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Pricing and coordination of remanufacturing supply chain considering remanufacturing capacity and preferences under government mechanisms

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A B S T R A C T

The management of recycling and remanufacturing supply chains, which can help enterprises achieve low pollution, low energy consumption and sustainable development, has become a new strategy of modern enterprises. The factors of supply chain and government mechanisms will have an impact on enterprise decisions for recycling, remanufacturing and social welfare. In order to promote the sustainable operation of the supply chain, considering the coordination role of government mechanisms and supply chain, a recycling and remanufacturing supply chain model composed of a manufacturer, retailer and recycler is constructed. This paper discusses the pricing decision of new/remanufactured products, supply chain performance level, such as remanufacturing effort, publicity service efforts and profit, and social welfare in five models of three situations: centralized situation, including non-government mechanisms and non-supply chain coordination; manufacturer-led situation, including non-government mechanisms and non-supply chain coordination, government mechanisms and non-supply chain coordination, government mechanisms and supply chain coordination; government-led situation, including government mechanisms and non-supply chain coordination. It is found that under manufacturer-led situations, the government subsidy and bonus-penalty mechanisms can encourage manufacturer and retailer to actively participate in the recycling and remanufacturing activities. The supply chain coordination contract can further enhance the role of the consumer market and promote the implementation of government mechanisms. Manufacturer adopts a cost-sharing contract to encourage recyclers to carry out recycling activities. Under certain conditions, the contract can effectively improve the benefits and social welfare. The research conclusions have important theoretical and practical application value for the coordination and cooperation among enterprises in the supply chain and the formulation of government mechanisms.

1. Introduction

With the development of economy, science and technology, society pays more attention to the protection of the ecological environment and the recycling of resources (Ranjan & Jha, 2019). Recycling and remanufacturing industry has gradually become an important way to realize the sustainable development strategy of all countries in the world. The implementation of recycling and remanufacturing management in enterprises cannot only reduce the environmental pollution and resource consumption in the production process, but also realize the sustainable development of the economy and enhance the

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competitiveness of enterprises (Savaskan & Wassenhove, 2006; Hong & Guo, 2019). Compared with new products, remanufactured products have less impact on the environment. At present, the remanufacturing industry in many countries is still in its infancy. How to promote the rapid and healthy development of the recycling and remanufacturing industry has attracted extensive attention from the society. With the gradual enhancement of consumers' awareness of environmental protection, consumers are more willing to buy remanufactured products (Ismail & Hanafiah, 2019), and retailers are also willing to promote service remanufactured products. In this context, how the preferences of consumers and retailers affect the market demand of new/remanufactured products, and further affect the production and pricing of supply chain are urgent practical problems to be solved. On the other hand, in order to reduce the environmental burden and support the national sustainable development strategy, it is necessary for the government to promote the development of the recycling and remanufacturing industry through the mechanisms of environmental regulations and financial intervention (Ma et al., 2013). The government can promote the recycling and remanufacturing activities and affect social welfare by setting the basic remanufacturing capacity, the degree of publicity services for remanufactured products, implementing bonus-penalty measures, and providing subsidies for remanufactured products. At the same time, compared with centralized decision-making, when considering the role of government mechanisms and consumer market, the decision-makers in the supply chain system simply pursue their own profit maximization, which will lead to "double marginal effect". The recycling and remanufacturing supply chain is in a state of imbalance, which affects the overall benefits of the system and social welfare. Therefore, in decentralized decision-making, how to formulate the internal coordination contract of the supply chain to promote the cooperation between node enterprises and form a win-win relationship with the government has become one of the important contents of recycling and remanufacturing supply chain management. In view of the above considerations, this paper will focus on solving the following three problems:

I. How to make decisions in the recycling and remanufacturing supply chain system under the independent role of the consumer market?
II. Under the joint action of government mechanisms and consumer market, how do the government mechanisms affect the remanufacturing decision-making of enterprises, the remanufacturing capacity of manufacturers, the degree of product publicity services of retailers, the efficiency of supply chain and social welfare?
III. What kind of coordination contract should be designed to achieve the improvement of supply chain node enterprises profits, system benefits and social welfare under the joint action of government mechanisms and consumer market?
The remainder of this paper is organized as follows: Section 2 reviews the related literature. Section 3 presents notations and assumptions. In Section 4, four different game models are proposed, along with the optimal solutions. Section 5 studies the system coordination in the manufacturer-led model with government mechanisms. Numerical results and sensitivity analysis are presented in Section 6. Section 7 concludes the paper.

2. Literature review

This section aims to present a brief review of literature that is related to our context. The related literature is classified into two main streams: (1) the pricing problem of closed-loop supply chains only considering preferences and market demand, and (2) pricing problems involving government mechanisms.

2.1. Preferences and market demand

At present, some literature has studied the decision-making of recycling and remanufacturing from the perspective of consumer preferences and market demand. Some scholars believe that consumers have certain environmental preferences and are willing to pay extra for green products. On this basis, they have studied the decision-making behavior of manufacturer and retailer (Ghosh & Shah, 2015; Liu et al., 2012; Xiong et al., 2014; Yu et al., 2016). However, some literature (Ferrer & Swaminathan, 2010; Cheng et al., 2017; Heng et al., 2020) mentioned that due to different consumption tendencies and cognitive levels, consumers have different degrees of market demand for new/remanufactured products. Although there is no difference in quality between new/remanufactured products, consumers have different recognition and willingness to pay for remanufactured products of the same quality and function. Ho & Zhang (2008) constructed a fair utility function to study the secondary supply chain, and pointed out that consumers' bias towards remanufactured products will seriously affect the market demand for remanufactured products. Of course, if the remanufacturing process is very transparent, consumers will also prefer to buy remanufactured products at a higher price (Hazen et al., 2012). Considering the low-carbon awareness of consumers, Xia et al. (2018) established a game model led by manufacturers. The conclusion shows that consumers' low-carbon preference can effectively improve the enthusiasm of supply chain members to invest in low-carbon industries and increase their investment profits. Under the carbon trading system and low-carbon preferences of consumers, Liu et al. (2016) discussed the coordination of a low-carbon supply chain with supplier-led investment and emission reduction, proposed a quantity discount contract for retailers to share emission reduction costs for coordination, and gave the optimal combination of quantity discount strategies. At the same time, by building a centralized and decentralized model, Li et al. (2017) found that there were significant differences in the willingness to buy low-carbon products in terms of age, values, education level and income. Wang et al. (2021) claimed that altruistic preference would help to improve the profits and system efficiency of small and medium-sized manufacturer, but would reduce the profit of retailer, by building three decision-making models in the low-carbon supply chain: centralized, decentralized without altruistic preference and decentralized with altruistic preference. Ji et al. (2017) considered consumer preferences in a dual channel supply chain. Zhao & Xiao (2018) discussed the operation strategies of dual channel closed-loop supply chains under different recycling modes from the perspective of channel preference and risk aversion.

2.2. Government mechanisms

The government mechanisms to promote and intervene in recycling and remanufacturing decision-making is also a research hotspot (Wang et al., 2018). Mitra et al. (2008) and Sheu & Chen (2012) considered the impact of government subsidies to supply chain members on the closed-loop supply chain. The conclusion shows that the government subsidy mechanisms can effectively promote the development of the remanufacturing industry. Wan & Hong (2019) and Shu et al. (2017) found that government subsidies will stimulate market demand and benefit members of the closed-loop supply chain. It will play a good role in promoting the benefits of the supply chain and social benefits. Appropriate government subsidies can help promote the market demand for remanufactured products, and the government can adopt appropriate subsidy policies to encourage the recycling of returned goods (Guo et al., 2019). In order to study the problem of energy conservation and emission reduction in the supply chain, Yi and Li (2018) established the stackelberg model of retailer and manufacturer. They found that government subsidies can increase the profits of supply chain members. Heydari et al. (2017) found that the government should provide subsidies to manufacturers rather than retailers. Meanwhile, Ma et al. (2015) analyzed the impact of subsidies to consumers and subsidies to manufacturers on the closed-loop supply chain. The research shows that subsidies to consumers can bring more profits to manufacturers and retailers. Xia et al. (2018) analyzed the impact of government subsidies on consumer perceptions and found that not all consumers expect government subsidies. He et al. (2019) believes that manufacturers can distribute new products through independent retailers and sell remanufactured products through third-party companies or platforms (3P) where government subsidies may exist. Zhao et al. (2018) developed a decision-making model that takes into account consumer preferences for remanufactured products and the impact of government subsidies. The study found that if remanufacturers share a certain proportion of subsidies with consumers, they can obtain more profits due to the expansion of the market.

To sum up, many scholars have studied the management and decision-making of recycling and remanufacturing. In the construction of recycling and remanufacturing supply chain management systems, the influence of preferences and government mechanisms on enterprises and supply chains has become the focus of scholars. However, the current research on recycling and remanufacturing supply chains is mostly about consumer preferences and undifferentiated government subsidies for remanufactured products. In fact, in the complex recycling and remanufacturing supply chain system, retailers, as the main sales force, have sales efforts, especially the role of advertising (Qi et al., 2020; Xie et al., 2017; Ranjan & Jha, 2019). Retailers also have a preference for publicity services for new/remanufactured products. When the efforts of retailers to advertise services for remanufactured change, on the one hand, it will affect the product brand and corporate image of upstream manufacturers and affect the decision-making of manufacturers. On the other hand, it will affect the downstream consumer market demand for new/remanufactured products. At the same time, in addition to providing undifferentiated subsidies, the bonus-penalty mechanisms are also a research area. Therefore, in view of the cooperative operation of recycling and remanufacturing supply chain (upstream manufacturer's new/remanufactured products, midstream retailer's promotion of enterprises and products, and downstream recycler's recycling of used products), considering the role of consumer market and government mechanisms, this paper adopts game theory to establish a decision-making model of remanufacturing supply chain. In order to further improve the efficiency of the system, a supply chain internal coordination contract with cost-sharing contract is designed to improve the economic benefits and social welfare.

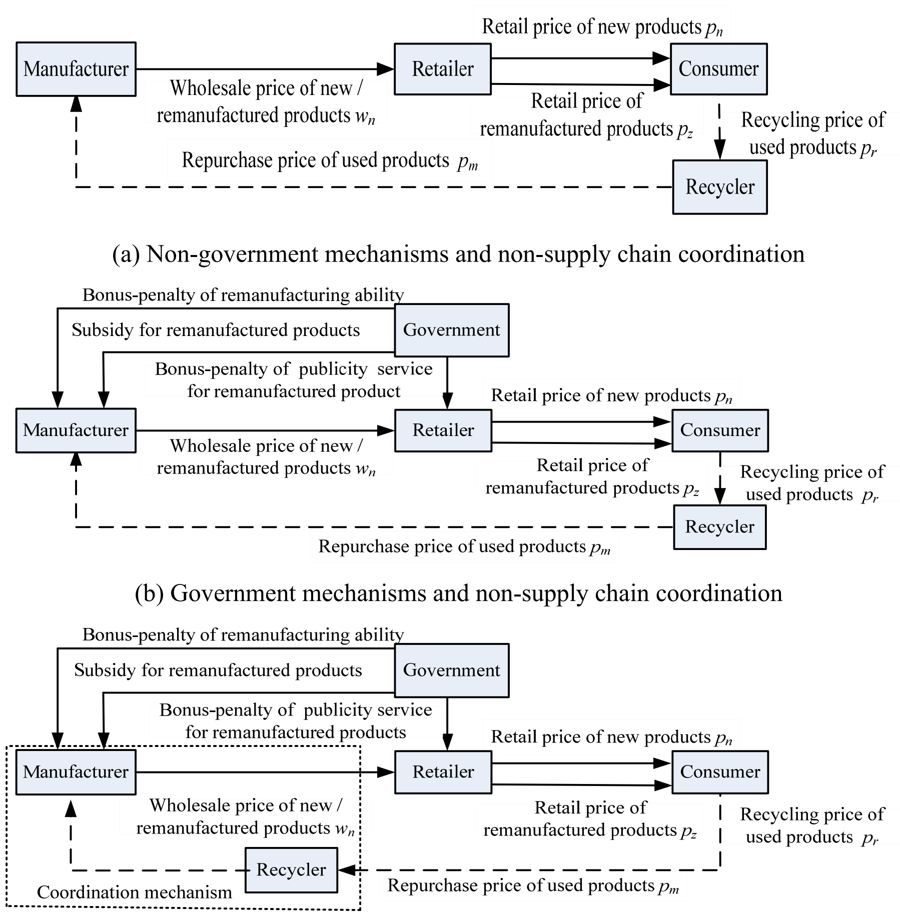
In short, this paper constructs a recycling and remanufacturing supply chain for manufacturer, retailer and recycler. The factors, such as product differences (new/remanufactured products), consumer preferences (ordinary/environmentally friendly consumers) and retailer publicity services preferences (new/remanufactured products publicity and services), are considered. The remanufacturing ability and goodwill level of the manufacturer are taken as decision variables. Five analytical decision models are studied: (a) concentration/manufacturer dominance based on the consumer market in which there is non-government mechanism and non-supply chain coordination,(b) manufacturer/government dominance based on government mechanisms in which there is government mechanisms without supply chain coordination, and (c) manufacturer dominance supply chain coordination in which there is government mechanisms with supply chain coordination. This paper discusses the impact of relevant market and government mechanisms parameters on the decision-making, efficiency and social welfare of the supply chain system. The conditions and effects of cost-sharing contracts in coordinating the supply chain are analyzed. The conclusions provide reference for enterprises to choose cooperation contracts, formulate and implement government mechanisms according to the actual situation.

3. Model background description

3.1. Problem description

Manufacturer is responsible for manufacturing new products, buying back used products from recyclers and carrying out remanufacturing activities. Retailer is responsible for the sale of new/remanufactured products, as well as providing publicity service for products and the brand of the manufacturer. Recyclers are responsible for recycling used products from consumers.

Firstly, this paper discusses the decision-making problem of recycling and remanufacturing supply chain without supply chain coordination (model and model) based on the role of consumer market, as shown in Fig.1(a). Further, the situation where there are government mechanisms without supply chain coordination (model and model), as shown in Fig.1(b), is considered. In this case, on the one hand, the government has subsidies for remanufactured products. On the other hand, the government carries out bonus-penalty activities for remanufacturing production and retailer's remanufacturing product publicity service. If the remanufacturing capacity of the manufacturer is higher than the basic remanufacturing capacity stipulated by the government, the manufacturer will be rewarded. Otherwise, it will be punished; If the efforts to publicity service remanufactured products of retailers are higher than the basic publicity service stipulated by the government, the retailer will be rewarded. Otherwise, it will be punished. Through analysis and comparison, this paper studies the impact of government mechanisms on manufacturer's remanufacturing strategy and supply chain pricing decision. Finally, the situation with government mechanisms and supply chain coordination (model) to improve the operation of remanufacturing supply chains based on government mechanisms is considered, as shown in Fig. 1(c). In this case, the manufacturer bears part of the recovery cost of the recycler. We discuss the decision-making problem with supply chain coordination and analyze the impact of cost-sharing contracts on enterprise operation efficiency and government mechanisms implementation by comparing model and model .



(a) Non-government mechanisms and non-supply chain coordination

(b) Government mechanisms and non-supply chain coordination

(c) Government mechanisms and supply chain coordination

Fig. 1. Structure of five models in a recycling and remanufacturing supply chain.

3.2. Symbol description

Based on the problem description, we employ the major index, parameters, variables and functions given in Table 1 throughout this paper.

Table 1
Index, parameters, variables and functions.

Index	
i	Index for model i , $i \in \{\text{I}C, \text{I}M, \text{II}M, \text{II}G, \text{III}\}$.
Parameters	
υ	The impact of remanufacturing effort on remanufacturing capacity, $0 < \upsilon < 1$;

ε	The impact of the retailer's publicity service efforts on the goodwill level of manufacturer, $0<\varepsilon<1$;
k	Proportion of retailer's publicity service efforts for remanufactured products, $0<k<1$;
φ	The quantity of potential consumers in the market, $\varphi>0$;
τ	The proportion of ordinary consumers thinks that new products are better, $0<\tau<1$;
α	The positive impact of remanufacturing capacity on the consumer market, $0<\alpha<1$;
ξ	The positive impact of manufacturer's goodwill on the consumer market, $0<\xi<1$;
ω	Cost coefficient of retailer's publicity service efforts, $\omega>0$;
c_n	Unit production cost of new products, $c_n>0$;
c_z	Unit production cost of remanufactured products, $c_z>0$;
c_r	Unit treatment cost of recycler to recycle used products, $c_r>0$;
c_h	Unit environmental treatment cost of unrecycled used products in the consumer market, $c_h>0$;
E_0	The basic remanufacturing capacity of manufacturer set by the government, $0<E_0<1$;
A_0	The basic publicity service efforts for remanufactured products set by the government , $0<A_0<1$;
s	Unit subsidy for remanufacturing products, $s>0$;
h_j	Bonus-penalty coefficient for the remanufacturing capacity of manufacturer, $h_j>0$;
h_z	Bonus-penalty coefficient for retailer's publicity service remanufactured products, $h_z>0$;
ψ	The quantity of used products voluntarily returned by consumers, $\psi>0$;
δ	Consumers' sensitivity to recycling prices, $\delta>0$;
μ	Sensitivity coefficient of ordinary/environmentally friendly consumers to the retail price of remanufactured/new products, $0<\mu<1$;
θ	Remanufacturing input cost coefficient, $\theta>0$;
σ	Manufacturer's share coefficient of recovery cost to recycler, $0<\sigma<1$;
Decision variables	
w_n^i	Unit wholesale price of new/remanufactured products in the model i, $w_n^i>0$;
p_n^i	Unit retail price of new products in the model i, $p_n^i>0$;
p_z^i	Unit retail price of remanufactured products in the model i , $p_z^i>0$;
p_r^i	Unit recycling price of used products in the model i, $p_r^i>0$;
p_m^i	Unit repurchase price for used products in the model i, $p_m^i>0$;
e^i	The remanufacturing effort of manufacturer in the model i, $e^i>0$;
A_r^i	The publicity service efforts of retailer for new/remanufactured products in the model i, $A_r^i>0$.
Functions	
E_m^i	The remanufacturing capacity of manufacturer in the model i;
G_r^i	The goodwill level of manufacturer in the model i;
d_n^i	Consumer demand for new products in the model i;
d_z^i	Consumer demand for remanufactured products in the model i;
c_{mz}^i	Input cost of manufacturer's remanufacturing capacity in the model i;
c_{rp}^i	The retailer's input cost of new/remanufactured product publicity service activities in the model i;
T^i	The quantity of used products recycled by recyclers in the model i;
f_m^i	The profit of manufacturer in the model i;
f_r^i	The profit of retailer in the model i;
f_R^i	The profit of recycler in the model i;
f^i	The overall profit of supply chain system in the model i;
f_g^i	Social welfare in the model i.

3.3 Problem assumptions

Assumption 1. New/remanufactured products have no difference in quality and function, so it is assumed that new/remanufactured products have the same wholesale price (Zheng et al., 2017). However, consumers have different preferences for new/remanufactured products. New/remanufactured products have different retail prices.

Assumption 2. The publicity service efforts of retailer for new/remanufactured products will affect the goodwill level of manufacturer. The two are linear, i.e. $G_r^i=\varepsilon A_r^i$.

Assumption 3. The publicity service efforts of retailer for remanufactured products is kA_r^i, and for new products is $(1-k)A_r^i$. Then the retailer's input cost for new/remanufactured product publicity service activities is $c_{rp}^i=\frac{\omega(kA_r^i)^2}{2}+\frac{\omega(1-k)^2(A_r^i)^2}{2}$.

Assumption 4. The remanufacturing capacity of manufacturer is linearly related to the remanufacturing effort, i.e. $E_m^i = \upsilon e^i$, and the input cost of the remanufacturing capacity is $c_{mz}^i = \frac{\theta(E_m^i)^2}{2}$.

Assumption 5. The demand function is linear and related to the quantity of potential consumers, retail price, remanufacturing capacity and the goodwill level of manufacturer. The market demand for new products is $d_n^i = \varphi\tau - \mu p_n^i + k\xi G_r^i$. The market demand for remanufactured products is $d_z^i = (1-\tau)\varphi - \mu p_z^i + \alpha E_m^i + (1-k)\xi G_r^i$.

Assumption 6. The total amount of government subsidies, bonus-penalty for manufacturer's remanufacturing activities is $sd_z^i + h_j(E_m^i - E_0)$. The total bonus-penalty of government for retailer's publicity service efforts for remanufactured products is $h_z(kA_r^i - A_0)$.

Assumption 7. The quantity of recycled by the recycler is related to the recycling price, i.e. $T^i = \psi + \delta p_r^i$. The government conducts environmental treatment on the unrecycled used products. The quantity of unrecycled used products is $d_n^i + d_z^i - T^i$.

Assumption 8. The market is completely open and all members of the multi-level closed-loop supply chain make decisions under the condition of symmetrical information.

Assumption 9. There are $p_n^i > p_z^i > w_n^i > c_n > c_z + c_r$ (Zhao et al., 2019; Huang & Chen, 2019).

Assumption 10. Referring to literature (Li et al., 2021), social welfare is given by

$$f_g^i = \begin{cases} f^i - c_h(d_n^i + d_z^i - T^i), i \in \{\text{IC}, \text{I}M\}; \\ f^i - [d_z^i s + h_j(E_m^i - E_0) + h_z(kA_r^i - A_0) + c_h(d_n^i + d_z^i - T^i)], i \in \{\text{II}M, \text{II}G, \text{III}\}. \end{cases}$$

4. Model construction and solution

4.1. Non-government mechanisms and non-supply chain coordination

In the market environment without government mechanisms, the decision-making of the supply chain is only affected by the role of consumer market. The profit functions of manufacturer, retailer and recycler are given by

$$f_m^i = d_n^i(w_n^i - c_n) + d_z^i(w_n^i - c_z) - c_{mz}^i + (c_n - c_z - p_m^i)T^i, i \in \{\text{I}C, \text{I}M\} \tag{1}$$

$$f_r^i = d_n^i(p_n^i - w_n^i) + d_z^i(p_z^i - w_n^i) - c_{rp}^i, i \in \{\text{I}C, \text{I}M\} \tag{2}$$

$$f_R^i = (p_m^i - p_r^i - c_r)T^i, i \in \{\text{I}C, \text{I}M\} \tag{3}$$

The overall profit and social welfare functions of the supply chain are given by

$$f^i = d_n^i(p_n^i - c_n) + d_z^i(p_z^i - c_z) - c_{mz}^i - c_{rp}^i + (c_n - c_z - p_r^i - c_r)T^i, i \in \{\text{I}C, \text{I}M\} \tag{4}$$

$$f_g^i = f^i - c_h(d_n^i + d_z^i - T^i), i \in \{\text{I}C, \text{I}M\} \tag{5}$$

4.1.1. Centralized decision-making without governmental mechanisms (model $\text{I}C$*)*

Under centralized decision-making, manufacturer, retailer and recycler no longer make decisions alone, but jointly determine strategies. The decision variables to be determined by the recycling and remanufacturing supply chain system are as follows: the recycling price, the remanufacturing effort of the manufacturer, the retail price of new/remanufactured products, and the publicity service efforts of the retailer. The repurchase price and the wholesale price of new/remanufactured products, which only affect the profits among distribution members, will not affect the profits of the entire supply chain. The manufacturer, retailer and recycler jointly determine the retail price of new/remanufactured products, the recycling price of used products, the remanufacturing effort of the manufacturer and the publicity service efforts of the retailer, so as to maximize the profits of the whole recycling and remanufacturing supply chain system. The decision variables in the overall profit function $f^{\text{IC}}(p_n^{\text{IC}}, p_z^{\text{IC}}, p_r^{\text{IC}}, e^{\text{IC}}, A_r^{\text{IC}})$ of the supply chain are p_n^{IC}, p_z^{IC}, p_r^{IC}, e^{IC} and A_r^{IC}.

$$f^{\text{IC}} = d_n^{\text{IC}}(p_n^{\text{IC}} - c_n) + d_z^{\text{IC}}(p_z^{\text{IC}} - c_z) - c_{mz}^{\text{IC}} - c_{rp}^{\text{IC}} + (c_n - c_z - p_r^{\text{IC}} - c_r)T^{\text{IC}} \tag{6}$$

By using the reverse induction, we obtain the optimal results for the model $\text{I}C$, which are described in the following Proposition 1.

Proposition1. In the model $\text{I}C$, if $\xi^2\varepsilon^2 < \mu\omega$ and $\mu\theta > \alpha^2$, the overall profit function $f^{\text{IC}}(p_n^{\text{IC}}, p_z^{\text{IC}}, p_r^{\text{IC}}, e^{\text{IC}}, A_r^{\text{IC}})$ of the supply chain is a concave function of p_n^{IC}, p_z^{IC}, p_r^{IC}, e^{IC} and A_r^{IC}. The equilibrium results of manufacturer, retailer and recycler are shown as

$p_n^{IC*}=\frac{1}{2\mu A_3}[(\mu A_3+k\varepsilon\xi B_5)c_n+k\varepsilon\xi B_6c_z+\varphi\tau A_3-k\varepsilon\xi(2\mu\theta D_3+k\alpha^2\varphi\tau\varepsilon\xi)]$, $p_r^{IC*}=\frac{c_n-c_z-c_r}{2}-\frac{\psi}{2\delta}$

$p_z^{IC*}=\frac{1}{\alpha A_3}[-\mu\theta G_1c_n+(\alpha A_3-\mu\theta G_2)c_z+\theta D_2]$, $e^{IC*}=\frac{1}{\upsilon A_3}(-\mu G_1c_n-\mu G_2c_z+D_2)$, $A_r^{IC*}=\frac{1}{A_3}(B_5c_n+B_6c_z-2\mu\theta D_3-k\alpha^2\varphi\tau\varepsilon\xi)$,

where

$A_3=-\varepsilon^2\xi^2[-2k^2\alpha^2+2\theta\mu(2k^2-2k+1)]+2\mu\omega(2\mu\theta-\alpha^2)(2k^2-2k+1)$, $B_5=-k\mu\varepsilon\xi(2\mu\theta-\alpha^2)$, $B_6=-2\mu^2\theta\varepsilon\xi(1-k)$,

$D_2=2\mu\omega\varphi(1-\tau)(2k^2-2k+1)+k\varphi\varepsilon^2\xi^2(-k+\tau)$, $D_3=\varepsilon\xi\varphi(k-1+\tau-2k\tau)$, $G_1=\alpha\varepsilon^2\xi^2k(1-k)$,

$G_2=\alpha[2\mu\omega(2k^2-2k+1)-k^2\varepsilon^2\xi^2]$.

Proof. Please see **Appendix A.**

Further analysis of Proposition 1 leads to Lemma 1.

Lemma 1.(1). $\frac{\partial p_n^{IC*}}{\partial c_n}>0,\frac{\partial p_z^{IC*}}{\partial c_n}<0,\frac{\partial p_r^{IC*}}{\partial c_n}>0,\frac{\partial e^{IC*}}{\partial c_n}<0,\frac{\partial A_r^{IC*}}{\partial c_n}<0.$

(2). $\frac{\partial p_n^{IC*}}{\partial c_z}<0,\frac{\partial p_z^{IC*}}{\partial c_z}>0,\frac{\partial p_r^{IC*}}{\partial c_z}<0,\frac{\partial e^{IC*}}{\partial c_z}<0,\frac{\partial A_r^{IC*}}{\partial c_z}<0.$

(3).If $k\in(0,1),\left|\frac{\partial e^{IC*}}{\partial c_n}\right|<\left|\frac{\partial e^{IC*}}{\partial c_z}\right|$.

(4).If $k\in(0,\frac{1}{2}),\left|\frac{\partial A_r^{IC*}}{\partial c_n}\right|<\left|\frac{\partial A_r^{IC*}}{\partial c_z}\right|$; If $k\in(\frac{2}{3},1),\left|\frac{\partial A_r^{IC*}}{\partial c_n}\right|>\left|\frac{\partial A_r^{IC*}}{\partial c_z}\right|$.

Proof. Please see **Appendix B.**

Lemma 1(1) shows that in the market environment without government mechanisms, if centralized decision-making is adopted, with the increase of the production cost of new products, the retail price of new products and the recycling price of used products increase, while the retail price of remanufactured products, the remanufacturing effort of manufacturer and the publicity service efforts of retailer decrease. Lemma 1(2) indicates that as the production cost of remanufactured products rise, the retail price of new products, the recycling price of used products, the remanufacturing effort of the manufacturer and the publicity service efforts of retailer decrease, while the retail price of remanufactured products increase.

4.1.2. Decentralized decision-making without government mechanisms (model IM *)*

In the decentralized decision-making of non-government mechanisms, manufacturer dominate, retailer and recycler are followers. The sequence of decisions is described as follows: (a)the manufacturer determines the wholesale price w_n^{IM}, the remanufacturing effort e^{IM}, and the repurchase price p_m^{IM}, (b) the retailer sets the retail price p_n^{IM}, p_z^{IM} and the publicity service efforts A_r^{IM}, and(c) the recycler decides the recycling price p_r^{IM}.

By using the reverse induction, we obtain the optimal results for the model IM, which are described in the following Proposition 2.

Proposition 2. In the model IM, the recycler's profit function $f_R^{IM}(p_r^{IM})$ is a concave function of p_r^{IM}. If $\xi^2\varepsilon^2<2\mu\omega$, retailer's profit function $f_r^{IM}(p_n^{IM},p_z^{IM},A_r^{IM})$ is a concave function of p_n^{IM}, p_z^{IM} and A_r^{IM}. If $\mu\theta>\alpha^2$, the manufacturer's profit function $f_m^{IM}(w_n^{IM},e^{IM},p_m^{IM})$ is a concave function of w_n^{IM}, e^{IM} and p_m^{IM}. The equilibrium results of manufacturer, retailer and recycler are given as

$w_n^{IM*}=-\frac{1}{A_1}(\theta B_1c_n+B_3c_z+\theta B_2)$, $e^{IM*}=-\frac{1}{\upsilon A_1}(\alpha B_1c_n+B_4c_z+\alpha B_2)$, $p_m^{IM*}=\frac{c_n+c_r-c_z}{2}-\frac{\psi}{2\delta}$, $p_r^{IM*}=\frac{c_n-c_z-c_r}{4}-\frac{3\psi}{4\delta}$,

$p_n^{IM*}=\frac{1}{A_1A_2}[-(B_1-2k\mu\varepsilon^2\xi^2)(\theta B_1c_n+B_3c_z+\theta B_2)-G_1(\alpha B_1c_n+B_4c_z+\alpha B_2)+A_1D_1]$,

$p_z^{IM*}=\frac{1}{A_1A_2}[-(B_1-\mu\varepsilon^2\xi^2)(\theta B_1c_n+B_3c_z+\theta B_2)-G_2(\alpha B_1c_n+B_4c_z+\alpha B_2)+A_1D_2]$,

$A_r^{IM*}=\frac{2\mu}{A_1A_2}[\mu\varepsilon\xi(\theta B_1c_n+B_3c_z+\theta B_2)-\alpha\varepsilon\xi(1-k)(\alpha B_1c_n+B_4c_z+\alpha B_2)-A_1D_3]$.

where

$A_1=\varepsilon^2\xi^2(1-2k)[\alpha^2k+2\theta\mu(1-2k)]+2\mu\omega(\alpha^2-4\mu\theta)(2k^2-2k+1)$, $A_2=2\mu(2k^2-2k+1)(2\mu\omega-\varepsilon^2\xi^2)$,

$B_1=\mu[2\mu\omega(2k^2-2k+1)-\varepsilon^2\xi^2(2k^2-3k+1)]$, $B_2=2\mu\omega\varphi(2k^2-2k+1)+\varphi\varepsilon^2\xi^2(1-2k)(k-\tau)$,

$B_3 = (\mu\theta - \alpha^2)[2\mu\omega(2k^2 - 2k + 1) - k\varepsilon^2\xi^2(2k-1)]$, $B_4 = \mu\alpha[-6\mu\omega(2k^2 - 2k + 1) + \varepsilon^2\xi^2(6k^2 - 7k + 2)]$, $G_1 = \alpha\varepsilon^2\xi^2 k(1-k)$,
$G_2 = \alpha[2\mu\omega(2k^2 - 2k + 1) - k^2\varepsilon^2\xi^2]$, $D_1 = 2\mu\omega\varphi\tau(2k^2 - 2k + 1) + \varphi\varepsilon^2\xi^2(1-k)(k-\tau)$,
$D_2 = 2\mu\omega\varphi(1-\tau)(2k^2 - 2k + 1) + k\varphi\varepsilon^2\xi^2(-k+\tau)$, $D_3 = \varepsilon\xi\varphi(k - 1 + \tau - 2k\tau)$.

Proof. Please see **Appendix C.**

Further analysis of Proposition 2 leads to Lemma 2-4.

Lemma 2.(1). $\frac{\partial w_n^{IM*}}{\partial c_n} > 0, \frac{\partial e^{IM*}}{\partial c_n} > 0, \frac{\partial p_m^{IM*}}{\partial c_n} > 0, \frac{\partial A_r^{IM*}}{\partial c_n} < 0, \frac{\partial p_r^{IM*}}{\partial c_n} > 0.$

(2). $\frac{\partial w_n^{IM*}}{\partial c_z} > 0, \frac{\partial e^{IM*}}{\partial c_z} < 0, \frac{\partial p_m^{IM*}}{\partial c_z} < 0, \frac{\partial A_r^{IM*}}{\partial c_z} < 0, \frac{\partial p_r^{IM*}}{\partial c_z} < 0.$

(3). If $2\mu\omega \geq \frac{3+\sqrt{2}}{2}\varepsilon^2\xi^2, \frac{\partial p_n^{IM*}}{\partial c_n} > 0, \frac{\partial p_z^{IM*}}{\partial c_n} > 0.$

Proof. Please see **Appendix D1 .**

From Lemma 2, we find that with the increase of the production cost of new products, the wholesale price, the recycling price and repurchase price of used products, and the remanufacturing effort of the manufacturer increase, while the retailer's publicity service efforts for new/remanufactured products decrease. With the increase of production cost of remanufactured products, the wholesale price increases, while the recycling price and repurchase price of used products, the retailer's publicity service efforts for new/remanufactured products, and the remanufacturing effort of the manufacturer all decrease. Under certain circumstances, the retail price of new/remanufactured products increases with the increase of the production cost of new products.

Lemma 3. If $k \in (0, \frac{1}{2})$, $\frac{\partial w_n^{IM*}}{\partial \tau} < 0, \frac{\partial e^{IM*}}{\partial \tau} < 0, \frac{\partial A_r^{IM*}}{\partial \tau} < 0.$ If $k \in (\frac{1}{2}, 1)$, $\frac{\partial w_n^{IM*}}{\partial \tau} > 0, \frac{\partial e^{IM*}}{\partial \tau} > 0, \frac{\partial A_r^{IM*}}{\partial \tau} > 0.$
If $k \in (0,1)$, $\frac{\partial p_n^{IM*}}{\partial \tau} > 0, \frac{\partial p_z^{IM*}}{\partial \tau} < 0, \frac{\partial p_m^{IM*}}{\partial \tau} = \frac{\partial p_r^{IM*}}{\partial \tau} = 0.$

Proof. Please see **Appendix D2.**

From Lemma 3, it is found that if a retailer prefers the publicity service efforts for new products, with the quantity of ordinary consumers raises, the wholesale price, remanufacturing effort of manufacturer, the retailer's publicity service efforts for new/remanufactured products decrease. If the retailer prefers the publicity service efforts for remanufactured products, with the quantity of ordinary consumers raises, the wholesale price, remanufacturing effort of manufacturer, the retailer's publicity service efforts for new/remanufactured products increase. No matter whether retailers prefer publicity service efforts for new or remanufactured products, the retail price of new products increase, the retail price of remanufactured products decrease, the recycling price and repurchase price of used products do not change with the quantity of ordinary consumers.

lemma 4. (1). If $k \in (0, \frac{1}{2})$, $\frac{\partial w_n^{IM*}}{\partial c_n} < \frac{\partial w_n^{IM*}}{\partial c_z}$.

(2). $\forall k \in (0,1), \left|\frac{\partial p_m^{IM*}}{\partial c_n}\right| = \left|\frac{\partial p_m^{IM*}}{\partial c_z}\right|, \left|\frac{\partial p_r^{IM*}}{\partial c_n}\right| = \left|\frac{\partial p_r^{IM*}}{\partial c_z}\right|$.

Proof. Please see **Appendix D3.**

Lemma 4(1) shows that if the retailer prefers the publicity service efforts for new products, the change rate of wholesale price with the production cost of remanufactured products is higher than that of wholesale price with the production cost of new products. Lemma 4(2) indicates that no matter whether retailers prefer the publicity service efforts for new or remanufactured products, the change rate of the recycling price and repurchase price of used products with the production cost of remanufactured products is the same as that with the production cost of new products.

4.2. With government mechanisms

To guide and encourage enterprises to actively participate in recycling and remanufacturing activities, the government adopts certain subsidies and bonus-penalty mechanisms to affect the decisions of manufacturer and retailer, which indirectly affect the pricing decisions of the supply chain. Finally, the goal of improving product remanufacturing level and environmental performance will be realized. The profit functions of manufacturer, retailer and recycler are formulated as

$$f_m^i = d_n^i(w_n^i - c_n) + d_z^i(w_n^i - c_z) - c_{mz}^i + (c_n - c_z - p_m^i)T^i + d_z^i s + h_j(E_m^i - E_0), i \in \{\text{II}M, \text{II}G\} \tag{7}$$

$$f_r^i = d_n^i(p_n^i - w_n^i) + d_z^i(p_z^i - w_n^i) - c_{rp}^i + h_z(kA_r^i - A_0), i \in \{\text{II}M, \text{II}G\} \tag{8}$$

$$f_R^i = (p_m^i - p_r^i - c_r)T^i, i \in \{\text{II}M, \text{II}G\} \quad (9)$$

The overall profit and social welfare functions of supply chain are formulated as

$$f^i = f_m^i + f_r^i + f_R^i = d_n^i(p_n^i - c_n) + d_z^i(p_z^i - c_z) - c_{mz}^i - c_{rp}^i + (c_n - c_z - p_r^i - c_r)T^i + d_z^i s + h_j(E_m^i - E_0) + h_z(kA_r^i - A_0), i \in \{\text{II}M, \text{II}G\} \quad (10)$$

$$f_g^i = f^i - [d_z^i s + h_j(E_m^i - E_0) + h_z(kA_r^i - A_0) + c_h(d_n^i + d_z^i - T^i)], i \in \{\text{II}M, \text{II}G\} \quad (11)$$

4.2.1. Manufacturer-led decentralized decision (model $\text{II}M$)

In the model $\text{II}M$, the market environment is dominated by manufacturer with government mechanisms. Hence, retailer and recycler are followers. The decision sequence is as follows: (a)the manufacturer determines the wholesale price $w_n^{\text{II}M}$, the repurchase price $p_m^{\text{II}M}$, and the remanufacturing effort $e^{\text{II}M}$, (b)the retailer sets the retail price $p_n^{\text{II}M}$, $p_z^{\text{II}M}$ and the publicity service efforts $A_r^{\text{II}M}$, and (c)the recycler decides the recycling price $p_r^{\text{II}M}$.

By using the reverse induction, we obtain the optimal results for the model $\text{II}M$, which are described in the following Proposition 3.

Proposition 3. In the model $\text{II}M$, the recycler's profit function $f_R^{\text{II}M}(p_r^{\text{II}M})$ is a concave function of $p_r^{\text{II}M}$. If $\xi^2\varepsilon^2 < 2\mu\omega$, the retailer's profit function $f_r^{\text{II}M}(p_n^{\text{II}M}, p_z^{\text{II}M}, A_r^{\text{II}M})$ is a concave function of $p_n^{\text{II}M}$, $p_z^{\text{II}M}$ and $A_r^{\text{II}M}$. If $\mu\theta > \alpha^2$, the manufacturer's profit function $f_m^{\text{II}M}(w_n^{\text{II}M}, e^{\text{II}M}, p_m^{\text{II}M})$ is a concave function of $w_n^{\text{II}M}$, $p_m^{\text{II}M}$ and $e^{\text{II}M}$. The equilibrium decisions of manufacturer, retailer and recycler are given by

$$w_n^{\text{II}M*} = -\frac{1}{A_1}(\theta B_1 c_n + B_3 c_z + \theta B_2 + \alpha C_1 h_j + C_2 s + \theta C_3 h_z), \; e^{\text{II}M*} = -\frac{1}{\upsilon A_1}[\alpha B_1 c_n + B_4 c_z + \alpha B_2 + 2\mu(2C_1 - \varepsilon^2\xi^2)h_j - B_4 s + \alpha C_3 h_z],$$

$$p_m^{\text{II}M*} = \frac{c_n + c_r - c_z}{2} - \frac{\psi}{2\delta}, \; p_r^{\text{II}M*} = \frac{c_n - c_z - c_r}{4} - \frac{3\psi}{4\delta},$$

$$p_n^{\text{II}M*} = \frac{1}{A_1 A_2}\{-(B_1 - 2k\mu\varepsilon^2\xi^2)(\theta B_1 c_n + B_3 c_z + \theta B_2 + \alpha C_1 h_j + C_2 s + \theta C_3 h_z) - G_1[\alpha B_1 c_n + B_4 c_z + \alpha B_2 + 2\mu(2C_1 - \varepsilon^2\xi^2)h_j - B_4 s + \alpha C_3 h_z] + A_1(D_1 + kC_3 h_z)\},$$

$$p_z^{\text{II}M*} = \frac{1}{A_1 A_2}\{-(B_1 - \mu\varepsilon^2\xi^2)(\theta B_1 c_n + B_3 c_z + \theta B_2 + \alpha C_1 h_j + C_2 s + \theta C_3 h_z) - G_2[\alpha B_1 c_n + B_4 c_z + \alpha B_2 + 2\mu(2C_1 - \varepsilon^2\xi^2)h_j - B_4 s + \alpha C_3 h_z] + A_1[D_2 + (1-k)C_3 h_z]\},$$

$$A_r^{\text{II}M*} = \frac{2\mu}{A_1 A_2}\{\mu\varepsilon\xi(\theta B_1 c_n + B_3 c_z + \theta B_2 + \alpha C_1 h_j + C_2 s + \theta C_3 h_z) - \alpha\varepsilon\xi(1-k)[\alpha B_1 c_n + B_4 c_z + \alpha B_2 + 2\mu(2C_1 - \varepsilon^2\xi^2)h_j - B_4 s + \alpha C_3 h_z] - A_1(D_3 - 2k\mu h_z)\},$$

where

$C_1 = 2\mu\omega(2k^2 - 2k + 1) + k\varepsilon^2\xi^2(1-2k)$, $C_2 = 2\mu\omega(\alpha^2 - \mu\theta - 2k^2\mu\theta + 2\alpha^2 k^2 - 2\alpha^2 k) + k\varepsilon^2\xi^2(\alpha^2 - \mu\theta)(1-2k)$, $C_3 = 2k\mu\varepsilon\xi$.

Proof. Please see **Appendix E.**

By analyzing Proposition 3, Lemma 5 and Lemma 6 are obtained.

Lemma 5. (1). $\frac{\partial w_n^{\text{II}M*}}{\partial h_j} > 0, \frac{\partial e^{\text{II}M*}}{\partial h_j} > 0, \frac{\partial p_m^{\text{II}M*}}{\partial h_j} = \frac{\partial p_r^{\text{II}M*}}{\partial h_j} = 0.$

(2). $\frac{\partial w_n^{\text{II}M*}}{\partial h_z} > 0, \frac{\partial e^{\text{II}M*}}{\partial h_z} > 0, \frac{\partial p_m^{\text{II}M*}}{\partial h_z} = \frac{\partial p_r^{\text{II}M*}}{\partial h_z} = 0.$

(3). $\frac{\partial w_n^{\text{II}M*}}{\partial s} < 0, \frac{\partial e^{\text{II}M*}}{\partial s} > 0, \frac{\partial A_r^{\text{II}M*}}{\partial s} > 0, \frac{\partial p_m^{\text{II}M*}}{\partial s} = \frac{\partial p_r^{\text{II}M*}}{\partial s} = 0.$

Proof. Please see **Appendix E1.**

Lemma 5 shows that with the increase of the government's bonus-penalty coefficient for manufacturer's remanufacturing capacity, the wholesale price and the manufacturer's remanufacturing effort increase, but the recycling price and repurchase price of used products remain unchanged. With the increase of bonus-penalty coefficient of the government's promotional service activities for retailer's remanufactured products, the wholesale price and the remanufacturing effort of manufacturers increase, but the recycling price and repurchase price of used products remain unchanged. As the government's subsidies for basic units of remanufactured products rise, the manufacturer's remanufacturing effort and the retailer's publicity service efforts increase, but the wholesale prices decrease, while the recycling price and repurchase price of used products remain unchanged.

Lemma 6. $\frac{\partial e^{\text{II}M*}}{\partial h_j} > \frac{\partial e^{\text{II}M*}}{\partial s}$.

Proof. Please see **Appendix E2.**

Lemma 6 shows that in the model $\text{II}M$, the change rate of manufacturer's remanufacturing effort with government bonus-penalty is higher than that with government's subsidies. The bonus-penalty mechanisms are more effective than the subsidy policy in the model $\text{II}M$.

Further analysis of Proposition 2 and Proposition 3 leads to Lemma 7 and Lemma 8.

Lemma 7. (1). $e^{\text{I}M*} < e^{\text{II}M*}$, $A_r^{\text{I}M*} < A_r^{\text{II}M*}$.

(2). $p_r^{\text{I}M*} = p_r^{\text{II}M*}$, $p_m^{\text{I}M*} = p_m^{\text{II}M*}$.

(3).The relationship of $w_n^{\text{I}M*}$ and $w_n^{\text{II}M*}$, $p_n^{\text{I}M*}$ and $p_n^{\text{II}M*}$, $p_z^{\text{I}M*}$ and $p_z^{\text{II}M*}$ are related to h_j, s and h_z.

Proof. Please see **Appendix F.**

Lemma 7 shows that if the government adopts a series of bonus-penalty and subsidy mechanisms to manufacturer and retailer, it encourages the node enterprises in the supply chain to actively participate in the production and sales of remanufactured products. Compared with model $\text{I}M$, in model $\text{II}M$, the remanufacturing effort is increased, and the remanufacture level is increased, $e^{\text{I}M*} < e^{\text{II}M*}, E_m^{\text{I}M*} < E_m^{\text{II}M*}$. Retailer's publicity service efforts for new/remanufactured products has increased, and the level of corporate goodwill has increased, $A_r^{\text{I}M*} < A_r^{\text{II}M*}, G_r^{\text{I}M*} < G_r^{\text{II}M*}$. Due to the direct effect of the government mechanisms on upstream manufacturer and retailer, there is a lack of incentives for recycler, and it is unable to increase the recycling price and repurchase price of used products. The recycling quantity of used products has not changed.

Lemma 8. (1). $f_R^{\text{II}M*} = f_R^{\text{I}M*}$.

(2). If $E_m^{\text{II}M} > E_0$, $f_m^{\text{II}M*} > f_m^{\text{I}M*}$. If $E_m^{\text{II}M} < E_0$, $f_m^{\text{II}M*}$ is related to the government bonus-penalty coefficient h_j and the manufacturer's actual remanufacture level $E_m^{\text{II}M}$.

(3). If $kA_r^{\text{II}M} > A_0$, $f_r^{\text{II}M*} > f_r^{\text{I}M*}$. If $kA_r^{\text{II}M} < A_0$, $f_r^{\text{II}M*}$ is related to the government bonus-penalty coefficient h_z and the retailer's actual publicity service level $kA_r^{\text{II}M}$ for remanufactured products.

Lemma 8 (1) shows that the government mechanisms do not act on the recycler, so there is no difference in the profit of the recycler in model $\text{I}M$ and model $\text{II}M$. Lemma 8 (2) indicates that if $E_m^{\text{II}M} > E_0$, the basic remanufacture level and the bonus-penalty coefficient set by the government as incentive measures promote the increase of manufacturer's profit. If $E_m^{\text{II}M} < E_0$, the government takes punitive measures. Manufacturer's profit is affected by many factors. Therefore, in the early stage of the development of remanufacturing industry, the government set too high a basic remanufacturing level, which led to the decline of manufacturer's profit. It was not conducive to the improvement of the overall benefits of the supply chain. From Lemma 8 (3), we find that if $kA_r^{\text{II}M} > A_0$, the basic retailer's publicity service efforts for new/remanufactured products and the bonus-penalty coefficient set by the government for remanufactured products can be used as incentive measures to promote the improvement of retailer's profit. If $kA_r^{\text{II}M} < A_0$, the government will take punitive measures. Retailer's profit is affected by many factors. Therefore, in the early stage of the development of remanufacturing industry, the government set too high the effort level of basic publicity service, which led to the decline of retailer's profit. That was not conducive to the improvement of the overall benefits of the supply chain.

Comparing model and model, manufacturer and retailer will adopt different product production and sales strategies in the environment of whether there is a government mechanism. Compared with the model , the remanufacturing level of the manufacturer, the publicity service efforts for new/remanufactured products and the level of corporate goodwill under the model are effectively improved. Both manufacturer and retailer's profits increased. No matter what pricing strategy is adopted in remanufacturing supply chains, the profit of recyclers remains unchanged, indicating that the incentive effect of government mechanisms on recyclers is invalid.

To sum up, the government mechanisms can effectively promote the development of the remanufacture industry, guide manufacturers to improve product remanufacture level, promote retailers to improve their efforts in product publicity and service, and finally achieve the purpose of improving the level of corporate goodwill. At the same time, the government mechanisms are conducive to the improvement of the overall benefits of enterprises and supply chains.

4.2.2 Government-led decentralized decision(model $\text{II}G$ *)*

In the model $\text{II}G$, the government is the market leader, and manufacturer, retailer and recycler are the followers. The order of decision-making is as follows:(a) the government makes decisions on the remanufacturing effort $e^{\text{II}G}$ and the retailer's publicity service efforts $A_r^{\text{II}G}$ with the goal of maximizing social welfare, (b)the manufacturer decides the wholesale price $w_n^{\text{II}G}$

and the repurchase price $p_m^{\mathrm{II}G}$, (c) the retailer determines the retail price $p_n^{\mathrm{II}G}$, $p_z^{\mathrm{II}G}$, and (d) the recycler sets the recycling price $p_r^{\mathrm{II}G}$.

By using the reverse induction, we obtain the optimal results for the model $\mathrm{II}G$, which are described in the following Proposition 4.

Proposition 4. In the model $\mathrm{II}G$, the recycler's profit function $f_R^{\mathrm{II}G}(p_r^{\mathrm{II}G})$ is a concave function of $p_r^{\mathrm{II}G}$. The retailer's profit function $f_r^{\mathrm{II}G}(p_n^{\mathrm{II}G}, p_z^{\mathrm{II}G})$ is a concave function of $p_n^{\mathrm{II}G}$ and $p_z^{\mathrm{II}G}$. The manufacturer's profit function $f_m^{\mathrm{II}G}(w_n^{\mathrm{II}G}, p_m^{\mathrm{II}G})$ is a concave function of $w_n^{\mathrm{II}G}$ and $p_m^{\mathrm{II}G}$. If $\mu\theta > \alpha^2$ and $\xi^2\varepsilon^2 < \mu\omega$, the social welfare function $f_g^{\mathrm{II}G}(A_r^{\mathrm{II}G}, e^{\mathrm{II}G})$ is a concave function of $A_r^{\mathrm{II}G}$ and $e^{\mathrm{II}G}$. The equilibrium results of government, manufacturer, retailer and recycler decisions are as follows.

$$w_n^{\mathrm{II}G*} = \frac{1}{4\mu A_4}[\upsilon\varepsilon\xi(B_9c_n + B_{10}c_z + C_5 - 4C_6c_h + C_6s) + \alpha\upsilon(B_7c_n + B_8c_z + \alpha B_2 + C_4 - 4\mu\alpha C_1c_h + \mu\alpha C_1s) + A_4(\varphi + \mu c_n + \mu c_z - \mu s)],$$

$$e^{\mathrm{II}G*} = \frac{1}{A_4}(B_7c_n + B_8c_z + \alpha B_2 + C_4 - 4\mu\alpha C_1c_h + \alpha\mu C_1s),\ p_m^{\mathrm{II}G*} = \frac{c_n + c_r - c_z}{2} - \frac{\psi}{2\delta},\ p_r^{\mathrm{II}G*} = \frac{c_n - c_z - c_r}{4} - \frac{3\psi}{4\delta},$$

$$p_n^{\mathrm{II}G*} = \frac{1}{8A_4\mu}[(1+4k)\upsilon\varepsilon\xi(B_9c_n + B_{10}c_z + C_5 - 4C_6c_h + C_6s) + A_4(\varphi + \mu c_n + \mu c_z - \mu s) + 4A_4\varphi\tau + \alpha\upsilon(B_7c_n + B_8c_z + \alpha B_2 + C_4 - 4\mu\alpha C_1c_h + \mu\alpha C_1s)],$$

$$p_z^{\mathrm{II}G*} = \frac{1}{8\mu A_4}[5\alpha\upsilon(B_7c_n + B_8c_z + \alpha B_2 + C_4 - 4\mu\alpha C_1c_h + \mu\alpha C_1s) + A_4(\varphi + \mu c_n + \mu c_z - \mu s) + 4A_4\varphi(1-\tau) + (5-4k)\varepsilon\xi\upsilon(B_9c_n + B_{10}c_z + C_5 - 4C_6c_h + C_6s)],$$

$$A_r^{\mathrm{II}G*} = \frac{\upsilon}{A_4}(B_9c_n + B_{10}c_z + C_5 - 4C_6c_h + C_6s),$$

where

$$A_4 = 2\upsilon[\mu\omega(16\mu\theta - 7\alpha^2)(2k^2 - 2k + 1) + 3k^2\alpha^2\varepsilon^2\xi^2 - \mu\theta\varepsilon^2\xi^2(16k^2 - 16k + 7)] > 0,\ B_9 = \mu\varepsilon\xi[2\mu\theta(1-8k) + 6\alpha^2k],$$

$$B_{10} = \mu^2\theta\varepsilon\xi(16k - 14),\ B_7 = 2\mu\alpha[\mu\omega(2k^2 - 2k + 1) + 3\varepsilon^2\xi^2(k^2 - k)] > 0,\ B_8 = 2\mu\alpha[7\mu\omega(-2k^2 + 2k - 1) + 3\varepsilon^2\xi^2k^2],$$

$$C_4 = 2\mu\omega\alpha\varphi(2k^2 - 2k + 1)(7 - 8\tau) + 6\alpha\varepsilon^2\xi^2\varphi k(\tau - k),\ C_5 = \varepsilon\xi[2\mu\varphi\theta(7 - 8\tau - 8k + 16k\tau) - 6\alpha^2k\varphi\tau],\ C_6 = \mu\varepsilon\xi(2\mu\theta - k\alpha^2).$$

Proof. Please see **Appendix G.**

Analyzing Proposition 4 leads to Lemma 9.

Lemma 9.

(1). $\frac{\partial e^{\mathrm{II}G*}}{\partial c_h} < 0, \frac{\partial A_r^{\mathrm{II}G*}}{\partial c_h} < 0.$

(2). $\frac{\partial e^{\mathrm{II}G*}}{\partial s} > 0, \frac{\partial A_r^{\mathrm{II}G*}}{\partial s} > 0.$

Proof. Please see **Appendix H.**

Lemma 9 indicates that in model IIG, as the government increases the unit environmental treatment cost of our recycled used products in the consumer market, the remanufacturing effort of manufacturer and the publicity service efforts of retailer for new/remanufactured products decrease. With the increase of government's subsidies for remanufactured products, the remanufacturing effort of manufacturers and publicity service efforts of retailers for new/remanufactured products have increased.

5. Government mechanisms and supply chain coordination model

In the model $\mathrm{II}M$, on the one hand, the government mechanisms lack incentives for recycler. On the other hand, decentralized decision-making has double marginal effects, which leads to the reduction of system efficiency, the failure to reach the Pareto optimal solution. At the same time, the profits of each node enterprise in the supply chain and social welfare are also low. The purpose of establishing coordination mechanisms is to improve the performance of remanufacturing supply chains and further improve social welfare. According to the extended producer responsibility system, this section adopts the recycling cost-sharing contract to improve the recycling and remanufacturing supply chain operation based on government participation. The manufacturer helps the recycler to share the recycling cost of σ and the recycler to share the recycling cost of $1-\sigma$. After adopting the coordination mechanism, the profit functions of manufacturer, retailer and recycler are as follows:

$$f_m^{\mathrm{III}} = d_n^{\mathrm{III}}(w_n^{\mathrm{III}} - c_n) + d_z^{\mathrm{III}}(w_n^{\mathrm{III}} - c_z) - c_{mz}^{\mathrm{III}} + (c_n - c_z - p_m^{\mathrm{III}})T^{\mathrm{III}} + d_z^{\mathrm{III}}s + h_j(E_m^{\mathrm{III}} - E_0) - \sigma(p_r^{\mathrm{III}} + c_r)T^{\mathrm{III}} \tag{12}$$

$$f_r^{\mathrm{III}} = d_n^{\mathrm{III}}(p_n^{\mathrm{III}} - w_n^{\mathrm{III}}) + d_z^{\mathrm{III}}(p_z^{\mathrm{III}} - w_n^{\mathrm{III}}) - c_{rp}^{\mathrm{III}} + h_z(kA_r^{\mathrm{III}} - A_0) \tag{13}$$

$$f_R^{\mathrm{III}} = [p_m^{\mathrm{III}} - (1-\sigma)(p_r^{\mathrm{III}} + c_r)]T^{\mathrm{III}} \tag{14}$$

The overall profit and social welfare functions of the supply chain are as follows:

$$f^{\text{III}} = f_m^{\text{III}} + f_r^{\text{III}} + f_R^{\text{III}} = d_n^{\text{III}}(p_n^{\text{III}} - c_n) + d_z^{\text{III}}(p_z^{\text{III}} - c_z) - c_{mz}^{\text{III}} - c_{rp}^{\text{III}} + (c_n - c_z - p_r^{\text{III}} - c_r)T^{\text{III}} + d_z^{\text{III}} s + h_j(E_m^{\text{III}} - E_0) + h_{z(}(kA_r^{\text{III}} - A_0) \quad (15)$$

$$f_g^{\text{III}} = f^{\text{III}} - [d_z^{\text{III}} s + h_j(E_m^{\text{III}} - E_0) + h_z(kA_r^{\text{III}} - A_0) + c_h(d_n^{\text{III}} + d_z^{\text{III}} - T^{\text{III}})] \quad (16)$$

If the coordination mechanism is adopted, in order to achieve the optimal solution for each entity in the supply chain, the profit function of the recycler is first analyzed, which can make $p_m^{\text{III}*} = p_m^{\text{II}M*}$. According to the optimal response function of the recycler's recycling price with respect to the manufacturer's repurchase price, the following can be obtained:

$$p_r^{\text{III}*} = \frac{\psi(1-\sigma) + \delta[p_m^{\text{III}*} - c_r(1-\sigma)]}{2\delta(1-\sigma)} \quad (17)$$

Then the retailer's profit function is analyzed, $A_r^{\text{III}*} = A_r^{\text{I}C*}, e^{\text{III}*} = e^{\text{II}M*}$. According to the optimal response function of the retailer's publicity service efforts on the wholesale price and remanufacturing effort, the temporary value $w_n^{\text{III}O}$ of the wholesale price can be obtained.

$$w_n^{\text{III}O} = \frac{\psi(1-\sigma) + \delta[p_m^{\text{III}*} - c_r(1-\sigma)]}{2\delta(1-\sigma)} \quad (18)$$

Finally, the optimal solutions of the retail prices p_n^{III} and p_z^{III} are obtained.

$$p_n^{\text{III}*} = \frac{\psi(1-\sigma) + \delta[p_m^{\text{III}*} - c_r(1-\sigma)]}{2\delta(1-\sigma)} \text{ and } p_z^{\text{III}*} = \frac{\psi(1-\sigma) + \delta[p_m^{\text{III}*} - c_r(1-\sigma)]}{2\delta(1-\sigma)} \quad (19)$$

Thus, the profit function of manufacturer is analyzed so that the equilibrium solution of the wholesale price is $w_n^{\text{III}*} = w_n^{\text{II}G*}$. After adopting the recycling cost-sharing contract, the profit functions of manufacturer, retailer and recycler are as follows:

$$f_m^{\text{III}*} = (\varphi\tau - \mu p_n^{\text{III}*} + k\varepsilon\xi A_r^{\text{III}*})(w_n^{\text{III}*} - c_n) + [(1-\tau)\varphi - \mu p_z^{\text{III}*} + \alpha v e^{\text{III}*} + (1-k)\varepsilon\xi A_r^{\text{III}*}](w_n^{\text{III}*} - c_z) - \frac{\theta(ve^{\text{III}*})^2}{2} \quad (20)$$
$$+ (c_n - c_z - p_m^{\text{III}*} - \sigma p_r^{\text{III}*})(\psi + \delta p_r^{\text{III}*}) + [(1-\tau)\varphi - \mu p_z^{\text{III}*} + \alpha v e^{\text{III}*} + (1-k)\varepsilon\xi A_r^{\text{III}*}]s + h_j(ve^{\text{III}*} - E_0)$$

$$f_r^{\text{III}*} = (\varphi\tau - \mu p_n^{\text{III}*} + k\varepsilon\xi A_r^{\text{III}*})[p_n^{\text{III}*} - w_n^{\text{III}*}] + [(1-\tau)\varphi - \mu p_z^{\text{III}*} + \alpha v e^{\text{III}*} + (1-k)\varepsilon\xi A_r^{\text{III}*}][p_z^{\text{III}*} - w_n^{\text{III}*}] \quad (21)$$
$$- \frac{(\omega A_r^{\text{III}*})^2(2k^2 - 2k + 1)}{2} + h_z(kA_r^{\text{III}*} - A_0)$$

$$f_R^{\text{III}*} = [p_m^{\text{III}*} - (1-\sigma)p_r^{\text{III}*} - c_r](\psi + \delta p_r^{\text{III}*}) \quad (22)$$

The overall profit and social welfare functions of the supply chain are as follows:

$$f^{\text{III}*} = d_n^{\text{III}*}(p_n^{\text{III}*} - c_n) + d_z^{\text{III}*}(p_z^{\text{III}*} - c_z) - c_{mz}^{\text{III}*} - c_{rp}^{\text{III}*} + (c_n - c_z - p_r^{\text{III}*} - c_r)T^{\text{III}*} + d_z^{\text{III}*} s + h_j(E_m^{\text{III}*} - E_0) + h_{z(}(kA_r^{\text{III}*} - A_0) \quad (23)$$

$$f_g^{\text{III}*} = f^{\text{III}*} - [d_z^{\text{III}*} s + h_j(E_m^{\text{III}*} - E_0) + h_z(kA_r^{\text{III}*} - A_0) + c_h(d_n^{\text{III}*} + d_z^{\text{III}*} - T^{\text{III}*})] \quad (24)$$

Only in the case of individual rationality constraints, that is, the profits of all entities in the supply chain are higher than the profits before coordination, they will be willing to adopt the coordination mechanism. The proportional parameter σ needs to satisfy the conditions $f_m^{\text{III}*} \ge f_m^{\text{II}*}$, $f_r^{\text{III}*} \ge f_r^{\text{II}*}, f_R^{\text{III}*} \ge f_R^{\text{II}*}, f_g^{\text{III}*} \ge f_g^{\text{II}*}$, from which the value range of σ can be obtained. The specific size is determined by the negotiation ability of both parties. As the above expressions are complex, it will be discussed in the numerical analysis.

Proposition 5. After adopting the recycling cost-sharing contract, if σ takes a certain range of values, we get $f_m^{\text{III}*} > f_m^{\text{II}*}$, $f_r^{\text{III}*} > f_r^{\text{II}*}, f_R^{\text{III}*} > f_R^{\text{II}*}, f^{\text{III}*} > f^{\text{II}*}, f_g^{\text{III}*} > f_g^{\text{II}M*}$.

6. Numerical analysis and discussion

In order to further illustrate the correctness of the above conclusions and the effectiveness of the coordination contract, a specific example is given below. This paper takes the closed-loop supply chain of a household electrical appliance as the research object and gives the data in Table 2 according to the symbols and assumptions.

Table 2
Parameter values

υ	ε	k	φ	τ	α	ξ	c_n	c_z	c_r	E_0
8	0.5	0.4	100	0.6	0.2	0.12	40	20	3	3.5
A_0	s	h_j	h_z	ψ	δ	μ	θ	ω	c_h	
1.1	1	2.5	3	10	2	0.4	1	1.2	1.5	

The optimal solutions under centralized decision and decentralized decision before the coordination mechanism is adopted are shown in Table 3. It is found that only in the case of the role of the consumer market, the retail price under centralized decision-making is lower than that under decentralized decision-making, but the recycling price, recycling quantity, product publicity service ability, the overall profit and social welfare of the supply chain are better than those under decentralized decision-making, which are more conducive to the development of the closed-loop supply chain. Under centralized decision-making, on the one hand, consumers can buy products at a cheaper price and recycle used products at a higher price, which increases the motivation for consumers to participate in recycling activities; On the other hand, the goodwill level of enterprises is also high, which has a higher utility than decentralized decision-making. However, compared with decentralized decision-making, the remanufacturing effort under centralized decision-making is lower. The enthusiasm of manufacturers to participate in remanufacturing activities is not high. If the manufacturer is the leader of the supply chain, with the participation of the government, the remanufacturing effort of the manufacturer and the publicity service efforts of the retailer are much higher than those in the consumer market. Meanwhile, the wholesale price and retail price have little change, and the recycling price and repurchase price have not changed. Therefore, the profits of manufacturers and retailers have greatly increased. However, the quantity of recycled used products has not changed, and the government also needs to pay certain subsidies to manufacturers and retailers, so social welfare is reduced. If the government is the leader of the supply chain, the government incentives do not work for recyclers. Therefore, compared with the situation dominated by manufacturers, the quantity of recycled used products has not changed. However, in order to improve social welfare, the government will try its best to reduce government subsidies and bonus-penalty, for that manufacturer's remanufacturing effort and retailer's publicity service efforts drop considerably. This is not conducive to the development of the recycling and remanufacturing industry.

Table 3
Optimal solutions

Model i	w_n^i	p_n^i	p_z^i	p_r^i	p_m^i	e^i	A_r^i	T^i	f_m^i	f_r^i	f_R^i	f^i	f_g^i
$i = \text{I}C$	-	95.1	62.3	6	-	1.1	4.6	22	-	-	-	2132	2107
$i = \text{I}M$	79.0	114.6	92.6	0.5	9.0	1.5	2.2	11	924.2	578.4	60.5	1563.2	1550.2
$i = \text{II}M$	79.1	114.7	93.4	0.5	9.0	1.8	4.1	11	942.4	583.4	60.5	1586.3	1538.0
$i = \text{II}G$	78.1	114.2	90.7	0.5	9.0	0.7	3.3	11	956.1	580.1	60.5	1596.9	1572.5

Assuming that the values of other basic parameters remain unchanged, then we analyze the effects of preferences parameters (the proportion of ordinary consumers and the proportion of retailer's publicity service efforts for remanufactured products) and government mechanisms parameters (unit subsidies and bonus-penalty coefficients) on decisions, enterprise profits and social welfare of remanufacturing supply chain.

6.1. Effects of preferences (τ and k) on supply chain

6.1.1 Effects of the proportion of ordinary consumers (τ)

The effects of the proportion of ordinary consumers τ on the remanufacturing effort e^i is shown in Fig. 2. It is found that e^i decrease with the increase of τ whether there are government mechanisms or not. Since the remanufacturing capacity E_m^i is positively correlated with e^i, E_m^i decreases. In the consumer market, if consumers prefer new products, it means that consumers' acceptance of remanufactured products is not high, and manufacturer lacks the enthusiasm of remanufacturing activities. In addition, we find that, (a) $\forall \tau \in (0,0.29)$, $e^{\text{IC}*} > e^{\text{IIG}*} > e^{\text{IIM}*} > e^{\text{IM}*}$, (b) $\forall \tau \in (0.29,0.38)$, $e^{\text{IC}*} > e^{\text{IIM}*} > e^{\text{IIG}*} > e^{\text{IM}*}$, (c) $\forall \tau \in (0.38,0.5)$, $e^{\text{IIM}*} > e^{\text{IC}*} > e^{\text{IM}*} > e^{\text{IIG}*}$ and (d) $\forall \tau \in (0.5,1)$, $e^{\text{IIM}*} > e^{\text{IM}*} > e^{\text{IC}*} > e^{\text{IIG}*}$. Comparing the four models, if there are fewer ordinary consumers, the remanufacturing effort in model $\text{I}C$ is the highest. However, with the increase of the proportion of ordinary consumers, the remanufacturing effort in model $\text{I}C$ and $\text{II}G$ have decreased rapidly. If there are many ordinary consumers, the remanufacturing effort in model $\text{II}M$ is the highest. And with the increase of the proportion of ordinary consumers, the degree of remanufacturing effort in model $\text{II}M$ decreases particularly slowly. So, considering the remanufacturing effort of the manufacturer, the model $\text{II}M$ is optimal.

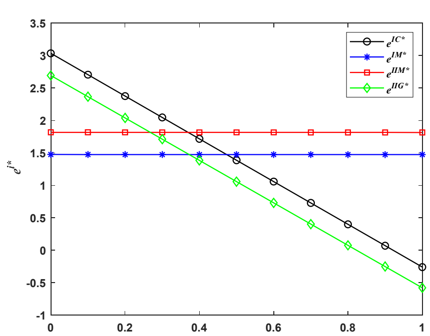


Fig. 2. The effects *of* τ on e^i.

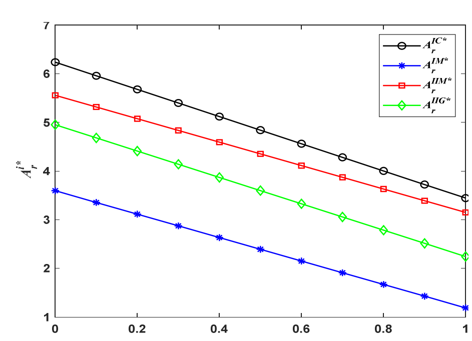
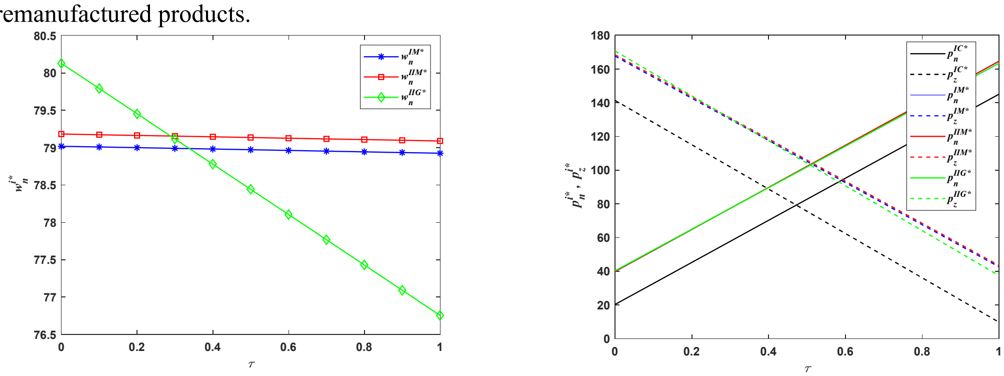


Fig. 3. The effects *of* τ on A_r^i.

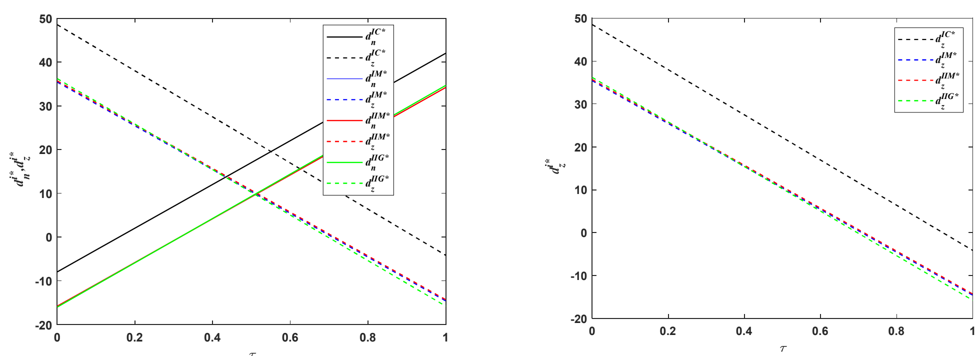
Fig.3 illustrates the effects of the proportion of ordinary consumers τ on the retailer's publicity service efforts A_r^i. With the increase of τ, A_r^i decrease whether there is a government mechanism or not. In addition, $\forall \tau \in (0,1)$, $A_r^{IC*} > A_r^{IIM*} > A_r^{IIG*} > A_r^{IM*}$. However, in the four models, with the increase of the proportion of ordinary consumers, the retailer's publicity service efforts in model IIM decreased the slowest. Based on the above, considering the retailer's publicity service efforts, the model IIM is optimal. Fig. 4 demonstrates the effects of the proportion of ordinary consumers τ on the wholesale price w_n^i, the retail price of new products p_n^i and the retail price of remanufactured products p_z^i. Some interesting conclusions were found. (a)With the increase of τ, w_n^{IM} and w_n^{IIM} decrease slowly, but w_n^{IIG} rapidly decreases. In addition, $\forall \tau \in (0,1)$, $w_n^{IIM*} > w_n^{IM*}$. From the perspective of the manufacturer, model IIM is optimal. No matter what consumers' preferences are, the wholesale prices of manufacturer change little, and meet the price needs of two kinds of consumers at the same time. (b)With the increase of τ, p_n^i increases, but p_z^i decreases. The higher the value of τ, the higher the preference of consumers for new products and the willingness to pay higher fees for new products. Retailer chooses to raise the retail price of new products and reduce the retail price of remanufactured products.



(a) The effects *of* τ on w_n^i. (b) The effects *of* τ on p_n^i and p_z^i.

Fig. 4. The effects *of* τ on w_n^i, p_n^i and p_z^i.

Fig. 5 depicts that in the four models, with the increase of the proportion of ordinary consumers τ, the demand for new products d_n^i increases and the demand for remanufactured products d_z^i decreases. The higher the value of τ, the higher the preference of consumers for new products. The market demand for new products has been expanded. Retailer can make more profit by selling new products.

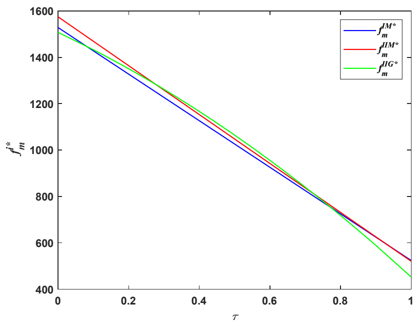


(a) The effects *of* τ on d_n^i and d_z^i. (b) The effects *of* τ on d_z^i.

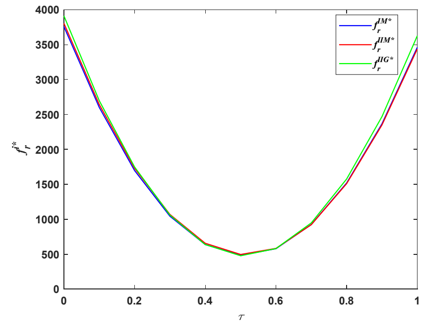
Fig. 5. The effects *of* τ on d_n^i and d_z^i.

Combining Fig. 4 and Fig. 5, we find that in model IIM, if ordinary consumers increase by a certain amount, manufacturer can sell new/remanufactured products to retailer at a higher price, while retailer can sell remanufactured products to consumers at a higher price. Moreover, the market demand for remanufactured products has also maintained a high level. Therefore, model IIM is more conducive to the development of remanufacturing supply chain. Fig. 6 depicts the effects of the proportion of ordinary consumers τ on manufacturer's profit f_m^i, retailer's profit f_r^i, overall supply chain profit f^i and social welfare f_g^i. In the four models, f_m^i decreases with the increase of τ. This is mainly because the retailer's publicity service efforts A_r^i is negatively correlated with τ, while A_r^i is linearly positively correlated with the manufacturer's goodwill level G_r^i. If τ increases, G_r^i decreases, and the manufacturer's profit from selling new/remanufactured products decreases. From the size of f_m^i, model IIM is the best. In the four models, f_r^i decrease first and then increase. Therefore, $\exists \tau_0 = 0.5$, f_r^i takes the minimum value.When the proportion of consumers who prefer new products is high, retailer can make profits by selling new products.

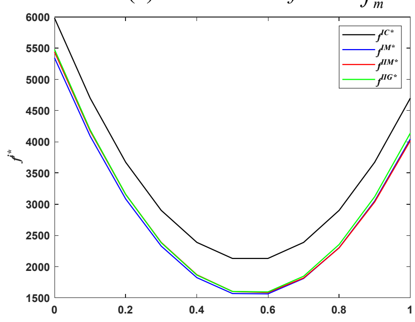
On the contrary, retailer can make profits by selling remanufactured products. In model $\mathrm{II}M$ and $\mathrm{II}G$, retailer can obtain government incentives by publicity service remanufactured products. As a result, when $\tau < 0.5$, $f_r^{\mathrm{II}G} > f_r^{\mathrm{II}M} > f_r^{\mathrm{I}M}$ .With the participation of the government, retailer should augment the publicity service for remanufactured products and encourage more consumers to buy remanufactured products. In the four models, f^i and f_g^i decrease first and then increase. Therefore, $\exists \tau_0 \in [0.5,0.6]$, f^i and f_g^i take the minimum value. In the decentralized model, no matter which product the consumers preference, model $\mathrm{II}G$ is optimal from the perspective of the retailer's profit , the overall profit of the supply chain and social welfare.



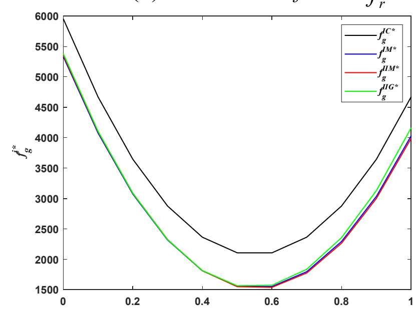
(a) The effects *of* τ on f_m^i.



(b) The effects *of* τ on f_r^i.



(c) The effects *of* τ on f^i.



(d) The effects *of* τ on f_g^i.

Fig. 6. The effects *of* τ on f_m^i, f_r^i, f^i and f_g^i.

6.1.2 Effects of proportion of retailer's publicity service efforts for remanufactured products(k *)*

The effects of k on remanufacturing effort e^i are shown in Fig. 7. In the four models, e^i increases with the increase of k. This shows that retailer increases the publicity service efforts for remanufactured products, expand the sales market of remanufactured products, and manufacturer can obtain greater profit through the production of remanufactured products. Therefore, manufacturer is willing to invest more financial and material resources to improve remanufacturing technology. On the other hand, $\forall k \in (0,1)$, $e^{\mathrm{II}M*} > e^{\mathrm{I}M*} > e^{\mathrm{I}C*} > e^{\mathrm{II}G*}$. In the decentralized decision-making led by manufacturer, the government mechanisms are more conducive to improve the remanufacturing effort of manufacturer. The effects of k on the retailer's publicity service efforts A_r^i are shown in Fig. 8. In the four models, with the increase of k, A_r^i first increases and then decreases. In addition, we found some interesting phenomena. (a) $\forall k \in (0,0.38)$, $A_r^{\mathrm{I}C*} > A_r^{\mathrm{II}G*} > A_r^{\mathrm{II}M*} > A_r^{\mathrm{I}M*}$. (b) $\forall k \in (0.38,0.5)$, $A_r^{\mathrm{I}C*} > A_r^{\mathrm{II}M*} > A_r^{\mathrm{II}G*} > A_r^{\mathrm{I}M*}$. (c) $\forall k \in (0.5,1)$, $A_r^{\mathrm{II}M*} > A_r^{\mathrm{I}C*} > A_r^{\mathrm{II}G*} > A_r^{\mathrm{I}M*}$. (d)In terms of growth rate, $A_r^{\mathrm{II}M}$ has the fastest growth. In terms of the reduction speed, $A_r^{\mathrm{II}M}$ decreases the slowest. Thus, in the decentralized decision-making led by manufacturer, the government mechanisms are more conducive to improve the retailer's publicity service efforts.

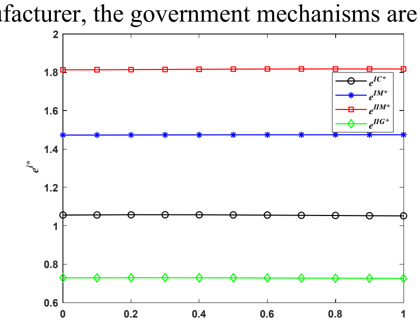


Fig. 7. The effects *of* k on e^i.

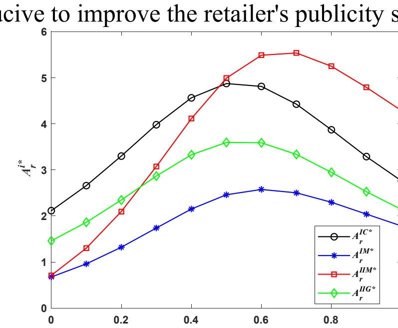
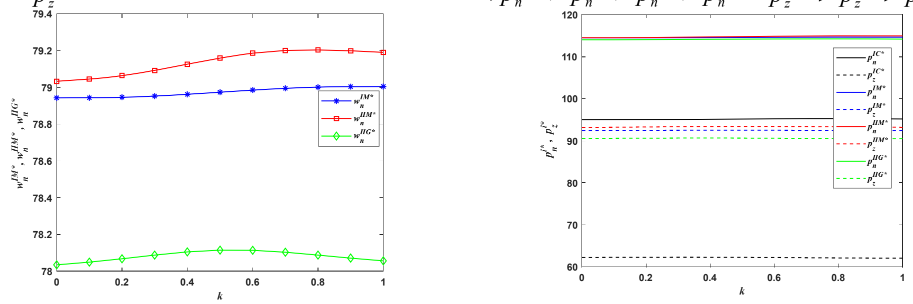


Fig. 8. The effects *of* k on A_r^i.

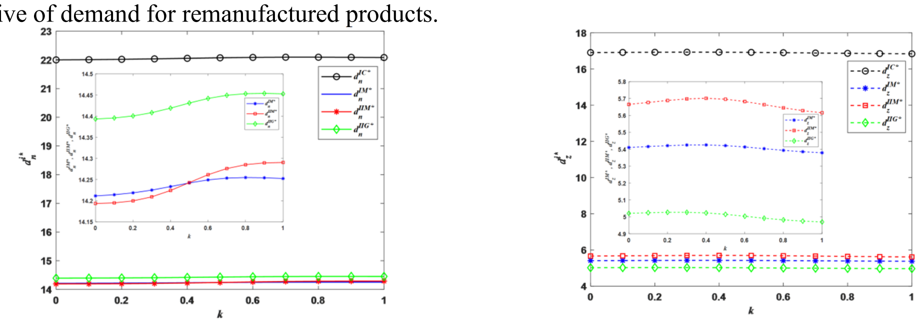
The effects of k on the wholesale price w_n^i are shown in Fig. 9(a). In the decentralized decision-making led by manufacturer, with the increase of k, w_n^i first increases and then decreases. Meanwhile, $\forall k \in (0,1)$, $w_n^{\mathrm{II}M} > w_n^{\mathrm{I}M} > w_n^{\mathrm{II}G}$. The effects of k on the retail price of new products p_n^i and the retail price of remanufactured products p_z^i are shown in Fig.9(b). In the four models, both p_n^i and p_z^i increase with the increase of k. $\forall k \in (0,1)$, $p_n^{\mathrm{II}M} > p_n^{\mathrm{I}M} > p_n^{\mathrm{II}G} > p_n^{\mathrm{I}C}$, $p_z^{\mathrm{II}M} > p_z^{\mathrm{I}M} > p_z^{\mathrm{II}G} > p_z^{\mathrm{I}C}$.



(a) The effects *of* k on w_n^i. (b) The effects *of* k on p_n^i and p_z^i.

Fig. 9. The effects *of* k on w_n^i, p_n^i and p_z^i.

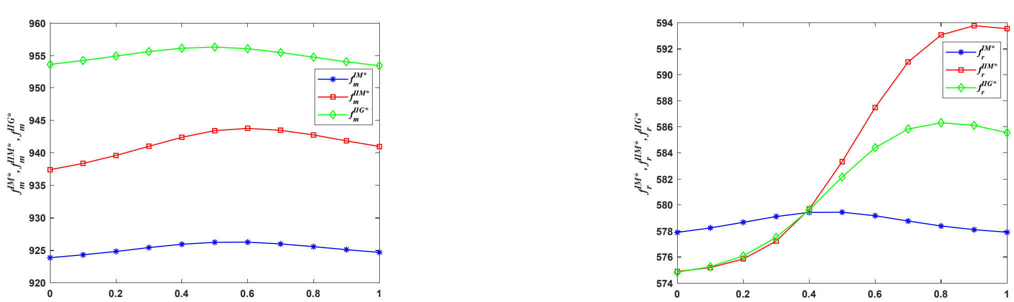
The effects of k on the demand for new products d_n^i and the demand for remanufactured products d_z^i are shown in Fig.10. $\forall k \in (0,0.5)$, $d_n^{\mathrm{I}C} > d_n^{\mathrm{II}G} > d_n^{\mathrm{I}M} > d_n^{\mathrm{II}M}$. $\forall k \in (0.5,1)$, $d_n^{\mathrm{I}C} > d_n^{\mathrm{II}G} > d_n^{\mathrm{II}M} > d_n^{\mathrm{I}M}$. $\forall k \in (0,1)$, $d_z^{\mathrm{I}C} > d_z^{\mathrm{II}M} > d_z^{\mathrm{I}M} > d_z^{\mathrm{II}G}$. In the decentralized model, no matter which product the retailer chooses to publicity service, model IIM is optimal from the perspective of demand for remanufactured products.



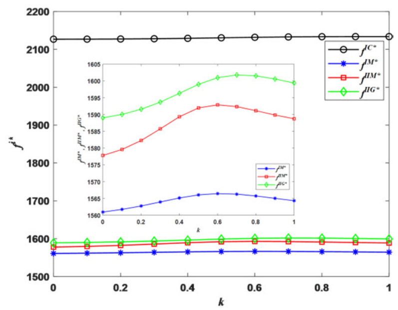
(a) The effects *of* k on d_n^i. (b) The effects *of* k on d_z^i.

Fig. 10. The effects *of* k on d_n^i and d_z^i.

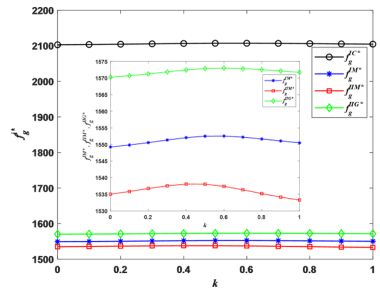
The effects of k on manufacturer's profit f_m^i, retailer's profit f_r^i, overall supply chain profit f^i and social welfare f_g^i are shown in Fig.11. We find that:(a)With the increase of k, f_m^i, f^i and f_g^i all increase first and then decrease. Hence, the proportion of retailer's publicity service efforts for remanufactured products should not be too large or toosmall. However, with the participation of the government, f_r^i keeps increasing as k raises. Retailer can expand the consumer market of remanufactured products by promoting remanufactured products and obtain government incentives. (b) $\forall k \in (0,1)$, $f_m^{\mathrm{II}G*} > f_m^{\mathrm{II}M*} > f_m^{\mathrm{I}M*}$, $f^{\mathrm{I}C*} > f^{\mathrm{II}G*} > f^{\mathrm{II}M*} > f^{\mathrm{I}M*}$ and $f_g^{\mathrm{I}C*} > f_g^{\mathrm{II}G*} > f_g^{\mathrm{I}M*} > f_g^{\mathrm{II}M*}$. Combining Fig. 9 and Fig.10, in the decentralized model, no matter which product the retailer chooses to publicity service, model IIG is optimal from the perspective of the manufacturer's profit , the overall profit of the supply chain and social welfare. Simultaneously, consumers can buy new/remanufactured products at low prices. (c) $\forall k \in (0,0.4)$, $f_r^{\mathrm{I}M*} > f_r^{\mathrm{II}G*} > f_r^{\mathrm{II}M*}$. $\forall k \in (0.4,1)$, $f_r^{\mathrm{II}M*} > f_r^{\mathrm{II}G*} > f_r^{\mathrm{I}M*}$. In the remanufacturing supply chain with the participation of the government, if the manufacturer is the leader of the supply chain and the retailer tends to publicity service for remanufactured products, the manufacturer can obtain more profits.



(a) The effects *of* k on f_m^i. (b) The effects *of* k on f_r^i.



(c) The effects *of* k on f^i.



(d) The effects *of* k on f_g^i.

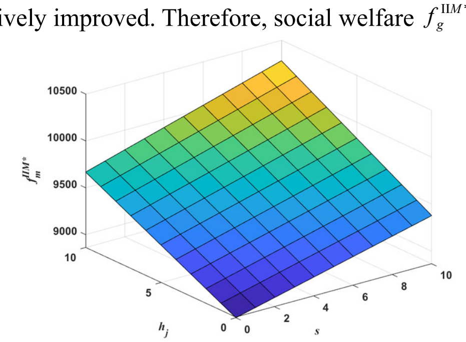
Fig. 11. The effects *of* k on f_m^i, f_r^i, f^i and f_g^i.

6.2. Effects of government mechanisms parameters on supply chain

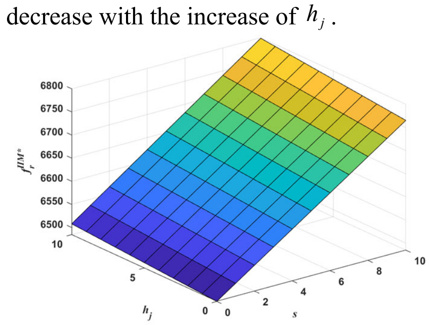
Based on the analysis in the previous sections, government mechanisms (including subsidies for remanufactured products and bonus-penalty mechanisms) can effectively promote the development of remanufactured industry and is conducive to the improvement of profits and social welfare. In order to deeply analyze the effects of government mechanisms parameters on profits and social welfare of the supply chain, the following figures are drawn respectively.

6.2.1 Effects of s and h_j on profits and social welfare

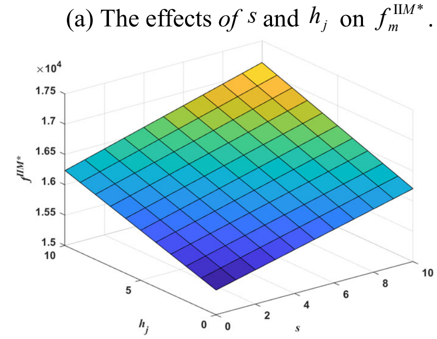
In model $\text{II}M$, the effects of unit subsidy for remanufacturing products s and bonus-penalty coefficient for the remanufacturing capacity of manufacturer h_j on manufacturer's profit $f_m^{\text{II}M*}$, retailer's profit $f_r^{\text{II}M*}$, overall profit of supply chain system $f^{\text{II}M*}$ and social welfare $f_g^{\text{II}M*}$ are shown in Fig.12. With the increase of s, $f_m^{\text{II}M*}$, $f_r^{\text{II}M*}$, $f^{\text{II}M*}$ and $f_g^{\text{II}M*}$ increased. With the increase of h_j, $f_m^{\text{II}M*}$, $f_r^{\text{II}M*}$ and $f^{\text{II}M*}$ increased, while $f_g^{\text{II}M*}$ decreased. Although the government's subsidy for remanufactured products and the bonus-penalty mechanisms for remanufacture capability do not directly encourage retailer to actively participate in the publicity service activities, by encouraging manufacturer to participate in remanufacture activities, retailer's income from selling remanufactured products will increase, thereby improving the profit of retailer. Under the positive guidance of the government mechanisms to the remanufacturing industry, it can effectively improve the benefits of manufacturer, retailer and the whole supply chain system. However, the government mechanisms do not motivate the recycler, which leads to the low enthusiasm of the recycler to participate in the recycling of used products, and the quantity of used products recycled has not been effectively improved. Therefore, social welfare $f_g^{\text{II}M*}$ will decrease with the increase of h_j.



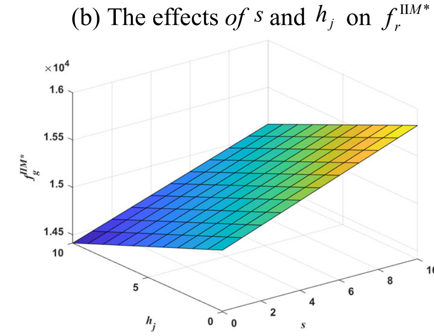
(a) The effects *of* s and h_j on $f_m^{\text{II}M*}$.



(b) The effects *of* s and h_j on $f_r^{\text{II}M*}$.



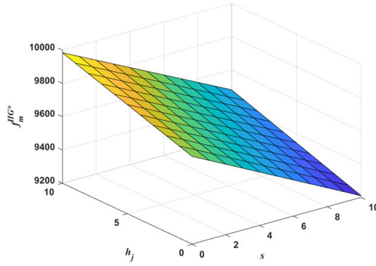
(c) The effects *of* s and h_j on $f^{\text{II}M*}$.



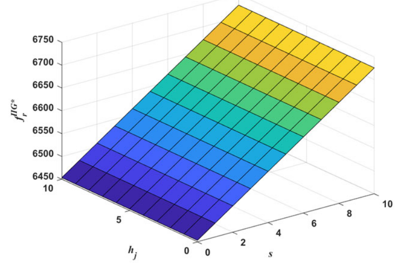
(d) The effects *of* s and h_j on $f_g^{\text{II}M*}$.

Fig. 12. The effects *of* s and h_j on $f_m^{\text{II}M*}$, $f_r^{\text{II}M*}$, $f^{\text{II}M*}$ and $f_g^{\text{II}M*}$ in model $\text{II}M$.

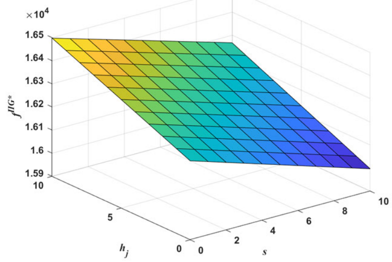
In model $\text{II}G$, the effects of unit subsidy for remanufacturing products s and bonus-penalty coefficient for the remanufacturing capacity of manufacturer h_j on manufacturer's profit f_m^{IIG*}, retailer's profit f_r^{IIG*}, overall profit of supply chain system f^{IIG*} and social welfare f_g^{IIG*} are shown in Fig.13. With the increase of s, f_r^{IIG*} increased, while f_m^{IIG*}, f^{IIG*} and f_g^{IIG*} decreased. With the increase of h_j, f_m^{IIG*}, f^{IIG*} and f_g^{IIG*} increase, while f_r^{IIG*} is almost unchanged.



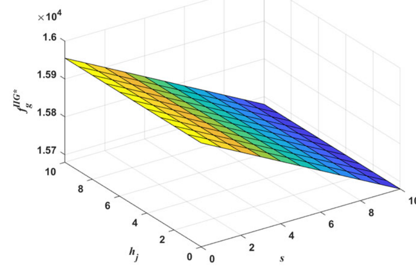
(a) The effects *of* s and h_j on f_m^{IIG*}.



(b) The effects *of* s and h_j on f_r^{IIG*}.



(c) The effects *of* s and h_j on f^{IIG*}.



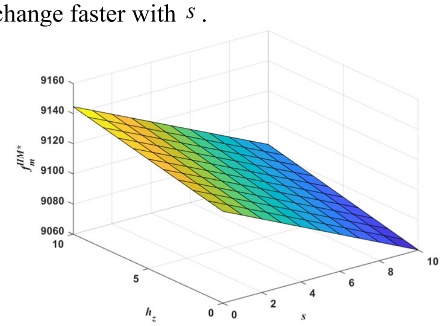
(d) The effects *of* s and h_j on f_g^{IIG*}.

Fig. 13. The effects *of* s and h_j on f_m^{IIG*}, f_r^{IIG*}, f^{IIG*} and f^{IIG*} in model $\text{II}G$.

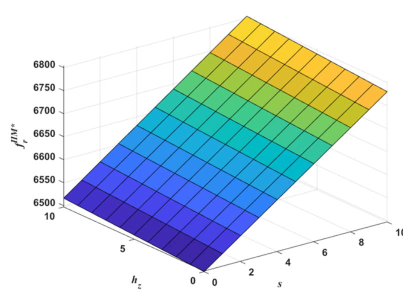
As illustrated in Fig.12 and Fig.13, at the initial stage of the development of remanufacture industry, the government needs to guide enterprises' remanufacture activities. If the manufacturer is the leader of the supply chain, the subsidy policy are more effective than the bonus-penalty mechanisms. However, manufacturer prefers to choose the onus-penalty mechanism. This is mainly because f_m^{IIM*} increases faster with h_j. If the government is the leader of the supply chain, the bonus-penalty mechanisms are more effective than the subsidy policy. However, retailer prefers to choose the subsidy policy. This is mainly because f_r^{IIG*} increases faster with s.

6.2.2 Effects of s and h_z on profits and social welfare

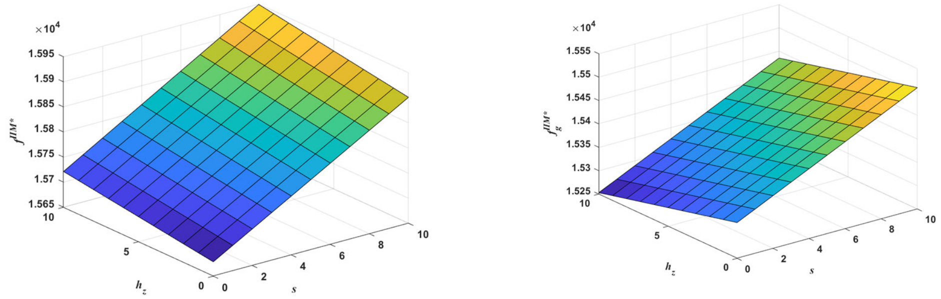
In model $\text{II}M$, the effects of unit subsidy for remanufacturing products s and bonus-penalty coefficient for retailer's publicity service efforts for remanufactured products h_z on manufacturer's profit f_m^{IIM*}, retailer's profit f_r^{IIM*}, overall supply chain profit f^{IIM*} and social welfare f_g^{IIM*} are shown in Fig.14. With the increase of s, f_r^{IIM*}, f^{IIM*} and f_g^{IIM*} increased, while f_m^{IIM*} decreased. With the increase of h_z, f_m^{IIM*}, f_r^{IIM*} and f^{IIM*} increase, while f_g^{IIM*} decreased. However, the profits and social welfare change faster with s.



(a)The effects *of* s and h_z on f_m^{IIM*}.



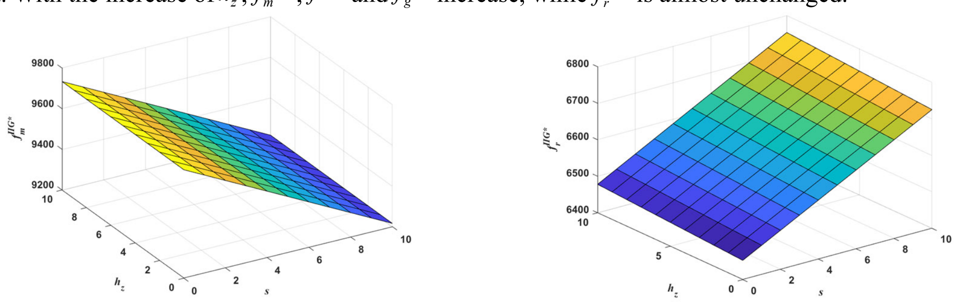
(b)The effects *of* s and h_z on f_r^{IIM*}.



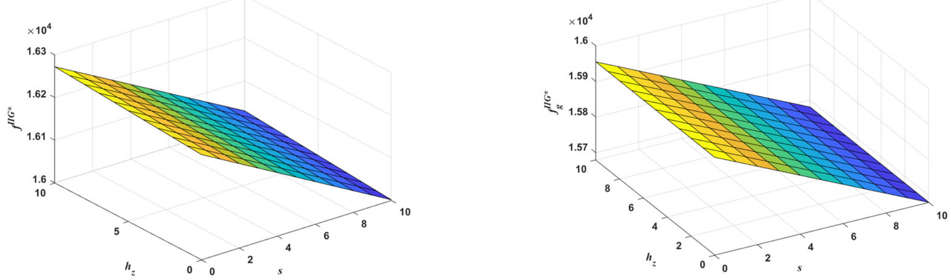
(c)The effects *of* s and h_z on $f^{\mathrm{II}M*}$.　　(d)The effects *of* s and h_z on $f_g^{\mathrm{II}M*}$.

Fig. 14. The effects *of* s and h_z on $f_m^{\mathrm{II}M*}$, $f_r^{\mathrm{II}M*}$, $f^{\mathrm{II}M*}$ and $f_g^{\mathrm{II}M*}$ in model $\mathrm{II}M$.

In model $\mathrm{II}G$, the effects of unit subsidy for remanufacturing products s and bonus-penalty coefficient for retailer's publicity service efforts for remanufactured products h_z on manufacturer's profit $f_m^{\mathrm{II}G*}$, retailer's profit $f_r^{\mathrm{II}G*}$, overall profit of supply chain system $f^{\mathrm{II}G*}$ and social welfare $f_g^{\mathrm{II}G*}$ are shown in Fig.15. With the increase of s, $f_r^{\mathrm{II}G*}$ increased, while $f_m^{\mathrm{II}G*}$, $f^{\mathrm{II}G*}$ and $f_g^{\mathrm{II}G*}$ decreased. With the increase of h_z, $f_m^{\mathrm{II}G*}$, $f^{\mathrm{II}G*}$ and $f_g^{\mathrm{II}G*}$ increase, while $f_r^{\mathrm{II}G*}$ is almost unchanged.



(a)The effects *of* s and h_z on $f_m^{\mathrm{II}G*}$.　　(b)The effects *of* s and h_z on $f_r^{\mathrm{II}G*}$.



(c)The effects *of* s and h_z on $f^{\mathrm{II}G*}$.　　(d)The effects *of* s and h_z on $f_g^{\mathrm{II}G*}$.

Fig. 15. The effects *of* s and h_z on $f_m^{\mathrm{II}G*}$, $f_r^{\mathrm{II}G*}$, $f^{\mathrm{II}G*}$ and $f^{\mathrm{II}G*}$ in model $\mathrm{II}G$.

As illustrated in Fig.14 and Fig. 15, at the initial stage of the development of the remanufacture industry, the government needs to guide enterprises' remanufacture activities. If the manufacturer is the leader of the supply chain, the subsidy policies are more effective than the bonus-penalty mechanisms. However, manufacturers prefer to choose the onus-penalty mechanism. If the government is the leader of the supply chain, the bonus-penalty mechanisms are more effective than the subsidy policy. However, retailer prefers to choose the subsidy policy. This is mainly because $f_r^{\mathrm{II}G*}$ increases faster with s.

6.3. Analysis on coordination mechanism of supply chain

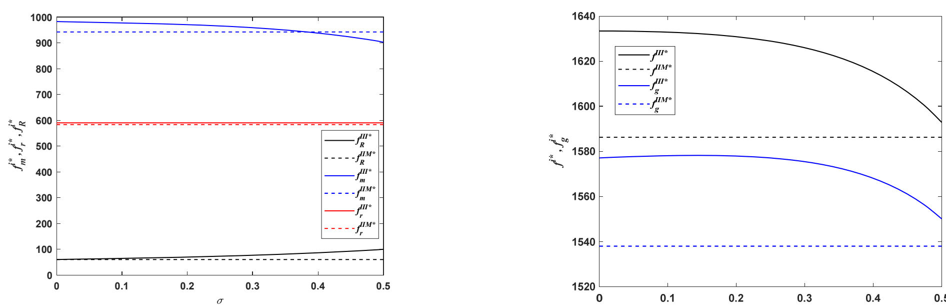
Under the condition of satisfying individual rational constraints, the value range of manufacturer's share coefficient of recovery cost to recycler σ is calculated as $\sigma \in (0,0.38)$ according to the data given in the example. We take $\sigma = 0.2$ and $\sigma = 0.3$ as coordination mechanism 1 and 2 respectively. After adopting the coordination mechanism, the optimal decision-making, profits and social welfare are shown in Table 4. From Table 4, it is found that after adopting the cost-sharing contract, if the value of σ is within the range of individual rational constraints, it can effectively adjust the remanufacturing supply chain compared with that before the coordination. The profits of all entities in the supply chain have been improved. The quantity of recycling has increased significantly. The overall profits and social welfare of the supply chain have been effectively improved. This shows that in the remanufacturing supply chain considering the government mechanisms, the cost-sharing contract can effectively coordinate the interests of supply chain members and improve social welfare. The proportion parameter in the contract needs to meet the constraints of individual rationality. At the same time, after the internal coordination of the supply chain, consumers can buy products at a lower price and recycle used products to recyclers at a higher price, which improves the utility and enthusiasm of consumers. The coordination of remanufacturing supply chain is not only conducive to the development and improvement of the closed-loop recycling system, but also realizes the multi-win of consumers, supply chain members and the government.

Table 4
Optimal solutions under different coordination mechanisms

	w_n^i	p_n^i	p_z^i	p_r^i	p_m^i	e^i	A_r^i	T^i	f_m^i	f_r^i	f_R^i	f^i	f_g^i
Coordination mechanism 1	78.1	110.0	88.7	1.6	9.0	1.8	4.6	13.3	970.3	590.3	70.2	1630.9	1578.0
Coordination mechanism 2	78.1	110.0	88.7	3.5	9.0	1.8	4.6	14.9	958.4	590.3	77.3	1626.0	1575.5
Model IIM	79.1	114.7	93.4	0.5	9.0	1.8	4.1	11	942.4	583.4	60.5	1586.3	1538.0

6.4. Effects of σ on profits and social welfare

Compared with model IIM, the enterprise decisions, supply chain profits and social welfare in model III are optimal. After adopting the cost-sharing contract, the change trends of the profits and social welfare are shown in Fig. 16. With manufacturer's share coefficient of recovery cost to recycler σ increases, the manufacturer's profit $f_m^{\mathrm{III}*}$ decreases, and the retailer's profit $f_r^{\mathrm{III}*}$ has no effect, but it is beneficial to improve the recycler's profit $f_R^{\mathrm{III}*}$. If $\sigma \in (0,0.38)$, the internal coordination of the supply chain can effectively increase the profits of manufacturer, retailer and recycler, thus improving the overall profit $f^{\mathrm{III}*}$ and social welfare $f_g^{\mathrm{III}*}$. Because the change trend of $f_m^{\mathrm{III}*}$ is greater than that of $f_R^{\mathrm{III}*}$, $f^{\mathrm{III}*}$ decreases with the increase of σ. However, due to the increase of the recycling quantity of used products, the $f_g^{\mathrm{III}*}$ showed a trend of first increasing and then decreasing. If $\sigma > 0.38$, the greater the σ, the faster the $f_m^{\mathrm{III}*}$, $f^{\mathrm{III}*}$ and $f_g^{\mathrm{III}*}$ decrease. If σ is too low or too high, it is not conducive to the increase of enterprise profits, which leads to the failure of supply chain coordination contract. Of cause, the improvement of overall profits and social welfare are hindered. Finally, the system profits and social welfare are significantly reduced. Therefore, enterprises should set a reasonable σ when entering a cost-sharing contract. Under certain conditions, the contract can effectively improve the profits(manufacturer, retailer, recycler and supply chain) and social welfare.



(a) The effects *of* σ on f_m^{i*}, f_R^{i*} and f_r^{i*} (b) The effects *of* σ on f^{i*} and f_g^{i*}.

Fig. 16. The effects *of* σ on f_m^{i*}, f_R^{i*}, f_r^{i*}, f^{i*} and f_g^{i*}

7. Conclusions

Based on considering the preferences, this paper constructs an optimization model for manufacturer, retailer and recycler under government mechanisms and supply chain coordination. By comparing and analyzing the remanufacturing efforts of manufacturer, the publicity service efforts of retailer, and the pricing, output and profit of products under the three situations: centralized situation, manufacturer-led situation and government-led situation, the effects of consumers market, government mechanisms and supply chain coordination parameters on remanufacturing supply chain decision-making and profits are obtained.

(1) At the initial stage of the development of the remanufacture industry, the role of consumer market and the internal coordination contract of supply chain have limited incentive effect on the remanufacture activities of enterprises. The remanufacture level, publicity service efforts for remanufactured products and the overall benefit of the supply chain cannot be improved by the independent behavior of enterprises alone. However, the government financial subsidies and bonus-penalty mechanisms are conducive to manufacturer's and retailer's active participation in remanufacture activities, which can greatly improve the remanufacture level of products and publicity service efforts for remanufactured products.

(2) The government should set a moderate basic level for manufacturer's remanufacture. If the setting is too high, it will lead to the decline of manufacturer's profit and the reduction of manufacturer's enthusiasm to participate in remanufacturing activities, which is not conducive to the improvement of the overall efficiency of the supply chain; On the other hand, bonus-

penalty mechanisms are more conducive to encouraging manufacturer to actively participate in remanufacturing activities than subsidy measures.

(3) The government should set an appropriate basic level for retailer's publicity service efforts for remanufactured products. If the setting is too high, it will lead to the decline of retailer's profit, reduce the enthusiasm of retailers to participate in the publicity service activities of remanufactured products, and is not conducive to the improvement of the overall efficiency of the supply chain. On the other hand, the subsidy measures are more conducive to encouraging retailers to actively participate in the publicity service activities of remanufactured products than bonus-penalty mechanisms.

(4) In the absence of supply chain coordination, government measures have a direct impact on manufacturers and retailers, unable to enhance the enthusiasm of recyclers to recycle used products, resulting in low social welfare. In the case of supply chain coordination, the incentives of government measures to manufacturers are partially transferred to recyclers through cost-sharing contracts, to improve the enthusiasm of recyclers to recycle used products and increase the quantity of recycled. If the cost-sharing rate is within a certain range, the contract can effectively improve the interests of system members and greatly improve social welfare.

(5) Compared with the role of external consumer markets and government mechanisms, the role of supply chain internal coordination contracts in improving enterprise efficiency and social welfare is more significant. Under the coordination of supply chain, the level of product remanufactures and enterprise goodwill, the profits of supply chain and social welfare are optimal. Coordination and cooperation among supply chain members can effectively enhance the role of the consumer market and promote the implementation of government measures.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

Data availability statement

The data that support the findings are available from the corresponding author upon reasonable request.

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Appendix

Appendix A. Proof of Proposition 1

The overall profit function $f^{IC}(p_n^{IC}, p_z^{IC}, p_r^{IC}, e^{IC}, A_r^{IC})$ of the supply chain is analyzed, and the corresponding Hessian matrix is H_1^{IC}.

$$H_1^{IC}=\begin{bmatrix}-\theta\upsilon^2 & 0 & 0 & \alpha\upsilon & 0\\ 0 & -2\delta & 0 & 0 & 0\\ 0 & 0 & -2\mu & 0 & k\xi\varepsilon\\ \alpha\upsilon & 0 & 0 & -2\mu & (1-k)\xi\varepsilon\\ 0 & 0 & k\xi\varepsilon & (1-k)\xi\varepsilon & -\omega[k^2+(1-k)^2]\end{bmatrix}$$

First order principal minor is $-\theta\upsilon^2<0$. Second order principal minor is $2\theta\upsilon^2\delta>0$. Third order principal minor is $-4\mu\theta\upsilon^2\delta<0$. If $\mu\theta>\alpha^2$, Fourth order principal minor is $8\theta\upsilon^2\mu^2\delta-4\alpha^2\upsilon^2\mu\delta>0$. If $\xi^2\varepsilon^2<\mu\omega$ and $\mu\theta>\alpha^2$, fifth order principal minor is $2\mu\omega[k^2+(1-k)^2](\alpha^2-2\mu\theta)+2\mu\theta\xi^2\varepsilon^2[k^2+(1-k)^2]-\alpha^2k^2\xi^2\varepsilon^2<0$. Therefore, if $\xi^2\varepsilon^2<\mu\omega$ and $\mu\theta>\alpha^2$, $f^{IC}(p_n^{IC},p_z^{IC},p_r^{IC},e^{IC},A_r^{IC})$ is a strictly jointly concave function in p_n^{IC}, p_z^{IC}, p_r^{IC}, e^{IC} and A_r^{IC}. By solving first-order partial derivatives of $f^{IC}(p_n^{IC},p_z^{IC},p_r^{IC},e^{IC},A_r^{IC})$ is 0, the optimal solution is as follows.

$$p_n^{IC*}=\frac{1}{2\mu A_3}[(\mu A_3+k\varepsilon\xi B_5)c_n+k\varepsilon\xi B_6c_z+\varphi\tau A_3-k\varepsilon\xi(2\mu\theta D_3+k\alpha^2\varphi\tau\varepsilon\xi)]$$

$$p_z^{IC*}=\frac{1}{\alpha A_3}[-\mu\theta G_1c_n+(\alpha A_3-\mu\theta G_2)c_z+\theta D_2]$$

$$p_r^{IC*}=\frac{c_n-c_z-c_r}{2}-\frac{\psi}{2\delta}$$

$$e^{IC*}=\frac{1}{\upsilon A_3}(-\mu G_1c_n-\mu G_2c_z+D_2)$$

$$A_r^{IC*}=\frac{1}{A_3}(B_5c_n+B_6c_z-2\mu\theta D_3-k\alpha^2\varphi\tau\varepsilon\xi)$$

Where,

$$A_3=-\varepsilon^2\xi^2[-2k^2\alpha^2+2\theta\mu(2k^2-2k+1)]+2\mu\omega(2\mu\theta-\alpha^2)(2k^2-2k+1)$$

$$B_5=-k\mu\varepsilon\xi(2\mu\theta-\alpha^2)$$

$$B_6=-2\mu^2\theta\varepsilon\xi(1-k)$$

$$D_2=2\mu\omega\varphi(1-\tau)(2k^2-2k+1)+k\varphi\varepsilon^2\xi^2(-k+\tau)$$

$$D_3=\varepsilon\xi\varphi(k-1+\tau-2k\tau)$$

$$G_1=\alpha\varepsilon^2\xi^2k(1-k)$$

$$G_2=\alpha[2\mu\omega(2k^2-2k+1)-k^2\varepsilon^2\xi^2]$$

This proves Proposition 1.

Appendix B. Proof of Lemma 1

Due to $\xi^2\varepsilon^2<\mu\omega$ and $\mu\theta>\alpha^2$,

$$A_3=-\varepsilon^2\xi^2[-2k^2\alpha^2+2\theta\mu(2k^2-2k+1)]+2\mu\omega(2\mu\theta-\alpha^2)(2k^2-2k+1)$$
$$>\varepsilon^2\xi^2[(2\mu\theta-\alpha^2)(4k^2-4k+2)+2k^2\alpha^2-2\theta\mu(2k^2-2k+1)]=\varepsilon^2\xi^2[2\mu\theta(2k^2-2k+1)-\alpha^2(2k^2-4k+2)].$$
$$>2\varepsilon^2\xi^2\alpha^2k^2>0$$

Similarly, $B_5=-k\mu\varepsilon\xi(2\mu\theta-\alpha^2)<0$, $G_2=\alpha[2\mu\omega(2k^2-2k+1)-k^2\varepsilon^2\xi^2]>0$, $\mu A_3+k\varepsilon\xi B_5>0$, $\alpha A_3-\mu\theta G_2>0$. Obviously, $G_1=\alpha\varepsilon^2\xi^2k(1-k)>0$, $B_6=-2\mu^2\theta\varepsilon\xi(1-k)<0$.

So,(1) $\frac{\partial p_n^{IC*}}{\partial c_n}=\frac{\mu A_3+k\varepsilon\xi B_5}{2\mu A_3}>0$, $\frac{\partial p_z^{IC*}}{\partial c_n}=\frac{-\mu\theta G_1}{\alpha A_3}<0$, $\frac{\partial p_r^{IC*}}{\partial c_n}=\frac{1}{2}>0$, $\frac{\partial e^{IC*}}{\partial c_n}=\frac{-\mu G_1}{\upsilon A_3}<0$, $\frac{\partial A_r^{IC*}}{\partial c_n}=\frac{B_5}{A_3}<0$.

(2) $\frac{\partial p_n^{IC*}}{\partial c_z}=\frac{k\varepsilon\xi B_6}{2\mu A_3}<0$, $\frac{\partial p_z^{IC*}}{\partial c_z}=\frac{\alpha A_3-\mu\theta G_2}{\alpha A_3}>0$, $\frac{\partial p_r^{IC*}}{\partial c_z}=-\frac{1}{2}<0$, $\frac{\partial e^{IC*}}{\partial c_z}=\frac{-\mu G_2}{\upsilon A_3}<0$, $\frac{\partial A_r^{IC*}}{\partial c_z}=\frac{B_6}{A_3}<0$.

Lemma 1 is thus confirmed.

Appendix C. Proof of Proposition 2

The first step is to analyze the recycler profit function $f_R^{IM}(p_r^{IM})$. Since $\frac{\partial^2 f_R^{IM}}{\partial(p_r^{IM})^2}=-2\delta<0$, $f_R^{IM}(p_r^{IM})$ is a strictly jointly concave function in p_r^{IM}. By solving first-order partial derivatives of $f_R^{IM}(p_r^{IM})$ is 0, the optimal solution is obtained as follows.

$$p_r^{IM*}=\frac{-\psi+\delta p_m^{IM*}-\delta c_r}{2\delta}.$$

The second step is to analyze the retailer profit function $f_r^{IM}(p_n^{IM},p_z^{IM},A_r^{IM})$. The corresponding Hessian matrix is H_1^{IM}.

$$H_1^{IM}=\begin{bmatrix}-2\mu & 0 & k\xi\varepsilon\\ 0 & -2\mu & (1-k)\xi\varepsilon\\ k\xi\varepsilon & (1-k)\xi\varepsilon & -\omega[k^2+(1-k)^2]\end{bmatrix}$$

First order principal minor is $-2\mu<0$. Second order principal minor is $4\mu^2>0$. If $\xi^2\varepsilon^2<2\mu\omega$, third order principal minor is $-4\omega\mu^2[k^2+(1-k)^2]+2\mu\xi^2\varepsilon^2(1-k)^2+2\mu\xi^2\varepsilon^2k^2<0$. Therefore, if $\xi^2\varepsilon^2<2\mu\omega$, $f_r^{IM}(p_n^{IM},p_z^{IM},A_r^{IM})$ is a strictly jointly concave function in p_n^{IM}, p_z^{IM} and A_r^{IM}. By solving first-order partial derivatives of $f_r^{IM}(p_n^{IM},p_z^{IM},A_r^{IM})$ is 0, the optimal solution of p_n^{IM}, p_z^{IM} and A_r^{IM} are obtained as follows.

$$p_n^{IM*}=\frac{1}{A_2}[(B_1-2k\mu\varepsilon^2\xi^2)w_n^{IM*}+\upsilon G_1e^{IM*}+D_1],$$

$$p_z^{IM*}=\frac{1}{A_2}[(B_1-\mu\varepsilon^2\xi^2)w_n^{IM*}+\upsilon G_2e^{IM*}+D_2],$$

$$A_r^{IM*}=\frac{2\mu}{A_2}[-\mu\varepsilon\xi w_n^{IM*}+\alpha\varepsilon\xi\upsilon(1-k)e^{IM*}-D_3].$$

The third step is to analyze the manufacturer profit function $f_m^{IM}(w_n^{IM},e^{IM},p_m^{IM})$. The corresponding Hessian matrix is H_2^{IM}.

$$H_2^{IM}=\begin{bmatrix}-\delta & 0 & 0\\ 0 & -\theta\upsilon^2 & \alpha\upsilon\\ 0 & \alpha\upsilon & \dfrac{\mu\varepsilon^2\xi^2(2k-1)^2-4\mu^2\omega(2k^2-2k+1)}{(2\mu\omega-\varepsilon^2\xi^2)(2k^2-2k+1)}\end{bmatrix}$$

First order principal minor is $-\delta<0$. Second order principal minor is $\delta\theta\upsilon^2>0$. If $\mu\theta>\alpha^2$, $\frac{\theta[-\mu\varepsilon^2\xi^2(2k-1)^2+4\mu^2\omega(2k^2-2k+1)]}{(2\mu\omega-\varepsilon^2\xi^2)(2k^2-2k+1)}>\mu\theta$, thus $\frac{\theta[-\mu\varepsilon^2\xi^2(2k-1)^2+4\mu^2\omega(2k^2-2k+1)]}{(2\mu\omega-\varepsilon^2\xi^2)(2k^2-2k+1)}>\alpha^2$. Third order principal minor is $\delta[\frac{\theta\upsilon^2\mu\varepsilon^2\xi^2(2k-1)^2-4\theta\mu^2\omega(2k^2-2k+1)}{(2\mu\omega-\varepsilon^2\xi^2)(2k^2-2k+1)}+\alpha^2\upsilon^2]<0$. Therefore, if $\mu\theta>\alpha^2$, $f_m^{IM}(w_n^{IM},e^{IM},p_m^{IM})$ is a strictly jointly concave function in w_n^{IM}, e^{IM} and p_m^{IM}. So, by solving first-order partial derivatives of $f_m^{IM}(w_n^{IM},e^{IM},p_m^{IM})$ is 0, the optimal solution of w_n^{IM}, e^{IM} and p_m^{IM} are obtained as follows.

$$w_n^{IM*}=-\frac{1}{A_1}(\theta B_1c_n+B_3c_z+\theta B_2),$$

$$e^{IM*}=-\frac{1}{\upsilon A_1}(\alpha B_1c_n+B_4c_z+\alpha B_2),$$

$$p_m^{IM*}=\frac{c_n+c_r-c_z}{2}-\frac{\psi}{2\delta}.$$

Where,

$$A_1=\varepsilon^2\xi^2(1-2k)[\alpha^2k+2\theta\mu(1-2k)]+2\mu\omega(\alpha^2-4\mu\theta)(2k^2-2k+1),$$

$$A_2=2\mu(2k^2-2k+1)(2\mu\omega-\varepsilon^2\xi^2),$$

$$B_1=\mu[2\mu\omega(2k^2-2k+1)-\varepsilon^2\xi^2(2k^2-3k+1)],$$

$$B_2=2\mu\omega\varphi(2k^2-2k+1)+\varphi\varepsilon^2\xi^2(1-2k)(k-\tau),$$

$$B_3=(\mu\theta-\alpha^2)[2\mu\omega(2k^2-2k+1)-k\varepsilon^2\xi^2(2k-1)],$$

$$B_4=\mu\alpha[-6\mu\omega(2k^2-2k+1)+\varepsilon^2\xi^2(6k^2-7k+2)],$$

$$D_1=2\mu\omega\varphi\tau(2k^2-2k+1)+\varphi\varepsilon^2\xi^2(1-k)(k-\tau),$$

$$D_2=2\mu\omega\varphi(1-\tau)(2k^2-2k+1)+k\varphi\varepsilon^2\xi^2(-k+\tau),$$

$$D_3=\varepsilon\xi\varphi(k-1+\tau-2k\tau),$$

$$G_1=\alpha\varepsilon^2\xi^2k(1-k),$$

$$G_2=\alpha[2\mu\omega(2k^2-2k+1)-k^2\varepsilon^2\xi^2].$$

We then complete the proof of Proposition 2.

Appendix D1. Proof of Lemma 2

$(1-2k)[\alpha^2k+2\theta\mu(1-2k)]-(-\alpha^2+4\mu\theta)(2k^2-2k+1)=\alpha^2(1-k)-2\mu\theta$. If $\mu\theta>\alpha^2$, $\alpha^2(1-k)-2\mu\theta<0$. So, $(1-2k)[\alpha^2k+2\theta\mu(1-2k)]<(-\alpha^2+4\mu\theta)(2k^2-2k+1)$ and $(-\alpha^2+4\mu\theta)(2k^2-2k+1)>0$.

Because $0<\xi^2\varepsilon^2<2\mu\omega$, we can get $A_1=\varepsilon^2\xi^2(1-2k)[\alpha^2k+2\theta\mu(1-2k)]+2\mu\omega(\alpha^2-4\mu\theta)(2k^2-2k+1)<0$,

$A_2=2\mu(2k^2-2k+1)(2\mu\omega-\varepsilon^2\xi^2)>0$,

$B_1=\mu[2\mu\omega(2k^2-2k+1)-\varepsilon^2\xi^2(2k^2-3k+1)]>0$,

$B_3=(\mu\theta-\alpha^2)[2\mu\omega(2k^2-2k+1)-k\varepsilon^2\xi^2(2k-1)]>0$,

$B_4=\mu\alpha[-6\mu\omega(2k^2-2k+1)+\varepsilon^2\xi^2(6k^2-7k+2)]<0$,

$G_1=\alpha\varepsilon^2\xi^2k(1-k)>0$,

$G_2=\alpha[2\mu\omega(2k^2-2k+1)-k^2\varepsilon^2\xi^2]>0$,

We have

(1) $\frac{\partial w_n^{IM*}}{\partial c_n}=-\frac{\theta B_1}{A_1}>0, \frac{\partial e^{IM*}}{\partial c_n}=-\frac{\alpha B_1}{\upsilon A_1}>0, \frac{\partial p_m^{IM*}}{\partial c_n}=\frac{1}{2}>0, \frac{\partial A_r^{IM*}}{\partial c_n}=\frac{2\mu B_1\varepsilon\xi}{A_1A_2}[\mu\theta-(1-k)\alpha^2]<0, \frac{\partial p_r^{IM*}}{\partial c_n}=\frac{1}{4}>0.$

(2) $\frac{\partial w_n^{IM*}}{\partial c_z}=-\frac{B_3}{A_1}>0, \frac{\partial e^{IM*}}{\partial c_z}=-\frac{B_4}{\upsilon A_1}<0, \frac{\partial p_m^{IM*}}{\partial c_z}=-\frac{1}{2}<0, \frac{\partial A_r^{IM*}}{\partial c_z}=\frac{1}{A_1A_2}[B_3\mu\varepsilon\xi-(1-k)B_4\alpha\varepsilon\xi]<0,$

$\frac{\partial p_r^{IM*}}{\partial c_z}=-\frac{1}{4}<0.$

(3) $\frac{\partial p_n^{IM*}}{\partial c_n}=-\frac{B_1}{A_1A_2}\{\theta\mu[2\mu\omega(2k^2-2k+1)-\varepsilon^2\xi^2(2k^2-k+1)]+\alpha^2\varepsilon^2\xi^2(-k^2+k)\}$.

If $2\mu\omega(2k^2-2k+1)-\varepsilon^2\xi^2(2k^2-k+1)\geq 0$ is constant, i.e. $\frac{2\mu\omega}{\varepsilon^2\xi^2}\geq\frac{2k^2-k+1}{2k^2-2k+1}=1+\frac{1}{2k+\frac{1}{k}-2}\leq\frac{3+\sqrt{2}}{2},\ \forall k\in(0,1)$.

Therefore, If the condition $2\mu\omega\geq\frac{3+\sqrt{2}}{2}\varepsilon^2\xi^2$ is met,

$\theta\mu[2\mu\omega(2k^2-2k+1)-\varepsilon^2\xi^2(2k^2-k+1)]+\alpha^2\varepsilon^2\xi^2(-k^2+k)\geq 0$ .Thus, $\frac{\partial p_n^{IM*}}{\partial c_n}>0$.

$\frac{\partial p_z^{IM*}}{\partial c_n}=-\frac{B_1}{A_1A_2}(\theta B_1-\theta\mu\varepsilon^2\xi^2+\alpha G_2)$. If $\theta B_1-\theta\mu\varepsilon^2\xi^2\geq 0$ is constant, i.e.

$\frac{2\mu\omega}{\varepsilon^2\xi^2}\geq\frac{2k^2-3k+2}{2k^2-2k+1}=1+\frac{1}{2(1-k)+\frac{1}{1-k}-2}\leq\frac{3+\sqrt{2}}{2},\forall k\in(0,1)$.

If the condition $2\mu\omega\geq\frac{3+\sqrt{2}}{2}\varepsilon^2\xi^2$ is met, $\theta B_1-\theta\mu\varepsilon^2\xi^2+\alpha G_2\geq 0$. Thus, $\frac{\partial p_z^{IM*}}{\partial c_n}>0$.

Lemma 2 is thus proved.

Appendix D2. Proof of Lemma 3

(1) $\frac{\partial w_n^{IM*}}{\partial\tau}=\frac{\theta\varphi\varepsilon^2\xi^2(1-2k)}{A_1}$, $\frac{\partial e^{IM*}}{\partial\tau}=\frac{\alpha\varphi\varepsilon^2\xi^2(1-2k)}{\upsilon A_1}$. From the proof of Lemma 1,we have obtain $A_1<0$.When $1-2k>0$, that is, $k\in(0,\frac{1}{2})$, $\frac{\partial w_n^{IM*}}{\partial\tau}<0$, $\frac{\partial e^{IM*}}{\partial\tau}<0$. When $1-2k<0$, that is, $k\in(\frac{1}{2},1)$, $\frac{\partial w_n^{IM*}}{\partial\tau}>0$, $\frac{\partial e^{IM*}}{\partial\tau}>0$. In the same way, other conclusions of Lemma 3 (1) and Lemma 3(2) can be proved to be true.

(3) $\frac{\partial p_m^{IM*}}{\partial\tau}=\frac{\partial p_r^{IM*}}{\partial\tau}=0$ is obviously true.

This proves Lemma 3.

Appendix D3. Proof of Lemma 4

If $k\in(0,\frac{1}{2})$, $\frac{\partial w_n^{IM*}}{\partial c_n}-\frac{\partial w_n^{IM*}}{\partial c_z}=-\frac{1}{A_1}(\theta B_1-B_3)=\mu\theta\varepsilon^2\xi^2(2k-1)-\alpha^2[2\mu\omega(2k^2-2k+1)-\varepsilon^2\xi^2(2k^2-k)]<0$. So $\frac{\partial w_n^{IM*}}{\partial c_n}<\frac{\partial w_n^{IM*}}{\partial c_z}$.The proof of Lemma 4 is thus completed.

Appendix E. Proof of Proposition 3

The first step is to analyze the recycler profit function $f_R^{IIM}(p_r^{IIM})$. Since $\frac{\partial^2 f_R^{IIM}}{\partial (p_r^{IIM})^2} = -2\delta < 0$, $f_R^{IIM}(p_r^{IIM})$ is a strictly jointly concave function in p_r^{IIM}. When the first derivative is 0, the optimal solution is obtained as follows.

$$p_r^{IIM*} = \frac{-\psi + \delta p_m^{IIM*} - \delta c_r}{2\delta}$$

The second step is to analyze the retailer profit function $f_r^{IIM}(p_n^{IIM}, p_z^{IIM}, A_r^{IIM})$, and the corresponding Hessian matrix is H_1^{IIM}.

$$H_1^{IIM} = \begin{bmatrix} -2\mu & 0 & k\xi\varepsilon \\ 0 & -2\mu & (1-k)\xi\varepsilon \\ k\xi\varepsilon & (1-k)\xi\varepsilon & -\omega[k^2+(1-k)^2] \end{bmatrix}$$

First order principal minor is $-2\mu < 0$. Second order principal minor is $4\mu^2 > 0$. If $\xi^2\varepsilon^2 < 2\mu\omega$, third order principal minor is $-4\omega\mu^2[k^2+(1-k)^2] + 2\mu\xi^2\varepsilon^2(1-k)^2 + 2\mu\xi^2\varepsilon^2 k^2 < 0$. Therefore, if $\xi^2\varepsilon^2 < 2\mu\omega$, $f_r^{IIM}(p_n^{IIM}, p_z^{IIM}, A_r^{IIM})$ is a strictly jointly concave function in p_n^{IIM}, p_z^{IIM} and A_r^{IIM}. Therefore, taking the first-order derivatives of $f_r^{IIM}(p_n^{IIM}, p_z^{IIM}, A_r^{IIM})$ with respect to p_n^{IIM}, p_z^{IIM} and A_r^{IIM}, and setting them to zero .The optimal solution of p_n^{IIM}, p_z^{IIM} and A_r^{IIM} are as follows.

$$p_n^{IIM*} = \frac{1}{A_2}[(B_1 - 2k\mu\varepsilon^2\xi^2)w_n^{IIM*} + \upsilon G_1 e^{IIM*} + D_1 + kC_3 h_z]$$

$$p_z^{IIM*} = \frac{1}{A_2}[(B_1 - \mu\varepsilon^2\xi^2)w_n^{IIM*} + \upsilon G_2 e^{IIM*} + D_2 + (1-k)C_3 h_z]$$

$$A_r^{IIM*} = \frac{2\mu}{A_2}[-\mu\varepsilon\xi w_n^{IIM*} + \alpha\varepsilon\xi\upsilon(1-k)e^{IIM*} - D_3 + 2k\mu h_z]$$

The third step is to analyze the manufacturer profit function $f_m^{IIM}(w_n^{IIM}, e^{IIM}, p_m^{IIM})$. The corresponding Hessian matrix is H_2^{IIM}.

$$H_2^{IIM} = \begin{bmatrix} -\delta & 0 & 0 \\ 0 & -\theta\upsilon^2 & \alpha\upsilon \\ 0 & \alpha\upsilon & \frac{\mu\varepsilon^2\xi^2(2k-1)^2 - 4\mu^2\omega(2k^2-2k+1)}{(2\mu\omega - \varepsilon^2\xi^2)(2k^2-2k+1)} \end{bmatrix}$$

First order principal minor is $-\delta < 0$. Second order principal minor is $\delta\theta\upsilon^2 > 0$. If $\mu\theta > \alpha^2$, $\frac{\theta[-\mu\varepsilon^2\xi^2(2k-1)^2 + 4\mu^2\omega(2k^2-2k+1)]}{(2\mu\omega - \varepsilon^2\xi^2)(2k^2-2k+1)} > \mu\theta$, thus $\frac{\theta[-\mu\varepsilon^2\xi^2(2k-1)^2 + 4\mu^2\omega(2k^2-2k+1)]}{(2\mu\omega - \varepsilon^2\xi^2)(2k^2-2k+1)} > \alpha^2$. Third order principal minor is $\delta[\frac{\theta\upsilon^2\mu\varepsilon^2\xi^2(2k-1)^2 - 4\theta\mu^2\omega(2k^2-2k+1)}{(2\mu\omega - \varepsilon^2\xi^2)(2k^2-2k+1)} + \alpha^2\upsilon^2] < 0$. Therefore, if $\mu\theta > \alpha^2$, $f_m^{IIM}(w_n^{IIM}, e^{IIM}, p_m^{IIM})$ is a strictly jointly concave function in w_n^{IIM}, e^{IIM} and p_m^{IIM}. So, when the first derivative of $f_m^{IIM}(w_n^{IIM}, e^{IIM}, p_m^{IIM})$ is 0, the optimal solution of w_n^{IIM}, e^{IIM} and p_m^{IIM} are as follows.

$$w_n^{IIM*} = -\frac{1}{A_1}(\theta B_1 c_n + B_3 c_z + \theta B_2 + \alpha C_1 h_j + C_2 s + \theta C_3 h_z)$$

$$e^{IIM*} = -\frac{1}{\upsilon A_1}[\alpha B_1 c_n + B_4 c_z + \alpha B_2 + 2\mu(2C_1 - \varepsilon^2\xi^2)h_j - B_4 s + \alpha C_3 h_z]$$

$$p_m^{IIM*} = \frac{c_n + c_r - c_z}{2} - \frac{\psi}{2\delta}$$

Where,

$$C_1 = 2\mu\omega(2k^2 - 2k + 1) + k\varepsilon^2\xi^2(1-2k)$$

$$C_2 = 2\mu\omega(\alpha^2 - \mu\theta - 2k^2\mu\theta + 2\alpha^2 k^2 - 2\alpha^2 k) + k\varepsilon^2\xi^2(\alpha^2 - \mu\theta)(1-2k)$$

$$C_3 = 2k\mu\varepsilon\xi$$

The proof of Proposition 3 is thus completed.

Appendix E1. Proof of Lemma 5

$$C_1 = 2\mu\omega(2k^2 - 2k + 1) + k\varepsilon^2\xi^2(1-2k) > 0$$

$$C_2 = 2\mu\omega(\alpha^2 - \mu\theta - 2k^2\mu\theta + 2\alpha^2 k^2 - 2\alpha^2 k) + k\varepsilon^2\xi^2(\alpha^2 - \mu\theta)(1-2k) < 0$$

$$C_3 = 2k\mu\varepsilon\xi > 0$$

From Proposition 3, we can obtain

$$(1)\frac{\partial w_n^{\mathrm{II}M*}}{\partial h_j}=-\frac{\alpha C_1}{A_1}>0,\frac{\partial e^{\mathrm{II}M*}}{\partial h_j}=-\frac{2\mu(2C_1-\varepsilon^2\xi^2)}{\upsilon A_1}>0,\frac{\partial p_m^{\mathrm{II}M*}}{\partial h_j}=\frac{\partial p_r^{\mathrm{II}M*}}{\partial h_j}=0;$$

$$(2)\frac{\partial w_n^{\mathrm{II}M*}}{\partial h_z}=-\frac{\theta C_3}{A_1}>0,\frac{\partial e^{\mathrm{II}M*}}{\partial h_z}=-\frac{\alpha C_3}{\upsilon A_1}>0,\frac{\partial p_m^{\mathrm{II}M*}}{\partial h_z}=\frac{\partial p_r^{\mathrm{II}M*}}{\partial h_z}=0;$$

$$(3)\frac{\partial w_n^{\mathrm{II}M*}}{\partial s}=-\frac{C_2}{A_1}<0,\frac{\partial e^{\mathrm{II}M*}}{\partial s}=\frac{B_4}{\upsilon A_1}>0,\frac{\partial A_r^{\mathrm{II}M*}}{\partial s}=\frac{2\mu\varepsilon\xi}{A_1A_2}[\mu C_2+\alpha(1-k)B_4]>0,\frac{\partial p_m^{\mathrm{II}M*}}{\partial s}=\frac{\partial p_r^{\mathrm{II}M*}}{\partial s}=0.$$

This proves Lemma 5.

Appendix E2. Proof of Lemma 6

From Proposition 3, proving Lemma 6 is proving $-2\mu(2C_1-\varepsilon^2\xi^2)<B_4$ .That is ,
$-2\mu[2\mu\omega(4k^2-4k+2)+\varepsilon^2\xi^2(2k-4k^2)-\varepsilon^2\xi^2]<\mu\alpha[-6\mu\omega(2k^2-2k+1)+\varepsilon^2\xi^2(6k^2-7k+2)]$,i.e
$\mu\omega(2k^2-2k+1)(6\alpha-8)<\varepsilon^2\xi^2[\alpha(6k^2-7k+2)+4k-8k^2-2]$.Since $\mu\omega(2k^2-2k+1)(6\alpha-8)<\varepsilon^2\xi^2(2k^2-2k+1)(3\alpha-4)$,
$\varepsilon^2\xi^2(2k^2-2k+1)(3\alpha-4)<\varepsilon^2\xi^2[\alpha(6k^2-7k+2)+4k-8k^2-2]$ is obviously true.
The proof of Lemma 6 is thus completed.

Appendix F. Proof of Lemma 7

From Proposition 2 and Proposition 3, we can get

$$e^{\mathrm{II}M*}-e^{\mathrm{I}M*}=-\frac{1}{\upsilon A_1}[2\mu(2C_1-\varepsilon^2\xi^2)h_j-B_4s+\alpha C_3h_z]>0,$$

$$A_r^{\mathrm{II}M*}-A_r^{\mathrm{I}M*}=\frac{2\mu}{A_1A_2}\{\mu\varepsilon\xi(\alpha C_1h_j+C_2s+\theta C_3h_z)-\alpha\varepsilon\xi(1-k)[2\mu(2C_1-\varepsilon^2\xi^2)h_j-B_4s+\alpha C_3h_z]+2A_1k\mu h_z\}>0$$

From Proposition 2 and proposition 3, we can conclude that 7 (2) and 7 (3) are obviously true.
This proves Lemma 7.

Appendix G. Proof of Proposition 4

The first step is to analyze the recycler profit function $f_R^{\mathrm{II}G}(p_r^{\mathrm{II}G})$. Since $\frac{\partial^2 f_R^{\mathrm{II}G}}{\partial (p_r^{\mathrm{II}G})^2}=-2\delta<0$, $f_R^{\mathrm{II}G}(p_r^{\mathrm{II}G})$ is a strictly jointly concave function in $p_r^{\mathrm{II}G}$.When the first derivative is 0, the optimal solution is obtained as follows.

$$p_r^{\mathrm{II}G*}=\frac{-\psi+\delta p_m^{\mathrm{II}G*}-\delta c_r}{2\delta}$$

The second step is to analyze the retailer profit function $f_r^{\mathrm{II}G}(p_n^{\mathrm{II}G},p_z^{\mathrm{II}G})$, and the corresponding Hessian matrix is $H_1^{\mathrm{II}G}$.

$$H_1^{\mathrm{II}G}=\begin{bmatrix}-2\mu & 0\\ 0 & -2\mu\end{bmatrix}$$

First order principal minor is $-2\mu<0$. Second order principal minor is $4\mu^2>0$. $f_r^{\mathrm{II}G}(p_n^{\mathrm{II}G},p_z^{\mathrm{II}G})$ is a strictly jointly concave function in $p_n^{\mathrm{II}G}$ and $p_z^{\mathrm{II}G}$. Therefore, when the first derivative of $f_r^{\mathrm{II}G}(p_n^{\mathrm{II}G},p_z^{\mathrm{II}G})$ is 0, the optimal solution of $p_n^{\mathrm{II}G}$ and $p_z^{\mathrm{II}G}$ are as follows.

$$p_n^{\mathrm{II}G*}=\frac{1}{2\mu}[\mu w_n^{\mathrm{II}G*}+\varphi\tau+k\varepsilon\xi A_r^{\mathrm{II}G*}]$$

$$p_z^{\mathrm{II}G*}=\frac{1}{2\mu}[\mu w_n^{\mathrm{II}G*}+(1-k)\varepsilon\xi A_r^{\mathrm{II}G*}+\alpha\upsilon e^{\mathrm{II}G*}+\varphi(1-\tau)]$$

The third step is to analyze the manufacturer profit function $f_m^{\mathrm{II}G}(w_n^{\mathrm{II}G},p_m^{\mathrm{II}G})$, and the corresponding Hessian matrix is $H_2^{\mathrm{II}G}$.

$$H_2^{\mathrm{II}G}=\begin{bmatrix}-2\mu & 0\\ 0 & -\delta\end{bmatrix}$$

First order principal minor is $-2\mu<0$. Second order principal minor is $2\mu\delta>0$. $f_m^{\mathrm{II}G}(w_n^{\mathrm{II}G},p_m^{\mathrm{II}G})$ is a strictly jointly concave function in $w_n^{\mathrm{II}G}$ and $p_m^{\mathrm{II}G}$. Therefore, when the first derivative of $f_m^{\mathrm{II}G}(w_n^{\mathrm{II}G},p_m^{\mathrm{II}G})$ is 0, there is an optimal solution. The optimal solution of $w_n^{\mathrm{II}G}$ and $p_m^{\mathrm{II}G}$ are as follows.

$$w_n^{\mathrm{II}G*}=\frac{1}{4\mu}(\varepsilon\xi A_r^{\mathrm{II}G*}+\alpha\upsilon e^{\mathrm{II}G*}+\varphi+\mu c_n+\mu c_z-\mu s)$$

$$p_m^{\mathrm{II}G*}=\frac{c_n+c_r-c_z}{2}-\frac{\psi}{2\delta}$$

The fourth step is to analyze the social welfare function $f_g^{\mathrm{II}G}(A_r^{\mathrm{II}G},e^{\mathrm{II}G})$, and the corresponding Hessian matrix is $H_3^{\mathrm{II}G}$.

$$H_3^{IIG} = \begin{bmatrix} \frac{\upsilon^2(7\alpha^2-16\mu\theta)}{16\mu} & \frac{\alpha\upsilon\varepsilon\xi(7-8k)}{16\mu} \\ \frac{\alpha\upsilon\varepsilon\xi(7-8k)}{16\mu} & \frac{\varepsilon^2\xi^2(16k^2-16k+7)}{16\mu}-\omega(2k^2-2k+1) \end{bmatrix}$$

If $\mu\theta > \alpha^2$, first order principal minor is $\frac{\upsilon^2(7\alpha^2-16\mu\theta)}{16\mu} < 0$. If $\mu\theta > \alpha^2$ and $\xi^2\varepsilon^2 < \mu\omega$, second order principal minor is $\frac{\upsilon^2(7\alpha^2-16\mu\theta)}{16\mu}[\frac{\varepsilon^2\xi^2(16k^2-16k+7)}{16\mu}-\omega(2k^2-2k+1)]-[\frac{\alpha\upsilon\varepsilon\xi(7-8k)}{16\mu}]^2 > 0$. Therefore, if $\mu\theta > \alpha^2$ and $\xi^2\varepsilon^2 < \mu\omega$, $f_g^{IIG}(A_r^{IIG}, e^{IIG})$ is a strictly jointly concave function in A_r^{IIG} and e^{IIG}. So, when the first derivative of $f_g^{IIG}(A_r^{IIG}, e^{IIG})$ is 0, the optimal solution of A_r^{IIG} and e^{IIG} are as follows.

$$A_r^{IIG*} = \frac{\upsilon}{A_4}(B_9c_n + B_{10}c_z + C_5 - 4C_6c_h + C_6s)$$

$$e^{IIG*} = \frac{1}{A_4}(B_7c_n + B_8c_z + \alpha B_2 + C_4 - 4\mu\alpha C_1c_h + \alpha\mu C_1s)$$

Where,

$A_4 = 2\upsilon[\mu\omega(16\mu\theta - 7\alpha^2)(2k^2-2k+1) + 3k^2\alpha^2\varepsilon^2\xi^2 - \mu\theta\varepsilon^2\xi^2(16k^2-16k+7)] > 0$;

$B_7 = 2\mu\alpha[\mu\omega(2k^2-2k+1) + 3\varepsilon^2\xi^2(k^2-k)] > 0$;

$B_8 = 2\mu\alpha[7\mu\omega(-2k^2+2k-1) + 3\varepsilon^2\xi^2k^2]$;

$C_4 = 2\mu\omega\alpha\varphi(2k^2-2k+1)(7-8\tau) + 6\alpha\varepsilon^2\xi^2\varphi k(\tau-k)$;

$C_5 = \varepsilon\xi[2\mu\varphi\theta(7-8\tau-8k+16k\tau) - 6\alpha^2k\varphi\tau]$

$C_6 = \mu\varepsilon\xi(2\mu\theta - k\alpha^2)$

$B_9 = \mu\varepsilon\xi[2\mu\theta(1-8k) + 6\alpha^2k]$

$B_{10} = \mu^2\theta\varepsilon\xi(16k-14)$

This proves Proposition 4.

Appendix H. Proof of Lemma 9

(1) $\frac{\partial e^{IIG*}}{\partial c_h} = \frac{-4\mu\alpha C_1}{A_4} < 0, \frac{\partial A_r^{IIG*}}{\partial c_h} = \frac{-4\upsilon C_6}{A_4} < 0$; (2) $\frac{\partial e^{IIG*}}{\partial s} = \frac{\mu\alpha C_1}{A_4} > 0, \frac{\partial A_r^{IIG*}}{\partial s} = \frac{\upsilon C_6}{A_4} > 0$;

This proves Lemma 9.