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A dynamic decision-making framework for a hybrid production system for decayed merchandise with shortages in traditional and electronic markets

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A B S T R A C T

This research proposes a dynamic decision-making framework for a hybrid production system that incorporates manufacturing and remanufacturing procedures into a closed-loop supply chain network with merchandise substitution and shortages within traditional markets (TM) and electronic markets (EM). In particular, we develop models of profit maximization and equilibrium analysis by using calculus with dynamic programming under four business schemes, including a manufacturing-only model within TM/EM and a hybrid remanufacturing model within TM/EM. Dynamic decision-making planning was taken for brand-new and like-new decayed merchandise in hybrid production systems. The results demonstrate that solutions generated within EMs surpass those within TMs in terms of maximizing profits. Further, the hybrid remanufacturing model did not surpass the manufacturing-only model under a general setting, but had better performance under certain conditions, including intense competition, a smaller remanufacturing cost, a larger brand-new merchandise market size, and a smaller like-new merchandise market size.

1. Introduction

With the increasingly fierce competition in the global market, supply chain management (SCM) has come to play a crucial role in enhancing a firm's competitive edge (Li et al., 2006; Flynn et al., 2010; Liao et al., 2017). SCM can improve profits by efficiently establishing a dynamic supply-and-demand network interconnecting upstream suppliers and downstream buyers, thus forming a competitive and coordinated supply chain (Ketchen & Hult, 2007). Indeed, the supply chain is a dynamic system that evolves with changes in the supply network and customer needs (Simchi-Levi et al., 2009). Developing appropriate strategies for optimizing marketing and production decisions is therefore imperative for effectively leveraging supply chains in different business environments.

In response to increasing concern about the impact of business activity on the environment, an increasing number of firms have begun to engage in green SCM (GSCM), which strives to meet the requirements of environmental protection. GSCM has become a popular topic in both business and academia during the last few decades, and firms have been adjusting their strategies and supply chain designs to take into account environmental regulations, consumer environmental awareness, the increasingly complex green business environment, and the growing market for green products (He, 2017; Khorshidvand et al., 2021). Traditional supply chain designs focus on the forward flow of raw materials from suppliers to end consumers; however, to extract reusable materials from returned products while reducing negative environmental effects, many firms have started adopting reverse logistics coupled with forward logistics, which is known as a "closed-loop supply chain" (CLSC) (Jabbarzadeh et al., 2018). CLSC management simultaneously takes into account both forward and reverse logistics. Forward logistics involves a series of activities, including the development and design of new merchandise, procurement, production, marketing and sales, and distribution; while reverse logistics involves repairing, reconditioning, remanufacturing, recycling,

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and disposing, so as to extend a product's lifespan and facilitate the return of end-of-life items from customers so that the manufacturer can recover valuable materials and return them to the supply chain (Ullah et al., 2021). In the last decade, CLSC has become a primary research subject of SCM (Govindan & Soleimani, 2017). Several analytical and quantitative studies have been carried out in various areas of CLSC, such as price policy (Modak et al., 2018; Liu et al., 2020; Khorshidvand et al., 2021), inventory management (Mitra, 2013; Bhatia & Srivastaba, 2019; Wang et al., 2019 ; Assid et al., 2021), production planning (Benedito & Corominas, 2013; Zhang et al., 2011; Assid et al., 2019; Assid et al., 2021), subsidy policy (Wang & Hong, 2019; Zhu et al., 2019), acquisition management (Hong et al., 2015; He, 2017), and CLSC network design (Fallah et al., 2015; Tsao et al., 2017; Haddadsisakht & Ryan, 2018; Sahebjamnia et al., 2018). CLSC couples the forward supply chain network with reverse logistics, and in recent years has become one of the most significant strategies for achieving sustainable manufacturing while staying competitive (Li et al., 2016; Battini et al., 2017; Govindan & Soleimani, 2017; Zhang & Chen, 2021). In recent years, various large firms, including Ford, Hewlett-Packard, FujiFilm, IBM, and Xerox, have implemented CLSC management to effectively minimize waste and to maximize resource recovery, which in turn enhances their profitability and social image (Mitra & Webster, 2008; Qiang et al., 2013). In addition, firms have begun to adopt remanufacturing and dynamic pricing activities to improve their level of GSCM.

Additionally, advances in information technology (IT) have revolutionized traditional SCM (Gunasekaran et al., 2017) and enabled supply chain members to gradually shift from traditional markets (TMs) to electronic markets (EMs), thereby gaining significant economic benefits (Ren & Zhang, 2014; Alsaad et al., 2017). EMs represent internet-based transactions for sellers and buyers, and can be viewed as an inter-organizational information system that has key functions (e.g., cataloging, matching, and aggregating) which improve the efficiency of transactions between channel members (Jean, 2014) and reduce uncertainty (Chang & Graham, 2012; Chandak et al., 2019). In comparison with the TM, the EM provides a virtual marketplace that allows the participating sellers and buyers to exchange information about product offerings and prices, which in turn brings such benefits as lower transaction costs, increased competition, and reduced search and coordination costs (Chang & Wong, 2010; Koch, 2010). Indeed, the increased efficiency of the EM reduces procurement costs by an average of 15% (Simchi-Levi et al., 2009), thus attracting numerous firms into this market. According to Chandak et al. (2019), the e-business process consists of operation efficiency, inventory management, supply chain flexibility, logistics performance, and supply chain integration. Through implementing this process, a firm can improve its supply chain performance by reducing costs and increasing customer satisfaction. The EM provides a coordinated platform which allows networks of sellers and buyers to conduct their business more effectively, and electronic commerce (EC) has profoundly changed the business models and consumption patterns of numerous firms in recent years (Gao et al., 2018). Indeed, EC has already become a major business model globally because of its advantages in terms of paying, pricing, marketing, and replenishing services and goods.

On the other hand, to enhance inter-organizational interactions in terms of marketing, production, and remanufacturing, firms have implemented various information systems (IS) or e-marketing services (Chong et al., 2016), so as to integrate internal and external business operations, such as customer relationship management, the CLSC system, and the enterprise resource planning (ERP) system. Moreover, the current prevalence of such EC tools as social media and digital media has greatly impacted the operation of the business-to-business (B2B) supply chain. Social media can provide B2B partners with abundant information, better communication channels, data-based optimization, and improved coordination and collaboration (Chae et al., 2020). According to Krings et al. (2021), digital media can facilitate identifying potential buyers, sharing information and maintaining knowledge, managing existing relations, and generating opportunities. The use of IS in SCM and the CLSC not only enables the integration of processes, but also synthesizes a wide variety of data exchange between channel partners, which in turn enhances the performance of the forward or reverse logistics network, so that it responds rapidly to changes in market demand (Rai et al., 2006; Lee & Lam, 2012; Hu et al., 2014; Khor et al., 2015). Nevertheless, the benefits of IS cannot be fully realized without a fine-tuned reconciliation and alignment among core business processes, system configurations, and organizational requirements (Al-Mashari et al., 2003). The configurations of scheduling and planning in an IS are based on static and fixed settings (Petty et al., 2000), such that the system may only provide sub-optimal solutions for lot size/scheduling and pricing problems. Although earlier studies have proposed various application models and tactics (e.g., dynamic pricing strategies, inventory control management, and production planning) for solving optimization problems in the supply chain (Benedito & Corominas, 2013; Mitra 2013; Modak et al., 2018; özelkan et al., 2018; Wang et al., 2019; Zhang et al., 2011), very little attention has been paid to the dynamic strategies of a hybrid manufacturing/remanufacturing mechanism in different trading markets. Thus the purpose of this research is to elucidate the effect joint dynamic pricing-replenishing and decision-making has on profits with shortages during a particular multi-period horizon, and to simultaneously present four business schemes for dynamic manufacturing-only and hybrid manufacturing/remanufacturing systems in different trading markets (TMs or EMs).

This research contributes to the literature in several ways. To our knowledge, the present research represents the first attempt to clarify the dynamic manufacturing-only and hybrid manufacturing/remanufacturing decision problems for decayed merchandise. At the same time, this study deals with joint dynamic pricing-replenishing planning by using calculus with dynamic programming (DP) under the dynamic manufacturing-only model and the hybrid manufacturing/remanufacturing model with merchandise substitution. Moreover, the models considered in this research can make a pricing strategy more responsive to changes in the market demand during a multi-period horizon in different trading markets, including TMs and EMs. Finally, because shortages can be a valid cost control method for supervising decayed inventory, in this study we examine the profit-maximization issue while considering the shortages approach.

The remainder of this research is organized as follows. Section 2 outlines the problem considered and its context. The development of the mathematical model is described in section 3. Section 4 consists of a comprehensive comparative investigation of the solutions generated by the four policies, and reports the results of a sensitivity experiment conducted with respect to key parameters. Finally, section 5 summarizes the findings of this study, discusses its limitations and managerial implications, and proposes directions for future study.

2. Problem description

This research examines the new and remanufactured versions of decayed merchandise, with the production schedule/quantity and selling price reviewed at time t periodically, where t = 0, 1, 2,…, H, at which H denotes the multi-period time horizon. This research aims to optimize production order z_{k-1}, where k = 1, 2,…, n, at which stock depletion time/service level, selling price, and lot size are decided at the same time to maximize the total profit during the multi-period horizon. Each period begins with replenishment, and the inventory is held during [z_{k-1}, χ], followed by the shortage during [χ, z_k]. Furthermore, this research assumes that shortages are completely backordered and that the production rate is infinite. Under the dynamic manufacturing-only model, the demand function considered in this research satisfies the following assumptions: (i) $D_{\text{mo}}(p,t) > 0$ and is continuous for $p > 0$ and $t \geq 0$; (ii) $D_{\text{mo}}(p,t) < +\infty$ for $p > 0$; moreover, (iii) $D_{\text{mo}}(p,t)$ is non-increasing in p (Rajan et al., 1992). At the same time, under the dynamic hybrid manufacturing/remanufacturing model, we relate the subscript N to brand-new merchandise produced by the manufacturing organization, and relate the subscript R to like-new merchandise produced by the remanufacturing organization. The demand function considered in the dynamic hybrid model satisfies the following assumptions: (iv) $D_{\text{N}}(p_{\text{N}}, p_{\text{R}}, t) > 0$, $D_{\text{R}}(p_{\text{R}}, p_{\text{N}}, t) > 0$, and are continuous for $p_{\text{N}} > 0$, $p_{\text{R}} > 0$ and $t \geq 0$; (v) $D_{\text{N}}(p_{\text{N}}, p_{\text{R}}, t) < +\infty$ and $D_{\text{R}}(p_{\text{R}}, p_{\text{N}}, t) < +\infty$ for $p_{\text{N}} > 0$ and $p_{\text{R}} > 0$; moreover, (vi) $D_{\text{N}}(p_{\text{N}}, p_{\text{R}}, t)$ is non-increasing in p_{N} and non-decreasing in p_{R} , while $D_{\text{R}}(p_{\text{R}}, p_{\text{N}}, t)$ is non-increasing in p_{R} and non-decreasing in p_{N} . In addition, $C_{\text{N}} = c$ represents the manufacturing inputs cost from original raw materials and $C_{\text{R}} = \delta c$, $0 \leq \delta \leq 1$ represents the remanufacturing inputs cost from returned cores. Furthermore, ξ , $0 \leq \xi \leq 1$ represents the returned rate of the end-of-period merchandise. In the EM setting, firms perform commercial transactions online. The transaction fee per unit of merchandise ($\rho, 0 \leq \rho \leq 1$) in the EM may be a transaction-based payment, and generally is a fraction of revenue. This research uses the following notations.

$\Phi =$ Production setup cost per run, $\Phi \geq 0$
$c =$ Production cost per unit of merchandise, $c \geq 0$
$h =$ Holding cost per unit of merchandise per unit of time, $h \geq 0$
$s =$ Shortage cost per unit of merchandise, $s \geq 0$
$\varphi =$ Transaction cost per unit of merchandise in the TM, $\varphi \geq 0$
$\rho =$ Transaction fee per unit of merchandise in the EM, $0 \leq \rho \leq 1$
$\xi =$ Returned rate of the end-of-period merchandise, $0 \leq \xi \leq 1$
$\theta(\tau(t)) =$ Merchandise decayed rate for stock on hand over $\tau(t)$, for $t \in$ [z_{k-1}, χ]
$I(p,t) =$ Inventory at time t given price p for $t \in$ [z_{k-1}, χ]
$S(p,t) =$ Shortage at time t given price p for $t \in$ [χ, z_k]
$\theta(\tau(t))I(p,t) =$ Decayed rate at time t when the merchandise lifetime is $\tau(t)$ and the price is p
$\Pi_{z_k} =$ Profit generated over period [0, z_k]

3. The model

This section shows the mathematical development of the decision model, which considers pricing, stock depletion time, and lot size joint decisions for the dynamic strategies of a hybrid manufacturing/remanufacturing mechanism in different trading markets over [z_{k-1}, z_k].

3.1 The manufacturing-only system

3.1.1 The base scenario: Manufacturing-only TM (policy 1)

A base scenario was set up for benchmarking system performance, involving a brand-new version of a decayed piece of merchandise under a manufacturing-only system in a TM. The cycle in the periodical stock review system begins with supply at z_{k-1} and stock is held until χ, followed by a time interval of shortages until the next stock supplementation at z_k. Therefore, the production quantity $Q_{\mathrm{mo}}(p,\chi)$ under the manufacturing-only TM scenario consists of the stock level over time period [z_{k-1},χ] and the shortage level over the period [χ, z_k].

$$Q_{\mathrm{mo}}(p,\chi) = I_{\mathrm{mo}}(p,z_{k-1}) + S_{\mathrm{mo}}(p,z_k)$$
$$= \int_{z_{k-1}}^{\chi} D_{\mathrm{mo}}(p,t) e^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)} dt + \int_{\chi}^{z_k} D_{\mathrm{mo}}(p,t)dt \,. \tag{1}$$

The associated cost $\psi_{\mathrm{mo,TM}z_{k-1}}(p,\chi)$ over the length of time [z_{k-1} , z_k] for the manufacturing-only TM scenario consists of variable production costs, transaction costs, costs due to merchandise holding and decay, shortage costs, and production setup costs, as follows:

$$\psi_{\mathrm{mo,TM}z_{k-1}}(p,\chi) = \int_{z_{k-1}}^{\chi} \left((c+\varphi) e^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)} + h\int_{z_{k-1}}^{t} e^{\int_{u}^{t}\theta(\tau(v))d\tau(v)} du\right) D_{\mathrm{mo}}(p,t)dt$$
$$+\int_{\chi}^{z_k} \left(c+\varphi+s(z_k-t)\right) D_{\mathrm{mo}}(p,t)dt + \Phi \,. \tag{2a}$$

Combining the terms of Eq. (2a) yields the expression:

$$\psi_{\mathrm{mo,TM}z_{k-1}}(p,\chi) = \int_{z_{k-1}}^{\chi} C_{\mathrm{mo,TM,1}}(t) D_{\mathrm{mo}}(p,t)dt + \int_{\chi}^{z_k} C_{\mathrm{mo,TM,2}}(t) D_{\mathrm{mo}}(p,t)dt + \Phi \,, \tag{2b}$$

where

$$C_{\mathrm{mo,TM,1}}(t) = (c+\varphi) e^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)} + h\int_{z_{k-1}}^{t} e^{\int_{u}^{t}\theta(\tau(v))d\tau(v)} du \,, \tag{2c}$$

$$C_{\mathrm{mo,TM,2}}(t) = c+\varphi+s\left(z_k - t\right). \tag{2d}$$

Eqs. (2c) and (2d) indicate the cost function during the selling period when net inventory is respectively positive and negative. The selling revenue over the length of time [z_{k-1} , z_k] for the manufacturing-only TM scenario is as follows:

$$\upsilon_{\mathrm{mo,TM}z_{k-1}}(p,\chi) = \int_{z_{k-1}}^{z_k} pD_{\mathrm{mo}}(p,t)dt \,. \tag{3}$$

Based on above Eqs. (2b)-(2d) and (3), the profit function over the length of time [z_{k-1} , z_k] is as follows:

$$\pi_{\mathrm{mo,TM}z_{k-1}}(p,\chi) = \upsilon_{\mathrm{mo,TM}z_{k-1}}(p,\chi) - \psi_{\mathrm{mo,TM}z_{k-1}}(p,\chi)$$
$$= \int_{z_{k-1}}^{\chi} (p - C_{\mathrm{mo,TM,1}}(t)) D_{\mathrm{mo}}(p,t)dt + \int_{\chi}^{z_k} (p - C_{\mathrm{mo,TM,2}}(t)) D_{\mathrm{mo}}(p,t)dt - \Phi \,. \tag{4}$$

The optimal selling price p and depletion time χ for the manufacturing-only TM scenario over the length of time [z_{k-1} , z_k] can be gained through resolving the first order differential equation of Eq. (4) with respect to p and χ separately as follows:

$$\frac{\partial}{\partial p}\pi_{\mathrm{mo,TM}z_{k-1}}(p,\chi) = \int_{z_{k-1}}^{\chi} \left(D_{\mathrm{mo}}(p,t) + (p - C_{\mathrm{mo,TM,1}}(t))\frac{\partial}{\partial p} D_{\mathrm{mo}}(p,t)\right)dt$$
$$+\int_{\chi}^{z_k} \left(D_{\mathrm{mo}}(p,t) + (p - C_{\mathrm{mo,TM,2}}(t))\frac{\partial}{\partial p} D_{\mathrm{mo}}(p,t)\right)dt = 0, \tag{5}$$

and

$$\frac{\partial}{\partial \chi}\pi_{\mathrm{mo,TM}z_{k-1}}(p,\chi) = (C_{\mathrm{mo,TM,2}}(\chi) - C_{\mathrm{mo,TM,1}}(\chi)) D_{\mathrm{mo}}(p,\chi) = 0. \tag{6a}$$

From Eq. (6a) and assumption (i): $D_{\mathrm{mo}}(p,\chi) > 0$, we have

$$C_{\mathrm{mo,TM,2}}(\chi)-C_{\mathrm{mo,TM,1}}(\chi)=0. \tag{6b}$$

To show the uniqueness of the solution for the manufacturing-only TM setting requires proving that the profit function $\pi_{\mathrm{mo,TM}z_{k-1}}(p,\chi)$ given in Eq. (4) is concave on p and χ. Next, the propositions 1 to 3 are listed below and the proofs are mentioned in the appendix.

Proposition 1 The function $\pi_{\mathrm{mo,TM}z_{k-1}}(p,\chi)$ given in Eq. (4) is concave in p for the manufacturing-only TM scenario, given the market demand form: $D_{\mathrm{mo}}(p,t)=(M-p)e^{\lambda t}$, in which $M>0$ and $p\in[0,M)$.

This market demand form was chosen since it is regularly applied in related research.

Proposition 2 The function $\pi_{\mathrm{mo,TM}z_{k-1}}(p,\chi)$ given in Eq. (4) is concave in χ for the manufacturing-only TM scenario.

Proposition 3 The function $\pi_{\mathrm{mo,TM}z_{k-1}}(p,\chi)$ given in Eq. (4) is concave in p and χ jointly for the manufacturing-only TM scenario, given the market demand form: $D_{\mathrm{mo}}(p,t)=(M-p)e^{\lambda t}$, in which $M>0$ and $p\in[0,M)$.

The single period model is based on a given cycle over the length of time [z_{k-1}, z_k]. The optimal track of selling price and depletion point during a multi-period time horizon can be decided by using a DP method:

$$\Pi^{**}_{\mathrm{mo,TM}z_k}=\max_{z_{k-1}}\left\{\Pi^{*}_{\mathrm{mo,TM}z_{k-1}}+\pi^{*}_{\mathrm{mo,TM}z_{k-1}}(p,\chi):0\le z_{k-1}<z_k\le H\right\}, \tag{7}$$

with boundary condition $\Pi_{\mathrm{mo,TM,0}}=0$. The recursive series of steps operate in a forward fashion to decide the maximal profit during the premeditated time horizon. The final phase of this series of steps yields the maximal total profit $\Pi^{**}_{\mathrm{mo,TM},H}$ during the multi-period planning horizon. Following the track backwards from times H to 0 determines the optimal replenishment sequence z^{*}_{k-1} and associated depletion point and price over the planning horizon.

3.1.2 Manufacturing-only EM scenario (policy 2)

Under the manufacturing-only system in the EM, the associated cost $\psi_{\mathrm{mo,EM}z_{k-1}}(p,\chi)$ consists of variable production costs, costs due to merchandise holding and decay, shortage costs, transaction fees, and setup costs over the length of time [z_{k-1}, z_k] as follows:

$$\psi_{\mathrm{mo,EM}z_{k-1}}(p,\chi)=\int_{z_{k-1}}^{\chi}\Big(ce^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)}+h\int_{z_{k-1}}^{t}e^{\int_{u}^{t}\theta(\tau(v))d\tau(v)}du\Big)D_{\mathrm{mo}}(p,t)dt$$
$$+\int_{\chi}^{z_k}\big(c+s(z_k-t)\big)D_{\mathrm{mo}}(p,t)dt+\int_{z_{k-1}}^{z_k}\rho pD_{\mathrm{mo}}(p,t)dt+\Phi. \tag{8a}$$

$$=\int_{z_{k-1}}^{\chi}C_{\mathrm{mo,EM,1}}(t)D_{\mathrm{mo}}(p,t)dt+\int_{\chi}^{z_k}C_{\mathrm{mo,EM,2}}(t)D_{\mathrm{mo}}(p,t)dt+\Phi, \tag{8b}$$

where

$$C_{\mathrm{mo,EM,1}}(t)=\rho p+ce^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)}+h\int_{z_{k-1}}^{t}e^{\int_{u}^{t}\theta(\tau(v))d\tau(v)}du, \tag{8c}$$

$$C_{\mathrm{mo,EM,2}}(t)=\rho p+c+s(z_k-t). \tag{8d}$$

The revenue during the length of time [z_{k-1}, z_k] for the manufacturing-only EM scenario is as follows:

$$\upsilon_{\mathrm{mo,EM}z_{k-1}}(p,\chi)=\int_{z_{k-1}}^{z_k}pD_{\mathrm{mo}}(p,t)dt. \tag{9}$$

Based on above Eqs. (8b)–(8d) and (9), the profit function over the length of time [z_{k-1}, z_k] is as follows:

$$\pi_{\mathrm{mo,EM}z_{k-1}}(p,\chi)=\upsilon_{\mathrm{mo,EM}z_{k-1}}(p,\chi)-\psi_{\mathrm{mo,EM}z_{k-1}}(p,\chi)$$

$$= \int_{z_{k-1}}^{\chi} (p - C_{\mathrm{mo,EM,1}}(t)) D_{\mathrm{mo}}(p,t)dt + \int_{\chi}^{z_k} (p - C_{\mathrm{mo,EM,2}}(t)) D_{\mathrm{mo}}(p,t)dt - \Phi \ . \tag{10}$$

As for the manufacturing-only EM scenario, the optimal price p and depletion time χ can be calculated by resolving the first order differential equation of Eq. (10) with respect to p and χ separately over the length of time [z_{k-1} , z_k] as follows:

$$\frac{\partial}{\partial p}\pi_{\mathrm{mo,EM},z_{k-1}}(p,\chi) = \int_{z_{k-1}}^{\chi} ((1-\rho) D_{\mathrm{mo}}(p,t) + (p - C_{\mathrm{mo,EM,1}}(t)) \frac{\partial}{\partial p} D_{\mathrm{mo}}(p,t))dt$$

$$+ \int_{\chi}^{z_k} ((1-\rho) D_{\mathrm{mo}}(p,t) + (p - C_{\mathrm{mo,EM,2}}(t)) \frac{\partial}{\partial p} D_{\mathrm{mo}}(p,t))dt = 0, \tag{11}$$

and

$$\frac{\partial}{\partial \chi}\pi_{\mathrm{mo,EM},z_{k-1}}(p,\chi) = (C_{\mathrm{mo,EM,2}}(\chi) - C_{\mathrm{mo,EM,1}}(\chi)) D_{\mathrm{mo}}(p,\chi) = 0. \tag{12a}$$

From Eq. (12a) and assumption (i): $D(p,\chi) > 0$, we have

$$C_{\mathrm{mo,EM,2}}(\chi) - C_{\mathrm{mo,EM,1}}(\chi) = 0. \tag{12b}$$

Demonstrating the uniqueness of the solution for the manufacturing-only EM scenario requires proving that the profit function $\pi_{\mathrm{mo,EM},z_{k-1}}(p,\chi)$ given in Eq. (10) is concave on p and χ in a fashion similar to that used for the manufacturing-only TM scenario (policy 1). The propositions 4 to 6 are stated forthwith and the proofs are presented in the appendix.

Proposition 4 The function $\pi_{\mathrm{mo,EM},z_{k-1}}(p,\chi)$ given in Eq. (10) is concave in p for the manufacturing-only EM scenario, given the market demand form: $D_{\mathrm{mo}}(p,t) = (M-p)e^{\lambda t}$, in which $M > 0$ and $p \in [0, M)$.

Proposition 5 The function $\pi_{\mathrm{mo,EM},z_{k-1}}(p,\chi)$ given in Eq. (10) is concave in χ for the manufacturing-only EM scenario.

Proposition 6 The function $\pi_{\mathrm{mo,EM},z_{k-1}}(p,\chi)$ given in Eq. (10) is concave in p and χ jointly for the manufacturing-only system EM scenario, given the market demand form: $D_{\mathrm{mo}}(p,t) = (M-p)e^{\lambda t}$, in which $M > 0$ and $p \in [0, M)$.

The optimal track of selling price and depletion point during a planning time horizon for the manufacturing-only EM scenario can be decided by applying a DP method in a similar dynamic manner:

$$\Pi^{**}_{\mathrm{mo,EM},z_k} = \max_{z_{k-1}} \left\{ \Pi^{*}_{\mathrm{mo,EM},z_{k-1}} + \pi^{*}_{\mathrm{mo,EM},z_{k-1}}(p,\chi) : 0 \le z_{k-1} < z_k \le H \right\}, \tag{13}$$

with boundary condition $\Pi_{\mathrm{mo,EM},0} = 0$.

3.2 The hybrid system with remanufacturing

3.2.2 Hybrid remanufacturing TM scenario (policy 3)

The profit under the hybrid system with a remanufacturing TM scenario for the new merchandise over the length of time [z_{k-1} , z_k] is selling revenue minus related costs (including merchandise holding and decay costs, variable production costs, and setup, shortage, and transaction costs) and is described as:

$$\pi_{\mathrm{N,hr,TM},z_{k-1}}(p_{\mathrm{N}},\chi_{\mathrm{N}}) = \int_{z_{k-1}}^{\chi_{\mathrm{N}}} (p_{\mathrm{N}} - C_{\mathrm{N,hr,TM,1}}(t)) D_{\mathrm{N}}(p_{\mathrm{N}},t)dt + \int_{\chi_{\mathrm{N}}}^{1} (p_{\mathrm{N}} - C_{\mathrm{N,hr,TM,3}}(t)) D_{\mathrm{N}}(p_{\mathrm{N}},t)dt$$

$$+ \int_{1}^{z_k} (p_{\mathrm{N}} - C_{\mathrm{N,hr,TM,3}}(t)) D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)dt - \Phi \text{ , for } z_{k-1} = 0 \text{ and } \chi_{\mathrm{N}} < 1; \tag{14a1}$$

$$= \int_{z_{k-1}}^{1} (p_{\mathrm{N}} - C_{\mathrm{N,hr,TM,1}}(t)) D_{\mathrm{N}}(p_{\mathrm{N}},t)dt + \int_{1}^{\chi_{\mathrm{N}}} (p_{\mathrm{N}} - C_{\mathrm{N,hr,TM,2}}(t)) D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)dt$$

$$+\int_{\chi_{\rm N}}^{z_k}(p_{\rm N}-C_{\rm N,hr,TM,3}(t))D_{\rm N}(p_{\rm N},p_{\rm R},t)dt-\Phi \text{, for } z_{k-1}=0 \text{ and } \chi_{\rm N}>1; \tag{14a2}$$

$$=\int_{z_{k-1}}^{\chi_{\rm N}}(p_{\rm N}-C_{\rm N,hr,TM,1}(t))D_{\rm N}(p_{\rm N},p_{\rm R},t)dt+\int_{\chi_N}^{z_k}(p_{\rm N}-C_{\rm N,hr,TM,3}(t))D_{\rm N}(p_{\rm N},p_{\rm R},t)dt$$
$$-\Phi \text{, for } z_{k-1}=1, 2,\ldots, H\text{-}1 \text{ and } z_{k-1}<\chi_{\rm N}<z_k, \tag{14b}$$

where

$$C_{\rm N,hr,TM,1}(t)=(c_{\rm N}+\varphi)e^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)}+h\int_{z_{k-1}}^{t}e^{\int_u^t\theta(\tau(v))d\tau(v)}du, \tag{14c}$$

$$C_{\rm N,hr,TM,2}(t)=(c_{\rm N}+\varphi)e^{\int_1^t\theta(\tau(u))d\tau(u)}+h\int_1^t e^{\int_u^t\theta(\tau(v))d\tau(v)}du \text{, and} \tag{14d}$$

$$C_{\rm N,hr,TM,3}(t)=c_{\rm N}+\varphi+s(z_k-t). \tag{14e}$$

The profit under the hybrid system with a remanufacturing TM scenario for the remanufactured merchandise over the length of time [z_{k-1}, z_k] is selling revenue minus related costs (including merchandise holding and decay costs, variable production costs, and setup, shortage, and transaction costs) according to the following equation:

$$\pi_{{\rm R,hr,TM},z_{k-1}}(p_{\rm R},\chi_{\rm R})=\int_1^{\chi_{\rm R}}(p_{\rm R}-C_{\rm R,hr,TM,2}(t))D_{\rm R}(p_{\rm R},p_{\rm N},t)dt+\int_{\chi_{\rm R}}^{z_k}(p_{\rm R}-C_{\rm R,hr,TM,3}(t))D_{\rm R}(p_{\rm R},p_{\rm N},t)dt$$
$$-\Phi \text{, for } z_{k-1}=0 \text{ and } \chi_{\rm R}>1; \tag{15a}$$

$$=\int_{z_{k-1}}^{\chi_{\rm R}}(p_{\rm R}-C_{\rm R,hr,TM,1}(t))D_{\rm R}(p_{\rm R},p_{\rm N},t)dt+\int_{\chi_{\rm R}}^{z_k}(p_{\rm R}-C_{\rm R,hr,TM,3}(t))D_{\rm R}(p_{\rm R},p_{\rm N},t)dt$$
$$-\Phi \text{, for } z_{k-1}=1, 2,\ldots, H\text{-}1 \text{ and } z_{k-1}<\chi_{\rm R}<z_k, \tag{15b}$$

where

$$C_{\rm R,hr,TM,1}(t)=(c_{\rm R}+\varphi)e^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)}+h\int_{z_{k-1}}^{t}e^{\int_u^t\theta(\tau(v))d\tau(v)}du, \tag{15c}$$

$$C_{\rm R,hr,TM,2}(t)=(c_{\rm R}+\varphi)e^{\int_1^t\theta(\tau(u))d\tau(u)}+h\int_1^t e^{\int_u^t\theta(\tau(v))d\tau(v)}du \text{, and} \tag{15d}$$

$$C_{\rm R,hr,TM,3}(t)=c_{\rm R}+\varphi+s(z_k-t). \tag{15e}$$

Based on the above definitions, the profit generated via the new and remanufactured merchandise over the length of time [z_{k-1}, z_k] for the hybrid remanufacturing TM scenario can be calculated as:

$$\max\Phi_{{\rm hr,TM},z_{k-1}}(p_{\rm N},p_{\rm R},\chi_{\rm N},\chi_{\rm R})=\pi_{{\rm N,hr,TM},z_{k-1}}(p_{\rm N},\chi_{\rm N})+\pi_{{\rm R,hr,TM},z_{k-1}}(p_{\rm R},\chi_{\rm R}), \tag{16a}$$

subject to

$$Q_{{\rm R,hr,TM},[1,z_k]}(p_{\rm R},p_{\rm N},t)\le\xi*D_{{\rm N,hr,TM},[z_{k-1},1]}(p_{\rm N},p_{\rm R},t) \text{, for } z_{k-1}=0; \tag{16b}$$

$$Q_{{\rm R,hr,TM},[z_{k-1},z_k]}(p_{\rm R},p_{\rm N},t)\le\xi*(D_{{\rm N,hr,TM},[(2z_{k-1}-z_k),z_{k-1}]}(p_{\rm N},p_{\rm R},t)+D_{{\rm R,hr,TM},[(2z_{k-1}-z_k),z_{k-1}]}(p_{\rm R},p_{\rm N},t)),$$
$$\text{for } z_{k-1}=1, 2,\ldots, H\text{-}1 \text{ and if } (2z_{k-1}-z_k)<0 \text{ then } (2z_{k-1}-z_k)=0. \tag{16c}$$

The optimal selling prices p_N and p_R, and the depletion time points χ_N and χ_R for the hybrid remanufacturing TM scenario over the length of time [z_{k-1}, z_k] can be calculated by resolving the first order differential equation of Eq. (16a) with respect to p_N, p_R, χ_N, and χ_R separately, as follows:

$$\frac{\partial}{\partial p_{\mathrm{N}}}\Phi_{\mathrm{hr,TM},z_{k-1}}(p_{\mathrm{N}},p_{\mathrm{R}},\chi_{\mathrm{N}},\chi_{\mathrm{R}}) = \int_{z_{k-1}}^{\chi_{\mathrm{N}}}(D_{\mathrm{N}}(p_{\mathrm{N}},t)+(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},1}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{N}}(p_{\mathrm{N}},t))dt$$
$$+\int_{\chi_{\mathrm{N}}}^{1}(D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)+(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},3}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{N}}(p_{\mathrm{N}},t))dt$$
$$+\int_{1}^{z_k}(D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)+(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},3}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt=0,$$

for $z_{k-1}=0$ and $\chi_{\mathrm{N}}<1$; (17a1)

$$=\int_{z_{k-1}}^{1}(D_{\mathrm{N}}(p_{\mathrm{N}},t)+(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},1}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{N}}(p_{\mathrm{N}},t))dt$$
$$+\int_{1}^{\chi_{\mathrm{N}}}(D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)+(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},2}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt$$
$$+\int_{\chi_{\mathrm{N}}}^{z_k}(D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)+(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},3}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt$$
$$+\int_{1}^{\chi_{\mathrm{R}}}((p_{\mathrm{R}}-C_{\mathrm{R,hr,TM},2}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t))dt$$
$$+\int_{\chi_{\mathrm{R}}}^{z_k}((p_{\mathrm{R}}-C_{\mathrm{R,hr,TM},3}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t))dt=0,$$

for $z_{k-1}=0$, $\chi_{\mathrm{N}}>1$ and $\chi_{\mathrm{R}}>1$; (17a2)

$$=\int_{z_{k-1}}^{\chi_{\mathrm{N}}}(D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)+(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},1}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt$$
$$+\int_{\chi_{\mathrm{N}}}^{z_k}(D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)+(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},3}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt$$
$$+\int_{z_{k-1}}^{\chi_{\mathrm{R}}}((p_{\mathrm{R}}-C_{\mathrm{R,hr,TM},1}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t))dt$$
$$+\int_{\chi_{\mathrm{R}}}^{z_k}((p_{\mathrm{R}}-C_{\mathrm{R,hr,TM},3}(t))\frac{\partial}{\partial p_{\mathrm{N}}}D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t))dt=0,$$

for $z_{k-1}=1,2,\ldots,H\text{-}1$, $z_{k-1}<\chi_{\mathrm{N}}<z_k$ and $z_{k-1}<\chi_{\mathrm{R}}<z_k$, (17b)

$$\frac{\partial}{\partial p_{\mathrm{R}}}\Phi_{\mathrm{hr,TM},z_{k-1}}(p_{\mathrm{N}},p_{\mathrm{R}},\chi_{\mathrm{N}},\chi_{\mathrm{R}}) = \int_{1}^{z_k}(p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},3}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)dt=0,$$

for $z_{k-1}=0$, $\chi_{\mathrm{N}}<1$ and $\chi_{\mathrm{R}}>1$; (18a1)

$$=\int_{1}^{\chi_{\mathrm{N}}}((p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},2}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt$$
$$+\int_{\chi_{\mathrm{N}}}^{z_k}((p_{\mathrm{N}}-C_{\mathrm{N,hr,TM},3}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt$$
$$+\int_{1}^{\chi_{\mathrm{R}}}(D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t)+(p_{\mathrm{R}}-C_{\mathrm{R,hr,TM},2}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t))dt$$

$$+\int_{\chi_{\mathrm{R}}}^{z_k}(D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t)+(p_{\mathrm{R}}-C_{\mathrm{R,hr,TM,3}}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t))dt=0,$$

for $z_{k-1}=0$, $\chi_{\mathrm{N}}>1$ and $\chi_{\mathrm{R}}>1$; (18a2)

$$=\int_{z_{k-1}}^{\chi_{\mathrm{N}}}((p_{\mathrm{N}}-C_{\mathrm{N,hr,TM,1}}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt$$

$$+\int_{\chi_{\mathrm{N}}}^{z_k}((p_{\mathrm{N}}-C_{\mathrm{N,hr,TM,3}}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t))dt$$

$$+\int_{z_{k-1}}^{\chi_{\mathrm{R}}}(D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t)+(p_{\mathrm{R}}-C_{\mathrm{R,hr,TM,1}}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t))dt$$

$$+\int_{\chi_{\mathrm{R}}}^{z_k}(D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t)+(p_{\mathrm{R}}-C_{\mathrm{R,hr,TM,3}}(t))\frac{\partial}{\partial p_{\mathrm{R}}}D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t))dt=0,$$

for $z_{k-1}=1,2,\ldots,H\text{-}1$, $z_{k-1}<\chi_{\mathrm{N}}<z_k$ and $z_{k-1}<\chi_{\mathrm{R}}<z_k$, (18b)

and

$$\frac{\partial}{\partial\chi_{\mathrm{N}}}\Phi_{\mathrm{hr,TM},z_{k-1}}(p_{\mathrm{N}},p_{\mathrm{R}},\chi_{\mathrm{N}},\chi_{\mathrm{R}})=(C_{\mathrm{N,hr,TM,3}}(\chi_{\mathrm{N}})-C_{\mathrm{N,hr,TM,1}}(\chi_{\mathrm{N}}))D_{\mathrm{N}}(p_{\mathrm{N}},\chi_{\mathrm{N}})=0,$$

for $z_{k-1}=0$ and $\chi_{\mathrm{N}}<1$; (19a1)

$$=(C_{\mathrm{N,hr,TM,3}}(\chi_{\mathrm{N}})-C_{\mathrm{N,hr,TM,2}}(\chi_{\mathrm{N}}))D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},\chi_{\mathrm{N}})=0,$$

for $z_{k-1}=0$ and $\chi_{\mathrm{N}}>1$; (19a2)

$$=(C_{\mathrm{N,hr,TM,3}}(\chi_{\mathrm{N}})-C_{\mathrm{N,hr,TM,1}}(\chi_{\mathrm{N}}))D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},\chi_{\mathrm{N}})=0,$$

for $z_{k-1}=1,2,\ldots,H\text{-}1$ and $z_{k-1}<\chi_{\mathrm{N}}<z_k$. (19b)

From Eq. (19) and assumption (iv): $D_{\mathrm{N}}(p_{\mathrm{N}},t)>0$ and $D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)>0$, we have

$$C_{\mathrm{N,hr,TM,3}}(\chi_{\mathrm{N}})-C_{\mathrm{N,hr,TM,2}}(\chi_{\mathrm{N}})=0,$$

for $z_{k-1}=0$ and $\chi_{\mathrm{N}}>1$; (19c)

$$C_{\mathrm{N,hr,TM,3}}(\chi_{\mathrm{N}})-C_{\mathrm{N,hr,TM,1}}(\chi_{\mathrm{N}})=0,$$

for $z_{k-1}=0$ and $\chi_{\mathrm{N}}<1$, and for $z_{k-1}=1,2,\ldots,H\text{-}1$ and $z_{k-1}<\chi_{\mathrm{N}}<z_k$. (19d)

In the same manner, we have

$$C_{\mathrm{R,hr,TM,3}}(\chi_{\mathrm{R}})-C_{\mathrm{R,hr,TM,2}}(\chi_{\mathrm{R}})=0,$$

for $z_{k-1}=0$ and $\chi_{\mathrm{R}}>1$; (20a)

$$C_{\mathrm{R,hr,TM,3}}(\chi_{\mathrm{R}})-C_{\mathrm{R,hr,TM,1}}(\chi_{\mathrm{R}})=0,$$

for $z_{k-1}=1,2,\ldots,H\text{-}1$ and $z_{k-1}<\chi_{\mathrm{R}}<z_k$. (20b)

In view of the fact that the profit function $\max\Phi_{\mathrm{hr,TM},z_{k-1}}(p_{\mathrm{N}},p_{\mathrm{R}},\chi_{\mathrm{N}},\chi_{\mathrm{R}})$ given in Eq. (16a) under the hybrid remanufacturing TM scenario is the high-power expression and the sufficient condition depends to a large extent on the parameter values, the closed-form solution to the profit maximization cannot be analytically verified (Wang et al., 2004). Instead, numerical experiments and examples are used to show the model's characteristic for the proposed scenarios. The optimal track of selling price and depletion point for the total profit generated via the new and remanufactured merchandise for the hybrid remanufacturing TM scenario over a planning time horizon can be decided in a similar dynamic manner by applying the DP method:

$$\Pi^{**}_{\mathrm{hr,TM},z_k}=\max_{z_{k-1}}\left\{\Pi^{*}_{\mathrm{hr,TM},z_{k-1}}+\Phi^{*}_{\mathrm{hr,TM},z_{k-1}}(p_{\mathrm{N}},p_{\mathrm{R}},\chi_{\mathrm{N}},\chi_{\mathrm{R}}):0\le z_{k-1}<z_k\le H\right\}, \tag{21}$$

with boundary condition $\Pi_{\mathrm{hr,TM},0}=0$.

3.2.3 Hybrid remanufacturing EM scenario (policy 4)

Under the hybrid system with remanufacturing in the EM setting, the profit over the length of time [z_{k-1}, z_k] for the new merchandise is the selling revenue minus the related costs, including variable production costs, costs due to merchandise holding and decay, shortage costs, transaction fees, and setup costs, as follows:

$$\pi_{\mathrm{N,hr,EM},z_{k-1}}(p_{\mathrm{N}},\chi_{\mathrm{N}}) = \int_{z_{k-1}}^{\chi_{\mathrm{N}}}(p_{\mathrm{N}}-C_{\mathrm{N,hr,EM,1}}(t))D_{\mathrm{N}}(p_{\mathrm{N}},t)dt+\int_{\chi_{\mathrm{N}}}^{1}(p_{\mathrm{N}}-C_{\mathrm{N,hr,EM,3}}(t))D_{\mathrm{N}}(p_{\mathrm{N}},t)dt$$

$$+\int_{1}^{z_k}(p_{\mathrm{N}}-C_{\mathrm{N,hr,EM,3}}(t))D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)dt-\Phi\text{ , for } z_{k-1}=0 \text{ and } \chi_{\mathrm{N}}<1; \tag{22a1}$$

$$=\int_{z_{k-1}}^{1}(p_{\mathrm{N}}-C_{\mathrm{N,hr,EM,1}}(t))D_{\mathrm{N}}(p_{\mathrm{N}},t)dt+\int_{1}^{\chi_{\mathrm{N}}}(p_{\mathrm{N}}-C_{\mathrm{N,hr,EM,2}}(t))D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)dt$$

$$+\int_{\chi_{\mathrm{N}}}^{z_k}(p_{\mathrm{N}}-C_{\mathrm{N,hr,EM,3}}(t))D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)dt-\Phi\text{ , for } z_{k-1}=0 \text{ and } \chi_{\mathrm{N}}>1; \tag{22a2}$$

$$=\int_{z_{k-1}}^{\chi_{\mathrm{N}}}(p_{\mathrm{N}}-C_{\mathrm{N,hr,EM,1}}(t))D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)dt+\int_{\chi_{N}}^{z_k}(p_{\mathrm{N}}-C_{\mathrm{N,hr,EM,3}}(t))D_{\mathrm{N}}(p_{\mathrm{N}},p_{\mathrm{R}},t)dt$$

$$-\Phi\text{ , for } z_{k-1}=1, 2,\ldots, H\text{-}1 \text{ and } z_{k-1}<\chi_{\mathrm{N}}<z_k, \tag{22b}$$

where

$$C_{\mathrm{N,hr,EM,1}}(t)=\rho p_{\mathrm{N}}+c_{\mathrm{N}}e^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)}+h\int_{z_{k-1}}^{t}e^{\int_{u}^{t}\theta(\tau(v))d\tau(v)}du, \tag{22c}$$

$$C_{\mathrm{N,hr,EM,2}}(t)=\rho p_{\mathrm{N}}+c_{\mathrm{N}}e^{\int_{1}^{t}\theta(\tau(u))d\tau(u)}+h\int_{1}^{t}e^{\int_{u}^{t}\theta(\tau(v))d\tau(v)}du\text{, and} \tag{22d}$$

$$C_{\mathrm{N,hr,EM,3}}(t)=\rho p_{\mathrm{N}}+c_{\mathrm{N}}+s(z_k-t). \tag{22e}$$

Under the hybrid remanufacturing EM scenario, the profit over the length of time [z_{k-1}, z_k] for the remanufactured merchandise is the selling revenue minus the related costs, as follows:

$$\pi_{\mathrm{R,hr,EM},z_{k-1}}(p_{\mathrm{R}},\chi_{\mathrm{R}}) = \int_{1}^{\chi_{\mathrm{R}}}(p_{\mathrm{R}}-C_{\mathrm{R,hr,EM,2}}(t))D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t)dt+\int_{\chi_{\mathrm{R}}}^{z_k}(p_{\mathrm{R}}-C_{\mathrm{R,hr,EM,3}}(t))D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t)dt$$

$$-\Phi\text{ , for } z_{k-1}=0 \text{ and } \chi_{\mathrm{R}}>1; \tag{23a}$$

$$=\int_{z_{k-1}}^{\chi_{\mathrm{R}}}(p_{\mathrm{R}}-C_{\mathrm{R,hr,EM,1}}(t))D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t)dt+\int_{\chi_{\mathrm{R}}}^{z_k}(p_{\mathrm{R}}-C_{\mathrm{R,hr,EM,3}}(t))D_{\mathrm{R}}(p_{\mathrm{R}},p_{\mathrm{N}},t)dt$$

$$-\Phi\text{ , for } z_{k-1}=1, 2,\ldots, H\text{-}1 \text{ and } z_{k-1}<\chi_{\mathrm{R}}<z_k, \tag{23b}$$

where

$$C_{\mathrm{R,hr,EM,1}}(t)=\rho p_{\mathrm{R}}+c_{\mathrm{R}}e^{\int_{z_{k-1}}^{t}\theta(\tau(u))d\tau(u)}+h\int_{z_{k-1}}^{t}e^{\int_{u}^{t}\theta(\tau(v))d\tau(v)}du, \tag{23c}$$

$$C_{\mathrm{R,hr,EM,1}}(t)=\rho p_{\mathrm{R}}+c_{\mathrm{R}}e^{\int_{1}^{t}\theta(\tau(u))d\tau(u)}+h\int_{1}^{t}e^{\int_{u}^{t}\theta(\tau(v))d\tau(v)}du\text{, and} \tag{23d}$$

$$C_{\mathrm{R,hr,EM,2}}(t)=\rho p_{\mathrm{R}}+c_{\mathrm{R}}+s(z_k-t), \tag{23e}$$

The profit generated via the new and remanufactured merchandise for the hybrid remanufacturing EM scenario over the length of time [z_{k-1}, z_k] can be calculated as follows:

$$\max\Phi_{\mathrm{hr,EM},z_{k-1}}(p_{\mathrm{N}},p_{\mathrm{R}},\chi_{\mathrm{N}},\chi_{\mathrm{R}})=\pi_{\mathrm{N,hr,EM},z_{k-1}}(p_{\mathrm{N}},\chi_{\mathrm{N}})+\pi_{\mathrm{R,hr,EM},z_{k-1}}(p_{\mathrm{R}},\chi_{\mathrm{R}}), \tag{24a}$$

subject to

$$Q_{\mathrm{R,hr,EM},[1,z_k]}(p_{\mathrm{R}}, p_{\mathrm{N}}, t) \le \xi * D_{\mathrm{N,hr,EM},[z_{k-1},1]}(p_{\mathrm{N}}, p_{\mathrm{R}}, t), \text{ for } z_{k-1} = 0; \tag{24b}$$

$$Q_{\mathrm{R,hr,EM},[z_{k-1},z_k]}(p_{\mathrm{R}}, p_{\mathrm{N}}, t) \le \xi * (D_{\mathrm{N,hr,EM},[(2z_{k-1}-z_k),z_{k-1}]}(p_{\mathrm{N}}, p_{\mathrm{R}}, t) + D_{\mathrm{R,hr,EM},[(2z_{k-1}-z_k),z_{k-1}]}(p_{\mathrm{R}}, p_{\mathrm{N}}, t)),$$
$$\text{for } z_{k-1} = 1, 2, \ldots, H\text{-}1 \text{ and if } (2z_{k-1} - z_k) < 0 \text{ then } (2z_{k-1} - z_k) = 0. \tag{24c}$$

Similarly, the optimal prices p_N and p_R, and depletion time points χ_N and χ_R over the length of time [z_{k-1}, z_k] for the hybrid remanufacturing EM scenario can be calculated by solving the first order differential equation of Eq. (24a) with respect to p_N, p_R, χ_N and χ_R separately as follows:

$$\begin{aligned}\frac{\partial}{\partial p_{\mathrm{N}}}\Phi_{\mathrm{hr,EM},z_{k-1}}(p_{\mathrm{N}}, p_{\mathrm{R}}, \chi_{\mathrm{N}}, \chi_{\mathrm{R}}) = &\int_{z_{k-1}}^{\chi_{\mathrm{N}}} ((1-\rho) D_{\mathrm{N}}(p_{\mathrm{N}}, t) + (p_{\mathrm{N}} - C_{\mathrm{N,hr,EM},1}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{N}}(p_{\mathrm{N}}, t)) dt \\ &+ \int_{\chi_{\mathrm{N}}}^{1} ((1-\rho) D_{\mathrm{N}}(p_{\mathrm{N}}, t) + (p_{\mathrm{N}} - C_{\mathrm{N,hr,EM},3}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{N}}(p_{\mathrm{N}}, t)) dt \\ &+ \int_{1}^{z_k} ((1-\rho) D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t) + (p_{\mathrm{N}} - C_{\mathrm{N,hr,EM},3}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t)) dt = 0, \\ &\text{for } z_{k-1} = 0 \text{ and } \chi_{\mathrm{N}} < 1;\end{aligned} \tag{25a1}$$

$$\begin{aligned}= &\int_{z_{k-1}}^{1} ((1-\rho) D_{\mathrm{N}}(p_{\mathrm{N}}, t) + (p_{\mathrm{N}} - C_{\mathrm{N,hr,EM},1}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{N}}(p_{\mathrm{N}}, t)) dt \\ &+ \int_{1}^{\chi_{\mathrm{N}}} ((1-\rho) D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t) + (p_{\mathrm{N}} - C_{\mathrm{N,hr,N,EM},2}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t)) dt \\ &+ \int_{\chi_{\mathrm{N}}}^{z_k} ((1-\rho) D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t) + (p_{\mathrm{N}} - C_{\mathrm{N,hr,EM},3}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t)) dt \\ &+ \int_{1}^{\chi_{\mathrm{R}}} ((p_{\mathrm{R}} - C_{\mathrm{R,hr,EM},2}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{R}}(p_{\mathrm{R}}, p_{\mathrm{N}}, t)) dt \\ &+ \int_{\chi_{\mathrm{R}}}^{z_k} ((p_{\mathrm{R}} - C_{\mathrm{R,hr,EM},3}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{R}}(p_{\mathrm{R}}, p_{\mathrm{N}}, t)) dt = 0, \\ &\text{for } z_{k-1} = 0,\ \chi_{\mathrm{N}} > 1 \text{ and } \chi_{\mathrm{R}} > 1;\end{aligned} \tag{25a2}$$

$$\begin{aligned}= &\int_{z_{k-1}}^{\chi_{\mathrm{N}}} ((1-\rho) D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t) + (p_{\mathrm{N}} - C_{\mathrm{N,hr,EM},1}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t)) dt \\ &+ \int_{\chi_{\mathrm{N}}}^{z_k} ((1-\rho) D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t) + (p_{\mathrm{N}} - C_{\mathrm{N,hr,EM},3}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t)) dt \\ &+ \int_{z_{k-1}}^{\chi_{\mathrm{R}}} ((p_{\mathrm{R}} - C_{\mathrm{R,hr,EM},1}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{R}}(p_{\mathrm{R}}, p_{\mathrm{N}}, t)) dt \\ &+ \int_{\chi_{\mathrm{R}}}^{z_k} ((p_{\mathrm{R}} - C_{\mathrm{R,hr,R,EM},3}(t)) \frac{\partial}{\partial p_{\mathrm{N}}} D_{\mathrm{R}}(p_{\mathrm{R}}, p_{\mathrm{N}}, t)) dt = 0, \\ &\text{for } z_{k-1} = 1, 2, \ldots, H\text{-}1,\ z_{k-1} < \chi_{\mathrm{N}} < z_k \text{ and } z_{k-1} < \chi_{\mathrm{R}} < z_k,\end{aligned} \tag{25b}$$

$$\begin{aligned}\frac{\partial}{\partial p_{\mathrm{R}}}\Phi_{\mathrm{hr,EM},z_{k-1}}(p_{\mathrm{N}}, p_{\mathrm{R}}, \chi_{\mathrm{N}}, \chi_{\mathrm{R}}) = &\int_{1}^{z_k} (p_{\mathrm{N}} - C_{\mathrm{N,hr,TM},3}(t)) \frac{\partial}{\partial p_{\mathrm{R}}} D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t) dt = 0, \\ &\text{for } z_{k-1} = 0,\ \chi_{\mathrm{N}} < 1 \text{ and } \chi_{\mathrm{R}} > 1;\end{aligned} \tag{26a1}$$

$$= \int_{1}^{\chi_{\rm N}} ((p_{\rm N} - C_{\rm N,hr,TM,2}(t))\frac{\partial}{\partial p_{\rm R}} D_{\rm N}(p_{\rm N}, p_{\rm R}, t))dt$$

$$+ \int_{\chi_{\rm N}}^{z_k} ((p_{\rm N} - C_{\rm N,hr,TM,3}(t))\frac{\partial}{\partial p_{\rm R}} D_{\rm N}(p_{\rm N}, p_{\rm R}, t))dt$$

$$+ \int_{1}^{\chi_{\rm R}} ((1-\rho)D_{\rm R}(p_{\rm R}, p_{\rm N}, t) + (p_{\rm R} - C_{\rm R,hr,TM,2}(t))\frac{\partial}{\partial p_{\rm R}} D_{\rm R}(p_{\rm R}, p_{\rm N}, t))dt$$

$$+ \int_{\chi_{\rm R}}^{z_k} ((1-\rho)D_{\rm R}(p_{\rm R}, p_{\rm N}, t) + (p_{\rm R} - C_{\rm R,hr,TM,3}(t))\frac{\partial}{\partial p_{\rm R}} D_{\rm R}(p_{\rm R}, p_{\rm N}, t))dt = 0,$$

for $z_{k-1} = 0$, $\chi_{\rm N} > 1$ and $\chi_{\rm R} > 1$; (26a2)

$$= \int_{z_{k-1}}^{\chi_{\rm N}} ((p_{\rm N} - C_{\rm N,hr,TM,1}(t))\frac{\partial}{\partial p_{\rm R}} D_{\rm N}(p_{\rm N}, p_{\rm R}, t))dt$$

$$+ \int_{\chi_{\rm N}}^{z_k} ((p_{\rm N} - C_{\rm N,hr,TM,3}(t))\frac{\partial}{\partial p_{\rm R}} D_{\rm N}(p_{\rm N}, p_{\rm R}, t))dt$$

$$+ \int_{z_{k-1}}^{\chi_{\rm R}} ((1-\rho)D_{\rm R}(p_{\rm R}, p_{\rm N}, t) + (p_{\rm R} - C_{\rm R,hr,TM,1}(t))\frac{\partial}{\partial p_{\rm R}} D_{\rm R}(p_{\rm R}, p_{\rm N}, t))dt$$

$$+ \int_{\chi_{\rm R}}^{z_k} ((1-\rho)D_{\rm R}(p_{\rm R}, p_{\rm N}, t) + (p_{\rm R} - C_{\rm R,hr,TM,3}(t))\frac{\partial}{\partial p_{\rm R}} D_{\rm R}(p_{\rm R}, p_{\rm N}, t))dt = 0,$$

for $z_{k-1} = 1, 2, \ldots, H\text{-}1$, $z_{k-1} < \chi_{\rm N} < z_k$ and $z_{k-1} < \chi_{\rm R} < z_k$, (26b)

and

$$\frac{\partial}{\partial \chi_{\rm N}} \Phi_{{\rm hr,EM},z_{k-1}}(p_{\rm N}, p_{\rm R}, \chi_{\rm N}, \chi_{\rm R}) = (C_{\rm N,hr,EM,3}(\chi_{\rm N}) - C_{\rm N,hr,EM,1}(\chi_{\rm N}))D_{\rm N}(p_{\rm N}, \chi_{\rm N}) = 0,$$

for $z_{k-1} = 0$ and $\chi_{\rm N} < 1$; (27a1)

$$= (C_{\rm N,hr,EM,3}(\chi_{\rm N}) - C_{\rm N,hr,EM,2}(\chi_{\rm N}))D_{\rm N}(p_{\rm N}, p_{\rm R}, \chi_{\rm N}) = 0,$$

for $z_{k-1} = 0$ and $\chi_{\rm N} > 1$; (27a2)

$$= (C_{\rm N,hr,EM,3}(\chi_{\rm N}) - C_{\rm N,hr,EM,1}(\chi_{\rm N}))D_{\rm N}(p_{\rm N}, p_{\rm R}, \chi_{\rm N}) = 0,$$

for $z_{k-1} = 1, 2, \ldots, H\text{-}1$ and $z_{k-1} < \chi_{\rm N} < z_k$. (27b)

From Eq. (27) and assumption (iv): $D_{\rm N}(p_{\rm N}, t) > 0$ and $D_{\rm N}(p_{\rm N}, p_{\rm R}, t) > 0$, we have

$$(C_{\rm N,hr,EM,3}(\chi_{\rm N}) - C_{\rm N,hr,EM,2}(\chi_{\rm N})) = 0,$$

for $z_{k-1} = 0$ and $\chi_{\rm N} > 1$; (27c)

$$(C_{\rm N,hr,EM,3}(\chi_{\rm N}) - C_{\rm hr,N,EM,1}(\chi_{\rm N})) = 0,$$

for $z_{k-1} = 0$ and $\chi_{\rm N} < 1$, and for $z_{k-1} = 1, 2, \ldots, H\text{-}1$ and $z_{k-1} < \chi_{\rm N} < z_k$. (27d)

In the same way, we have

$$(C_{\rm R,hr,EM,3}(\chi_{\rm R}) - C_{\rm R,hr,EM,2}(\chi_{\rm R})) = 0,$$

for $z_{k-1} = 0$ and $\chi_{\rm R} > 1$; (28a)

$$(C_{\rm R,hr,EM,3}(\chi_{\rm R}) - C_{\rm R,hr,EM,1}(\chi_{\rm R})) = 0,$$

for $z_{k-1} = 1, 2, \ldots, H\text{-}1$ and $z_{k-1} < \chi_{\rm R} < z_k$. (28b)

As discussed in policy 3 (the hybrid remanufacturing TM setting), the concave property of the profit function

$\Phi_{\mathrm{hr,EM},z_{k-1}}(p_{\mathrm{N}}, p_{\mathrm{R}}, \chi_{\mathrm{N}}, \chi_{\mathrm{R}})$ given in Eq. (24a) for the hybrid remanufacturing EM scenario cannot be analytically confirmed. As an alternative, a numerical analysis was conducted.

The optimal track of selling price and depletion point for the total profit generated via the new and remanufactured merchandise under the hybrid remanufacturing EM scenario over a planning time horizon can also be decided by applying the DP approach in a similar way:

$$\Pi^{**}_{\mathrm{hr,EM},z_k} = \max_{z_{k-1}}\left\{\Pi^{*}_{\mathrm{hr,EM},z_{k-1}} + \Phi^{*}_{\mathrm{hr,EM},z_{k-1}}(p_{\mathrm{N}}, p_{\mathrm{R}}, \chi_{\mathrm{N}}, \chi_{\mathrm{R}}) : 0 \le z_{k-1} < z_k \le H\right\}, \tag{29}$$

with boundary condition $\Pi_{\mathrm{hr,EM},0} = 0$.

3. Numerical study

The performance and properties of the DP models developed in section 3 were determined in numerical experiments. It was found that the DP models developed in section 3 are valid for all general decayed merchandise and demand functions, provided that conditions (i)-(vi) in section 2 are met. However, for simplicity, in the remainder of this study only a constant decay rate of $\theta(\tau(t)) = \alpha$ will be considered. Additionally, demand functions in dynamic scenarios are specified to be additive-deterministic, time-variant, price-dependent, and substitutable between both brand-new and like-new decayed merchandise. The demand function forms are: $D_{\mathrm{mo}}(p,t) = (M - p)e^{\lambda t}$ for the manufacturing-only system, in which M represents the total potential market; $D_{\mathrm{N}}(p_{\mathrm{N}},t) = (M - p_{\mathrm{N}})e^{\lambda t}$ with only brand-new decayed merchandise being offered over the first period, and $D_{\mathrm{N}}(p_{\mathrm{N}}, p_{\mathrm{R}}, t) = (M_{\mathrm{N}} - p_{\mathrm{N}} + \beta p_{\mathrm{R}})e^{\lambda t}$ and $D_{\mathrm{R}}(p_{\mathrm{R}}, p_{\mathrm{N}}, t) = (M_{\mathrm{R}} - p_{\mathrm{R}} + \beta p_{\mathrm{N}})e^{\lambda t}$ with brand-new and like-new decayed merchandise being produced for the hybrid remanufacturing system over subsequent periods, in which M_{N} (i.e., αM) and M_{R} (i.e., $(1-\alpha)M$), $0 \le \alpha \le 1$, denote the market sizes of brand-new and like-new decayed merchandise, respectively, and β denotes the demand cross-sensitivity coefficient, $0 \le \beta < 1$, employed by Ingene and Parry (1995). The demand functions satisfy the conditions (i)–(vi) in section 2 because $e^{\lambda t}$ is non-negative. In this research, a number of trials were developed to qualitatively understand the structures of the proposed scenarios and their sensitivity with respect to major parameters. The dynamic solution procedures were implemented using Windows 10 running Mathematica for the four scenarios.

4.1. A demonstrative example

During the experimentation, the base settings were specified as follows: demand parameters $M_{\mathrm{N}} = 75$, $M_{\mathrm{R}} = 50$, β = 0.3 and 0.6, and $\lambda = -0.2$; cost parameters $\Phi = 200$, $c = 40$, $h = 0.1$, $s = 0.25$, $\varphi = 5$, $\rho = 0.005$, δ = 0.3 and 0.6; decay rate $\theta = 0.25$; returned rate $\xi = 0.5$; and number of periods $H = 12$. Tables 1–4 list the numerical results generated by policies 1–4. Under a manufacturing-only system (Tables 1–2), the EM (policy 2) increased the total profit by 12.63% increments, compared to the TM (policy 1) (7577.9 *vs*. 6728.1). Under the hybrid remanufacturing system (Tables 3–4), the numerical results were similar. The performance improvements in the hybrid remanufacturing EM setting (policy 4) increased the total profit to 17.35%, 21.74%, 6.3%, and 6.1%, respectively, increments comparable to the hybrid remanufacturing TM setting (policy 3) when β = 0.3 and 0.6, and δ = 0.3 and 0.6 (i.e., 4803.3 vs. 4093.2 for β = 0.3 and δ = 0.3; 4403.2 vs. 3616.8 for β = 0.3 and δ = 0.6; 10615.1 vs. 9984.7 for β = 0.6 and δ = 0.3; 9985.3 vs. 9413.8 for β = 0.6 and δ = 0.6). On comparing the outcomes generated by the four policies (Tables 1–4), the hybrid model with remanufacturing under a general setting does not surpass the manufacturing-only model, but attains better performance under certain conditions with a larger substitute rate (i.e., intensity of competition) between both brand-new and like-new merchandise (β) and/or a smaller remanufacturing cost.

Table 1
Joint decisions and profits of the manufacturing-only TM setting (policy 1) under representative scenarios

$[z^{**}_{k-1}, z^{**}_k]$	p^{**}	χ^{**}	Q^{**}	π^{**}
[0, 5]	85.353	0.106	125.4	4768.0
[5, 12]	85.519	5.148	54.8	1960.1
$\sum$			180.2	6728.1

Table 2

Joint decisions and profits of the manufacturing-only EM setting (policy 2) under representative scenarios

$[z^{**}_{k-1}, z^{**}_{k}]$	p^{**}	χ^{**}	Q^{**}	π^{**}
[0,5]	82.954	0.119	133.0	5359.5
[5,12]	83.120	5.166	58.1	2218.4
$\sum$			191.1	7577.9

Table 3

Joint decisions and profits of the hybrid remanufacturing TM setting (policy 3) under representative scenarios

	δ=0.3										
	$[z^{**}_{N,k-1}, z^{**}_{N,k}]$	p^{**}_{N}	χ^{**}_{N}	Q^{**}_{N}	π^{**}_{N}	$[z^{**}_{R,k-1}, z^{**}_{R,k}]$	p^{**}_{R}	χ^{**}_{R}	Q^{**}_{R}	π^{**}_{R}	Π^{**}_{H}
β=0.3	[0,4]	76.132	0.085	77.1	2150.9	[1,4]	63.043	1.036	18.1	625.8	4093.2
	[4,7]	72.102	4.062	17.7	272.2	[4,7]	48.493	4.158	23.5	529.6	
	[7,12]	72.304	7.106	13.5	158.6	[7,12]	48.673	7.263	18.0	356.1	
	$\sum$			108.3	2581.7				59.6	1511.5	
β=0.6	[0,2]	91.143	0.042	54.5	2298.6	[1,2]	80.299	1.023.	18.1	943.2	9984.7
	[2,3]	104.106	2.021	12.7	547.4	[2,3]	83.272	2.045	17.7	973.2	
	[3,5]	104.531	3.042	18.2	877.5	[3,5]	82.711	3.106	27.2	1577.0	
	[5,6]	104.480	5.019	6.7	198.2	[5,6]	82.665	5.052	10.0	456.1	
	[6,7]	104.411	6.019	5.5	125.9	[6,7]	82.584	6.051	8.2	337.2	
	[7,11]	104.660	7.084	13.6	604.5	[7,11]	82.841	7.209	20.4	1128.4	
	[11,12]	104.605	11.017	2.0	-80.1	[11,12]	82.809	11.053	3.0	-2.4	
	$\sum$			113.2	4572				104.6	5412.7	

	δ=0.6										
	$[z^{**}_{N,k-1}, z^{**}_{N,k}]$	p^{**}_{N}	χ^{**}_{N}	Q^{**}_{N}	π^{**}_{N}	$[z^{**}_{R,k-1}, z^{**}_{R,k}]$	p^{**}_{R}	χ^{**}_{R}	Q^{**}_{R}	π^{**}_{R}	Π^{**}_{H}
β=0.3	[0,6]	75.646	0.127	94.2	2589.9	[1,6]	65.704	1.072	18.1	451.1	3616.8
	[6,12]	72.354	6.125	20.1	331.6	[6,12]	54.755	6.191	17.9	244.2	
				114.1	2921.5				36.0	695.3	
β=0.6	[0,3]	93.727	0.067	78.3	3582.4	[1,3]	92.839	1.022	18.1	949.6	9413.8
	[3,4]	104.468	3.020	11.8	500.4	[3,4]	88.661	3.031	11.9	511.3	
	[4,6]	104.661	4.043	17.5	840.4	[4,6]	88.846	4.065	17.7	857.1	
	[6,8]	104.610	6.040	11.8	497.4	[6,8]	88.794	6.062	11.9	508.6	
	[8,12]	104.677	8.084	13.2	577.5	[8,12]	88.885	8.130	13.3	589.1	
				132.6	5998.1				72.9	3415.7	

Table 4

Joint decisions and profits of the hybrid remanufacturing EM setting (policy 4) under representative scenarios.

	δ=0.3										
	$[z^{**}_{N,k-1}, z^{**}_{N,k}]$	p^{**}_{N}	χ^{**}_{N}	Q^{**}_{N}	π^{**}_{N}	$[z^{**}_{R,k-1}, z^{**}_{R,k}]$	p^{**}_{R}	χ^{**}_{R}	Q^{**}_{R}	π^{**}_{R}	Π^{**}_{H}
β=0.3	[0,3]	74.692	0.071	69.6	2153.9	[1,3]	58.193	1.030	19.2	675.6	4803.3
	[3,5]	69.681	3.048	17.3	302.8	[3,5]	45.988	3.147	22.6	555.4	
	[5,8]	69.705	5.070	15.9	259.0	[5,8]	46.018	5.215	20.7	490.3	
	[8,12]	69.792	8.093	10.6	105.9	[8,12]	46.120	8.288	13.8	260.4	
	$\sum$			113.4	2821.6				76.3	1981.7	
β=0.6	[0,2]	88.758	0.048	57.1	2543.9	[1,2]	77.414	1.022	19.2	1044.5	10615.1
	[2,3]	101.678	2.023	13.1	596.6	[2,3]	80.265	2.062	18.7	1065.3	
	[3,4]	102.111	3.023	10.5	442.7	[3,4]	80.214	3.073	15.5	845.6	
	[4,6]	102.253	4.048	15.5	754.7	[4,6]	80.363	4.147	23.0	1354.3	
	[6,7]	102.151	6.021	5.7	152.9	[6,7]	80.276	6.074	8.5	373.7	
	[7,9]	102.215	7.046	8.5	323.9	[7,9]	80.316	7.146	12.6	653.1	
	[9,11]	102.209	9.045	5.7	151.2	[9,11]	80.311	9.145	8.5	371.8	
	[11,12]	102.064	11.022	2.1	-69.7	[11,12]	80.300	11.076	3.1	10.6	
	$\sum$			118.2	4896.2				109.1	5718.9	

	δ=0.6										
	$[z^{**}_{N,k-1}, z^{**}_{N,k}]$	p^{**}_{N}	χ^{**}_{N}	Q^{**}_{N}	π^{**}_{N}	$[z^{**}_{R,k-1}, z^{**}_{R,k}]$	p^{**}_{R}	χ^{**}_{R}	Q^{**}_{R}	π^{**}_{R}	Π^{**}_{H}
β=0.3	[0,2]	77.110	0.048	53.9	1762.2	[1,2]	54.176	1.039	14.1	218.9	4403.2
	[2,6]	69.604	2.095	39.0	920.1	[2,6]	52.485	2.136	34.0	740.1	
	[6,12]	69.968	6.140	21.8	427.1	[6,12]	52.307	6.227	19.7	334.8	
				114.7	3109.4				67.8	1293.8	
β=0.6	[0,3]	91.468	0.071	81.6	3929.8	[1,3]	90.666	1.028	19.2	1065.3	9985.3
	[3,4]	102.075	3.023	12.3	553.9	[3,4]	86.242	3.037	12.4	567.2	
	[4,6]	102.150	4.046	18.2	919.6	[4,6]	86.294	4.076	18.5	940.6	
	[6,8]	102.149	6.045	12.2	550.5	[6,8]	86.294	6.075	12.4	564.5	
	[8,11]	102.280	8.072	11.2	487.5	[8,11]	86.461	8.114	11.4	499.2	
	[11,12]	102.209	11.019	2.5	-47.9	[11,12]	86.333	11.036	2.5	-44.9	
				138.0	6393.4				76.4	3591.9	

4.2. Sensitivity analysis

A sensitivity experiment was conducted to determine the percentage differences between profits in a hybrid remanufacturing EM setting (policy 4) and a manufacturing-only EM setting (policy 2) with respect to key factors: M_N, M_R, β, δ, c, ρ and ξ. The base parameter settings for the seven key factors were as follows: $M_N = 75$, $M_R = 50$, $\beta = 0.5$, $\delta = 0.5$, $c = 40$, $\rho = 0.005$, and $\xi = 0.5$. These parameters varied from their basic settings by up to $\pm 25\%$. Figure 1 demonstrates that policy 4 yields a larger pie and more profit than policy 2 under larger β, c and M_N, and smaller M_R and δ. Briefly, the hybrid remanufacturing strategy is preferable if there is intense competition for brand-new and like-new merchandise, the manufacturing inputs cost is large, the brand-new merchandise has a large market size, the like-new merchandise has a smaller market size, and/or the remanufacturing inputs cost is smaller.

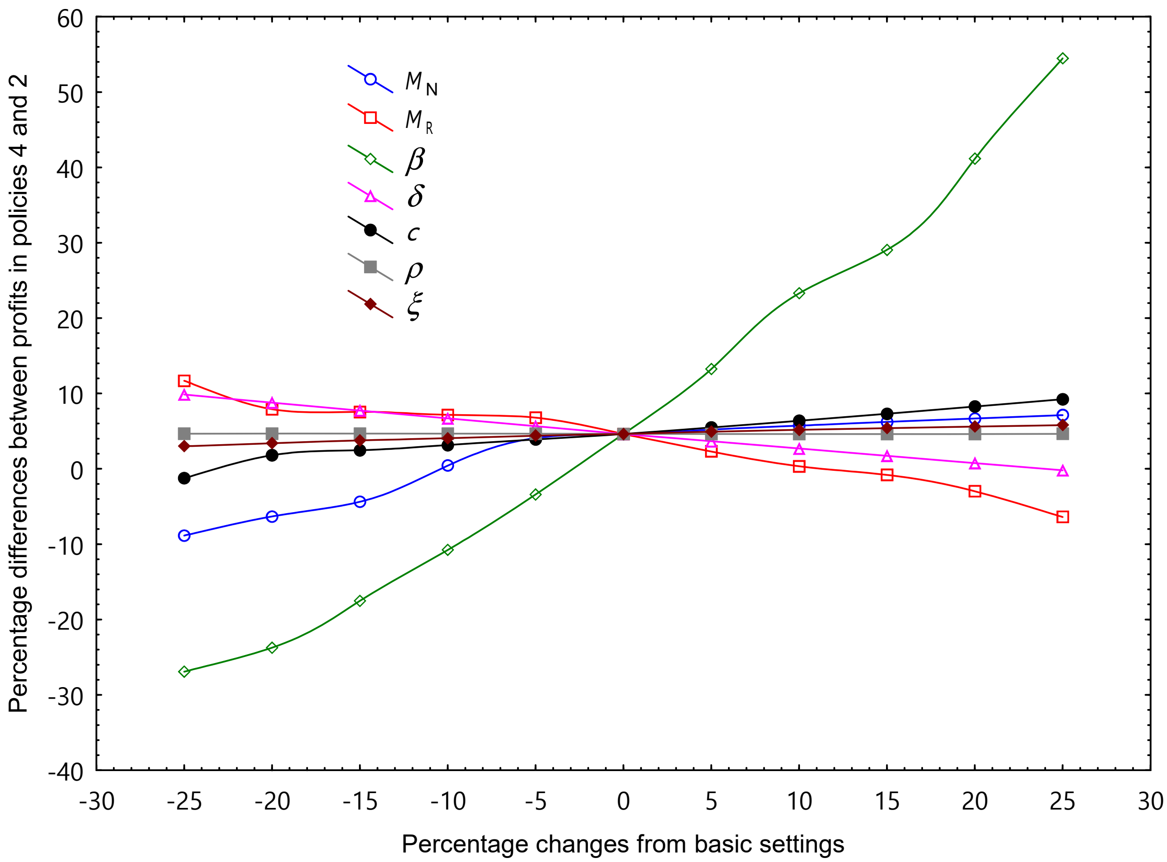

Fig. 1. Impact of key parameters on profit growth: policy 4 vs. policy 2

5. Concluding remarks

We summarize here the results of this study on the dynamic decision-making planning of a hybrid production system incorporating manufacturing and remanufacturing procedures for decayed merchandise in EMs and TMs, respectively. Furthermore, in this research we propose four decision-making models that represent the manufacturing-only and hybrid remanufacturing systems as multivariable profit maximization problems under dynamic schemes. The results indicate that solutions generated within EMs surpass those within TMs in terms of maximizing profits. Additionally, comparing solutions generated by the four policies, the hybrid model with remanufacturing under a general setting does not surpass the manufacturing-only model, but attains better performance under certain conditions with intense competition (i.e., substitute rate) between brand-new and like-new merchandise, a smaller remanufacturing cost, a larger brand-new merchandise market size, and/or a smaller like-new merchandise market size.

Although earlier studies have developed various application tactics and models to solve the optimization problems in the supply chain (i.e., Modak et al. 2018; özelkan et al. 2018; Wang et al. 2019), very little attention has been paid to the dynamic strategy of hybrid manufacturing/remanufacturing mechanisms in different trading markets. This research is the first attempt to propose a model to investigate the joint decision-making effect of dynamic pricing combined with shortages on profit maximization during the considered multi-period horizon for decayed merchandise. At the same time, this study also proposes several solutions by using calculus coupled with DP based on different pricing strategies for the dynamic manufacturing-only and hybrid manufacturing/remanufacturing systems with merchandise substitution in different trading markets (TMs or EMs). Furthermore, because shortages can be a powerful cost control for administering decayed merchandise, this research presents the profit-maximization issue by considering the manner of the shortage. In the future, the four decision models will be extended to consider closed-loop channel coordination and information asymmetry. Other future research directions can consider the development of more general demand functions, including uncertain and random demands.

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Appendix

Proof of Proposition 1

The second derivative of the function $\pi_{\text{mo,TM}z_{k-1}}(p,\chi)$ given in Eq. (4) with respect to p for the manufacturing-only system in the TM can be expressed as follows:

$$\frac{\partial^2}{\partial p^2}\pi_{\text{mo,TM},z_{k-1}}(p,\chi) = \int_{z_{i-1}}^{\chi}(2\frac{\partial}{\partial p}D_{\text{mo}}(p,t)+(p-C_{\text{mo,TM},1}(t))\frac{\partial}{\partial p^2}D_{\text{mo}}(p,t))dt$$
$$+\int_{\chi}^{z_i}(2\frac{\partial}{\partial p}D_{\text{mo}}(p,t)+(p-C_{\text{mo,TM},2}(t))\frac{\partial}{\partial p^2}D_{\text{mo}}(p,t))dt\,. \tag{A1a}$$

Substituting demand function $D_{\text{mo}}(p,t)=(M-p)e^{\lambda t}$ into Eq. (A1a) yields:

$$\frac{\partial^2}{\partial p^2}\pi_{\text{mo,TM},z_{k-1}}(p,\chi) = -2\int_{z_{k-1}}^{z_k} e^{\lambda t}dt$$
$$= -2(e^{\lambda z_k}-e^{\lambda z_{k-1}})/\lambda\,. \tag{A1b}$$

Since $(e^{\lambda z_k}-e^{\lambda z_{k-1}})/\lambda>0$ for $z_k>z_{k-1}$, regardless of the value of λ, Eq. (A1b) is negative.

Proof of Proposition 2

The second derivative of the function $\pi_{\text{mo,TM}z_{k-1}}(p,\chi)$ given in Eq. (4) with respect to χ for the manufacturing-only system with a TM setting can be expressed as:

$$\frac{\partial^2}{\partial \chi^2}\pi_{\text{mo,TM},z_{k-1}}(p,\chi)$$
$$=\frac{\partial}{\partial \chi}D_{\text{mo}}(p,\chi)(C_{\text{mo,TM},2}(\chi)-C_{\text{mo,TM},1}(\chi))$$
$$-D_{\text{mo}}(p,\chi)(s+(c+\varphi)\theta(\tau(\chi))e^{\int_{z_{k-1}}^{\chi}\theta(\tau(t))d\tau(t)}+h(1+\theta(\tau(\chi)))\int_{z_{k-1}}^{\chi}e^{\int_t^{\chi}\theta(\tau(u))d\tau(u)}dt))\,. \tag{A2a}$$

From Eq. (6b), the Eq. (A2a) can be simplified into:

$$\frac{\partial^2}{\partial \chi^2}\pi_{\text{mo,TM},z_{k-1}}(p,\chi)$$
$$=-D_{\text{mo}}(p,\chi)(s+(c+\varphi)\theta(\tau(\chi))e^{\int_{z_{k-1}}^{\chi}\theta(\tau(t))d\tau(t)}+h(1+\theta(\tau(\chi)))\int_{z_{k-1}}^{\chi}e^{\int_t^{\chi}\theta(\tau(u))d\tau(u)}dt))\,. \tag{A2b}$$

Since $D_{\text{mo}}(p,\chi)>0$ for $p>0$, $\int_{z_{k-1}}^{\chi}e^{\int_t^{\chi}\theta(\tau(u))d\tau(u)}dt>0$ for $\chi>z_{k-1}$, and all parameters: s, c, φ, h, and $\theta(\tau(\chi))$ are non-negative, Eq. (A2b) is negative.

Proof of Proposition 3

To prove that the function $\pi_{\text{mo,TM}z_{k-1}}(p,\chi)$ given in Eq. (4) is jointly concave in p and χ for the manufacturing-only system with a TM setting, the determinant of the Hessian matrix needs to be examined further. The second derivative of the function $\pi_{\text{mo,TM}z_{k-1}}(p,\chi)$ given in Eq. (4) with respect to p and χ can be expressed as follows:

$$\frac{\partial^2}{\partial p \partial \chi}\pi_{\text{mo,TM},z_{k-1}}(p,\chi) = \frac{\partial^2}{\partial \chi \partial p}\pi_{\text{mo,TM},z_{k-1}}(p,\chi)$$
$$= (C_{\text{mo,TM},2}(\chi) - C_{\text{mo,TM},1}(\chi))\frac{\partial}{\partial p}D(p,\chi). \tag{A3a}$$

From Eq. (6b), we have

$$\frac{\partial^2}{\partial p \partial \chi}\pi_{\text{mo,TM},z_{k-1}}(p,\chi) = 0. \tag{A3b}$$

From propositions 1 and 2, it can be easily shown that the determinant of the Hessian matrix is: $((\frac{\partial^2}{\partial p^2}\pi_{\text{mo,TM},z_{k-1}}(p,\chi))(\frac{\partial^2}{\partial \chi^2}\pi_{\text{mo,TM},z_{k-1}}(p,\chi)) - (\frac{\partial^2}{\partial p \partial \chi}\pi_{\text{mo,TM},z_{k-1}}(p,\chi))^2) > 0$. Hence, the function $\pi_{\text{mo,TM},z_{k-1}}(p,\chi)$ is jointly concave in both p and χ.

Proof of Proposition 4

The second derivative of the function $\pi_{\text{mo,EM},z_{k-1}}(p,\chi)$ given in Eq. (10) with respect to p for the manufacturing-only system in the EM can be expressed as below:

$$\frac{\partial^2}{\partial p^2}\pi_{\text{mo,EM},z_{k-1}}(p,\chi) = \int_{z_{i-1}}^{\chi}((2-\rho)\frac{\partial}{\partial p}D_{\text{mo}}(p,t) + (p - C_{\text{mo,EM},1}(t))\frac{\partial}{\partial p^2}D_{\text{mo}}(p,t))dt$$
$$+\int_{\chi}^{z_i}((2-\rho)\frac{\partial}{\partial p}D_{\text{mo}}(p,t) + (p - C_{\text{mo,EM},2}(t))\frac{\partial}{\partial p^2}D_{\text{mo}}(p,t))dt. \tag{A4a}$$

Substituting demand function $D_{\text{mo}}(p,t) = (M-p)e^{\lambda t}$ into Eq. (A4a) yields

$$\frac{\partial^2}{\partial p^2}\pi_{\text{mo,TM},z_{k-1}}(p,\chi) = -(2-\rho)\int_{z_{k-1}}^{z_k} e^{\lambda t}dt$$
$$= -(2-\rho)(e^{\lambda z_k} - e^{\lambda z_{k-1}})/\lambda. \tag{A4b}$$

Since $0 \le \rho \le 1$, and $(e^{\lambda z_k} - e^{\lambda z_{k-1}})/\lambda > 0$ for $z_k > z_{k-1}$, regardless of the value of λ, Eq. (A4b) is negative.

Proof of Proposition 5

The second derivative of the function $\pi_{\text{mo,EM},z_{k-1}}(p,\chi)$ given in Eq. (10) with respect to χ for the manufacturing-only system with an EM setting can be expressed as:

$$\frac{\partial^2}{\partial \chi^2}\pi_{\text{mo,EM},z_{k-1}}(p,\chi)$$
$$= \frac{\partial}{\partial \chi}D_{\text{mo}}(p,\chi)(C_{\text{mo,EM},2}(\chi) - C_{\text{mo,EM},1}(\chi))$$
$$-D_{\text{mo}}(p,\chi)(s + c\theta(\tau(\chi))e^{\int_{z_{k-1}}^{\chi}\theta(\tau(t))d\tau(t)} + h(1+\theta(\tau(\chi)))\int_{z_{k-1}}^{\chi} e^{\int_t^{\chi}\theta(\tau(u))d\tau(u)}dt)). \tag{A5a}$$

From Eq. (12b), the Eq. (A5a) can be simplified into:

$$\frac{\partial^2}{\partial \chi^2}\pi_{\text{mo,EM},z_{k-1}}(p,\chi)$$
$$= -D_{\text{mo}}(p,\chi)(s + c\theta(\tau(\chi))e^{\int_{z_{k-1}}^{\chi}\theta(\tau(t))d\tau(t)} + h(1+\theta(\tau(\chi)))\int_{z_{k-1}}^{\chi} e^{\int_t^{\chi}\theta(\tau(u))d\tau(u)}dt)). \tag{A5b}$$

Since $D_{\text{mo}}(p,\chi) > 0$ for $p > 0$, $\int_{z_{k-1}}^{\chi} e^{\int_t^{\chi}\theta(\tau(u))d\tau(u)}dt > 0$ for $\chi > z_{k-1}$, and all parameters *s*, *c*, *h,* and $\theta(\tau(\chi))$ are non-negative, Eq. (A5b) is negative.

Proof of Proposition 6

Proposition 6 can be proved in a similar way to Proposition 3.