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A matheuristic based solution approach for the general lot sizing and scheduling problem with sequence dependent changeovers and back ordering

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A B S T R A C T

This paper considers the general lot sizing and scheduling problem (GLSP) in single level capacitated environments with sequence dependent item changeovers. The proposed model simultaneously determines the production sequence of multiple items with capacity-constrained dynamic demand and lot size to minimize overall costs. First, a mixed-integer programming (MIP) model for the GLSP is developed in order to solve smaller size problems. Afterwards, a matheuristic algorithm that integrates Simulated Annealing (SA) algorithm and the proposed MIP model is devised for solving larger size problems. The proposed matheuristic approach decomposes the GLSP into sub-problems. The proposed SA algorithm plays the controller role. It guides the search process by determining values for some of the decision variables and calls the MIP model to identify the optimal values for the remaining decision variables at each iteration. Extensive numerical experiments on randomly generated test instances are performed in order to evaluate the performance of the proposed matheuristic method. It is observed that the proposed matheuristic based solution method outperforms the MIP and SA, if they are used alone for solving the present GLSP.

1. Introduction

The simultaneous lot sizing and scheduling (SLS) problem is an essential aspect of production planning. Production planning, among other things, defines the number of production lots created when inputs are transformed into final products. Production system of many companies comprises one of the stages of this problem. The main objective is to satisfy the best trade-off between the customer request and financial objectives (Pochet, 2001). Lot sizing and scheduling literature has several versions of the problem containing different models, mathematical formulations and solution approaches. Especially in the process industry, time-consuming changeovers become important when utilizing the existing capacity. The integration of lot sizing and sequencing decisions has received considerable attention in the related literature as it considerably improves usage of high-cost production resources by efficiently controlling final inventories and managing available capacity (Maravelias & Sung, 2009). General Lot Sizing and Scheduling problem (GLSP) is known for its computational complexity. Fleischmann and Meyr (1997) have shown that the GLSP with a non-zero minimum lot size is a challenging combinatorial optimization problem. Therefore, the lot sizing literature tends to create solution approaches that are computationally efficient, such as matheuristics, metaheuristics, and greedy heuristics to cope with the combinatorial complexity of the problem. Metaheuristics have received significant attention in lot sizing and scheduling problems among these techniques. Various metaheuristics techniques have been used to solve the GLSP effectively. Recently, exact methods and metaheuristic algorithms have been combined in order to form a new class of algorithms, which are known as matheuristics (Santos et al., 2010). Capability of identifying the optimum balance between the efficacy of exact methods and the efficiency of metaheuristics is the most

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significant benefit of matheuristics (Guimarães et al., 2013; Guimarães et al., 2014). These algorithms have several advantages over classical heuristics, such as needing fewer parameters to tune and requiring only minor revisions in response to changes in the mathematical formulation. The following studies are some of the good examples that present the advantages of combining metaheuristics with mathematical programming methodologies for handling lot sizing or lot sizing and scheduling problems.

Figueira et al. (2013) proposed an efficient solution method that integrates variable neighborhood search (VNS) algorithm and an exact method for solving lot-sizing problems in pulp and paper industry. Toledo et al. (2014) successfully solved a specific two-level lot sizing and scheduling problem by integrating Genetic Algorithms (GA) with mathematical programming techniques in the soft drink industry. Furlan et al. (2015) solved the lot sizing and scheduling problem of pulp and paper mills by applying a hybrid approach based on a customized GA with an embedded residual linear programming model. Goerler et al. (2020) studied the extended GLSP problem with rework and lifetime constraints by developing a late acceptance hill-climbing based matheuristic. Lee and Lee (2022) proposed a new optimization model with sequence dependent setups for the GLSP problem. They proposed a set of valid inequalities to compare the tightness of the linear programming relaxations of their model. Carvalho and Nascimento (2022) solved the lot sizing and scheduling problem (LSP) for parallel machines with non-triangular sequence dependent setups and setup carryover. They proposed a hybrid matheuristic approach that integrates local search heuristics and mathematical programming.

In line with the aforementioned studies, the following contributions are sought for the current study: (*i*) a matheuristic approach is proposed for solving the GLSP better than MIP and SA, (*ii*) micro period analysis on the randomly generated test instances is presented, (*iii*) new valid inequalities are proposed for the GLSP.

The rest of this paper is organized as follows. In Section two, related literature is reviewed. Section three presents the mathematical formulation of the GLSP. Section four describes the proposed metaheuristic and matheuristic approaches. Section five presents the computational experiments and the obtained results. Finally, Section six concludes the paper and presents advices for further work.

2. Literature review

There are two types of model formulation for lot sizing and scheduling problems with big and small time buckets. Big bucket models allow for multiple setup operations in a single period, whereas small bucket models only allow one setup for each period. Big bucket models are better for problems with longer periods, whereas small bucket models are better for problems with shorter periods. The split of macro periods of big bucket problems into micro periods for dealing with small bucket problems considerably increases the mathematical complexity (Almada-Lobo et al., 2007). Small bucket model formulations are less common than big bucket versions (Copil et al., 2017). Fleischmann and Meyr (1997) developed a model for a single line based on GLSP. Meyr (2004) expanded GLSP to multiple production stages (GLSPMS). Koçlar and Süral (2005) investigated lot sizing and planning reformulations in GLSP. Multi-level GLSP with multiple machines (MLGLSP_MM) was proposed by Fandel and Stammen-Hegene (2006). Their model's objective is to minimize the total costs of sequence-dependent setup, production, inventory and maintaining the setup conditions of the machines. They proposed a non-linear programming model. Transchel et al. (2011) developed a hybrid mixed-binary optimization model for the GLSP. This model was customized to meet the two-stage production structure of a company that operates in the process industry. Ferreira et al. (2012) presented several single-step formulations for solving synchronized two-stage lot size and scheduling problem in the soft drink production. Most of the medium and large size lot sizing and scheduling problems are very challenging to solve. Addition of realistic constraints into models complicates the situation further. For instance, when dealing with the limited capacity, meeting all customer requests on time may be impossible. Backlogging is critical in this condition, especially in highly capacitated environments (Babaei et al., 2014). There is a rich literature that discusses all of these issues along with possible formulations and more. Readers can refer to Drexl (1997), Copil et al. (2017), Karimi et al. (2003), Jans and Degraeve (2008), Díaz-Madroñero et al. (2014) for a comprehensive review of the previously developed models. On the other hand, the need for effective solution approaches for the devised models is still in a very high demand.

3. Problem statement and mathematical formulation

In the GLSP products j=1,...,J are to be scheduled in the planning period T that consists of discrete macro periods t=1,…,T with a given length. In each macro period, t is represented by a set S_t of non-overlapping micro periods r where the number $|S_t|$ of micro periods must be predetermined. In a single micro period, only one product can be produced. The micro period lengths are decision variables determined by the quantity x_{jr} produced in each micro period r multiplied by the production coefficient p_j. The following assumptions are made in the GLSP model formulation: Each item's demand is deterministic; A shortage is referred as backlogging; Backlog and inventory amounts are calculated at the end of the planning horizon; The triangle inequality is valid in terms of setup cost and setup time, i.e. $s_{ik} \leq s_{ij} + s_{jk}$ and $bs_{ik} \leq bs_{ij} + bs_{jk}$ for all products i, j, and k.

3.1. Notation

Table 1 presents the symbols of the indices, parameters, and variables, which is used to formulate the GLSP model.

Table 1
Indices, parameters and variables that are used in the GLSP model.

Indices:
J= Number of products;
T= Number of macro-periods;
R= Number of micro-periods;
Sets:
S_t= Set of micro-periods r in each macro-period t;
Parameters:
d_{jt}= Demand of product j in macro-period t;
h_j= Inventory holding cost of product j;
g_j= Backorder cost of product j;
p_j= Production time to produce one unit of product j;
k_j= Minimum lot size amount of product j;
C_t= Available capacity (time) in macro-period t;
s_{ij}= Sequence dependent setup cost of production changeover from product i *to* j;
bs_{ij}= Setup time of production changeover from product i *to* j;
I_{jo}^{+}= Initial inventory amount of product j;
I_{jo}^{-}= Initial backorder amount of product j;
$y_{j0}=\begin{cases}1, \text{if there is an initial setup of product } j\\ 0, \text{otherwise}\end{cases}$
Variables:
I_{jt}^{+}= Inventory of product j in macro-period t;
I_{jt}^{-}= Backorder amount of product j in macro-period t;
x_{jr}= Production amount of product j in micro-period r;
$z_{ijr}=\begin{cases}1, \text{if production changeover occurs from product } i \text{ to } j \text{ in micro} - \text{period } r\\ 0, \text{otherwise}\end{cases}$
$y_{jr}=\begin{cases}1, \text{if machine is setup for production of product } j \text{ in micro} - \text{period } r\\ 0, \text{otherwise}\end{cases}$

$$\min \ Z=\sum_{j=1}^{J}\sum_{t=1}^{T}(h_j I_{jt}^{+}+g_j I_{jt}^{-})+\sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{\substack{j=1\\ j\neq i}}^{J}(s_{ij}z_{ijr}) \tag{1}$$

subject to

$$I_{j(t-1)}^{+}+I_{jt}^{-}+\sum_{r=1}^{S_t}x_{jr}-I_{jt}^{+}-I_{j(t-1)}^{-}=d_{jt} \qquad \forall j, t \tag{2}$$

$$\sum_{j=1}^{J}\sum_{r=1}^{S_t}p_j x_{jr}+\sum_{r=1}^{S_t}\sum_{i=1}^{I}\sum_{\substack{j=1\\ j\neq i}}^{J}(bs_{ij}z_{ijr}) \le c_t \qquad \forall t \tag{3}$$

$$x_{jr}\le \frac{c_t}{p_j}y_{jr} \qquad \forall j, t, r\epsilon S_t \tag{4}$$

$$\sum_{j=1}^{J}y_{jr}=1 \qquad \forall r \tag{5}$$

$$z_{ijr}\ge y_{i(r-1)}+y_{jr}-1 \qquad \forall r, j, i, \ j\neq i \tag{6}$$

$$x_{jr}\ge k_j(y_{jr}-y_{j(r-1)}) \qquad \forall r, j \tag{7}$$

$$I_{jt}^{+}, I_{jt}^{-}\ge 0 \ \ \forall j, t; \ \ x_{jr}\ge 0 \ \ \forall j, r; \ z_{ijr}, y_{jr} \ \in\{0, 1\} \qquad \forall i, j, r \tag{8}$$

The objective function (1) aims to minimize the sum of the cost of inventory, backorder, and setup of production changeover in periods. The inventory balance constraints are described in (2). Constraints (3) satisfy the capacity limits of each period. Constraints (4) ensure that the production of product j only occurs in case the setup of production is organized. According to constraints (5) the setup must be prepared for a single product for each micro period. Since both $y_{i(r-1)}$ and y_{jr} are binary, the objective function (1) and the constraints (6) limit the variable z_{ijr} to have value 1 if there is a switch from the product i to j. Constraints (7) satisfy the minimum lot sizes that prevent the incorrect evaluation of setup times. Constraints (8) specify the range of the variables. It is worth noting that zero-length micro periods (idle periods) can occur. The GLSP formulation (1-8) is only useful in real situations where the setup state is preserved. In addition, the GLSP model fulfills the setup carryover automatically.

3.2. Valid inequalities

In this section, valid inequalities (cuts) that strengthen the GLSP model is presented. Wolsey (2002) presented a categorization system for several small bucket and big bucket models. The lower bounds given by the LP-relaxation of small bucket models yield poor lower bounds. Lower limit can only be improved by using customized reformulations and adding valid inequalities to the problem formulation. On the other hand, most of the big bucket models give significantly better lower limits. These valid inequalities are not necessary for the model formulation but make it tighter. Belvaux and Wolsey (2001), Wolsey (2002), Pochet and Wolsey (2006), and Kaczmarczyk (2020) extended various valid inequalities for different lot sizing problems. The following constraints (9) are valid for satisfying the inventory and backorder constraint for all micro-periods *r* in macro-period *t*.

$$I_{j(t-1)}^{+}+I_{jt}^{-} \geq d_{jt}\ (1-\sum_{r=1}^{S_t} y_{jr}) \qquad \forall j, t \tag{9}$$

Proof.

Let $(\hat{x}, \hat{y})$ be a feasible solution for (2)-(8). Consider the case in which $\hat{y}_{jr}$=0 for every $j,\ r\epsilon S_t$. This implies that $\hat{x}_{jr}$=0 for every $j,\ r\epsilon S_t$, and thus constraint (4) ensure that $I_{j(t-1)}^{+}+I_{jt}^{-}-I_{jt}^{+}-I_{j(t-1)}^{-}=d_{jt}$ for every *j, t.* If the obtained equation $I_{j(t-1)}^{+}+I_{jt}^{-}=I_{jt}^{+}+I_{j(t-1)}^{-}+d_{jt}$ is replaced in equation (9), it is obtained that the non-negativity of the summation of inventory and backorder $I_{jt}^{+}+I_{j(t-1)}^{-} \geq 0$. Now assume that the $\hat{y}_{jr}$=1 for at least one of the $j,\ r\epsilon S_t$. Then the following equation is satisfied $I_{j(t-1)}^{+}+I_{jt}^{-}+(k-1)*d_{jt} \geq 0$ for every *j, t.* Where k shows the total number of decision variables that satisfy the $\hat{y}_{jr}$=1. Therefore, the inequality is valid.

4. The proposed solution approaches

4.1. Simulated annealing algorithm

Kirkpatrick et al. (1983) developed the stochastic hill climbing-based single-solution metaheuristic method that is known as the Simulated Annealing (SA) algorithm. The SA algorithm starts with a randomly generated initial solution, generates a neighbor solution based on the specified neighborhood structure at each iteration, and evaluates it by using the fitness function. Better moves are always acceptable. On the other hand, accepting a worse neighbor is possible with a probability, P=$e^{-\frac{\theta}{T}}$, where θ is the difference between the current and neighbor solution's fitness, *T* is the temperature (the main parameter of SA) that is iteratively reduced in relation to a predefined cooling schedule. The SA algorithm continues until the termination conditions are reached. In the following sub-sections, solution representation, initial solution generation process, neighborhood generation procedure, the fitness (cost) function of the proposed SA algorithm is presented. The total violation minimization approach (Baykasoglu and Akpinar, 2015) is employed for handling infeasibilities related to problem constraints.

4.1.1. Solution representation

In this study, Ponnambalam and Reddy (2003)'s solution representation scheme is used for both lot sizing and scheduling decisions. A solution contains paired values for each macro-period in this representation. The first number in a pair indicates the product type, while the second shows the product's lot size. The pair order specifies the order in which the corresponding lots need to be manufactured. An example is presented in Fig. 1, where four products need to be assigned into four micro periods and four macro periods. 10 units of product 1 is produced in the first micro period, 27 units of product 2 is produced in the second micro period, 33 units of product 3 is produced in the third micro period etc.

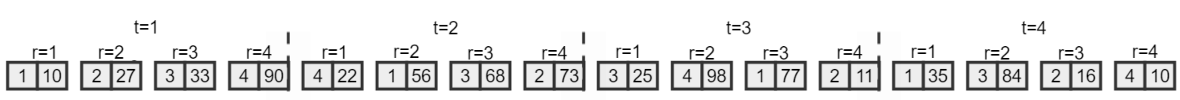

Fig. 1. An example for SA solution representation

4.1.2. Initialization

The initial solution is randomly generated. Lot size for every product in the initial solution is determined by using the following formula: "*lower bound for demand" +random(0,1)* × *("upper bound for demand" –"lower bound for demand").*

4.1.3. Neighbor solution generation

The current solution is tried to be improved through the neighbor generation process. Based on the solution structure as depicted in Fig 1., three neighborhood operators, namely arithmetic and zero operators (Ramezanian & Saidi-Mehrabad, 2013) are used for altering the lot sizes and swap operator for varying the sequence at each SA iteration.

Arithmetic operator: This operator decreases the quantity of a randomly selected product by the amount Δ and then adds it to another randomly selected product at each period. The value of Δ is set to *random(0,1)* × *max*(d_{jt}).

Zero operator: This operator reduces the lot size of a randomly selected product to zero and then adds its lot size to the lot size of another randomly selected product at each period.

Swap operator: Randomly selected products from randomly selected micro periods within macro periods are swapped.

4.1.4. Cooling schedule

One of main parameters of the SA algorithm is temperature, *T,* which steadily decreases as the algorithm progresses. *T* is initially set to a high level and reduced by using a cooling schedule. In this study, *T* is reduced with the following equation, $T_i = q * T_{i-1}$ (where q is the cooling factor that is usually set to a number between 0.9 and 1).

4.1.5. Termination condition

The termination criterion is defined as ($T > T_f$). As long as the termination criterion is not satisfied, the SA algorithm repeats its steps.

The pseudocode of the proposed SA algorithm is depicted in Fig. 2. The following abbreviations are used in the pseudocode:

$F_{(n)(o)(c)}$: Objective function value of the (neighbor) (initial) (current) solution
$X_{(n)(o)(c)}$: Solution vector of (neighbor) (initial) (current)
X_{best}: Solution vector with the minimum fitness value
F_{best}: Minimum fitness value

4.2. The proposed SA based matheuristic approach for solving GLSP

The GLSP is NP-Complete (Fleischmann & Meyr, 1997). Medium and large-scale problem instances cannot be solved by using commercial solvers within a reasonable time. Therefore, a SA-based matheuristic approach is proposed for the present GLSP. Actually, the proposed SA-based matheuristic approach uses the general framework of the SA algorithm, which calls a MIP model, performs fitness assessments and operates the swap/inversion operators to generate neighbor solutions. The simplified pseudocode of the proposed matheuristic approach is depicted in Fig. 3.

Initialization: Specify SA parameters
Build an initial solution and current solution
$X_{best} := X_{(o)}$; $F_{best} := F_{(o)}$; $X_{(c)} := X_{(o)}$; $F_{(c)} := F_{(o)}$;
$T := T_0$;
While ($T > T_f$)
    I:=0;
    While ($I < MaxI_{iter}$)
    Generate the neighboring solutions by selecting a random number between (1,3) and determine the neighboring solution $X_{(n)}$ with the objective function value $F_{(n)}$.
    If 1, apply the Arithmetic operator
    If 2, apply the Zero operator
    If 3, apply the Swap operator
    $\Delta := F_{(n)} - F_{(c)}$;
    If ($\Delta < 0$)
        $X_{(c)} := X_{(n)}$;
        $F_{(c)} := F_{(n)}$;
    Else

r_1:=random(0,1);
If (r_1 < exp(-Δ/T));
$X_{(c)}$:= $X_{(n)}$;
$F_{(c)}$:= $F_{(n)}$;
End
End
If ($F_{(c)}$< F_{best} and $X_{(c)}$ is feasible)
X_{best}:= $X_{(c)}$; F_{best}:= $F_{(c)}$;
End
I:= I+1;
End
T:= qT;
End While
Report X_{best}; F_{best}

Fig. 2. Pseudocode of the SA algorithm

The matheuristic algorithm begins by initializing solution, $X_{(o)}$, afterwards it computes the decision variables, x_{jr}, I_{jt}^{+} and I_{jt}^{-} for the corresponding solution $X_{(o)}$ via the MIP model. Then the result is used to calculate the fitness value, $F_{(o)}$. Subsequently, the matheuristic algorithm employs the inversion and swap operators to create $X_{(n)}$ from $X_{(o)}$. On the other hand, the SA process is started to produce $X_{(c)}$ from $X_{(n)}$. If a better solution is found, i.e., $F_{(c)} < F_{best}$, then X_{best} and F_{best} are updated. The matheuristic algorithm stops when it reaches to $MaxI_{iter}$.

Initialization: Specify *SA* parameters
Initialize $X_{(o)}$ →**Section 4.2.2**
Compute the lot sizing and inventory decisions for $X_{(o)}$ by LP model →**Section 4.2.4**
Evaluate $F_{(o)}$
X_{best}:= $X_{(o)}$; F_{best}:= $F_{(o)}$;
T:= T_0;
While ($T > T_f$)
I:=0;
While ($I < MaxI_{iter}$)
Generate neighboring solutions by selecting a random number between (1,2) and determine the neighboring solution $X_{(n)}$ with the objective function value $F_{(n)}$. →**Section 4.2.3**
If 1, apply Inversion
If 2, apply Swap
Δ:=$F_{(n)} - F_{(c)}$;
If (Δ < 0)
$X_{(c)}$:= $X_{(n)}$;
$F_{(c)}$:= $F_{(n)}$;
Else
r_1:=random(0,1);
If (r_1 < exp(-Δ/T));
$X_{(c)}$:= $X_{(n)}$;
$F_{(c)}$:= $F_{(n)}$;
End
End
If ($F_{(c)}$< F_{best} and $X_{(c)}$ is feasible)
X_{best}:= $X_{(c)}$; F_{best}:= $F_{(c)}$;
End
I:= I+1;
End
T:= qT ;
End While
Report X_{best}; F_{best}

Fig. 3. Pseudocode of the proposed SA-based matheuristic algorithm

4.2.1. Solution encoding

Since decisions to be made in the present problem entail sequencing of processes, it is logical to employ a permutation based encoding in the proposed matheuristic algorithm. In this encoding (solution representation), products are sequenced in each macro period. An example is shown in Fig. 4, where four products are sequenced in four micro periods of four macro periods.

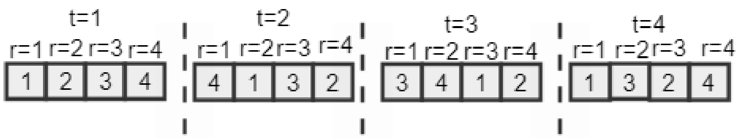

Fig. 4. An example of solution representation

4.2.2. Initialization

Solutions are randomly initialized.

4.2.3. Neighbor solution generation

Two operators are used to generate neighbor solutions, namely inversion and swap. Inversion operator *i*nverts randomly selected products from two randomly selected micro periods within macro periods. Swap operator swaps randomly selected products from randomly selected micro periods within macro periods.

4.2.4. Linear programming model

As aforementioned, the SA algorithm is responsible for determining y_{jr} and z_{ijr}, while the LP model is used to determine optimal values of decision variables related to lot sizing and inventory decisions. The LP model is given by Eqs. (10-13).

$$min\ \ Z=\sum_{j=1}^{J}\sum_{t=1}^{T}(h_j I_{jt}^{+} + g_j I_{jt}^{-})+\sum_{r=1}^{R}\sum_{i=1}^{I}\sum_{\substack{j=1\\ j\neq i}}^{J}(s_{ij}\overline{z_{ijr}}) \tag{10}$$

subject to

$$I_{j(t-1)}^{+}+I_{jt}^{-}+\sum_{r=1}^{S_t} x_{jr}-I_{jt}^{+}-I_{j(t-1)}^{-}=d_{jt} \qquad \forall j,\ t \tag{11}$$

$$\sum_{j=1}^{J}\sum_{r=1}^{S_t} p_j x_{jr}+\sum_{r=1}^{S_t}\sum_{i=1}^{I}\sum_{\substack{j=1\\ j\neq i}}^{J}(bs_{ij}\overline{z_{ijr}}) \le c_t \qquad \forall t \tag{12}$$

$$I_{jt}^{+}, I_{jt}^{-} \ge 0 \ \ \forall j, t;\ \ x_{jr} \ge 0 \qquad \forall j, r \tag{13}$$

The algorithm, which is shown in Fig. 5 is used for verification of model feasibility (capacity requirements, minimum lot size constraints and production changeovers) while solving the LP model.

forall *product j*, *micro period* rϵS_t and *macro period t*; **do**
    if y_{jr}=1 **then**
    add $x_{jr} \le \frac{c_t}{p_j}$
forall *product j* and *micro period* rϵS_t ; **do**
    if y_{jr}- $y_{j(r-1)}$=1 **then**
    add constraints $x_{jr} \ge k_j$;
forall *product i, j* and *micro period* rϵS_t where $i \neq j$; **do**
    if $y_{i(r-1)}$=1 and y_{jr}=1 **then**
      $z_{ijr} = 1$;
    Else
      $z_{ijr} = 0$;

Fig. 5. Algorithm for preserving feasibility in solving the LP model

4.3. Parameter tuning

In this section, it is aimed to determine good levels of parameters for the proposed SA and SA-based matheuristic algorithm. There are three parameters to be tuned and three levels are determined for each parameter as shown in Table 2. L9 is selected from the standard table of orthogonal arrays. For the preliminary trials, fifteen problems are used. The problems are constructed by selecting five random problems from each classes of A, B, C. As a result, (9x15) 135 runs are carried out in each analysis. All statistical analyses are performed with Minitab 20 software. "The Smaller, The Better" signal-to-noise ratio (S/N) is used within the Taguchi method in order to identify optimal parameter levels. The S/N ratio is computed via Eq. (14).

$$\frac{S}{N} = -10 * log\left(\frac{1}{n}\sum_{i=1}^{n} Y_i^2\right) \quad (14)$$

where n is the number of observations in each trial and Y_i is the objective function value. Since SA is a stochastic algorithm, the best values of 10 runs are selected in 10 experiments to obtain results that are more reliable. The optimal levels of the parameters are shown in Tables 2 and 3. The optimal levels of the parameters are determined as 20, 20000, 0.95 for SA and 30, 20, 0.95 for SA-based matheuristic.

Table 2
SA parameter levels.

Factors	*Levels*
Initial temperature (A)	20, 30, 50
Number of iterations at each temperature level (B)	5000, 10000, 20000
Cooling rate (C)	0.9, 0.95, 0.985

Table 3
SA-based matheuristic parameter levels.

Factors	*Levels*
Initial temperature (A)	20, 30, 50
Number of iterations at each temperature level (B)	10, 20, 30
Cooling rate (C)	0.9, 0.95, 0.985

5. Computational experiments

In the following sub-sections, performance analyses of the SA and the SA-based matheuristic algorithms are presented. Section 5.1 is devoted to randomly generated data instances. All computational tests are performed on the Windows 64-bit operating system and a Core(TM) i5-4590 CPU with 4 Gigabyte RAM and 3.30 Gigahertz speed. MATLAB R2020b is used for programming the SA algorithm. The SA-based matheuristic algorithm is programmed in Microsoft Visual Studio 2019 C++ by employing the callable library of ILOG CPLEX Optimization Studio Version 12.10 for solving the MIP model by the MIP solver of CPLEX.

5.1. Data instances

In order to analyze computational performance of the proposed algorithms, 150 randomly generated data instances within three data classes (A, B and C) are generated. Table 4 shows the parameter values for all data classes.

Table 4
Parameters for the data classes

Parameter	*Class A*	*Class B*	*Class C*
# of instances	50	50	50
# of products (j)	5	6	7
# of macro-periods (t)	4	3	2
# of micro-periods within t (| S_t |)	5,7,10	6,8,10	7,8,10
d_{jt}	[0;40,120]	[0;40,120]	[0;40,120]
s_{ij}	[100,400]	[100,400]	[100,400]
bs_{ij}	$s_{ij}/10$	$s_{ij}/10$	$s_{ij}/10$
p_j	1	1	1
k_j	10	10	10
h_j	[10,20]	[1,5]	[1,10]
g_j	[10,20]	[1,5]	[1,10]
C_t	$\sum_{j=1}^{J}\sum_{t=1}^{T} d_{jt} * 2$	$\sum_{j=1}^{J}\sum_{t=1}^{T} d_{jt} * 2$	$\sum_{j=1}^{J}\sum_{t=1}^{T} d_{jt} * 2$

5.2. Analysis of valid inequality

The valid inequalities further reduce the computational time in class A by 61,98%, in class B by 42,59%, and in class C by 68,81% compared to the one without valid inequalities. The detailed analysis in each class A, B, and C are shown in Figs. 6-

8 respectively. As a result, it can be concluded that the valid inequalities help to improve MIP formulation and reduce the computing effort required to resolve the large problem instances.

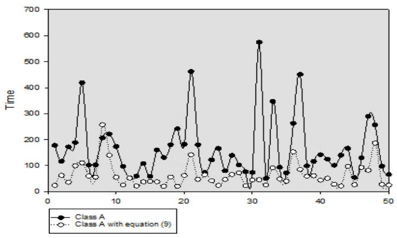

Fig. 6. Results of class A's validity analysis

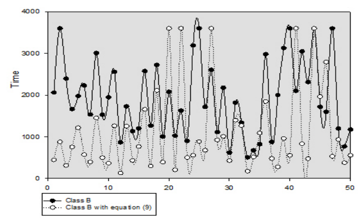

Fig. 7. Results of class B's validity analysis

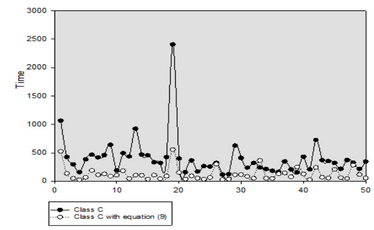

Fig. 8. Results of class C's validity analysis

5.3. Computational results for data instances

For testing the proposed algorithms, we set 3600s time limit for the GLSP model and the best solution obtained from the CPLEX 12.10 solver is used to compare the results. In Tables 5–7, all of the results for the three different data classes are shown. Tables 5-7 show the lower bounds (LBs) provided by CPLEX after running for 3600s. Hence, it is essential to provide a computable lower bound near to the optimal solution for evaluating the efficacy of the algorithms proposed in this research. In Table 5, we analyze Class A with micro periods equal to 5,7,10. In each case, the average CPLEX result is 3187.68. Overall, the average CPLEX time is equal to 165.27 s in micro period 5. For class A, instances with micro periods 7 and 10, CPLEX cannot identify optimum solutions for any case within the time limit of 3600s. In Class A, the proposed matheuristic algorithm can find the optimal solutions for 24 out of the 50 problems in micro period 5, for which GLSP provided the optimal results. For the micro periods 7 and 10 in Class A, the matheuristic algorithm can find the best solutions for 11 and 3 out of 50 problems, respectively, for which CPLEX is unable to identify optimum solutions for any case within the time limit of 3600s. For analyzing the performance of the SA algorithm, the average gap increases by 28.24%, 36.40%, and 38.02% compared to CPLEX, respectively, for micro periods 5,7,10. For analyzing the performance of the matheuristic the average gap increases through 0.56%, 1.42%, and 3.41% compared to CPLEX respectively for micro periods 5, 7, and 10. These results show that the average matheuristic performance outperforms the CPLEX and the SA algorithm in Class A. Furthermore, the average computation time of the SA algorithm is reduced by 91.45%, 99.60%, and 99.58% compared to CPLEX results, respectively, for micro periods 5, 7, and 10. The average computation time of the matheuristic algorithm reduces by 78.53%, 98.93%, and 98.79% compared to CPLEX results, respectively, for micro periods 5, 7, and 10. The SA and the matheuristic algorithm's performance decreases as the number of micro periods increases in Class A problems.

In Table 6, we analyze Class B problems with number of micro periods equal to 6, 8, 10. In Class B, the matheuristic algorithm can find 6 optimal solutions for micro period 6. Average CPLEX results are 2349.38, 2352.68, and 2359.76 for micro periods 6, 8, and 10. Average CPLEX time is equal to 1930.45s for micro periods 6. For the class B instances with micro periods 8 and 10, CPLEX cannot identify optimum solutions for any case within the time limit of 3600s. The average gap increases to 18.54%, 23.18%, and 25.77% compared to CPLEX for micro periods 6, 8, and 10 in the SA algorithm. The average gap increases by 3.26%, 3.78%, and 5.02% compared to CPLEX, for micro periods 6, 8, and 10 in the matheuristic algorithm. This results show that the average matheuristic performance outperforms CPLEX and SA for Class B problems. Furthermore, the average computation time of SA reduced by 99.41%, 99.68%, and 99.52% compared to CPLEX for micro periods 6, 8, and 10. Average computation time of the matheuristic algorithm reduces 98.01%, 98.80%, and 98.83% in comparison to CPLEX for micro period 6, 8, 10. Both the SA and the matheuristic algorithm's performance decreases as the number of micro periods increases in Class B problems. In Table 7, we analyze Class C problems with number of micro periods equal to 7, 8, 10. In Class C, the matheuristic algorithm can find 13 optimal solutions for micro period 7. Average CPLEX results are 2086.02 and average CPLEX time equals to 398.82s and 3511.80s for micro periods 7 and 8. CPLEX cannot identify optimum solutions for any case within the time limit of 3600s for the class C instances with micro period 10. The average CPLEX results are 2086.10 for the micro period 10. For analyzing the performance of SA, the average gap increases through 15.75%, 21.99% and 22.94% compared to CPLEX, respectively, for micro periods 7, 8, and 10. For analyzing the performance of the matheuristic, the average gap increases by 2.01%, 2.32% and 2.45% compared to CPLEX, respectively, for micro periods 7, 8, and 10. These results show that the average matheuristic performance outperforms CPLEX and the SA algorithm in Class C. Furthermore, the average computation time of the SA is reduced by 97.99%, 99.77%, and 99.77% compared to CPLEX results for micro periods 7, 8, and 10. The average computation time of the matheuristic algorithm is reduced by 90.45%, 98.92% and 98.75% compared to CPLEX results for micro periods 7, 8, 10. Both the SA and the matheuristic algorithm's performance decreases as the number of micro periods increases in Class C problems.

Table 5

Results of class A

	MIP/Micro Period=5				SA		Matheuristic		MIP/Micro Period=7				SA		Matheuristic		MIP/Micro Period=10				SA		Matheuristic	
Instance	Obj.	LB	Time	Gap	Obj.	Time	Obj.	Time	Obj.	LB	Time	Gap	Obj.	Time	Obj.	Time	Obj.	LB	Time	Gap	Obj.	Time	Obj.	Time
1	3320	3320	177,80	0,00	4104	14,37	**3320**	33,26	3320	1790,81	3600	46,06	4603	14,12	**3320**	38,85	3320	914,33	3600	72,46	4576	14,81	3520	42,76
2	3480	3480	116,59	0,00	4249	14,59	**3480**	34,41	3480	1712,51	3600	50,79	4491	14,12	3484	39,51	3480	731,50	3600	78,98	4287	14,70	3499	41,41
3	3172	3172	172,02	0,00	4213	14,72	**3172**	33,56	3172	1511,46	3600	52,35	4686	14,36	3184	39,86	3172	633,77	3600	80,02	4615	14,96	3220	41,27
4	3784	3784	188,83	0,00	4732	14,37	3804	36,98	3784	1784,91	3600	52,83	4685	14,38	3804	36,29	3784	1014,4	3600	73,19	4986	14,88	3892	43,65
5	3816	3816	418,67	0,00	4810	14,01	**3816**	34,70	3816	1669,88	3600	56,24	4928	14,20	3869	36,27	3816	667,04	3600	82,52	4818	14,78	3864	42,05
6	2996	2996	101,78	0,00	3999	14,19	**2996**	32,10	2996	1779,32	3600	40,61	4015	14,21	**2996**	35,74	2996	959,32	3600	67,98	4122	14,82	3180	44,96
7	3512	3512	102,81	0,00	4391	14,01	3528	35,34	3512	1757,05	3600	49,97	4529	14,29	3528	33,91	3512	1034,6	3600	70,54	4585	14,70	3609	40,65
8	3620	3620	206,55	0,00	4681	14,00	**3620**	35,92	3620	1445,83	3600	60,06	4621	14,48	**3620**	34,16	3620	924,19	3600	74,47	4841	14,65	3684	43,58
9	2988	2988	221,34	0,00	4020	14,27	3046	32,84	2988	1105,26	3600	63,01	3749	14,44	3030	34,24	2988	812,44	3600	72,81	4174	14,79	3155	41,40
10	3512	3512	173,41	0,00	4384	14,20	3520	32,27	3512	1326,48	3600	62,23	4922	14,15	3520	39,88	3512	919,44	3600	73,82	4913	14,36	3534	46,48
11	3292	3292	96,64	0,00	4138	14,06	3320	32,82	3292	1828,05	3600	44,47	4471	14,30	3393	36,20	3292	774,61	3600	76,47	4313	14,98	3328	41,28
12	2760	2760	52,56	0,00	3477	14,47	2795	35,69	2760	1259,66	3600	54,36	4101	14,10	2820	35,38	2760	832,14	3600	69,85	4122	14,87	2825	40,12
13	2884	2884	59,56	0,00	3804	13,87	**2884**	33,12	2884	1660,32	3600	42,43	4118	14,23	2985	36,22	2884	775,80	3600	73,10	4253	14,65	3009	41,30
14	2964	2964	107,97	0,00	4047	14,12	2988	36,24	2964	1482,00	3600	50,00	3936	14,32	3041	36,84	2964	765,60	3600	74,17	4084	14,70	3097	41,70
15	3088	3088	58,97	0,00	4078	14,04	3164	33,21	3088	1547,09	3600	49,90	4667	14,17	3164	39,13	3088	774,47	3600	74,92	4627	14,69	3194	43,12
16	3140	3140	160,45	0,00	4158	14,15	3161	46,21	3140	1313,46	3600	58,17	4104	14,38	3185	37,21	3140	659,71	3600	78,99	4232	14,85	3168	44,01
17	3460	3460	131,38	0,00	4365	14,12	**3460**	37,32	3460	2013,37	3600	41,81	4707	14,50	**3460**	36,61	3460	864,65	3600	75,01	4675	14,71	3644	47,25
18	3344	3344	180,00	0,00	4415	14,31	3384	37,57	3344	1737,88	3600	48,03	4551	14,02	3384	36,13	3344	783,83	3600	76,56	4484	14,90	**3344**	48,46
19	3232	3232	241,83	0,00	3819	14,03	**3232**	37,74	3232	1759,82	3600	45,55	4431	14,08	**3232**	36,31	3232	881,04	3600	72,74	4576	14,88	3349	42,48
20	3244	3244	182,00	0,00	3961	14,17	3251	36,62	3244	1554,20	3600	52,09	4410	14,12	3285	36,13	3244	781,16	3600	75,92	4405	14,60	3303	43,17
21	3348	3348	461,98	0,00	4147	14,16	**3348**	41,79	3348	1178,83	3600	64,79	4489	14,28	3357	37,92	3348	756,31	3600	77,41	4598	14,95	3435	46,92
22	3404	3404	180,11	0,00	4346	14,05	3432	36,50	3404	1576,73	3600	53,68	4696	14,32	3451	34,95	3404	781,90	3600	77,03	4684	15,02	3489	47,35
23	3016	3016	74,22	0,00	3846	13,99	3049	42,90	3016	1477,84	3600	51,00	4226	14,20	3046	34,48	3016	728,06	3600	75,86	4437	14,92	3148	44,60
24	3228	3228	122,00	0,00	4284	13,99	**3228**	37,75	3228	1469,06	3600	54,49	4118	14,15	3316	33,81	3228	500,66	3600	84,49	4139	14,88	3363	40,63
25	3364	3364	166,27	0,00	4240	14,06	3373	42,48	3364	1702,52	3600	49,39	4779	14,41	3382	48,81	3364	878,68	3600	73,88	4737	14,95	3555	44,55
26	2712	2712	80,38	0,00	3594	14,11	2748	36,64	2712	1460,41	3600	46,15	3898	14,38	2858	47,19	2712	749,60	3600	72,36	4068	14,93	2896	43,14
27	3124	3124	139,61	0,00	3739	14,12	3188	38,46	3124	1338,95	3600	57,14	4060	14,20	3273	47,07	3124	903,46	3600	71,08	4147	14,81	3320	42,36
28	2672	2672	102,39	0,00	3842	14,07	2693	39,14	2672	1443,95	3600	45,96	3511	14,00	2825	43,74	2672	629,26	3600	76,45	3747	14,82	2693	46,22
29	2516	2516	76,39	0,00	3676	14,09	**2516**	36,34	2516	1446,95	3600	42,49	3783	13,99	**2516**	4457	2516	776,94	3600	69,12	3699	14,93	2633	43,55
30	2940	2940	73,44	0,00	3649	13,99	**2940**	37,78	2940	1420,02	3600	51,70	4312	14,03	2981	38,59	2940	792,92	3600	73,03	4383	14,78	3007	41,12
31	3256	3256	574,28	0,00	3859	14,14	3287	37,15	3256	1199,51	3600	63,16	4320	14,36	**3256**	37,54	3256	797,39	3600	75,51	4471	14,99	3300	41,71
32	2400	2400	51,00	0,00	2891	14,01	**2400**	34,50	2400	1324,56	3600	44,81	3656	14,23	2402	38,69	2400	718,08	3600	70,08	3538	14,78	2458	42,72
33	2936	2936	347,11	0,00	3641	13,89	**2936**	32,52	2936	1675,58	3600	42,93	3708	14,41	**2936**	41,20	2936	552,26	3600	81,19	4072	14,92	**2936**	41,60
34	3684	3684	93,70	0,00	4646	14,08	3770	32,48	3684	1988,62	3600	46,02	4997	14,30	3712	40,96	3684	1067,2	3600	71,03	4945	15,01	3869	44,39
35	2760	2760	72,42	0,00	3724	14,23	2764	32,88	2760	1392,42	3600	49,55	4299	14,37	2768	37,54	2760	796,81	3600	71,13	4239	14,95	2768	44,74
36	3544	3544	262,52	0,00	4375	14,05	3558	33,03	3544	1798,23	3600	49,26	4775	14,06	3596	36,08	3544	833,55	3600	76,48	4674	14,74	3610	43,79
37	3668	3668	450,86	0,00	4408	14,15	**3668**	33,07	3668	1850,14	3600	49,56	4701	14,11	3712	38,54	3668	907,83	3600	75,25	4733	14,86	3735	46,16
38	3052	3052	98,98	0,00	4109	14,21	**3052**	38,46	3052	1769,85	3600	42,01	4259	14,25	3162	36,14	3052	993,73	3600	67,44	4435	14,92	3407	40,58
39	3516	3516	116,08	0,00	4497	14,10	**3516**	33,41	3516	1814,26	3600	48,40	4468	14,33	3607	39,23	3516	719,02	3600	79,55	4595	14,90	3630	43,97
40	3124	3124	141,98	0,00	3987	14,07	3160	31,76	3124	1745,38	3600	44,13	4335	14,39	3160	41,20	3124	602,93	3600	80,70	4315	14,83	3363	41,17
41	3376	3376	124,67	0,00	4413	13,96	**3376**	34,28	3376	1399,35	3600	58,55	4215	14,26	**3376**	40,59	3376	708,28	3600	79,02	4527	14,96	3545	46,99
42	2940	2940	101,28	0,00	3832	14,02	**2940**	34,44	2940	1795,46	3600	38,93	4303	14,02	**2940**	39,65	2940	422,77	3600	85,62	4368	14,87	3057	46,18
43	2916	2916	140,08	0,00	4000	14,10	3062	33,63	2916	1600,30	3600	45,12	4411	14,10	3084	40,98	2916	575,33	3600	80,27	4375	14,88	3062	43,27
44	3400	3400	166,58	0,00	4575	14,05	3424	32,25	3400	1846,88	3600	45,68	4458	14,41	3412	37,13	3400	603,50	3600	82,25	4540	14,75	3472	45,34
45	2676	2676	54,23	0,00	3561	14,25	**2676**	33,53	2676	1443,17	3600	46,07	3657	14,03	2828	40,57	2676	879,87	3600	67,12	3749	14,86	2930	44,84
46	3180	3180	130,48	0,00	4306	14,20	3208	33,07	3180	1312,70	3600	58,72	4375	14,30	3208	39,65	3180	902,17	3600	71,63	4282	14,75	3314	44,75
47	3480	3480	289,84	0,00	4623	14,08	**3480**	32,29	3480	1520,76	3600	56,30	4582	14,33	**3480**	37,56	3480	559,93	3600	83,91	4456	14,92	**3480**	42,01
48	3396	3396	256,36	0,00	4138	14,01	**3396**	37,22	3396	1772,03	3600	47,82	4570	14,38	3424	38,59	3396	609,24	3600	82,06	4588	14,89	3528	43,29
49	3292	3292	97,61	0,00	4087	14,07	3306	34,40	3292	1499,18	3600	54,46	4598	14,41	3313	35,12	3292	817,40	3600	75,17	4515	14,78	3447	45,01
50	2856	2856	65,98	0,00	3522	14,11	**2856**	32,38	2856	1609,64	3600	43,64	4129	14,28	2976	36,98	2856	831,67	3600	70,88	4239	14,92	2963	41,17
Average	3187,68	3187,68	165,27	0,00	4088,04	14,12	3205,82	35,48	3187,68	1578,26	3600	50,33	4348,26	14,24	3233,22	38,39	3187,6	778,02	3600	75,43	4399,66	14,83	3296,62	43,50
Gap to CPLEX (in %)					+28,24%	-91,45%	+0,56%	-78,53%					+36,40%	-99,60%	+1,42%	-98,93%					+38,02%	-99,58%	+3,41%	-98,79%

Table 6
Results of class B

	MIP/Micro Period=6				SA		Matheuristic		MIP/Micro Period=8				SA		Matheuristic		MIP/Micro Period=10				SA		Matheuristic	
Instance	Obj.	LB	Time	Gap	Obj.	Time	Obj.	Time	Obj.	LB	Time	Gap	Obj.	Time	Obj.	Time	Obj.	LB	Time	Gap	Obj.	Time	Obj.	Time
1	2237	2237	2060,95	0,00	2863	11,44	2438	33,81	2237	1345,78	3600	39,84	2968	11,60	2508	38,62	2237	602,65	3600	73,06	2991	11,24	2559	41,21
2	2563	1706,44	3600	33,42	2935	11,25	2668	34,27	2563	971,38	3600	62,10	3121	11,58	2668	44,62	2563	775,56	3600	69,74	3035	11,41	2722	40,78
3	2365	2365	2395,76	0,00	2899	11,37	**2365**	46,77	2365	1106,35	3600	53,22	3027	11,50	**2365**	44,30	2365	775,48	3600	67,21	3104	11,39	2382	50,33
4	2139	2139	1664,45	0,00	2517	11,20	2242	44,64	2139	1369,60	3600	35,97	2622	11,48	2242	40,26	2197	740,61	3600	66,29	2706	11,33	2242	50,30
5	2527	2527	1979,34	0,00	2948	11,18	2636	36,87	2527	1452,52	3600	42,52	3137	11,56	2664	39,37	2527	874,85	3600	65,38	3247	11,29	2678	40,34
6	2935	2935	2229,69	0,00	3311	11,29	**2935**	34,85	2935	1282,60	3600	56,30	3659	11,31	3024	42,89	2978	886,55	3600	70,23	3767	11,40	**2935**	41,41
7	1845	1845	1532,48	0,00	2223	11,30	1966	34,28	1845	1234,31	3600	33,1	2283	11,45	1966	41,70	1845	689,29	3600	62,64	2361	11,42	2028	40,28
8	2469	2469	3010,09	0,00	2954	11,36	2516	34,27	2476	1183,03	3600	52,22	3083	11,56	2516	41,30	2476	785,39	3600	68,28	3168	11,43	2564	50,97
9	2459	2459	1528,02	0,00	2904	11,28	2532	32,81	2459	1324,42	3600	46,14	3047	11,47	2567	44,71	2459	793,03	3600	67,75	3063	11,35	2589	41,33
10	2800	2800	1950,16	0,00	3267	11,34	2901	35,07	2800	1405,60	3600	49,80	3619	11,41	2937	43,82	2800	845,32	3600	69,81	3550	11,30	2999	41,16
11	2226	2226	2558,01	0,00	2747	11,28	2271	32,01	2226	1070,71	3600	51,90	2754	11,35	2271	44,15	2279	930,97	3600	59,15	2885	11,36	2334	45,53
12	2423	2423	866,33	0,00	2816	11,31	**2423**	39,50	2423	1290,25	3600	46,75	2956	11,32	**2423**	43,00	2423	966,53	3600	60,11	2828	11,31	2439	56,57
13	3038	3038	1730,31	0,00	3490	11,20	3169	38,11	3038	1214,89	3600	60,01	3551	11,16	3169	44,38	3038	954,54	3600	68,58	3580	11,36	3196	56,62
14	1976	1976	1133,49	0,00	2548	11,18	2070	47,83	1976	1295,86	3600	34,42	2507	11,49	2138	39,85	1976	824,58	3600	58,27	2484	11,36	2243	44,59
15	2189	2189	1200,05	0,00	2624	11,36	2286	38,53	2189	1378,41	3600	37,03	2628	11,51	2317	38,69	2189	646,41	3600	70,47	2629	11,26	2329	40,52
16	2573	2573	2576,83	0,00	2976	11,15	2612	40,37	2573	1017,36	3600	60,46	3125	11,42	2612	43,49	2616	790,82	3600	69,77	3095	11,20	2656	41,98
17	2515	2515	1265,72	0,00	2859	11,39	**2515**	38,12	2515	1233,36	3600	50,96	2963	11,35	**2515**	40,77	2515	635,79	3600	74,72	3074	11,23	2528	40,40
18	2715	2715	2725,36	0,00	3128	11,23	2792	37,17	2715	1340,67	3600	50,62	3297	11,25	2792	38,42	2776	1093,19	3600	60,62	3308	11,31	2866	40,32
19	2932	2932	1005,61	0,00	3418	11,13	3027	39,74	2948	1029,15	3600	65,09	3548	11,41	3041	46,20	2932	917,13	3600	68,72	3746	11,42	3027	42,30
20	2612	2612	2079,20	0,00	3150	11,23	2713	40,99	2612	1140,92	3600	56,32	3270	11,39	2713	44,75	2612	699,49	3600	73,22	3237	11,43	2744	41,95
21	2170	2170	1025,28	0,00	2795	11,36	2279	41,15	2170	1412,89	3600	34,89	2628	11,11	2310	37,74	2170	1065,90	3600	50,88	2982	11,38	2367	36,49
22	1907	1907	1629,95	0,00	2371	11,29	1967	45,73	1907	1067,73	3600	44,01	2527	11,40	1967	42,80	1907	776,15	3600	59,30	2448	11,25	2091	51,07
23	1765	1765	897,41	0,00	2273	11,16	1936	37,87	1765	1179,90	3600	33,15	2261	11,32	1936	40,80	1777	617,33	3600	65,26	2472	11,26	1936	49,21
24	2575	2575	3189,47	0,00	3182	11,28	2715	46,61	2575	1116,52	3600	56,64	3257	11,34	2715	49,12	2575	1104,68	3600	57,10	3361	11,32	2791	54,13
25	2292	2168,46	3600	5,39	2608	11,33	2396	38,11	2292	1166,63	3600	49,10	2776	11,24	2448	40,01	2292	880,59	3600	61,58	2730	11,39	2416	36,02
26	2219	2219	1718,09	0,00	2657	11,33	2301	38,08	2219	989,45	3600	55,41	2797	11,36	2355	44,67	2219	565,40	3600	74,52	2749	11,35	2359	36,75
27	2922	2922	2605,22	0,00	3321	11,20	2931	45,26	2922	1161,50	3600	60,25	3278	11,45	2931	40,34	2922	1147,47	3600	60,73	3444	11,40	2955	47,31
28	2046	2046	1111,50	0,00	2471	11,45	2091	37,49	2046	978,60	3600	52,17	2588	11,48	2091	44,18	2046	816,35	3600	60,10	2567	11,42	2091	35,10
29	2060	2060	2177,78	0,00	2571	11,40	2129	38,78	2068	906,40	3600	56,17	2503	11,46	2129	41,04	2068	284,97	3600	86,22	2696	11,39	2186	53,47
30	2413	2413	620,75	0,00	2955	11,49	2549	38,96	2452	1083,54	3600	55,81	2981	11,49	2569	41,52	2413	801,60	3600	66,78	2952	11,35	2589	35,85
31	2044	2044	1815,14	0,00	2290	11,35	2151	38,31	2044	813,31	3600	60,21	2507	11,35	2160	40,50	2062	701,08	3600	66,00	2504	11,30	2191	36,87
32	2055	2055	1337,52	0,00	2553	11,34	2158	37,65	2074	1157,71	3600	44,18	2598	11,41	2158	42,93	2055	851,80	3600	58,55	2454	11,26	2299	37,90
33	2015	2015	505,03	0,00	2477	11,25	2070	37,86	2015	1367,58	3600	32,13	2682	11,38	2070	43,96	2015	734,67	3600	63,54	2698	11,25	2243	37,92
34	1926	1926	675,44	0,00	2227	11,27	1969	38,07	1926	906,38	3600	52,94	2648	11,29	2039	40,61	1926	670,25	3600	65,20	2731	11,26	2058	44,77
35	2016	2016	819,13	0,00	2426	11,36	2138	37,97	2016	937,04	3600	53,52	2519	11,30	2138	48,74	2024	784,10	3600	61,26	2580	11,29	2199	50,46
36	2717	2717	2981,64	0,00	3177	11,20	2766	37,96	2717	1159,89	3600	57,31	3355	11,32	2766	46,07	2717	705,33	3600	74,04	3551	11,32	2864	39,29
37	2050	2050	878,86	0,00	2508	11,29	2168	38,16	2050	1183,06	3600	42,29	2566	11,35	2168	39,65	2050	947,92	3600	53,76	2699	11,38	2168	37,57
38	2540	2540	1999,49	0,00	3012	11,24	2586	37,32	2540	1083,06	3600	57,36	3156	11,28	2586	38,70	2563	718,15	3600	71,98	3353	11,37	2611	37,81
39	2397	2397	3125,33	0,00	2817	11,19	2494	38,18	2423	1082,59	3600	55,32	2917	11,25	2520	40,01	2423	685,95	3600	71,69	2975	11,35	2503	38,83
40	1774	1362,61	3600	23,19	2185	11,30	1853	37,95	1774	380,35	3600	78,56	2171	11,29	1853	44,30	1839	521,72	3600	71,63	2264	11,30	1911	36,23
41	2292	2292	2099,48	0,00	2768	11,15	2352	37,89	2292	1399,50	3600	38,94	2563	11,36	2406	43,35	2292	946,14	3600	58,72	2830	11,39	2529	36,16
42	2621	2621	3050,95	0,00	2982	11,27	**2621**	38,26	2621	1058,89	3600	59,60	3308	11,39	**2621**	49,17	2621	821,16	3600	68,67	3357	11,25	2665	35,80
43	2002	2002	2310,16	0,00	2319	11,28	2081	38,10	2002	1071,07	3600	46,50	2214	11,40	2081	43,55	2002	614,61	3600	69,30	2522	11,24	2081	36,41
44	2391	2078,25	3600	13,08	2614	11,23	**2391**	38,22	2391	1291,38	3600	45,99	2757	11,28	**2391**	49,18	2391	840,68	3600	64,84	2894	11,20	**2391**	36,29
45	2490	2490	1719,70	0,00	2899	11,34	2552	37,54	2490	1037,09	3600	58,35	3164	11,26	2611	51,31	2490	649,89	3600	73,9	3136	11,21	2621	38,33
46	3167	3167	1599,64	0,00	3687	11,19	3239	32,39	3167	1365,61	3600	56,88	3652	11,41	3255	43,96	3211	975,18	3600	69,63	3893	11,35	3216	36,42
47	2731	2582,98	3600	5,42	3095	11,16	2752	33,29	2731	1077,38	3600	60,55	3345	11,45	2765	42,75	2731	622,12	3600	77,22	3293	11,33	2752	36,31
48	2036	2036	1195,17	0,00	2419	11,29	2108	32,24	2060	1009,40	3600	51,00	2663	11,40	2108	42,96	2060	812,67	3600	60,55	2880	11,30	2220	36,13
49	2157	2157	768,13	0,00	2440	11,14	2257	39,85	2183	1108,53	3600	49,22	2686	11,22	2257	43,01	2183	693,76	3600	68,22	2824	11,27	2295	42,16
50	2141	2141	1174,83	0,00	2572	11,21	2229	38,19	2141	969,87	3600	54,70	2679	11,28	2229	43,42	2141	630,31	3600	70,56	2698	11,43	2262	39,38
Average	2349,38	2312,34	1930,45	1,61	2784,96	11,27	2426,12	38,26	2352,68	1144,20	3600	50,75	2898,22	11,37	2441,62	42,91	2359,76	784,32	3600	66,51	2967,9	11,32	2478,4	41,90
Gap to CPLEX (in %)					+18,54%	-99,41%	+3,26%	-98,01%					+23,18%	-99,68%	+3,78%	-98,80%					+25,77%	-99,52%	+5,02%	-98,83%

Table 7

Results of class C

Instance	MIP/Micro Period=7 Obj.	LB	Time	Gap	SA Obj.	Time	Matheuristic Obj.	Time	MIP/Micro Period=8 Obj.	LB	Time	Gap	SA Obj.	Time	Matheuristic Obj.	Time	MIP/Micro Period=10 Obj.	LB	Time	Gap	SA Obj.	Time	Matheuristic Obj.	Time
1	2145	2145	1065,9	0,00	2523	8,02	2219	41,61	2145	1788,93	3600	16,60	2636	7,98	2219	33,61	2145	1170,53	3600	45,43	2762	8,06	2219	42,25
2	2181	2181	426,81	0,00	2467	7,94	2222	38,18	2181	1707,72	3600	21,70	2666	7,88	2222	33,22	2181	1003,04	3600	54,01	2544	8,00	2222	41,67
3	1739	1739	295,30	0,00	2039	8,01	1773	42,83	1739	1739	1047,6	0,00	2097	8,00	1773	33,87	1739	1031,75	3600	40,67	2149	8,12	1833	47,14
4	1889	1889	156,75	0,00	2167	7,99	1912	40,33	1889	1889	2457,2	0,00	2484	8,03	1927	32,62	1889	1083,34	3600	42,65	2556	8,02	1942	46,84
5	2363	2363	386,19	0,00	2802	8,01	2450	49,64	2363	2363	1450,4	0,00	3053	8,06	2450	32,79	2363	1099,03	3600	53,49	3054	8,03	2473	41,20
6	2452	2452	467,64	0,00	3047	8,07	**2452**	45,77	2452	2183,51	3600	10,95	3114	8,10	**2452**	33,11	2452	995,27	3600	59,41	3019	8,04	**2452**	41,48
7	2023	2023	420,42	0,00	2313	8,01	2133	38,06	2023	1647,53	3600	18,56	2446	7,89	2133	38,21	2023	999,97	3600	50,57	2577	7,96	2133	45,06
8	2033	2033	458,20	0,00	2217	8,03	2044	33,89	2033	1371,26	3600	32,55	2525	8,02	2044	33,26	2033	926,84	3600	54,41	2683	7,98	2044	41,86
9	2161	2161	638,92	0,00	2447	8,04	2252	32,31	2161	1748,03	3600	19,11	2660	7,88	2252	38,61	2161	1007,46	3600	53,38	2724	7,99	2252	41,74
10	2023	2023	185,58	0,00	2473	8,06	2143	31,95	2023	1567,02	3600	22,54	2620	7,96	2143	41,35	2023	803,94	3600	60,26	2572	7,99	2143	44,97
11	2297	2297	492,94	0,00	2650	7,99	2362	33,24	2297	2249,91	3600	2,05	2742	7,98	2362	32,84	2297	1082,58	3600	52,87	2779	8,00	2362	42,57
12	2077	2077	436,63	0,00	2330	7,98	2093	38,95	2077	1612,38	3600	22,37	2407	8,08	2093	33,09	2077	1014,41	3600	51,16	2457	8,04	2093	49,60
13	2049	2049	922,95	0,00	2339	7,89	2062	38,06	2049	2049	2127,7	0,00	2346	8,01	2062	44,18	2049	1331,85	3600	35,00	2465	8,12	2062	43,86
14	2401	2401	467,64	0,00	2579	7,99	**2401**	35,13	2401	1483,10	3600	38,23	2702	8,10	**2401**	42,48	2401	1012,98	3600	57,81	2857	8,13	**2401**	41,65
15	2178	2178	455,05	0,00	2445	8,00	2178	38,24	2178	1867,20	3600	14,27	2755	8,09	2182	40,26	2182	1085,76	3600	50,24	2579	8,10	**2182**	43,20
16	1868	1868	334,11	0,00	2270	7,89	1909	45,94	1868	1466,75	3600	21,48	2209	7,99	1909	42,54	1868	884,12	3600	52,67	2252	8,15	1909	46,53
17	1985	1985	321,94	0,00	2138	8,06	2008	41,97	1985	1985	1802,1	0,00	2484	7,90	2008	41,46	1985	883,72	3600	55,48	2272	8,03	2010	45,66
18	2269	2269	422,61	0,00	2582	8,02	2321	34,13	2269	1678,61	3600	26,02	2674	7,89	2321	40,54	2269	874,25	3600	61,47	2818	8,03	2321	42,64
19	2345	2345	2406,0	0,00	2667	8,03	2345	41,32	2345	1962,77	3600	16,30	2748	8,09	2407	38,33	2345	1006,94	3600	57,06	2848	8,09	2407	40,59
20	2025	2025	401,73	0,00	2425	8,22	2115	46,90	2025	2025	947,59	0,00	2438	7,88	2115	37,40	2025	999,95	3600	50,62	2335	8,10	2148	45,79
21	2474	2474	158,70	0,00	2847	8,02	**2474**	47,65	2474	2146,44	3600	13,24	3006	7,90	**2474**	37,96	2474	1399,54	3600	43,43	3088	8,09	**2474**	46,01
22	2155	2155	365,06	0,00	2556	8,11	**2155**	43,38	2155	1759,13	3600	18,37	2600	7,91	2203	38,15	2155	1165,21	3600	45,93	2789	8,07	2207	43,15
23	2015	2015	171,91	0,00	2264	8,03	**2015**	47,28	2015	2015	2125,8	0,00	2440	7,93	2016	38,74	2015	1120,14	3600	44,41	2378	8,05	2016	43,60
24	1957	1957	266,67	0,00	2285	8,09	**1957**	36,09	1957	1503,95	36002	23,15	2520	7,95	**1957**	37,67	1957	992,00	3600	49,31	2456	8,17	**1957**	49,10
25	1771	1771	258,94	0,00	2198	8,08	1876	36,40	1771	1279,55	3600	27,75	2149	7,90	1876	38,46	1771	959,35	3600	45,83	2333	8,10	1876	41,36
26	2184	2184	323,92	0,00	2595	8,05	2276	33,64	2184	1627,08	3600	25,5	2805	8,01	2293	38,11	2184	1224,13	3600	43,95	2643	7,93	2293	41,45
27	1568	1568	116,38	0,00	1962	8,11	1653	33,11	1568	1568	1286,4	0,00	2057	8,03	1653	38,36	1568	929,20	3600	40,74	1957	8,11	1653	45,42
28	2398	2398	122,80	0,00	2847	8,03	2477	33,48	2398	2398	1302,0	0,00	2971	8,17	2477	38,46	2398	1370,46	3600	42,85	2852	8,16	2477	44,23
29	2186	2186	626,05	0,00	2511	8,00	2195	38,87	2186	1930,46	3600	11,69	2741	8,15	2195	37,79	2186	1171,70	3600	46,40	2716	8,12	2217	46,55
30	2164	2164	410,33	0,00	2435	7,99	**2164**	40,36	2164	1402,27	3600	35,20	2546	8,12	**2164**	37,71	2164	1133,94	3600	47,60	2536	7,98	**2164**	42,02
31	2067	2067	238,69	0,00	2452	8,01	2100	33,11	2067	2067	2549,0	0,00	2635	8,06	2100	38,72	2067	1071,74	3600	48,15	2783	7,99	2100	45,22
32	2144	2144	321,01	0,00	2500	8,07	2144	33,64	2144	1579,27	3600	26,34	2774	8,09	2144	38,20	2144	1036,62	3600	51,65	2568	8,15	2144	46,00
33	1912	1912	241,71	0,00	2227	8,06	1996	32,68	1912	1912	1341,9	0,00	2389	8,09	1996	38,41	1912	864,99	3600	54,76	2507	8,01	1996	46,26
34	1725	1725	213,08	0,00	2069	8,04	1830	32,75	1725	1725	3543,1	0,00	2072	8,04	1830	37,90	1725	701,56	3600	59,33	2109	8,06	1836	45,73
35	1970	1970	183,39	0,00	2163	8,02	2051	34,94	1970	1401,06	3600	28,88	2276	8,10	2051	39,84	1970	993,67	3600	49,56	2346	8,10	2051	45,50
36	1879	1879	163,16	0,00	2320	8,03	2008	36,29	1879	1879	1305,4	0,00	2468	8,01	2008	38,48	1879	977,46	3600	47,98	2453	8,18	2008	48,45
37	2155	2155	347,47	0,00	2467	8,05	2276	33,23	2155	2155	3113,0	0,00	2742	8,03	2276	37,74	2155	1102,93	3600	48,82	2712	8,08	2283	46,30
38	2001	2001	206,16	0,00	2347	8,01	2021	33,83	2001	1648,42	3600	17,62	2225	8,05	2021	38,21	2001	981,49	3600	50,95	2553	8,01	2021	42,20
39	1957	1957	155,95	0,00	2179	8,06	1979	33,55	1957	1957	2351,2	0,00	2236	8,15	1979	38,78	1957	1157,57	3600	40,85	2145	8,04	1979	47,85
40	2215	2215	430,16	0,00	2442	8,05	2240	33,17	2215	2215	2256,0	0,00	2500	8,01	2240	37,79	2215	1298,88	3600	41,36	2584	8,18	2240	48,05
41	2113	2113	209,14	0,00	2457	8,04	**2113**	38,75	2113	2113	957,61	0,00	2574	8,12	**2113**	38,01	2113	1149,68	3600	45,59	2648	8,22	**2113**	48,65
42	2239	2239	725,19	0,00	2524	8,01	2330	39,07	2239	2239	2568,2	0,00	2548	8,01	2330	38,35	2239	1126,44	3600	49,69	2581	8,01	2345	47,96
43	1956	1956	369,97	0,00	2166	8,00	**1956**	41,39	1956	1956	2598,5	0,00	2304	7,98	**1956**	37,81	1956	934,19	3600	52,24	2300	8,03	**1956**	46,92
44	2085	2085	353,76	0,00	2443	8,01	**2085**	38,66	2085	2085	2992,9	0,00	2529	7,99	**2085**	38,19	2085	1026,86	3600	50,75	2622	8,05	**2085**	45,75
45	2165	2165	320,01	0,00	2554	7,96	**2165**	38,17	2165	1802,80	3600	16,73	2698	8,01	2239	38,34	2165	874,44	3600	59,61	2607	8,07	**2165**	48,46
46	1895	1895	217,20	0,00	2185	7,98	**1895**	38,97	1895	1895	1670,8	0,00	2495	8,05	**1895**	38,56	1895	860,52	3600	54,59	2559	8,11	**1895**	49,69
47	2158	2158	371,66	0,00	2476	7,99	2181	38,51	2158	2158	2956,3	0,00	2439	8,15	2181	38,40	2158	1039,51	3600	51,83	2439	8,13	2189	43,02
48	2074	2074	327,31	0,00	2324	7,88	2129	38,22	2074	1791,94	3600	13,60	2494	8,12	2136	38,59	2074	1141,53	3600	44,96	2561	8,07	2136	42,18
49	1915	1915	216,39	0,00	2378	7,99	1962	38,16	1915	1915	1476,2	0,00	2462	8,11	2067	37,61	1915	918,43	3600	52,04	2517	8,08	2067	45,99
50	2301	2301	345,70	0,00	2641	7,84	**2301**	38,43	2301	2301	3360,3	0,00	2744	8,01	**2301**	38,65	2301	1352,07	3600	41,24	2593	8,02	2310	46,19
Average	2086,02	2086,02	398,82	0,00	2414,68	8,01	2127,9	38,08	2086,02	1856,20	3511,80	10,81	2544,9	8,02	2134,62	37,83	2086,1	1047,48	3600	49,70	2564,74	8,07	2137,22	44,91
Gap to CPLEX (in %)					+15,75%	-97,99%	+2,01%	-90,45%					+21,99 %	-99,77%	+2,32%	-98,92%					+22,94%	-99,77%	+2,45%	-98,75%

5.4. Computational results for various capacity settings

Different capacity (c_t) levels are analyzed in order to observe their effect on the quality of the solution. We divided the capacity into three categories: Tight ($0.8c_t$), Medium (c_t), and Loose ($1.2c_t$). Results for the average computational time of the solution are provided in Table 8, with 50 instances in each class and a total of 150 instances for each solution approach. The average computational time spent on the solution approaches increases from Tight to Loose in each class.

Table 8
Average computational time

Classes	MIP	SA	Matheuristic
Class A (*T*)	104,49	14,07	33,19
Class A (*M*)	165,27	14,12	35,48
Class A (*L*)	181,16	14,91	36,56
Class B (*T*)	1860,06	11,02	37,11
Class B (*M*)	1930,45	11,27	38,26
Class B (*L*)	2374,07	11,84	38,49
Class C (*T*)	254,84	7,87	37,72
Class C (*M*)	398,82	8,01	38,08
Class C (*L*)	443,73	8,27	38,64

6. Conclusions

In this study, the GLSP with sequence dependent changeovers and back-orders is investigated through mathematical modelling and computational approaches. SA and SA-based matheuristic algorithms are developed for solving the GLSP. Extensive computational tests are performed on randomly generated problem instances for comparing the computational performances of the proposed SA and SA-based matheuristic algorithms. The computational results demonstrate that the proposed SA-based matheuristic approach is superior to SA and classical MIP solver for solving the present GLSP. The computational time requirement of the proposed SA-based matheuristic algorithm is considerable less than the CPLEX solver. Moreover, in order to reveal the effect of valid inequalities on the overall time performance, all problem instances in Classes A, B, and C with micro periods 5, 6, and 7 are also solved by including valid inequality constraints into model formulations. It is observed that valid inequality constraints can decrease average computational times by 61,98%, 42,59%, and 68,81% in problem classes A, B, and C.

Future studies will be devoted to incorporating different real life constraints into the present GLSP model and enhancing the performance of the proposed matheuristic algorithm further by employing different local search strategies within SA.

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