$q(s) = x(s,q(s)) = \begin{cases} h, & \text{if } \kappa > \frac{u(s,h,h)}{\pi(h)}, \\ x_\kappa(s), & \text{if } \frac{u(s,m,m)}{\pi(m)} \le \kappa \le \frac{u(s,h,h)}{\pi(h)}, \\ m, & \text{if } 0 < \kappa < \frac{u(s,m,m)}{\pi(m)}. \end{cases}$$

(ii) *When* $\frac{u(s,m,m)}{\pi(m)} \le \varphi \le \frac{u(s,h,h)}{\pi(h)}$,

$$q(s) = x(s,q(s)) = \begin{cases} x_\varphi(s), & \text{if } \kappa > \varphi, \\ x_\kappa(s), & \text{if } \dfrac{u(s,m,m)}{\pi(m)} \le \kappa \le \varphi, \\ m, & \text{if } 0 < \kappa < \dfrac{u(s,m,m)}{\pi(m)}. \end{cases}$$

(iii) *When* $0 < \varphi < \frac{u(s,m,m)}{\pi(m)}$,

$$q(s) = x(s,q(s)) = m, \forall\, \kappa > 0.$$

Here, $x_\varphi(s)$ *is a unique solution to the equation* $\varphi * \pi(x) = u(s,x,x)$ *for* x *on the interval* $[m,h]$*, and* $x_\kappa(s)$ *is a unique solution to the equation* $\kappa * \pi(x) = u(s,x,x)$ *for* x *on the interval* $[m,h]$.

Theorem 4 shows the results of how a supplier makes his/her decisions under the positive evaluation system in a VMI program after identifying the focus at the first stage. It also illustrates that the optimal replenishment quantity must lie in the interval $[m,h]$ under the positive evaluation system. Actually, this conclusion is intuitive and well-understood in decision-making because the replenishment quantity m brings the highest likelihood and the highest satisfaction when the quantity q is confined to the interval $[l,m]$. As mentioned earlier, we know that the parameter κ can reflect the confidence level of the supplier when he/she makes decisions under the positive evaluation system. These results of Theorem 4 effectively interpret the behavioral patterns of the supplier's inventory decisions under the revenue-sharing contract mode. Based on the analysis of the above new supply chain model with the focus theory of choice, we achieve a perspective to understand how the supplier makes his/her replenishment decisions under the revenue-sharing contract mode by considering the optimism and confidence levels as well as the behavioral patterns. For any given sharing percentage $s \in [c/r, 1]$, we can specifically derive the optimal positive focus $x(s,q(s))$ and optimal replenishment quantity $q(s)$ for the supplier with focus theory of choice under the positive evaluation system with functions (2) and (3). The numerical results in Section 4 can help us to clearly understand how the supplier makes replenishment decisions by considering his/her optimism and confidence levels as well as the behavioral patterns.

3.2 Optimal sharing percentage for the retailer

In the Stackelberg game of the new revenue-sharing contract model, the retailer needs to decide his/her optimal sharing percentage by conjecting the supplier's replenishment decision. Thus, through maximizing his/her expected profits as the problem (6), we derive the optimal sharing percentage s^* for the retailer and the optimal replenishment quantity $q(s^*)$ for the supplier in the game. Then, we analyze the existence of the optimal sharing percentage by solving the first and second derivatives of the objective function of problem (6). We denote the objective function of problem (6) as $H(s)$. The first and second derivatives of $H(s)$ with respect to s can be given as follows:

$$H'(s) = r\left\{\int_l^{q(s)} F(x)dx - q(s) + (1-s)\,(q(s))'[1-F(q(s))]\right\}, \tag{7}$$
$$H''(s) = r[(1-s)(q(s))'' - (q(s))'][1-F(q(s))] - r(1-s)f(q(s))((q(s))')^2. \tag{8}$$

Based on the Eqs. (7-8), we mainly consider three possible situations to analyze the existence of the optimal sharing percentage as follows.

(i) If the objective function $H(s)$ is strictly increasing on the sharing percentage s, that is, its first derivative of $H(s)$ is strictly greater than 0 ($H'(s) > 0$), it illustrates that the retailer's expected profit increases as the sharing percentage increases. Thus, the retailer needs to provide a maximal sharing percentage to maximize his/her expected profit in this situation.

(ii) If the objective function $H(s)$ is strictly decreasing on the sharing percentage s, that is, its first derivative of $H(s)$ is strictly smaller than 0 ($H'(s) < 0$), it illustrates that the retailer's expected profit decreases as the sharing percentage increases. Thus, the retailer needs to provide a minimal sharing percentage to maximize his/her expected profit in this situation.

(iii) If the objective function $H(s)$ is a strictly concave function of the sharing percentage s, that is, its second derivative of $H(s)$ is smaller than 0 ($H''(s) < 0$), and there exists a sharing percentage s_0 such that $H'(s_0) = 0$, this situation illustrates that there exists a unique sharing percentage to maximize the retailer's expected profit.

4. Numerical examples and results analysis

A fashion clothing company (supplier), located in China, is planning to replenish a new design dress for the clothing store (retailer) according to the revenue-sharing contract in a VMI program. For such an innovative product, the procurement lead time is usually longer than the selling season so that there is only one opportunity for the supplier to replenish the new dress. The rights and responsibilities of both parties are stipulated in the agreement: the supplier takes the responsibility for the replenishment and inventory of the new dress, and the retailer is loss-free and does not bear inventory costs. Therefore, the supplier bears not only the production cost, but also the risk of leftover inventory in the VMI program.

4.1 Basic settings of the revenue-sharing contract model under the VMI program

The revenue-sharing contract also specific relevant parameters: the unit selling price of the retailer is $r = 60$ (RMB), the unit production cost of the supplier is $c = 15$ (RMB), and the revenue sharing percentage is in the interval of $[25\%, 100\%]$. We assume that the market demand is a random variable whose support set is $[100, 300]$ and the density function $f(x)$ is a symmetric triangular function and the most possible demand is $m = 200$. Among two participants, the retailer is perfectly rational and self-interested, the supplier has optimistic personalities, which is evaluated by the focus theory of choice under the positive evaluation system. These parameters and supplier's behavioral patterns are both common knowledge in this VMI program. Thus, the satisfaction level function of the supplier described by the Eq. (2) is

$$u(s,x,q) = \begin{cases} \dfrac{x-100}{200} + \dfrac{300-q}{800s}, & \text{if } x < q, \\ \dfrac{q-100}{200} + \dfrac{300-q}{800s}, & \text{if } x \geq q. \end{cases} \tag{9}$$

The probability density function is

$$f(x) = \begin{cases} 1\times 10^{-4}x - 0.008, & \text{if } 100 \leq x < 200, \\ -1\times 10^{-4}x + 0.032, & \text{if } 200 \leq x \leq 300. \end{cases}$$

According to Eq. (3), the relative likelihood function of the supplier is

$$\pi(x) = \begin{cases} \dfrac{x}{120} - 2/3, & \text{if } 100 \leq x < 200, \\ -\dfrac{x}{120} + 8/3, & \text{if } 200 \leq x \leq 300. \end{cases} \tag{10}$$

Referring to the function of Eq. (9) and Eq. (10), we can derive the optimal replenishment quantity for the supplier under the positive evaluation system with the focus theory of choice (Theorem 4). The optimal replenishment quantity $q(s)$ under the positive evaluation system and its corresponding optimal positive focus $x(s, q(s))$ can be expressed as follows:

(i) When $\varphi \geq 6$,

$$q(s) = x(s, q(s)) = \begin{cases} 300, & \text{if } \kappa > 6, \\ \dfrac{100(64\kappa s + 12s - 9)}{20\kappa s + 12s - 3}, & \text{if } \dfrac{1}{8s} + \dfrac{1}{2} \leq \kappa \leq 6, \\ 200, & \text{if } 0 < \kappa < \dfrac{1}{8s} + \dfrac{1}{2}. \end{cases}$$

(ii) When $\frac{1}{8s} + \frac{1}{2} \leq \varphi < 6$,

$$q(s) = x(s, q(s)) = \begin{cases} \dfrac{100(64\varphi s + 12s - 9)}{20\varphi s + 12s - 3}, & \text{if } \kappa > \varphi, \\ \dfrac{100(64\kappa s + 12s - 9)}{20\kappa s + 12s - 3}, & \text{if } \dfrac{1}{8s} + \dfrac{1}{2} \leq \kappa \leq \varphi, \\ 200, & \text{if } 0 < \kappa < \dfrac{1}{8s} + \dfrac{1}{2}. \end{cases}$$

(iii) When $0 < \varphi < \frac{1}{8s} + \frac{1}{2}$,

$$q(s) = x(s, q(s)) = 200, \qquad \forall\, \kappa > 0.$$

4.2 Optimal solutions of the revenue-sharing contract model under the VMI program

According to above basic settings, we can obtain the optimal replenishment quantity for the supplier at different optimistic and confident levels in a VMI program. In the following part, we will specifically solve the value of optimal sharing percentage s^*, optimal replenishment quantity $q(s^*)$, the profits of the retailer $H(s^*)$, the satisfaction level of the supplier $u(s^*, x(s^*, q(s^*)), q(s^*))$ and the relative likelihood level of the supplier $\pi(x(s^*, q(s^*)))$ by setting the specific values of φ and κ. Setting $\varphi = 0.5, 1.5, 3$ and 9, we can obtain the following results with κ being $0.5, 1.5, 3$ and 9 shown in Tables 1-4, respectively.

Table 1
Solutions of the proposed model with $\varphi = 0.5$

κ	s^*	$q(s^*)$	$H(s^*)$	$u(s^*, x(s^*, q(s^*)), q(s^*))$	$\pi(x(s^*, q(s^*)))$
0.5	25%	200	7800	1	1
1.5	25%	200	7800	1	1
3	25%	200	7800	1	1
9	25%	200	7800	1	1

Table 2
Solutions of the proposed model with $\varphi = 1.5$

κ	s^*	$q(s^*)$	$H(s^*)$	$u(s^*, x(s^*, q(s^*)), q(s^*))$	$\pi(x(s^*, q(s^*)))$
0.5	25%	200	7800	1	1
1.5	25%	240	11244	1	0.67
3	25%	240	11244	1	0.67
9	25%	240	11244	1	0.67

Table 3
Solutions of the proposed model with $\varphi = 3$

κ	s^*	$q(s^*)$	$H(s^*)$	$u(s^*, x(s^*, q(s^*)), q(s^*))$	$\pi(x(s^*, q(s^*)))$
0.5	25%	200	7800	1	1
1.5	25%	240	11244	1	0.67
3	25%	280	15264	1	0.33
9	25%	280	15264	1	0.33

Table 4
Solutions of the proposed model with $\varphi = 9$

κ	s^*	$q(s^*)$	$H(s^*)$	$u(s^*, x(s^*, q(s^*)), q(s^*))$	$\pi(x(s^*, q(s^*)))$
0.5	25%	200	9000	1	1
1.5	25%	240	11244	1	0.67
3	25%	280	15264	1	0.33
9	25%	300	17400	1	0.17

From Tables 1-4, we can solve the specific solutions to the new proposed revenue-sharing contract model under the VMI program. These results can also bring us the following three conclusions and management implications. The optimistic level φ and confidence level κ play an important role in determining the optimal replenishment quantity for the supplier under the positive evaluation system. For example, as shown in Table 1, if the supplier has a low optimistic level ($\varphi = 0.5$), no matter what the value of κ, he/she will take the most likely demand 200 as the optimal replenishment quantity. In this situation, the retailer can voluntarily compensate the supplier with an additional percentage of revenue after demand realization, which may improve the performance of both parties. There is also an extreme case that if the supplier is sufficiently optimistic and confident (both φ and κ take large values, as shown in Table 4), he/she will select the highest demand 300 as the optimal replenishment quantity. For such a positive and confident supplier, the retailer still needs to share a certain percentage of the revenue because of the inequality in the VMI program. If the supplier has a moderate optimistic level and confidence level (both φ and κ take the middle value) as Tables 2-3 shown, he/she will take the value between 200 and 300 as the optimal quantity. In general, these results are basically consistent with the analysis of Theorem 4, which is intuitive and understandable in real transactions. Meanwhile, they can also provide managerial insights into the retailer's strategic selections of optimal sharing percentage when he/she faces suppliers with different personalities. The optimal replenishment quantity generally increases as the optimal sharing percentage increases, excluding some extreme cases (the supplier has high or low optimistic and confident levels simultaneously). It illustrates that the retailer can offer a higher percentage to better motivate the supplier because of the inequality in a VMI program. Moreover, it should be noted that the profit of the retailer $H(s^*)$ also has a positive relationship with the optimal sharing percentage s^*. It can explain that if the retailer can offer a higher percentage, this action will benefit both parties of the supplier and the retailer, which effectively embodies the spirit of cooperation in a VMI program. In the literature of Zhao et al. (2019), it proposes that the retailer may present equality preference with adjustment, and the sharing percentage should be about 50%. Otherwise, it will lead to the breakdown of the cooperative relationship because the supplier may exhibit fairness concerns. The results of this paper are generally consistent with this experiment. And the difference of parameters in the transaction may affect the final equilibrium result.

4.3 Comparisons with traditional revenue-sharing contract model

In the classic revenue-sharing contract model, the supplier and retailer are both assumed to be perfectly rational and self-interested, the equilibrium of the game are mostly predicted by maximizing their own expected profits. Following the standard analysis, the optimal replenishment quantity of the supplier can be given as follows:

$$q^*(s) = \begin{cases} 0, & \text{if } s < 25\%, \\ F^{-1}\left(\dfrac{60s - 15}{60s}\right), & \text{if } s \geq 25\%. \end{cases}$$

Thus, similar to the table frame of above section, the specific results of the traditional revenue-sharing contract model can be given as follow.

Table 5
Solutions of traditional revenue-sharing contract model

s^*	$q(s^*)$	$H(s^*)$
25%	100	4500

Under the traditional revenue-sharing contract model, the expected profits are mainly predicted by equilibrium outcomes in game, with a perfectly rational and self-interested decision maker aiming to maximize their own profits. As shown in Table 5, the theoretical optimal replenishment quantity of the supplier is 100, which is inconsistent with our numerical results. Instead of considering all events of a lottery simultaneously, our research suppose that the supplier can choose one most salient event under the positive evaluation system with the focus theory of choice for his/her limited attention. By considering the relative likelihood function and satisfaction level simultaneously, the supplier can select one optimal quantity, which is most appropriate for him. Compared with traditional research of the revenue-sharing percentage, our research consider that the supplier may have different personalities and the positive and confidence index may affect their decision-making in a VMI program.

5. Conclusions

In this research, we construct a new revenue-sharing contract model with the focus theory of the choice framework in a VMI program. Unlike the traditional model with the expected utility where the optimal replenishment quantity and the optimal sharing percentage are obtained based on the weighted average of all payoffs, our decision-making is a procedural rational process and based on event-based thinking. This procedure can be divided into two steps: in the first step, for each potential replenishment quantity, the supplier selects the positive focus while considering the relative likelihood and satisfaction level; in the second step, based on relevant foci of all replenishment quantities, the supplier selects a most-preferred quantity. Meanwhile, we construct a Stackelberg game in a two-tier VMI supply chain consisting of one supplier and one retailer. Theoretical analysis provides a new perspective to the strategic selection of the retailer when he/she faces suppliers with different personalities, which may result in different Stackelberg equilibrium. There are several contributions that this research distinguishes from the previous work. First, we demonstrate that the optimal replenishment quantity of the supplier generally increases as the optimal sharing percentage increases under the positive evaluation system like under the traditional model mode. Second, the analysis also illustrates that optimistic and confident levels of the supplier both affect the determination of optimal replenishment quantity and optimal sharing percentage. We find that when the supplier is sufficiently optimistic and confident, he/she will replenish a large quantity although the retailer shares a low percentage of the revenue. In other extremes, when the supplier is sufficiently low optimistic and confident, he/she will replenish the most possible demand no matter what the value of sharing percentage. In these situations, we encourage the retailer to voluntarily compensate the supplier with an additional percentage of revenue in a VMI program after demand realization, which may improve the long-term and sustainable development of cooperation relationships. Third, our model employs the focus theory of choice framework in the revenue-sharing contract model and theoretical analysis provides new ideas for solving optimal solutions and shows some properties of them.

This research enriches the literature of revenue-sharing contract models and can be extended from several other directions. First, this research only considers the positive evaluation system with the focus theory of choice framework, the negative evaluation system can be also investigated. Second, this work considers the revenue-sharing contract model, which is only one form of other supply chain contract models. Thus, the application of the focus theory of choice framework can be extended to other forms of contracts or new contract design. Third, this research focuses on the two-tier supply chain with one supplier and one retailer, more complicated supply chain with multiple participants can be observed.

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Appendix A

Proof of Lemma 1. For any replenishment quantity $q \in [l,h]$, as $u(s,q,q) \geq u(s,m,q)$ and $0 < \pi(q) \leq \pi(m)$, we have $\frac{u(s,q,q)}{\pi(q)} \geq \frac{u(s,m,q)}{\pi(m)}$.

(i) If any replenishment quantity $q \in [l,m]$, we know that $u(s,x,q)$ stays the same on $[q,m]$ and $\varphi * \pi(x)$ strictly increase on $[q,m]$ for the monotonicity of $u(s,x,q)$ and $\pi(x)$. For any $x \neq m$, as $\varphi * \pi(q) > u(s,q,q)$, we have $\varphi * \pi(m) > \varphi * \pi(x)$, $\varphi * \pi(m) > u(s,m,q)$ and $u(s,m,q) = u(s,x,q)$ for $x \in [q,m]$. Hence, we can obtain that $\min\{\varphi * \pi(m), u(s,m,q)\} = u(s,m,q) \geq u(s,x,q) \geq \min\{\varphi * \pi(x), u(s,x,q)\}$. Based on (5) and the definition of the positive focus, we have $x(s,q) = m$. Then, if any replenishment quantity $q \in (m,h]$, for any $x \neq q$, as $\varphi * \pi(q) > u(s,q,q)$,we have $\min\{\varphi * \pi(q), u(s,q,q)\} = u(s,q,q) > u(s,x,q) \geq \min\{\varphi * \pi(x), u(s,x,q)\}$. Based on (5) and the definition of the positive focus, we have the positive focus $x(s,q) = q$.

(ii) If any replenishment quantity $q \in [l,m]$, for any $x \neq m$, as $\varphi * \pi(m) \geq u(s,m,q)$ and $\varphi * \pi(q) \leq u(s,q,q)$, we know that $u(s,x,q)$ stays the same on $[q,m]$ and $\varphi * \pi(x)$ strictly increase on $[q,m]$ for the monotonicity of $u(s,x,q)$ and $\pi(x)$. Thus, we have $\min\{\varphi * \pi(m), u(s,m,q)\} = u(s,m,q) \geq u(s,x,q) \geq \min\{\varphi * \pi(x), u(s,x,q)\}$. Based on (5) and the definition of the positive focus, we have $x(s,q) = m$. For any replenishment quantity $q \in (m,h]$, as $u(s,q,q) \geq \varphi * \pi(q)$ and $u(s,m,q) \leq \varphi * \pi(m)$, we know that $u(s,x,q)$ strictly increases on $(m,q]$ and $\varphi * \pi(x)$ strictly decreases on $(m,q]$. Thus, there exists a unique solution to the equation $u(s,x,q) = \varphi * \pi(x)$ for x on $[m,q]$, denoted by $x_r(s,q,\varphi)$. For any $x \neq x_r(s,q,\varphi)$, we have $\min\{\varphi * \pi(x_r(s,q,\varphi)), u_s(x_r(s,q,\varphi),q)\} > \min\{\varphi * \pi(x), u(s,x,q)\}$, which means $x(s,q) = x_r(s,q,\varphi)$.

(iii) For any demand $x \neq m$, as $\varphi * \pi(m) < u(s,m,q)$, we can have $\min\{\varphi * \pi(m), u(s,m,q)\} = \varphi * \pi(m) > \varphi * \pi(x) \geq \min\{\varphi * \pi(x), u(s,x,q)\}$. It means that $x(s,q) = m$.

Proof of Theorem 1. According to Lemma 1, for any $q \in [l,m]$, we know that $x(s,q) = m$ whatever $\varphi > 0$. Thus, the positive focus $x(s,q)$ is independent of replenishment quantity q in this case.

In the case of $q \in [m,h]$, we have $x(s,m) = m$ whenever $\varphi > 0$. let $q_1, q_2 \in [m,h]$ and $q_1 < q_2$. From Lemma 1, we know $x(s,q_i) \in [m,q_i]$ for $i = 1,2$. Then, contradictions can be used to show the proof in the following. Suppose $x(s,q_1) > x(s,q_2)$, then we have $m \leq x(s,q_2) < x(s,q_1) \leq q_1 < q_2 \leq h$. By the definitions of $v(s,x,q)$ and $u(s,x,q)$, it is easy to verify that

$$u(s,x(s,q_1),q_1) > u(s,x(s,q_1),q_2) \tag{A.1}$$

and

$$u(s,x(s,q_2),q_2) < u(s,x(s,q_1),q_2) \tag{A.2}$$

Considering $x(s,q_1) > m$, it follows from Lemma 1 that this situation is either $\varphi > \frac{u(s,q_1,q_1)}{\pi(q_1)}$ or $\frac{u(s,m,q_1)}{\pi(m)} \leq \varphi \leq \frac{u(s,q_1,q_1)}{\pi(q_1)}$ and if $\varphi > \frac{u(s,q_1,q_1)}{\pi(q_1)}$, we have $x(s,q_1) = q_1$, otherwise $x(s,q_1)$ satisfies the equation $u(s,x(s,q_1),q_1) = \varphi * \pi(x(s,q_1))$. In either of above situation, we have

$$u(s,x(s,q_1),q_1) \leq \varphi * \pi(x(s,q_1)) \tag{A.3}$$

Combining (A.3) and (A.1), it results in $u(s,x(s,q_1),q_2) < \varphi * \pi(x(s,q_1))$. Since $x(s,q_2) \in X(s,q_2)$ and $x(s,q_1) \neq x(s,q_2)$, we further have

$$u(s,x(s,q_2),q_2) \geq \min\{\varphi * \pi(x(s,q_2), u(s,x(s,q_2),q_2)\} > \min\{\varphi * \pi(x(s,q_1), u(s,x(s,q_1),q_2)\} = u(s,x(s,q_1),q_2)$$

It is clear that (A.3) and (A.2) contract each other. Thus, for any $q_1, q_2 \in [m,h]$, if $q_1 < q_2$, then $x(s,m) \leq x(s,q_1) \leq x(s,q_2)$. Over the above discussion, the Theorem 1 can be proved.

Proof of Lemma 2. According to the definitions of relative likelihood function and satisfaction functions, we know that $\pi(x)$ is strictly decreasing and $u(s,x,x)$ is strictly increasing on the $q \in [m,h]$.

(i) If $\varphi > \frac{u(s,h,h)}{\pi(h)}$ *and* $q \in [m,h]$, we have $\varphi > \frac{u(s,h,h)}{\pi(h)} \geq \frac{u(s,q,q)}{\pi(q)}$. According to the Lemma 1 (i), we know that $x(s,q) = q$. Thus, $u(s,x(s,q),q) = u(s,q,q)$.

(ii) If $\frac{u(s,m,m)}{\pi(m)} \leq \varphi \leq \frac{u(s,h,h)}{\pi(h)}$, then there is a unique solution $x_\varphi(s)$ for x on $[m,h]$ to the equation $\varphi * \pi(x) = u(s,x,x)$ for the monotonicity of $u(s,x,x)$ and $\pi(x)$. In the case of $q \in [m, x_\varphi]$, as $\varphi * \pi(q) > u(s,q,q)$, we have $x(s,q) = q$ as the result of Lemma 1(i). In the case of $q \in (x_\varphi(s), h]$, we have $\varphi = \frac{u(s,x_\varphi(s),x_\varphi(s))}{\pi(x_\varphi(s))} \leq \frac{u(s,q,q)}{\pi(q)}$ and $\varphi \geq \frac{u(s,m,m)}{\pi(m)} \geq \frac{u(s,m,q)}{\pi(m)}$. From Lemma 1(ii), we know that there is a unique solution $x_r(s)$ to the equation of $u(s,x,q) = \varphi * \pi(x)$ over $x \in [m,h]$. Then, we have $x(s,q) = x_r(s)$. In addition, $x(s, x_\varphi(s)) = x_\varphi(s)$.

(iii) If $\frac{u(s,m,h)}{\pi(m)} \leq \varphi < \frac{u(s,m,m)}{\pi(m)}$, since $u(s,m,x)$ is strictly decreasing over $q \in [m,h]$, then there exists a unique solution $q_\varphi(s)$ for q on $[m,h]$ to the equation $\varphi * \pi(m) = u(s,m,q)$. In the case of $q \in [m, q_\varphi(s)]$, as $\varphi * \pi(m) < u(s,m,q)$, it can be referred as Lemma 1(iii) that $x(s,q) = m$. In the case of the quantity $q \in (q_\varphi(s), h]$, as $\varphi * \pi(m) = u(s,m,q_\varphi(s))$, we can have $\varphi = \frac{u(s,m,q_\varphi(s))}{\pi(m)} \geq \frac{u(s,m,q)}{\pi(m)}$ and $\varphi = \frac{u(s,m,q_\varphi(s))}{\pi(m)} \leq \frac{u(s,m,m)}{\pi(m)} \leq \frac{u(s,q,q)}{\pi(q)}$. From Lemma 1(ii), we have $x(s,q) = x_r(s)$. Additionally, we have $x(s, q_\varphi(s)) = m$.

(iv) If $0 < \varphi < \frac{u(s,m,h)}{\pi(m)}$, then we have $\varphi * \pi(m) < u(s,m,h) \leq u(s,m,q)$ for $q \in [m,h]$. From Lemma 1(iii), we know that $x(s,q) = m$.

Proof of Theorem 3. According to (1), we know that $v(s,x(s,q),q) = \min\{r * s * x(s,q) - c * q, r * s * q - c * q\}$. Since $x(s,q)$ is continuous on $[l,h]$, then $v(s,x(q),q)$ is also continuous. Considering the definition of satisfaction function, we know that $u(s,x(q),q)$ is also a continuous function of q on $[l,h]$. In order to show the monotonicity of $u(s,x(q),q)$, we separate $[l,h]$ into two intervals: $[l,m]$ and $[m,h]$. In the following proof, we consider the two cases respectively.

Case 1. For $q \in [l,m]$, we have $u(s,x(s,q),q) = u(s,m,q)$ as Lemma 1 described. Thus, $u(s,x(s,q),q)$ is strictly increasing on $[l,m]$ whenever $\varphi > 0$ for the monotonicity of $v(s,x(s,q),q)$.

Case 2. It follows from Lemma 1 that $x(s,m) = m$ whenever $\varphi > 0$. In the following, as Lemma 2 described above, we considering the monotonicity of $u(s,x(s,q),q)$ for cases (i), (ii), (iii) and (iv), respectively.

(i) If $\varphi > \frac{u(s,h,h)}{\pi(h)}$ and $q \in [m,h]$, we have the focus demand $x(s,q) = q$. As noted earlier, $u(s,x(s,q),q) = u(s,q,q)$ is strictly increasing on $[m,h]$. Considering the continuity of $u(s,x(s,q),q)$ and Case 1, we know that $u(s,x(s,q),q)$ is strictly increasing on $[l,h]$.

(ii) Since $m < x_\varphi(s) \leq h$, we divide $[m,h]$ into the following two intervals: $[m, x_\varphi(s)]$ and $(x_\varphi(s), h]$. In what follows, we consider the two cases respectively.

(ii.a) For any $q \in [m, x_\varphi(s)]$, it follows from Lemma 2(ii) that $x(s,q) = q$. Clearly, $u(s,x(s,q),q) = u(s,q,q)$ is strictly increasing on $[m, x_\varphi]$.

(ii.b) Let $q_3, q_4 \in (x_\varphi(s), h]$ and $q_3 < q_4$. As per Lemma 1, we have $x(s,q_i) \leq q_i$ for $i = 3,4$. By the definition of (1), we know that (8) holds for $i = 3,4$. If $x(s,q_3) < x(s,q_4)$, as per Lemma 2(ii), we can have $u(s,x(s,q_i),q) = \varphi * \pi(x(s,q_i))fori = 3,4$ and hence $u(x(s,q_3),q_3) = \varphi * \pi(x(s,q_3))> \varphi * \pi(x(s,q_4))= u(x(s,q_4),q_4)$. If $x(s,q_3) = x(s,q_4)$, (8) results in $u(s,x(s,q_3),q_3) > u(s,x(s,q_4),q_4)$ due to $q_3 < q_4$ and hence, we have $u(s,x(s,q_3),q_3) > u(s,x(s,q_3),q_3)$.

In summary, Case 1, (i) and (ii.a) show that the function $u(s,x(s,q),q)$ is strictly increasing on $[l, x_\varphi(s)]$, case (ii.b) show that $u(s,x(s,q),q)$ is strictly decreasing on $[x_\varphi(s), h]$.

(iii) Since $m \leq q_\varphi(s) \leq h$, we divide $[m,h]$ into the following two intervals: $[m, q_\varphi(s)]$ and $(q_\varphi(s), h]$. In what follows, we consider the two cases respectively.

(iii.a) For any $q \in [m, q_\varphi(s)]$, as Lemma 2(iii) described, then $x(s,q) = m$. Thus, $u(s,m,q)$ is strictly decreasing on $[m, q_\varphi(s)]$ for the definition of $v(s,x,q)\ and\ u(s,x,q)$.

(iii.b) Let $q_5, q_6 \in (q_\varphi(s), h]$ and $q_5 < q_6$. As per Lemma 1, we have $x(s,q_i) \leq q_i$ for $i = 5,6$. By the definition of (1), for $i = 5,6$, we have

$$v(s,x(s,q_i),q) = r * s * x(s,q_i) - c * q. \tag{A.4}$$

If $x(s,q_5) < x(s,q_6)$, as per Lemma 2 (iii), we have $u(s,x(s,q_i),q) = \varphi * \pi(x(s,q_i))$ for $i = 5,6$ and hence $u(s,x(s,q_5),q_5) = \varphi * \pi\big(x(s,q_5)\big) > \varphi * \pi\big(x(s,q_6)\big) = u(s,x(s,q_6),q_6)$. If $x(s,q_5) = x(s,q_6)$, (A.4) results in $u(s,x(s,q_5),q_5) > u(s,x(s,q_6),q_6)$ due to $q_5 < q_6$ and hence, we have $u(s,x(s,q_5),q_5) > u(s,x(s,q_6),q_6)$.

In summary, Case 1 shows that the function $u(s,x(s,q),q)$ is strictly increasing on $[l,m]$, and (iii) shows that $u(s,x(s,q),q)$ is strictly decreasing on $[m,h]$.

(iv) For any $q \in [m,h]$, it follows from Lemma 2(iv) that $x(s,q) = m$. It is easy to verify from the definition of (1) that $v(s,x,q) = v(s,m,q)$ is strictly decreasing on $[m,h]$. Considering the definition of satisfaction function, we know that $u(s,m,q)$ is strictly decreasing on $[m,h]$. In summary, the function $u(s,x,q)$ is strictly increasing on $[l,m]$ and strictly decreasing on $[m,h]$.

Proof of Theorem 4. In view of above lemmas and theorems, we know that $\pi\big(x(s,m)\big) = \pi\big(x(s,q)\big)$ and $u(s,x(s,m),m) > u(s,x(s,q),q)$ whenever $\varphi > 0$. This means that $q(s)$ will not exist in the interval $[l,m)$. Thus, we only need to consider the interval $[m,h]$ in the following proof.

(i) When the parameter $\varphi > \frac{u(s,h,h)}{\pi(h)}$, we have $x(s,q) = q$ for any quantity $q \in [m,h]$ referring to Lemma 2(i). As described above, $\pi(x)$ is strictly decreasing and $u(s,x,x)$ is strictly increasing on the interval $[m,h]$ in this scenario.

(i.a) If $\kappa > \frac{u(s,h,h)}{\pi(h)}$, we have $\min\{\kappa * \pi\big(x(s,h)\big), u(s,x(s,h),h)\} = u(s,h,h) > u(s,x(s,q),q) \geq \min\{\kappa * \pi\big(x(s,q)\big), u(s,x(s,q),q)\}$. Based on Lemma 2, this means $q(s) = x(s,q(s)) = h$.

(i.b) If $\frac{u(s,m,m)}{\pi(m)} \leq \kappa \leq \frac{u(s,h,h)}{\pi(h)}$, there exists a unique solution $x_\kappa(s)$ to the equation $\kappa * \pi(x) = u(s,x,x)$ on $[m,h]$. For any replenishment quantity $q \in [m,x_\kappa(s)) \cup (x_\kappa(s),h]$, we can also have $\min\{\kappa * \pi\big(x(s,x_\kappa(s))\big), u(s,x(s,x_\kappa(s)),x_\kappa(s))\} = \min\{\kappa * \pi(x_\kappa(s)), u(s,x_\kappa(s),x_\kappa(s))\} > \min\{\kappa * \pi(q), u(s,q,q)\} = \min\{\kappa * \pi\big(x(s,q)\big), u(s,x(s,q),q)\}$. This means $q(s) = x(s,q(s)) = x_\kappa(s)$ in this situation.

(i.c) If $0 < \kappa < \frac{u(s,m,m)}{\pi(m)}$, for any replenishment quantity $q \in (m,h]$, we can have $\min\{\kappa * \pi\big(x(s,m)\big), u(s,x(s,m),m)\} = \kappa * \pi(m) > \kappa * \pi(q) \geq \min\{\kappa * \pi\big(x(s,q)\big), u(s,x(s,q),q)\}$. This means $q(s) = x(s,q(s)) = m$ in this situation.

(ii) When $\frac{u(s,m,m)}{\pi(m)} \leq \varphi \leq \frac{u(s,h,h)}{\pi(h)}$, we know that $\pi\big(x(s,q)\big)$ is decreasing on the interval $[m,h]$ as Theorem 2 described, and $u(s,x(s,q),q)$ is strictly increasing on $[m,x_\varphi(s)]$ and strictly decreasing on $[x_\varphi(s),h]$ as Theorem 3 described. Thus, this means that $q(s)$ will only lie in the interval $[m,x_\varphi(s)]$. We only consider this scenario in the following proof. Moreover, we have $x(s,q) = q$ for any $q \in [m,x_\varphi(s)]$ as Lemma 2(ii) described.

(ii.a) If the confidence index $\kappa > \varphi = \frac{u(s,x_\varphi(s),x_\varphi(s))}{\pi(x_\varphi(s))}$, we can have $\min\left\{\kappa * \pi\left(x\big(s,x_\varphi(s)\big)\right), u\big(s,x\big(s,x_\varphi(s)\big),x_\varphi(s)\big)\right\} = u\big(s,x\big(s,x_\varphi(s)\big),x_\varphi(s)\big) > u(s,x(s,q),q) \geq \min\{\kappa * \pi\big(x(s,q)\big), u(s,x(s,q),q)\}$ for any $q \in [m,x_\varphi(s))$. Thus, this means $q(s) = x(s,q(s)) = x_\varphi(s)$ in this situation.

(ii.b) If $\varphi \leq \kappa \leq \frac{u(s,m,m)}{\pi(m)}$, we have $x(s,q) = q$ for any $q \in \big[m,x_\varphi(s)\big]$ as Lemma 2(ii) described. Since $\kappa * \pi\big(x(s,m)\big) = \kappa * \pi(m) \geq u(s,m,m) = u(s,x(s,m),m)$ and $\kappa * \pi\left(x\big(s,x_\varphi(s)\big)\right) = \kappa * \pi\big(x_\varphi(s)\big) \leq \varphi * \pi\big(x_\varphi(s)\big) = u\big(s,x_\varphi(s),x_\varphi(s)\big) = u\big(s,x\big(s,x_\varphi(s)\big),x_\varphi(s)\big)$, there exists a unique solution to the equation $\kappa * \pi(x) = u(s,x,x)$, which can be denoted by $x_\kappa(s)$. Thus, we can have $\min\{\kappa * \pi\big(x(s,x_k(s))\big), u(s,x(s,x_k(s)),x_k(s))\} = \kappa * \pi\big(x(s,x_k(s))\big) = \kappa * \pi(x_k(s)) > \kappa * \pi(q) = \kappa * \pi\big(x(s,q)\big) \geq \min\{\kappa * \pi\big(x(s,q)\big), u(s,x(s,q),q)\}$ that holds for any quantity $q \in [m,x_k(s))$ and $\min\{\kappa * \pi\big(x(s,x_k(s))\big), u(s,x(s,x_k(s)),x_k(s))\} = u(s,x(s,x_k(s)),x_k(s)) > u(s,x(s,q),q) \geq \min\{\kappa * \pi\big(x(s,q)\big), u(s,x(s,q),q)\}$ that holds for any $q \in (x_k(s),x_\varphi(s)]$. This means $q(s) = x(s,q(s)) = x_\kappa(s)$ in this situation.

(ii.c) If $0 < \kappa < \frac{u(s,m,m)}{\pi(m)}$, we also have $x(s,q) = q$ for any $q \in \big[m,x_\varphi(s)\big]$ as Lemma 2(ii) described. Sincemin$\{\kappa * \pi\big(x(s,m)\big), u(s,x(s,m),m)\} = \kappa * \pi(m) > \kappa * \pi(q) = \kappa * \pi\big(x(s,q)\big) \geq \min\{\kappa * \pi\big(x(s,q)\big), u(s,x(s,q),q)\}$ holds for any $q \in (m,x_\varphi(s)]$, we have $q(s) = x(s,q(s)) = m$ in this situation.

(iii) When $0 < \varphi < \frac{u(s,m,m)}{\pi(m)}$, we have $\pi\big(x(s,m)\big) \geq \pi\big(x(s,q)\big)$ for any $q \in (m,h]$ as Theorem 2 and $u(s,x(s,m),m) > u(s,x(s,q),q)$ as Theorem 3 (i)-(ii). Thus, we have $q(s) = x(s,q(s)) = m$ for any $\kappa > 0$ in this case.