Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

Impact of dual uptime-reducing strategies, postponement, multi-delivery, and rework on a multiproduct fabrication-shipping problem

Yuan-Shyi Peter Chiu[a], Ting-Fang Yan[a], Singa Wang Chiu[b], Hui-Chi Wang[c*] and Tiffany Chiu[d]

[a]*Industrial Engineering & Management Department, Chaoyang University of Technology, Taiwan*
[b]*Business Administration Dept., Chaoyang University of Technology, Taiwan*
[c]*Social Work Dept., Chaoyang University of Technology, Taiwan*
[d]*School of Business, The State University of New York at New Paltz, NY, United States*

C H R O N I C L E

Article history:
Received October 10 2022
Accepted January 5 2023
Available online
January, 5 2023

Keywords:
Multiproduct production-shipping problem
Delayed differentiation
Rework
Multi-delivery
Overtime
Outsourcing

A B S T R A C T

This study examines the joint impact of outsourcing, overtime, multi-delivery, rework, and postponement on a multiproduct fabrication problem. A growing/clear trend in today's customer requirements turned into rapid response and desired quality of multi-merchandises and multiple fixed-amount deliveries in equal-interval time. To satisfy customers' expectations, current manufacturing firms must effectively design/plan their multiproduct production scheme with minimum fabrication-inventory-shipping expenses and under confined capacity. Motivated by assisting manufacturing firms in making the right production decision, this study develops a decision-support delayed-differentiation model considering multi-shipment, rework, and dual uptime-reducing strategies (namely, overtime and outsourcing). Our delayed-differentiation model comprises stage one, which makes all common/standard parts of multi-end-merchandises, and stage two, which produces multiple end merchandise. For cutting making times, the study proposes subcontracting a portion of the common/standard part's lot size and adopting overtime-making end merchandise in stage two. The screening and reworking tasks identify and repair faulty items to ensure customers' desired quality. The finished lots of end merchandise are divided into a few equal-amount shipments and distributed to customers in equal-interval time. We employ mathematical derivation and optimization methodology to derive the annual expected fabrication-inventory-shipping expense and the cost-minimized production-shipping policy. A numerical demonstration is presented to exhibit our research scheme's applicability and exposes the studied problem's critical managerial insights, which help the management make beneficial decisions.

1. Introduction

To cope with today's growing customer requirements trend of rapid response, quality, and multiple fixed-amount deliveries of multi-merchandises, current manufacturing firms must effectively design/plan multiproduct production schemes with the minimum overall expense and confined capacity. Inspired by helping manufacturing firms achieve their operational goals, this research exposes the impact of dual uptime-reducing strategies, multi-delivery, rework, and postponement on a multiproduct fabrication-shipping problem. In our proposed multiproduct postponement model, this study adopts the following two uptime-shortening strategies: (1) contracting out a proportion of the batch size of common/standard parts in the phase-1, and (2) using overtime-fabrication of the finished multi-merchandises batch. The literature relating to subcontracting and overtime is surveyed as follows: Dabhilkar and Bengtsson (2008) considered investing or divesting on outsourcing

* Corresponding author Tel.: +886-4-23323000 (ext. 4709)
E-mail: whc@cyut.edu.tw (H.-C. Wang)
ISSN 1923-2934 (Online) - ISSN 1923-2926 (Print)
2023 Growing Science Ltd.
doi: 10.5267/j.ijiec.2023.1.001

manufacturing. The researchers utilized a multiple regression approach on 267 manufacturing firms' sample data of Swedish to expose manufacturing outsourcing. They found that investing in improving production capability is more effective than outsourcing manufacturing to enhance a firm's operating performance. Although manufacturing outsourcing can positively improve operational performance, it may lead to a negative impact if solely considered a performance improvement strategy. However, a firm gains a significant performance improvement if outsourcing manufacturing is jointly applied with other production capabilities strategies. Therefore, the study concluded that upon investing in enhancing production capability, outsourcing is beneficial to freeing in-house resources and improving overall performance. Freeman et al. (2014) considered a parallel products scheduling problem featuring overtime, the waste cost for non-identical machines with setup times/costs all subject to different production sequences. The researchers explored the trade-offs of waste and overtime costs using mixed-integer programming (MIP) formulations. The study proposed an algorithm that derived the product to machine assignments vector that helped obtain the initial solution for their MIP. Then, the researchers presented a decomposition heuristic to solve a relaxed sub-problem that helped adjust their MIP's assignment parameters and find the solutions' bounds. The study used experimental analyses to show their research scheme outperformed traditional scheduling approaches concerning overtime labor and waste expenditures. They also conducted sensitivity analyses of problem parameters on overtime labor usage, total overtime labor expense, waste cost, and overall system performance. Taş et al. (2019) examined a production lot size system incorporating random setup times, limited capacity, and overtime options. For a setup time that follows Gamma distribution, the researchers developed a two-stage stochastic programming model comprising the ordinary fabrication, setup, stock-holding, and overtime capacity excessive usage expense. They used an approximation approach to gain the upper and lower bounds, and with the help of two additional heuristics, they conducted the capacity and setup-time sensitivity analyses. The study evaluated their heuristics' performance in comprehensive experimental investigations on well-known industrial examples. Via broadly interviewing the experts and reviewing the literature, Handley et al. (2022) explored the impact of vendor location, experiential learning, and emerging technologies on choosing single- or multi-sourcing IT services. The researchers developed knowledge-based concept hypotheses to link multi-sourcing related factors and studied these hypotheses on an extensive IT services contracts database. They categorized vendor location into domestic and offshore, experiential learning from vendor and client, and emerging technologies, including automation and cloud-based services. For more likely adopting multi-sourcing arrangements, the research results showed: (1) more experienced clients; (2) domestic firms; and (3) automation services rather than cloud-based services. Their findings facilitated vendor and client's single- or multi-sourcing decision making. Additional works (Friesen, 2001; Olson, 2007; Moon et al., 2012; Hofer et al., 2015; Chiu et al., 2021a; Khurosani, et al., 2021; Chiu, et al., 2022a,b; Çimen et al., 2022; Hidayat et al., 2022; Suharmono et al., 2022) studied various effects of dual uptime-reducing strategies (i.e., subcontracting and overtime) on the operations and optimization of different single-product, multiproduct, and supply-chain systems.

At the time needed to design a most beneficial multiproduct manufacturing scheme, management usually cautiously examines various alternatives when. The options include a postponement strategy separating standard common components' makings and the finished merchandise aiming to smooth the fabrication processes and save efforts/costs of logistical support. Bailey and Rabinovich (2006) studied how inventory speculation under the postponement strategy can reduce stock-holding expenses. The researchers developed hypotheses about inventory speculation and postponement and tested them using two marketplace retailers, Barnesandnoble.com and Amazon.com. For both retailers, the study found that inventory speculation has positively raised the merchandise's popularity but negatively impacted the vintage. Regarding the merchandise price, inventory speculation hurts Amazon.com but positively impacts Barnesandnoble.com (because Barnesandnoble.com operates on the Internet and physical stores). Loos, M.J., Rodriguez, C.M.T. (2015) applied a practical postponement policy to efficiently and efficiently (i.e., timely and less costly) serve their clients in textile industries compared to their competitors. The researchers presented a postponement strategy and its relevant analytical results utilizing a case study of a textile firm with various evidence collections. The study compared the leading performance indicators for the firm with or without applying the postponement strategy and evaluated the relevant impact on the textile firm. Budiman and Rau (2021) examined an uncertain-demand global supply chain (SC) system featuring conceptual modularization and speculation-postponement (SP) strategies. The study proposed a multi-period multiproduct two-stage stochastic supply-chain model considering modular items, their processing, and procurements to accommodate operations based on forecast and actual uncertain demand. The researchers used the sample mean approximation approach to derive a reliable solution proficiently. They found that the SP strategy can enhance the SC operations' efficiency and responsively deal with uncertain demand changes. The case study verified that an early postponement of the SP strategy could reduce unnecessary manufacturing processes, dead inventories, and lost sales. It can also boost the stock turnover rate, ease the risk of uncertainty in demand, and improve system performance. The study's outcomes showed that building the SC network using the right SP strategy could optimize production service and operations planning decisions and avoid offset of SP performance due to failure to adjust to uncertain demand. Additional works (Van Hoek, 2001; Granot and Yin, 2008; Bruneel et al., 2014; Chiu et al., 2020a; Tookanlou and Wong, 2020; Chiu et al., 2021b; Malladi et al., 2021; Prataviera et al., 2022; Sung et al., 2022) examined the impacts of production postponement/delayed differentiation on operations and optimization of various multiproduct fabrication and supply-chain systems.

Coping with the needs of clients' expectations on merchandise quality and multi-schedule shipments, management of manufacturing firms implements faulty items screening/repairing actions during fabrication processes and designs a multi-delivery policy for finished lots. Inderfurth et al. (2005) studied the coordination of fabrication and reworking tasks in a production system regarding operational lot sizing and timing to meet the client's demand. The scenario considered setup times, stock holding, and setup costs. The study also regarded potential deterioration during faulty goods awaiting rework tasks, which may increase the time and cost of the reworking. The study developed an economic production quantity (EPQ) model to depict the above mentioned features. The study developed optimization algorithms to address various planning situations and derive the problem's closed-form batch size solution. Besides, the study also conducted sensitivity analyses of batch-size considering product returns and deterioration. Taleizadeh et al. (2015) examined a supply chain featuring three-layer (the producer, distributor, and retailer), product pricing, and quality issues. The study's objective was to decide the distributor's, retailer's, and manufacturer's selling prices and order amounts that maximized the overall profits of the supply chain. The study developed various models based on the following scenarios concerning product quality matters, i.e., imperfect quality items are (i) all scrapped, (ii) all repairable (via rework), and (iii) sold at lower than ordinary price. Besides, the faulty items found by the distributor are sent back to the producer's external supplier with salvage value. The researchers used the Stackelberg approach and different mathematical theorems to help resolve their optimal profit functions and used numerical illustrations showing the study's research scheme's applicability. Mohammadi et al. (2022) examined a manufacturer- retailer integrated system featuring random yield, screening, and rework under unexpected demand situations. The study assumed the manufacturers face output of random faulty items and the retailers must deal with clients' price-dependent demands. Faulty items' screening and reworking processes are incorporated in centralized and decentralized supply-chain models to explore the fabrication lot size, retailer's sales price/order amount, and the anticipated profits for the manufacturer-retailer integrated system. The researchers extended their model by considering a buy-back contract in the studied system. They used numerical illustrations to show their findings regarding interactions among price, random demand/faulty rate, and rework expenses. Additional works (Guide Jr. and Srivastava, 1997; Ruiz-Torres and Mahmoodi, 2006; Hlioui et al., 2015; Quttineh and Lidestam, 2020; Suryanto and Mukhsin, 2020; Herrera et al., 2022; Purusotham et al., 2022; Siregar et al., 2022) studied the impact of retaining merchandise production quality and multi-schedule shipment issues on planning, management, and optimizing the supply chains and the manufacturing firms/units.

Additionally, the suggested references from the reviewer have been carefully studied, and an extra Table A (in Appendix A) is added to illustrate the distinction between this study's focus and the existing works mentioned by the reviewer. Since few previous works explored the combined impacts of dual uptime-reducing strategies, multi-delivery, rework, and postponement on a multiproduct fabrication-shipping problem, this study expects to fit the gap.

2. The proposed problem

This study investigates the joint influence of outsourcing, rework, multi-delivery, and overtime on a delayed differentiation multiproduct production-shipping problem. Multiple products have a standard, middle part in common. The study considers a two-stage postponement schedule to make all needed standard parts in stage one and produce end products in stage two. An external source supplies a portion of the standard components, and an overtime production of end products is used in the second stage; both aim to reduce the fabricating uptime. Poor quality exists in both processes; rework actions completely repair all faulty items made in both stages. The end products are delivered in equal-size multi-shipment to customers. We aim to decide the optimal manufacturing cycle and delivery frequency by minimizing such a multiproduct manufacturing-transportation coordination system's total operating expenses. Related notation and a more detailed problem description are given below.

2.1. Notation and description

T_Z = rotation fabricating cycle time,
n = equal-size shipping frequency in a cycle time,
λ_i = end merchandise i's annual demand ($i = 1, 2, \ldots, L$),
λ_0 = standard part's annual requirement,
Q_i = product i's lot-size,
Q_0 = in-house standard part's fabricating lot-size,
π_0 = outsourcing portion of λ_0,
$t_{1,i}$ = end merchandise i's uptime,
$t_{2,i}$ = end merchandise i's rework time,
t_i^* = optimal uptimes plus rework times of end merchandise i,
$t_{3,i}$ = end merchandise i's delivery time,
$t_{1,0}$ = standard part's uptime,
$t_{2,0}$ = standard part's rework time,
t_0^* = standard part's optimal uptime plus rework time,
$t_{3,0}$ = standard part's depleting time,
$H_{1,0}$ = stock level of standard part when uptime completion,
$H_{2,0}$ = standard part's stock level when rework completion,
$H_{3,0}$ = standard part's stock level when the outsourced items received,
$I(t)_i$ = time t's stock level ($i = 0, 1, 2, \ldots, L$),

K_0 = standard part's setup cost (in-house),
$K_{\pi 0}$ = outsourcing standard part's fixed cost,
$\beta_{1,0}$ = connecting factor of $K_{\pi 0}$ and K_0,
C_0 = standard part's unit fabricating cost (in-house),
$C_{\pi 0}$ = unit outsourcing expense of standard part,
C_i = end product i's unit fabricating cost,
$C_{T,i}$ = end product i's unit overtime fabricating cost,
$\alpha_{3,i}$ = connecting factor of $C_{T,i}$ and C_i,
$\beta_{2,0}$ = connecting factor of $C_{\pi 0}$ and C_0,
$P_{1,0}$ = standard part's annual producing rate,
x_0 = random faulty proportion of standard part,
x_i = end product i's random faulty proportion,
$d_{1,0}$ = faulty standard part's fabricating rate,
$d_{T1,i}$ = faulty end product i's overtime fabricating rate,
$P_{2,0}$ = faulty standard part's reworking rate,
$C_{R,0}$ = unit rework expense of faulty standard part,
$C_{R,i}$ = end product i's unit rework cost,
$C_{TR,i}$ = end product i's unit overtime rework cost,
$h_{1,0}$ = unit holding expense of standard part,
$h_{1,i}$ = unit holding expense of end merchandise i,
$h_{2,0}$ = reworked standard part's unit holding cost,
$h_{2,i}$ = end product i's unit holding cost during rework time,
i_0 = holding cost's connecting ratio ($h_{1,0} = i_0 C_0$),
γ = completion rate of standard part versus the finished merchandise,
H_i = standard part's stock level when product i's uptime ends,
$H_{1,i}$ = end product i's stock level when its uptime ends,
$H_{2,i}$ = end product i's stock level when its rework ends,
$\alpha_{1,i}$ = connecting factor of $P_{T1,i}$ and $P_{1,i}$ (also for $P_{T2,i}$ and $P_{2,i}$),
$P_{1,i}$ = end product i's regular annual fabricating rate,
$P_{T1,i}$ = end product i's overtime fabricating rate,
$P_{2,i}$ = end product i's regular annual rework rate,
$P_{T2,i}$ = end product i's overtime rework rate,
$\alpha_{2,i}$ = connecting factor of $K_{T,i}$ and K_i,
K_i = end product i's setup cost,
$K_{T,i}$ = end product i's overtime setup cost,
$I_d(t)_i$ = end product i's defective inventory level at time t,
$I_c(t)_i$ = end product i's stock level at the customer side at time t,
$t_{n,i}$ = end product i's fixed time-interval of shipmments,
$h_{3,i}$ = buyer side's unit holding cost,
D_i = end product i's fixed quantity per shipment,
I_i = end product i's leftover stocks when $t_{n,i}$ ends,
$K_{D,i}$ = end product i's fixed shipping cost,
$C_{D,i}$ = end product i's unit shipping cost,
S_0 = standard part's setup time,
S_i = end product i's setup time,
$TC(T_Z, n)$ = total system cost per cycle,
$E[TC(T_Z, n)]$ = the expected total operating expenses per cycle,
$E[T_Z]$ = the expected production cycle time,
$E[TCU(T_Z, n)]$ = the expected total operating expenses per unit time.

This study builds a precise math model to depict our delayed differentiation multiproduct production-shipping problem with outsourcing, rework, multi-delivery, and overtime. Our study considers the completion rate of standard part γ as a constant (i.e., if $\gamma = 0.5$, we will have the rates $P_{1,0}$ and $P_{1,i}$ twice their ordinary rates (in a single-stage system)). Fig. 1 shows the stock level in our model versus a problem without outsourcing nor overtime. The consequence of uptime reduction strategies appears as follows. In stage 1, an external source supplies a π_0 proportion of the standard part's needs. In stage 2, an overtime strategy is used to expedite L end products' fabrication; both aim to shorten the required manufacturing uptimes. Relating extra expenses and accelerating rates are exhibited as follows:

$$K_{\pi 0} = K_0\left(1+\beta_{1,0}\right) \tag{1}$$

$$C_{\pi 0} = C_0\left(1+\beta_{2,0}\right) \tag{2}$$

$$K_{T,i} = K_i\left(1+\alpha_{2,i}\right) \tag{3}$$

$$C_{\mathrm{T},i} = C_i\left(1+\alpha_{3,i}\right) \tag{4}$$

$$P_{\mathrm{T}1,i} = P_{1,i}\left(1+\alpha_{1,i}\right) \tag{5}$$

$$C_{\mathrm{T}R,i} = C_{R,i}\left(1+\alpha_{3,i}\right) \tag{6}$$

$$P_{\mathrm{T}2,i} = P_{2,i}\left(1+\alpha_{1,i}\right) \tag{7}$$

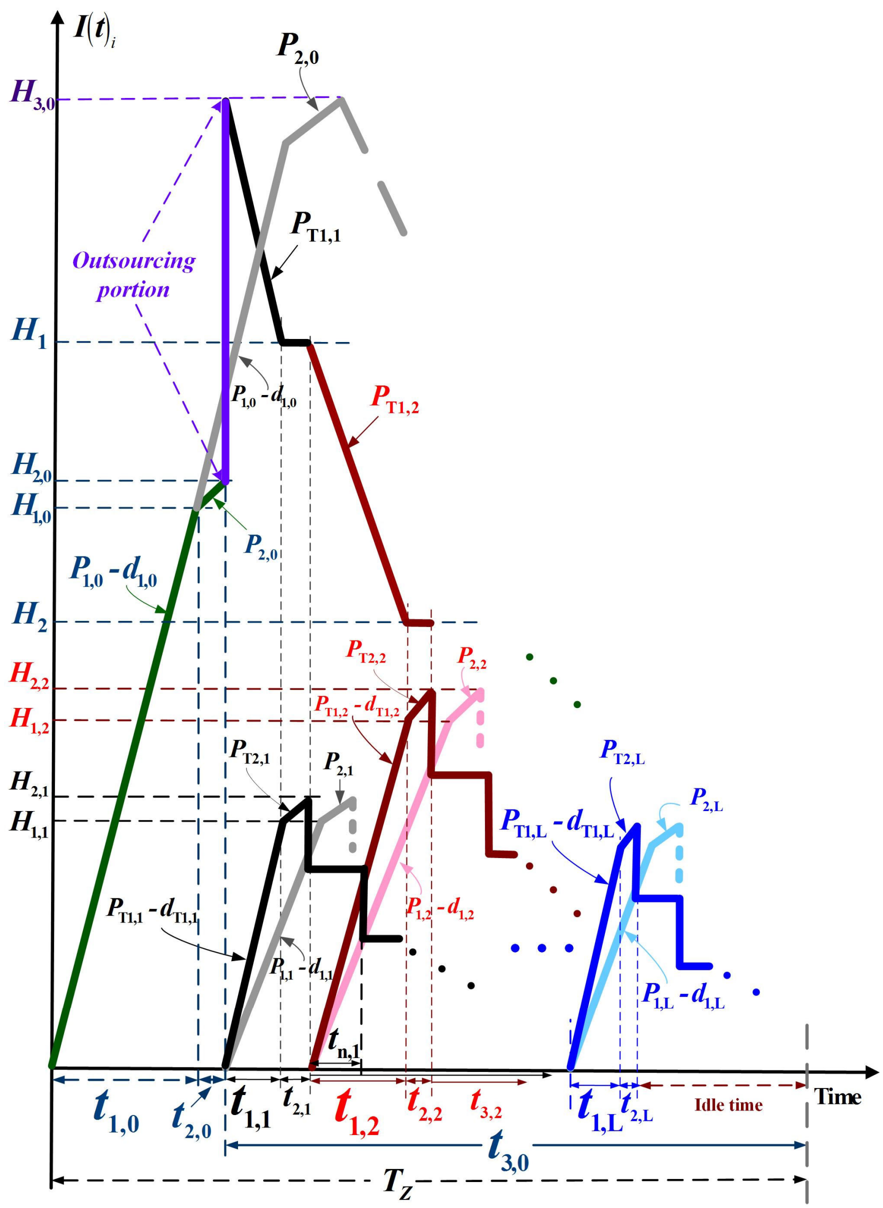

Fig. 1. The stock level in our model versus a problem without outsourcing nor overtime

Poor quality with random faulty rates x_0 and x_i happen in both fabricating stages; rework actions are used to fix all nonconforming items made entirely. Fig. 1 shows the stock level of standard part surges to $H_{1,0}$ when $t_{1,0}$ completes, and it reaches $H_{2,0}$ when $t_{2,0}$ completes. By receiving outsourcing parts, it further accumulates $H_{3,0}$ before stage 2 starts. In contrast, in stage 2, the stock level of end merchandise i surges to $H_{1,i}$ when $t_{1,i}$ ends, and it reaches $H_{2,i}$ when $t_{2,i}$ ends. Then, end merchandise i's inventories are distributed to the clients in equal-size multiple shipments (see Fig. 1).

Fig. 2 exhibits the faulty stock level of our model. It illustrates that the faulty standard part's inventory surges to ($d_{1,0}\, t_{1,0}$) when $t_{1,0}$ completes and gradually declines to zero when reworking $t_{2,0}$ ends. In stage 2, the faulty end merchandise i's inventory level has the same status. $(P_{1,0} - d_{1,0}) > 0$ and $(P_{\mathrm{T}1,i} - d_{\mathrm{T}1,i} - \lambda_i) > 0$ must hold to avoid the stock-out situations in both stages. The goal of this particular multiproduct manufacturing-transportation integration model is to decide the optimal policy of fabricating cycle time and delivery frequency through minimizing the total operating expenses.

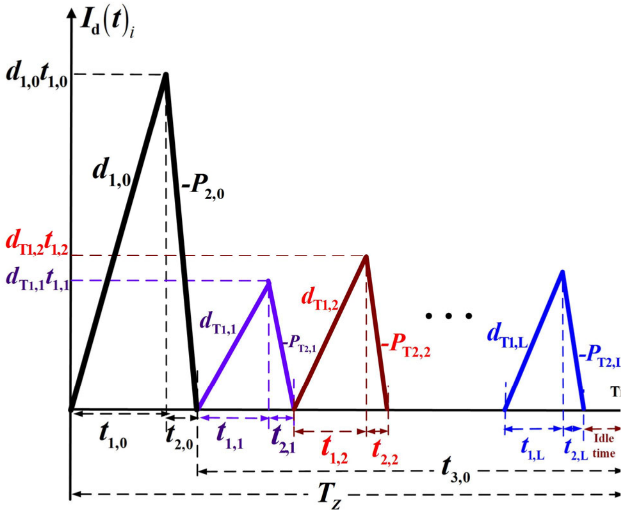

Fig. 2. The faulty stock level of this study

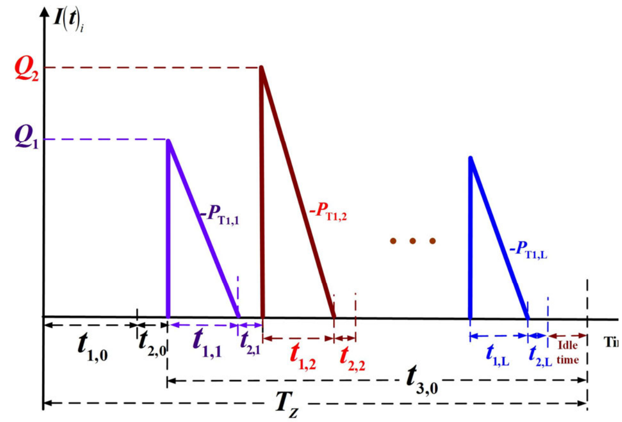

Fig. 3. Standard part's stock level when stage 2 starts

2.2. Formulations

The standard part's stock level is depleted gradually as stage 2 starts (see Figs. 1 and 3). Observing Fig. 1, as each end merchandise i's uptime completes, the standard part's inventory level decreases the amount of Q_i from the initial $H_{3,0}$ to H_i. Eq. (8) to Eq. (10) exhibit the details.

$$H_1 = H_{3,0} - Q_1 \tag{8}$$

$$H_i = H_{(i-1)} - Q_i \text{ , } for\ i = 2,\ 3,\ ...,L \tag{9}$$

$$H_L = H_{(L-1)} - Q_L = 0 \tag{10}$$

2.2.1. Formulations in stage 2

One can straightforwardly observe Eq. (11) to Eq. (17) from Figs. 1 to 3 relating to stage 2 of our proposed model.

$$T_Z = \frac{Q_i}{\lambda_i} \text{ or } Q_i = \lambda_i T_Z \tag{11}$$

$$T_Z = t_{1,i} + t_{2,i} + t_{3,i} \text{ where } i = 1,\ 2,\ \dots,\ L \tag{12}$$

$$t_{1,i} = \frac{H_{1,i}}{\left(P_{T1,i} - d_{T1,i}\right)} = \frac{Q_i}{P_{T1,i}} \tag{13}$$

$$t_{2,i} = \frac{Q_i x_i}{P_{T2,i}} = \frac{H_{2,i} - H_{1,i}}{P_{T2,i}} \tag{14}$$

$$t_{3,i} = T_Z - \left(t_{1,i} + t_{2,i}\right) \tag{15}$$

$$H_{1,i} = \left(P_{T1,i} - d_{T1,i}\right) t_{1,i} \tag{16}$$

$$H_{2,i} = H_{1,i} + P_{T2,i} t_{2,i} \tag{17}$$

Each end merchandise i's inventory level during $t_{3,i}$ is shown in Fig. 4. The total stocks in $t_{3,i}$ are displayed in Eq. (18).

$$\left(\frac{1}{n^2}\right) H_{2,i} \left(\sum_{i=1}^{n-1} i\right) t_{3,i} = \left(\frac{1}{n^2}\right) H_{2,i} \left[\frac{n(n-1)}{2}\right] t_{3,i} = \left(\frac{n-1}{2n}\right) H_{2,i} t_{3,i} \tag{18}$$

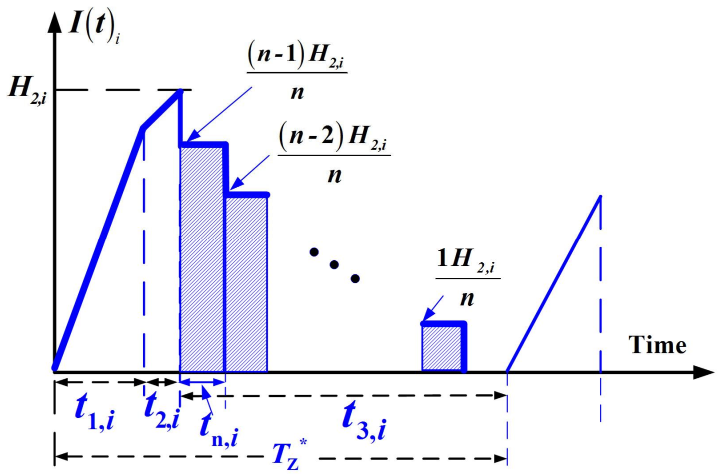

Fig. 4. Each end product i's inventory status during $t_{3,i}$

Fig. 5 depicts each end products' inventory level on the buyer side. Its total stocks are expressed in Eq. (19).

$$\left[\frac{n(D_i-I_i)t_{n,i}}{2}+\frac{nI_i(t_{1,i}+t_{2,i})}{2}+\frac{n(n+1)}{2}I_i t_{n,i}\right] \tag{19}$$

Where

$$t_{n,i}=\frac{t_{3,i}}{n} \tag{20}$$

$$I_i=D_i-\lambda_i(t_{n,i}) \tag{21}$$

$$D_i=\frac{H_{2,i}}{n} \tag{22}$$

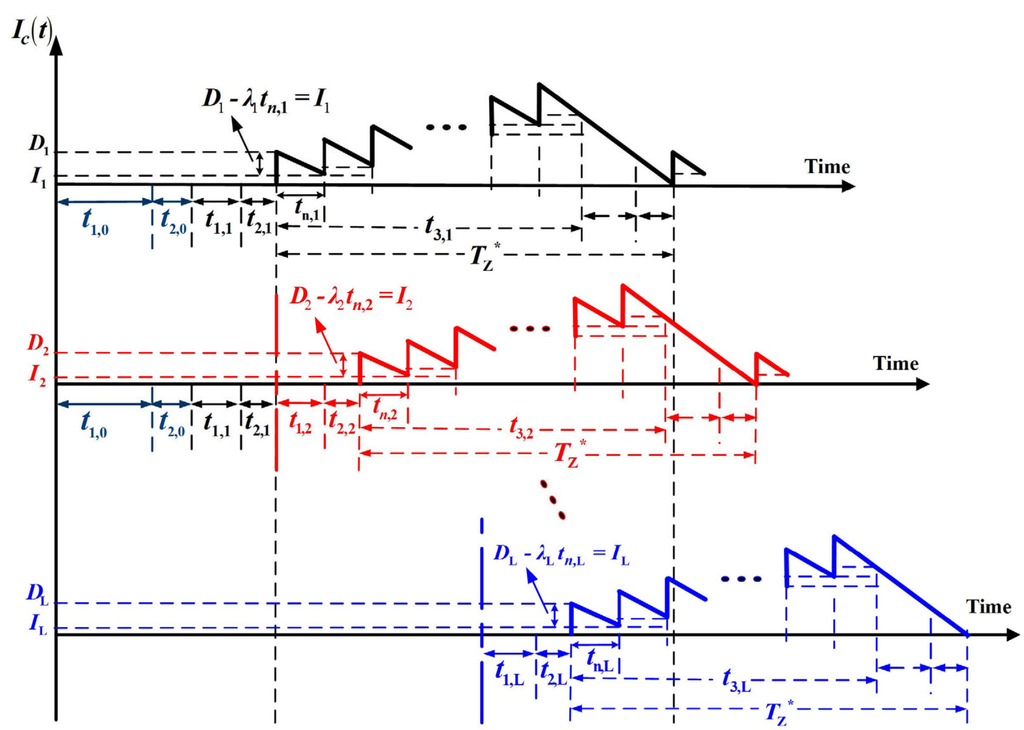

Fig. 5. End merchandise i's inventories status at client side

2.2.2. Formulations in stage 1

For stage 1, one can straightforwardly observe the following equations from Figs. 1 to 3 and the problem description. First, to meet the needs of the standard parts for fabricating L end products, one finds the common parts' annual requirement and in-house cycle lot size as follows:

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T_Z} \tag{23}$$

$$H_{3,0} = \sum_{i=1}^{L} Q_i = \sum_{i=1}^{L} \lambda_i T_Z \tag{24}$$

$$Q_0 = (1-\pi_0) H_{3,0} = (1-\pi_0)\left(\sum_{i=1}^{L} Q_i\right) = H_{2,0} \tag{25}$$

$$T_Z = t_{1,0} + t_{2,0} + t_{3,0} \tag{26}$$

$$t_{1,0} = \frac{Q_0}{P_{1,0}} = \frac{H_{1,0}}{\left(P_{1,0} - d_{1,0}\right)} \tag{27}$$

$$t_{2,0} = \frac{Q_0 x_0}{P_{2,0}} = \frac{H_{2,0} - H_{1,0}}{P_{2,0}} \tag{28}$$

$$H_{1,0} = \left(P_{1,0} - d_{1,0}\right) t_{1,0} \tag{29}$$

$$H_{2,0} = H_{1,0} + P_{2,0} t_{2,0} \tag{30}$$

3. Total system operating expenditures and optimization procedure

Total system operating expenditures $TC(T_Z, n)$ involve: (a) outsourcing and in-house setup and variable expenditures, in-house rework, and stock holding expenses; (b) the sum of end product i's overtime producing setup, variable, rework, inventory holding, distribution expenses; and (c) buyer's stock holding expense. Hence, $TC(T_Z, n)$ can be gained as follows:

$$\begin{aligned} TC\left(T_Z, n\right) &= K_{\pi 0} + C_{\pi 0}\pi_0\left(\sum_{i=1}^{L} Q_i\right) + K_0 + C_0 Q_0 + C_{R,0} Q_0 x_0 \\ &+ h_{1,0}\left[\frac{H_{1,0} t_{1,0}}{2} + \sum_{i=1}^{L}\left[\frac{Q_i}{2}\left(t_{1,i}\right) + H_i\left(t_{1,i} + t_{2,i}\right)\right] + \frac{H_{2,0} + H_{1,0}}{2}\left(t_{2,0}\right) + \frac{d_{1,0} t_{1,0}}{2}\left(t_{1,0}\right)\right] + h_{2,0}\left(t_{2,0}\right)\left(\frac{d_{1,0} t_{1,0}}{2}\right) \\ &+ \sum_{i=1}^{L}\left\{\begin{array}{l} Q_i C_{\mathrm{T},i} + K_{\mathrm{T},i} + h_{2,i}\left(\dfrac{d_{T1,i} t_{1,i}}{2}\right)\left(t_{2,i}\right) + C_{\mathrm{T}R,i} Q_i x_i + nK_{D,i} \\ + h_{1,i}\left[\dfrac{H_{1,i} t_{1,i}}{2} + \dfrac{H_{2,i} + H_{1,i}}{2}\left(t_{2,i}\right) + \dfrac{d_{T1,i} t_{1,i}}{2}\left(t_{1,i}\right) + \left(\dfrac{n-1}{2n}\right) H_{2,i}\left(t_{3,i}\right)\right] \\ + + C_{D,i} Q_i h_{3,i}\left[\dfrac{n\left(D_i - I_i\right) t_{n,i}}{2} + \dfrac{nI_i\left(t_{1,i} + t_{2,i}\right)}{2} + \dfrac{n(n+1)}{2} I_i t_{n,i}\right] \end{array}\right\} \end{aligned} \tag{31}$$

We employ $E[x_i]$ (for $i = 0, 1, 2, \ldots, L$) to deal with random faulty rates, substituting Eqs. (1) to (30) in Eq. (31), plus additional derivation, $E[TCU(T_Z, n)]$ becomes (see details in Appendix B):

$$\begin{aligned} E\left[TCU\left(T_Z, n\right)\right] &= \left\{\begin{array}{l} C_0\left(1-\pi_0\right)\lambda_0 + \dfrac{K_0\left(1+\beta_{1,0}\right)}{T_Z} + C_0\left(1+\beta_{2,0}\right)\pi_0\lambda_0 + \dfrac{h_{1,0}\lambda_0^{\,2} T_Z}{2}\left(1-\pi_0\right)^2 E_{0P} \\ + \dfrac{K_0}{T_Z} + C_{R,0}\left(1-\pi_0\right)\lambda_0 E\left[x_0\right] + \dfrac{h_{2,0}\lambda_0^{\,2}\left(1-\pi_0\right)^2}{2P_{2,0}} E\left[x_0\right]^2 T_Z \\ + h_{1,0}\displaystyle\sum_{i=1}^{L}\left\{\dfrac{\lambda_i^2 T_Z}{2\left[\left(1+\alpha_{1,i}\right)P_{1,i}\right]} + \left(\sum_{i=1}^{L}\left[\lambda_i T_Z\right] - \sum_{j=1}^{i}\left[\lambda_j T_Z\right]\right)\lambda_i E_{2i}\right\} \end{array}\right\} \\ &+ \sum_{i=1}^{L}\left\{\begin{array}{l} C_i\left(1+\alpha_{3,i}\right)\lambda_i + \dfrac{nK_{D,i}}{T_Z} + \dfrac{K_i\left(1+\alpha_{2,i}\right)}{T_Z} + C_{D,i}\lambda_i + C_{R,i}\left(1+\alpha_{3,i}\right)\lambda_i E\left[x_i\right] + h_{1,i} T_Z \lambda_i^{\,2} E_{3i} \\ + h_{2,i}\dfrac{T_Z}{2\left[\left(1+\alpha_{1,i}\right)P_{2,i}\right]}\left(\lambda_i E\left[x_i\right]\right)^2 + \left(\dfrac{\lambda_i^{\,2} T_Z}{2n}\right)\left[\dfrac{1}{\lambda_i} - E_{2i}\right]\left(h_{3,i} - h_{1,i}\right) + \dfrac{h_{3,i}}{2}\left(\lambda_i^2 T_Z\right) E_{2i} \end{array}\right\} \end{aligned} \tag{32}$$

3.1. Optimization procedure

The Hessian Matrix equations are applied to $E[TCU(T_Z, n)]$:

$$[T_Z \quad n]\cdot\begin{pmatrix}\dfrac{\partial^2 E\left[TCU\left(T_Z,\, n\right)\right]}{\partial T_Z^{\,2}} & \dfrac{\partial^2 E\left[TCU\left(T_Z,\, n\right)\right]}{\partial T_Z\partial n}\\ \dfrac{\partial^2 E\left[TCU\left(T_Z,\, n\right)\right]}{\partial T_Z\partial n} & \dfrac{\partial^2 E\left[TCU\left(T_Z,\, n\right)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}T_Z\\ n\end{bmatrix}=\left[\frac{2K_0}{T_Z}+\sum_{i=1}^{L}\left\{\frac{2\left(1+\alpha_{2,i}\right)K_i}{T_Z}\right\}+\frac{2\left(1+\beta_{1,0}\right)K_0}{T_Z}\right]>0 \tag{33}$$

Eq. (33) yields positive, since K_i, $(1 + \alpha_{2,i})$, T_Z, K_0, and $(1 + \beta_{1,0})$ are positive. So, $E[TCU(T_Z, n)]$ is strictly convex, and it exi sts the minimum for all n and T_Z values > 0. Letting first derivatives of $E[TCU(T_Z, n)]$ regarding n and T_Z equal to zero.

$$\frac{\partial E\left[TCU\left(T_Z,\, n\right)\right]}{\partial n}=\sum_{i=1}^{L}\left\{\frac{K_{D,i}}{T_Z}-\left(\frac{\lambda_i^{\,2}T_Z}{2n^2}\right)\left[\frac{1}{\lambda_i}-E_{2i}\right]\left(h_{3,i}-h_{1,i}\right)\right\}=0 \tag{34}$$

$$\frac{\partial E\left[TCU(T_Z,\, n)\right]}{\partial T_Z}=\left\{\begin{array}{l}-\dfrac{K_0}{T_Z^2}-\dfrac{K_0\left(1+\beta_{1,0}\right)}{T_Z^2}+\dfrac{h_{1,0}\lambda_0^{\,2}}{2}\left(1-\pi_0\right)^2E_{0P}+\dfrac{h_{2,0}\lambda_0^{\,2}\left(1-\pi_0\right)^2}{2P_{2,0}}E\left[x_0\right]^2\\ +h_{1,0}\displaystyle\sum_{i=1}^{L}\left\{\frac{\lambda_i^{\,2}}{2\left[\left(1+\alpha_{1,i}\right)P_{1,i}\right]}+\left(\sum_{i=1}^{L}\lambda_i-\sum_{j=1}^{i}\lambda_j\right)\lambda_iE_{2i}\right\}\end{array}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{array}{l}-\dfrac{K_i\left(1+\alpha_{2,i}\right)}{T_Z^2}-\dfrac{nK_{D,i}}{T_Z^2}+h_{2,i}\dfrac{\left(\lambda_iE\left[x_i\right]\right)^2}{2\left[\left(1+\alpha_{1,i}\right)P_{2,i}\right]}+h_{1,i}\lambda_i^2E_{3i}\\ +\left(\dfrac{\lambda_i^2}{2n}\right)\left[\dfrac{1}{\lambda_i}-E_{2i}\right]\left(h_{3,i}-h_{1,i}\right)+\dfrac{E_{2i}h_{3,i}}{2}\left(\lambda_i^2\right)\end{array}\right\}=0 \tag{35}$$

Then, solving Eq. (34) and Eq. (35) simultaneously, one can derive T_Z* and n*.

$$T_Z^{\;*}=\sqrt{\frac{2\left\{\left(2+\beta_{1,0}\right)K_0+\displaystyle\sum_{i=1}^{L}\left[nK_{D,i}+K_i\left(1+\alpha_{2,i}\right)\right]\right\}}{\begin{array}{l}\displaystyle\sum_{i=1}^{L}\left\{h_{1,i}\lambda_i^2E_{3i}+h_{2,i}E\left[x_i\right]^2\frac{\lambda_i^2}{\left(1+\alpha_{1,i}\right)P_{2,i}}+h_{3,i}\lambda_i^2E_{2i}+\left(\frac{\lambda_i^2}{n}\right)\left[\frac{1}{\lambda_i}-E_{2i}\right]\left(h_{3,i}-h_{1,i}\right)\right\}+h_{2,0}\frac{\left(1-\pi_0\right)^2\lambda_0^2}{P_{2,0}}E\left[x_0\right]^2\\ +h_{1,0}\left[E_{0P}\left(1-\pi_0\right)^2\lambda_0^{\,2}+\displaystyle\sum_{i=1}^{L}\left(\frac{\lambda_i^2}{\left(1+\alpha_{1,i}\right)P_{1,i}}\right)+2\sum_{i=1}^{L}\left(\lambda_i\right)\sum_{i=1}^{L}\lambda_iE_{2i}-2\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\left(\lambda_j\right)\right)\left(\lambda_iE_{2i}\right)\right]\right]\end{array}}} \tag{36}$$

and

$$n^*=\sqrt{\frac{\left[\left(2+\beta_{1,0}\right)K_0+\displaystyle\sum_{i=1}^{L}K_i\left(1+\alpha_{2,i}\right)\right]\cdot\sum_{i=1}^{L}\left\{\left(\frac{1}{\lambda_i}-E_{2i}\right)\lambda_i^{\,2}\left(h_{3,i}-h_{1,i}\right)\right\}}{\displaystyle\sum_{i=1}^{L}\left\{2K_{Di}\right\}\cdot\sum_{i=1}^{L}\left\{\begin{array}{l}\displaystyle\sum_{i=1}^{L}\left\{h_{3,i}\left(\lambda_i^2\right)E_{2i}+h_{2,i}E\left[x_i\right]^2\frac{\lambda_i^2}{\left(1+\alpha_{1,i}\right)P_{2,i}}+h_{1,i}\left[\lambda_i^2E_{3i}\right]\right\}+h_{2,0}\left(\frac{\left(1-\pi_0\right)^2\lambda_0^2}{P_{2,0}}\right)E\left[x_0\right]^2\\ +h_{1,0}\left[\displaystyle\sum_{i=1}^{L}\left(\frac{\lambda_i^2}{\left(1+\alpha_{1,i}\right)P_{1,i}}\right)+E_{0P}\left(1-\pi_0\right)^2\lambda_0^{\,2}+2\sum_{i=1}^{L}\left(\lambda_i\right)\sum_{i=1}^{L}\lambda_iE_{2i}-2\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\left(\lambda_j\right)\right)\left(\lambda_iE_{2i}\right)\right]\right]\end{array}\right\}}} \tag{37}$$

3.2. Discussion on the setup times

In a single-equipment multiple items production, if the sum of setup times S_i is over the cycle's idle time exhibited in Fig. 1; one must compute the $T_{\min}$. Then, select the max of T_Z* and $T_{\min}$ as the final solution for the cycle time to guarantee the equipment capacity to make the end merchandise and all standard parts.

$$T_{\min}=\frac{\displaystyle\sum_{i=0}^{L}\left(S_i\right)}{1-\left\{\lambda_0\left(\frac{1}{P_{1,0}}+\frac{E\left[x_0\right]}{P_{2,0}}\right)\left(1-\pi_0\right)+\displaystyle\sum_{i=1}^{L}\lambda_i\left[\frac{E\left[x_i\right]}{P_{T2,i}}+\frac{1}{P_{T1,i}}\right]\right\}} \tag{38}$$

3.3. Discussion on the prerequisite condition

Again, a single-equipment multiple items production, Eq. (39) is the prerequisite condition to guarantee the equipment capacity to make the end merchandise and all standard parts.

$$\left[\sum_{i=1}^{L}\left(t_{1,i}+t_{2,i}\right)+\left(t_{1,0}+t_{2,0}\right)\right]<T_Z \text{ or } \left[\sum_{i=1}^{L}Q_i\left(\frac{E[x_i]}{P_{T2,i}}+\frac{1}{P_{T1,i}}\right)+Q_0\left(\frac{E[x_0]}{P_{2,0}}+\frac{1}{P_{1,0}}\right)\right]<T_Z \tag{39}$$

or

$$\left\{\lambda_0\left(1-\pi_0\right)\left(\frac{1}{P_{1,0}}+\frac{E[x_0]}{P_{2,0}}\right)+\sum_{i=1}^{L}\lambda_i\left[\frac{1}{P_{T1,i}}+\frac{E[x_i]}{P_{T2,i}}\right]\right\}<\ 1 \tag{40}$$

4. Illustrating example

The variable values assumption for our multiproduct delayed differentiation model is given in Tables 1, 2(a), and 2(b). Conversely, Table C in Appendix C gives their corresponding variable values for a single-phase production scheme. An illustrating example shows how our obtained result can find the optimal production-shipping policies T_Z* and n* and explore various critical characteristics in the studied problem.

Table 1
Assumption of variable values for phase one of our delayed differentiation model

$P_{1,0}$	x_0	π_0	$h_{1,0}$	γ	$\beta_{2,0}$	$C_{R,0}$	$h_{2,0}$
120000	2.5%	0.4	\$8	0.5	0.4	\$25	\$8
$\beta_{1,0}$	λ_0	δ	i_0	C_0	$P_{2,0}$	K_0	
-0.7	17000	0.5	0.2	\$40	96000	\$8500	

Table 2(a)
Assumption of variable values for phase two of our delayed differentiation model (1 of 2)

Product i	$C_{D,i}$	$\alpha_{1,i}$	x_i	$\alpha_{2,i}$	λ_i	$\alpha_{3,i}$	$K_{D,i}$	K_i
1	\$0.1	0.5	2.5%	0.1	3000	0.25	\$1800	\$8500
2	\$0.2	0.5	7.5%	0.1	3200	0.25	\$1900	\$9000
3	\$0.3	0.5	12.5%	0.1	3400	0.25	\$2000	\$9500
4	\$0.4	0.5	17.5%	0.1	3600	0.25	\$2100	\$10000
5	\$0.5	0.5	22.5%	0.1	3800	0.25	\$2200	\$10500

Table 2(b)
Assumption of variable values for phase two of our delayed differentiation model (2 of 2)

Product i	$C_{R,i}$	C_i	$h_{3,i}$	$P_{1,i}$	$h_{2,i}$	$h_{1,i}$	i_i	$P_{2,i}$
1	\$25	\$40	\$70	112258	\$8	\$8	0.2	89806
2	\$30	\$50	\$75	116066	\$10	\$10	0.2	92852
3	\$35	\$60	\$80	120000	\$12	\$12	0.2	96000
4	\$40	\$70	\$85	124068	\$14	\$14	0.2	99254
5	\$45	\$80	\$90	128276	\$16	\$16	0.2	102621

By calculating formulas (37) and (36), we first find the problem's optimal operating solution in terms of shipping frequency and production cycle time: n* = 4 and T_Z* = 0.5660. We then calculate formula (32) with this solution and gain the optimal annual operating expense $E[TCU(T_Z^*, n^*)]$ = \$2,517,055. Figure 6 illustrates $E[TCU(T_Z, n)]$'s behavior vis-à-vis n and T_Z. It explicitly exposes the $E[TCU(T_Z, n)]$'s convexity as n and T_Z deviate from their optimal values n* and T_Z*. One may be curious about how the shipping frequency n influences transportation relevant expenses. As n increases, less amount of end products per shipment is transported at a time, so the producer's holding, and fixed transportation costs surge considerably. Conversely, the buyer's holding expense declines drastically. Fig. 7 displays the impact of shipping frequency n per cycle on various system expenses. This study assures the quality of standard parts and end products by reworking and repairing faulty items in both fabricating phases. As the ratio of needed mean rework cost over unit cost rises, the optimal annual operating expense $E[TCU(T_Z^*, n^*)]$ surges. Fig. 8 depicts $E[TCU(T_Z^*, n^*)]$'s performance concerning the mean rework cost over unit cost. It indicates that at our assumption when this ratio is 0.6, one confirms the optimal $E[TCU(T_Z^* = 0.566, n^* = 4)]$ = \$2,517,055.

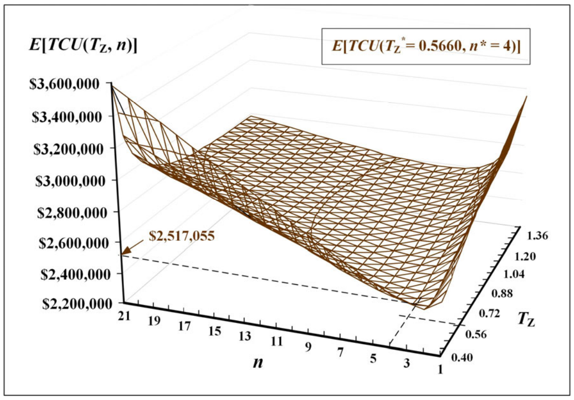

Fig. 6. $E[TCU(T_Z, n)]$'s behavior vis-à-vis n and T_Z

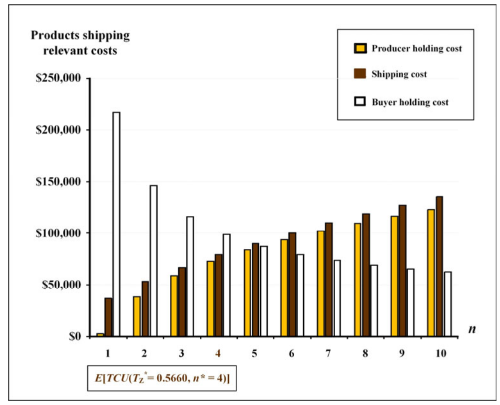

Fig. 7. The impact of shipping frequency n per cycle on various system expense

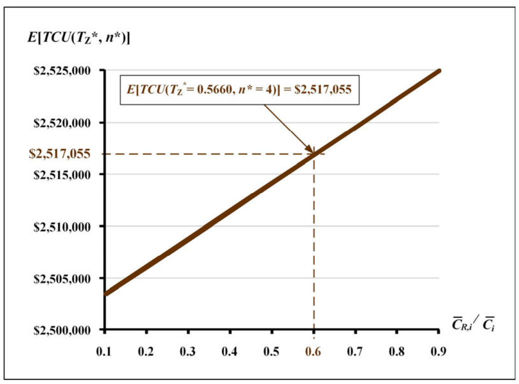

Fig. 8. $E[TCU(T_Z^*, n^*)]$'s performance concerning the mean rework cost over unit cost

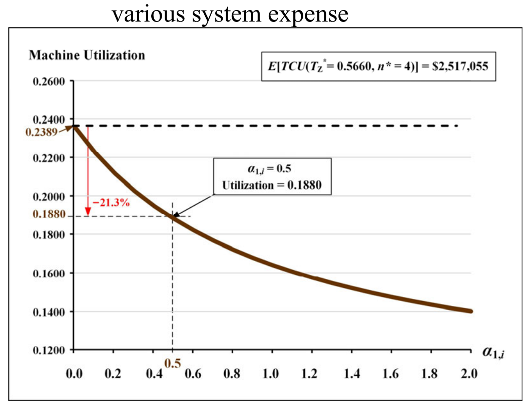

Fig. 9. The utilization's performance concerning $\alpha_{1,i}$

This study implements an overtime policy in phase 2 to increase the output of end products. As the added fabricating proportion $\alpha_{1,i}$ rises, machine utilization drops. Table D-1 (in Appendix D) demonstrates the analytical outcomes of different critical system features influenced by overtime factor $\alpha_{1,0}$. Fig. 9 illustrates utilization's performance concerning $\alpha_{1,i}$. It discovers that at our assumption $\alpha_{1,i} = 0.5$, a 21.3% decrease in utilization, i.e., dropping from 0.2389 to 0.1880. Furthermore, analytical outcomes present the price paid for the 21.3% decline in utilization (with $\alpha_{1,i} = 0.5$) is a rise of 12.14% in the optimal annual operating expense $E[TCU(T_Z^*, n^*)]$, i.e., surging from $2,244,654 (when $\alpha_{1,i} = 0$) to $2,517,055 (as shown in Fig. 10; also see Table D-1). Fig. 11 demonstrates the analytical outcome of the collective impact of mean faulty rate and $\alpha_{1,i}$ on overtime rework expense. As $\alpha_{1,i}$ goes up, overall rework expense increases slightly; if the mean faulty rate rises, overall rework expense surges severely.

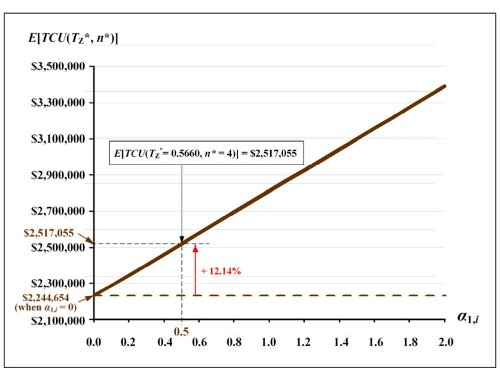

Fig. 10. $E[TCU(T_Z^*, n^*)]$'s performance concerning $\alpha_{1,i}$

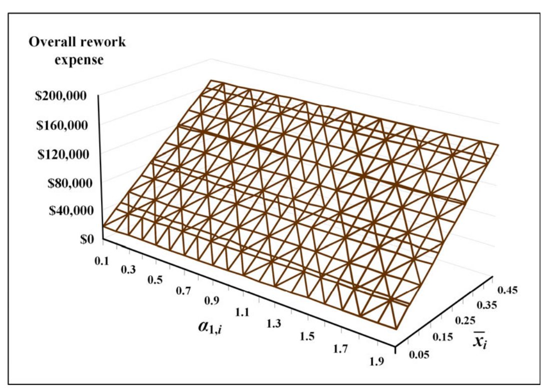

Fig. 11. The collective impact of mean faulty rate and $\alpha_{1,i}$ on overtime rework expense

The present work also adopts a partial outsourcing policy (i.e., a π_0 portion of all standard parts requirement in a cycle λ_0) to reduce the uptime required for making the common parts in phase one. As the outsourcing proportion π_0 rises, phase one's uptime declines; consequently, the utilization decreases drastically. Table D-2 displays the analytical outcomes of various critical system features effected by outsourcing factor π_0. Fig. 12 shows the machine utilization's response to the variation of π_0. It discovers that for the assumption $\pi_0 = 0.4$, a 23.45% decrease in utilization, i.e., dropping from 0.2456 to 0.1880.

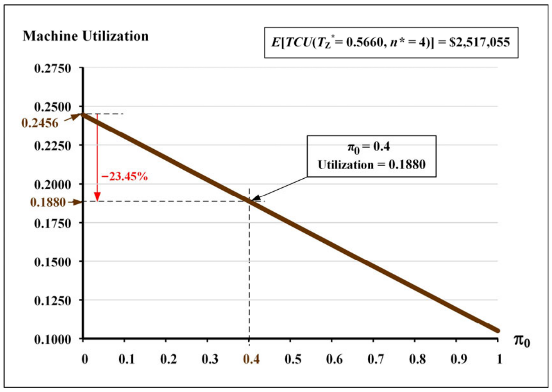

Fig. 12. Machine utilization's response to the variation of π_0

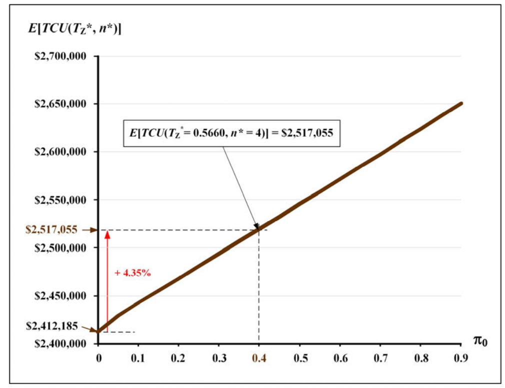

Fig. 13. $E[TCU(T_Z^*, n^*)]$'s performance concerning π_0

Additionally, investigative price paid consequences for bringing down a 23.45% utilization (with $\pi_0 = 0.4$) is an increase of 4.35% in the optimal annual operating expense $E[TCU(T_Z^*, n^*)]$, i.e., surging from \$2,412,185 (when $\pi_0 = 0$) to \$2,517,055 (as shown in Fig. 13; also see Table D-2). To reduce utilization, there is a different price paid concerning outsourcing and/or overtime strategies (recall Fig. 10 and Fig. 13). The further analysis exposes crucial managerial decisional information on effectively implementing outsourcing, overtime, or both strategies. Fig. 14 exhibits the findings about $E[TCU(T_Z^*, n^*)]$'s performance regarding these utilization-reduction options. The information in Figure 14 advises managers on a more cost-effective approach to reducing utilization: outsourcing a constant percentage of 40% ($\pi_0 = 0.4$) of a batch of standard parts and simultaneously adopting the overtime with an increasing $\alpha_{1,i}$ (as shown in the dashed line of Fig. 14). Once the overtime rate $\alpha_{1,i}$ reaches 0.5 or utilization declines to 0.188 (see the intersection of the dashed and solid lines), keep $\alpha_{1,i}$ at a constant rate of 0.5 and increase the outsourcing percentage π_0. That is the most beneficial approach for reducing utilization.

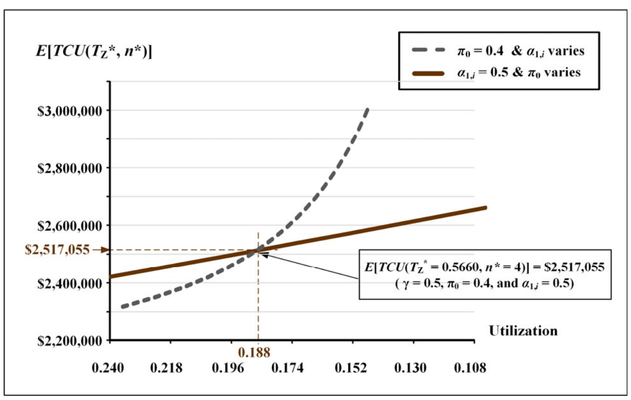

Fig. 14. $E[TCU(T_Z^*, n^*)]$'s performance regarding the utilization-reduction options

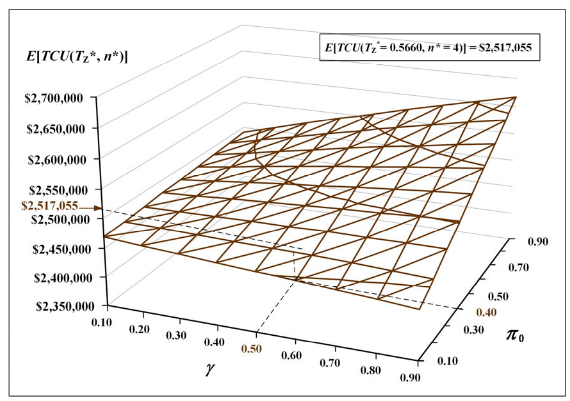

Fig. 15. $E[TCU(T_Z^*, n^*)]$'s behavior vis-à-vis the collective impacts of γ and π_0

In a multiproduct delayed differentiation fabricating system, the completion rate γ of standard part influences stage 1's uptime and the optimal expected annual operating expenditure $E[TCU(T_Z^*, n^*)]$. Fig. 15 demonstrates the explorative outcome of $E[TCU(T_Z^*, n^*)]$'s performance concerning the collective effect of γ and the outsourcing proportion π_0. As γ rises, $E[TCU(T_Z^*, n^*)]$ slightly declines when π_0.is lower; however, when π_0.is higher, $E[TCU(T_Z^*, n^*)]$ increases knowingly. Meantime, as π_0 rises because of the more expensive outsourcing unit cost, $E[TCU(T_Z^*, n^*)]$ surges considerably, especially when γ is high (i.e., more workloads stay on stage one, and they have been outsourced).

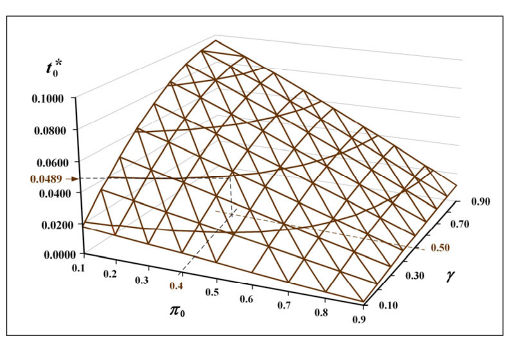

Fig. 16. t_0^*'s performance concerning the collective effects of π_0 and γ

Moreover, Fig. 16 depicts the investigative outcome of combined influences of γ and outsourcing proportion π_0 on t_0^*. As π_0 rises because the outsourcer is responsible for more common parts workloads, t_0^* declines significantly. On the contrary, as γ surges (the producer needs to devote more times (works) fabricating standard parts), t_0^* surges drastically; especially when π_0 is low, the in-house fabricating job becomes heavily loaded. For our example's assumption $\alpha_{1,i}$ at 0.5 and π_0 at 0.4, t_0^* is 0.0489 years.

5. Conclusions

This research targets establishing a decision-support delayed-differentiation model to assist manufacturing firms in making the right production plan to meet customers' current demands. The model incorporates the strategies of postponement, outsourcing, overtime, rework, and multi-delivery. We optimize the model with minimum fabrication-inventory-shipping expenses under a restricted in-house capacity. Complex problem features' modeling, formulation, and math derivation are cautiously presented in Section 2. Determination of the best production-shipping policy (see Eqs. (36) & (37)) using the optimization approach is iteratively presented in Section 3 and Appendix B. The model's prerequisite and setup times conditions are revealed in Subsections 3.2 and 3.3. Validation of the model's applicability and our research results' capacity by a numerical illustration is offered in Section 4.

Besides accomplishing a helpful decision-support delayed-differentiation model, this work contributes to the studied areas by revealing various significant managerial insights into the studying issues. Examples of such include but are not limited to the following: (1) The convexity of the expected system operating expense (See Fig. 6); (2) Individual system features' influence (such as shipping frequency *n* per cycle, added rework expense, additional overtime rate, subcontracting proportion) on shipping-relevant cost (see Fig. 7) or the expected system operating expenses (refer to Fig. 8, Fig. 10 and Fig. 13); (3) The collective system features' influence (such as the mean faulty rate and additional overtime rate) on overtime rework expense (Fig. 11); (4) The collective system features' influence (such as utilization-reduction strategies or subcontracting proportion) on the expected system operating expenses (Figs. (14-15)); (5) Individual system features' influence (such as overtime added output rate and subcontracting proportion) on machine utilization (Figs. (9-12)); and (6) The collective influence of subcontracting proportion and standard/common part's completing rate on phase 1's optimal uptime (see Fig. 16). For future research direction, examining the impact of stochastic finished products' requirements on the proposed model is a practical and worthy investigative subject.

References

Bailey, J.P., & Rabinovich, E. (2006). The adoption of inventory postponement and speculation: An empirical assessment of oligopolistic Internet retailers. *Transportation Research Part E: Logistics and Transportation Review, 42*(4), 258-271.

Bruneel, H., Maertens, T., & Walraevens, J. (2014). Class clustering destroys delay differentiation in priority queues. *European Journal of Operational Research, 235*(1), 149-158.

Budiman, S.D., & Rau, H. (2021). A stochastic model for developing speculation-postponement strategies and modularization concepts in the global supply chain with demand uncertainty. *Computers and Industrial Engineering, 158*, Art. No. 107392.

Chiu, Y.-S.P., Wang, Y., Chou, C.-L., & Chiu, T. (2020a). A two-stage multiproduct EMQ-based model with delayed differentiation and overtime option for common part's fabrication. *Journal of Applied Research and Technology, 18*(5), 259-268.

Chiu, Y. S. P., Jhan, J. H., Chiu, V., & Chiu, S. W. (2020b). Fabrication cycle time and shipment decision for a multiproduct intra-supply chain system with external source and scrap. *International Journal of Mathematical, Engineering and Management Sciences, 5*(4), 614-630.

Chiu, S.W., Wu, H.-Y., Yeh, T.-M., & Wang, Y. (2021a). Solving a hybrid batch production problem with unreliable equipment and quality reassurance. *International Journal of Industrial Engineering Computations, 12*(3), 235-248.

Chiu, S.W., Chiu, V., Hwang, M.-H., & Chiu, Y.-S.P. (2021b). A delayed differentiation multiproduct model with the outsourcing of common parts, overtime strategy for end products, and quality reassurance. *International Journal of Industrial Engineering Computations, 12*(2), 143-158.

Chiu, Y.-S.P., Wang, Y., Yeh, T.-M., & Chiu, S.W. (2022a). Fabrication runtime decision for a hybrid system incorporating probabilistic breakdowns, scrap, and overtime. *International Journal of Industrial Engineering Computations, 13*(3), 293-308.

Chiu, Y.-S.P., Chiu, S.W., Chiu, T., & Wang, H.-C. (2022b). The collective effect of rework, expedited-rate, external source, and machine failures on manufacturing runtime planning. *Decision Science Letters, 11* 4), 529-544.

Çimen, T., Baykasoğlu, A., & Akyol, S.D. (2022). Assembly line rebalancing and worker assignment considering ergonomic risks in an automotive parts manufacturing plant. *International Journal of Industrial Engineering Computations, 13*(3), 363-384.

Dabhilkar, M., & Bengtsson, L. (2008). Invest or divest? on the relative improvement potential in outsourcing manufacturing. *Production Planning and Control, 19*(3), 212-228.

Davila, T., & Wouters, M. (2007). An empirical test of inventory, service and cost benefits from a postponement strategy. *International Journal of Production Research, 45*(10), 2245-2267.

Freeman, N.K., Mittenthal, J., & Melouk, S.H. (2014). Parallel-machine scheduling to minimize overtime and waste costs. *IIE Transactions (Institute of Industrial Engineers), 46*(6), 601-618.

Friesen, J. (2001). Overtime pay regulation and weekly hours of work in Canada. *Labour Economics, 8*(6), 691-720.

Graman, G. A., & Magazine, M. J. (2006). Implementation issues influencing the decision to adopt postponement. *International Journal of Operations & Production Management, 26*(10), 1068-1083.

Granot, D., & Yin, S. (2008). Price and order postponement in a decentralized newsvendor model with multiplicative and price-dependent demand. *Operations Research, 56*(1), 121-139.

Guide Jr., V.D.R., & Srivastava, R. (1997) Repairable inventory theory: Models and applications. *European Journal of Operational Research, 102*(1), 1-20.

Handley, S., Skowronski, K., & Thakar, D. (2022). The single-sourcing versus multisourcing decision in information technology outsourcing. *Journal of Operations Management, 68*(6-7), 702-727.

Herrera, O., Jerez, V., Arrieta, N., & Camacho, G. (2022). Transshipment and routing model for multi-product planning and distribution at a refreshment company in Bogota, Colombia. *Journal of Applied Research and Technology, 20*(4), 408-417.

Hidayat, C.W., Sulistyo, Wilujeng, S., Chrismardani, Y., & Wahab (2022). Human resource competencies, supply chain management, and financial performance. *Uncertain Supply Chain Management, 10*(2), 471-476.

Hlioui, R., Gharbi, A., & Hajji, A. (2015). Replenishment, production and quality control strategies in three-stage supply chain. *International Journal of Production Economics, 166*, 90-102.

Hofer, A.R., Knemeyer, A.M., & Murphy, P.R. (2015). Achieving and exceeding logistics outsourcing expectations in Brazil: A replication study. *Transportation Journal, 54*(3), 339-367.

Khurosani, A., Nupus, H., & Ganika, G. (2021). The role of green human resource management in creating green supply chain culture in a service industry. *Uncertain Supply Chain Management, 9*(4), 973-982.

Loos, M.J., & Rodriguez, C.M.T. (2015). Practical application of postponement in a textile company. Producao, 25 (4), 894-910.

Malladi, S.S., Erera, A.L., & White, C.C., III (2021). Managing mobile production-inventory systems influenced by a modulation process. *Annals of Operations Research, 304*(1-2), 299-330.

Moon, I., Lee, J.-H., & Seong, J. (2012). Vehicle routing problem with time windows considering overtime and outsourcing vehicles. *Expert Systems with Applications, 39*(18), 13202-13213.

Olson, D.L. (2007). Evaluation of ERP outsourcing. *Computers and Operations Research, 34*(12), 3715-3724.

Prataviera, L.B., Moretti, E., & Tappia, E. (2022). Coping with the postponement boundary problem: an empirical investigation in global food supply chains. *International Journal of Logistics Management, 33*(2), 687-711.

Purusotham, S., Kumar, T.J., Vimala, T., & Ghanshyam, K.J. (2022). An efficient hybrid genetic algorithm for solving truncated travelling salesman problem. *Decision Science Letters, 11*(4), 473-484.

Quttineh, N.-H., & Lidestam, H. (2020). Applying heuristics in supply chain planning in the process industry. *International Journal of Industrial Engineering Computations, 11*(4), 585-606.

Ruiz-Torres, A.J., & Mahmoodi, F. (2006). A supplier allocation model considering delivery failure, maintenance and supplier cycle costs. *International Journal of Production Economics, 103*(2), 755-766.

Siregar, H., Suroso, A.I., Siregar, H., & Djohar, S. (2022). Development of efficient strategies to optimize production efficiency: Evidence from Pine chemical industry. *Decision Science Letters, 11*(4), 419-430.

Suharmono, M., Alexandri, M.B., Sumadinata, R.W.S., & Muhyi, H.A. (2022). Outsourcing in supply chain: A bibliometric analysis. *Uncertain Supply Chain Management, 10*(4), 1501-1508.

Sung, P.-C., Lai, C.-M., Wang, Y., & Chiu, Y.-S.P. (2022). Minimization of multiproduct fabrication cost featuring rework, commonality, external provider, and postponement. *Uncertain Supply Chain Management, 10*(2), 353-364.

Suryanto, T., & Mukhsin, M. (2020). Mediation of supply chain integration on the relationship between market orientation with company performance. *Uncertain Supply Chain Management, 8*(4), 739-744.

Taleizadeh, A. A., Jalali-Naini, S. G., Wee, H. M., & Kuo, T. C. (2013). An imperfect multi-product production system with rework. *Scientia Iranica, 20*(3), 811-823.

Taleizadeh, A. A., Cárdenas-Barrón, L. E., & Mohammadi, B. (2014). A deterministic multi product single machine EPQ model with backordering, scraped products, rework and interruption in manufacturing process. *International Journal of Production Economics, 150*, 9-27.

Taş, D., Gendreau, M., Jabali, O., & Jans, R. (2019) A capacitated lot sizing problem with stochastic setup times and overtime. *European Journal of Operational Research, 273*(1), pp. 146-159.

Tookanlou, P.B., & Wong, H. (2020). Determining the optimal customization levels, lead times, and inventory positioning in vertical product differentiation. *International Journal of Production Economics, 221*, Art. No. 107479.

Van Hoek, R.I (2001). Rediscovery of postponement a literature review and directions for research. *Journal of Operations Management, 19*(2), 161-184.

Yang, B., Burns, N. D., & Backhouse, C. J. (2004). Management of uncertainty through postponement. *International Journal of Production Research, 42*(6), 1049-1064.

Yu, P., Su, J., Gao, Q., Li, M., & Niou, C. (2007, June). Study of aluminum pad contamination sources during wafer fabrication, shipping, storage and assembly. *In 2007 International Symposium on High Density packaging and Microsystem Integration, pp. 1-3*. IEEE doi: 10.1109/HDP.2007.4283573.

Appendix - A

Table A
Distinction between this study's focuses and a few existing studies mentioned by the reviewer

Research's focuses/ features	Multi-product	Postponement strategies		Uptime reduction strategies		Multi-delivery	Imperfect items actions	Back-ordering & Interrup-tion	Services level & budget constraints	Optimal decision		Study of different scenarios with sugges-tions	Literature reviews (future research directions)	Empirical study - manufac-turer's data
		production	Purchasing & logistics	Outsourcing	Overtime					fabrication	fabrication & shipment			
The present study	X	X		X	X	X	X (rework)				X			
Chiu et al. (2020b)	X			X		X	X (scrapped)				X			
Yang et al. (2003)		X	X									X		
Van Hoek (2001)		X	X										X	
Davila & Wouters (2007)		X (focused on improving inventory & cost)	X (improving service quality)											X
Graman & Magazine (2006)		X (managerial issues & impacts)	X											X (via depth interview)
Taleizadeh et al. (2013)	X						X (rework)		X	X				
Taleizadeh et al. (2014)	X						X (rework & scrapped)			X				
Yu et al. (2007)						X				X				

X: stands for that this feature is included

Appendix - B

Detailed derivations of Eq. (32) are as follows:

We employ $E[x_i]$ (for i = 0, 1, 2, …, L) to deal with random faulty rates, substituting Eq. (1) to Eq. (30) in Eq. (31), plus additional derivation, $E[TCU(T_Z, n)]$ becomes:

$$E\big[TCU(T_Z,n)\big]=\left\{\begin{aligned}&\frac{K_0(1+\beta_{1,0})}{T_Z}+C_0\lambda_0(1-\pi_0)+C_{\pi 0}\pi_0\lambda_0(1+\beta_{2,0})+\frac{K_0}{T_Z}+h_{2,0}\left(\frac{E[x_0]^2(1-\pi_0)^2\lambda_0^2T_Z}{2P_{2,0}}\right)\\&+C_{R,0}(1-\pi_0)\lambda_0E[x_0]+h_{1,0}\left[\left[\frac{(1-\pi_0)^2\lambda_0^2T_Z}{2}\right]\left[\frac{E[x_0](2-E[x_0])}{P_{2,0}}+\frac{1}{P_{1,0}}\right]+\sum_{i=1}^{L}\left[\frac{\lambda_i^2T_Z}{2(1+\alpha_{1,i})P_{1,i}}\right]\right]\\&+h_{1,0}\left\{\left(\sum_{i=1}^{L}\lambda_iT_Z\right)\sum_{i=1}^{L}\left[\frac{\lambda_iE[x_i]}{(1+\alpha_{1,i})P_{2,i}}+\frac{\lambda_i}{(1+\alpha_{1,i})P_{1,i}}\right]+\sum_{i=1}^{L}\left[\left(\sum_{j=1}^{i}\lambda_jT_Z\right)\left(\frac{\lambda_iE[x_i]}{(1+\alpha_{1,i})P_{2,i}}+\frac{\lambda_i}{(1+\alpha_{1,i})P_{1,i}}\right)\right]\right\}\end{aligned}\right\}$$
$$+\sum_{i=1}^{L}\left\{\begin{aligned}&C_i(1+\alpha_{3,i})\lambda_i+E[x_i]\lambda_iC_{TR,i}+\frac{(1+\alpha_{2,i})K_i}{T_Z}+\frac{nK_{D,i}}{T_Z}+h_{2,i}\left(\frac{\lambda_i^2T_ZE[x_i]^2}{2(1+\alpha_{1,i})P_{2,i}}\right)+C_{D,i}\lambda_i\\&+h_{1,i}\left[\left(\frac{E[x_i](1-E[x_i])}{(1+\alpha_{1,i})P_{2,i}}+\frac{1}{\lambda_i}\right)\left(\frac{\lambda_i^2T_Z}{2}\right)\right]+\frac{h_{3,i}}{2}\left(\lambda_i^2T_Z\right)\left(\frac{E[x_i]}{(1+\alpha_{1,i})P_{2,i}}+\frac{1}{(1+\alpha_{1,i})P_{1,i}}\right)\\&+\left(\frac{\lambda_i^2T_Z}{2n}\right)\left[\frac{1}{\lambda_i}-\frac{E[x_i]}{(1+\alpha_{1,i})P_{2,i}}-\frac{1}{(1+\alpha_{1,i})P_{1,i}}\right](h_{3,i}-h_{1,i})\end{aligned}\right\} \tag{B-1}$$

Let E_{0P}, E_{2i}, and E_{3i} be the following:

$$E_{0P} = \left[\frac{\left[2 - E[x_0]\right] E[x_0]}{P_{2,0}} + \frac{1}{P_{1,0}} \right] \tag{B-2}$$

$$E_{2i} = \left[\frac{E[x_i]}{\left[(1+\alpha_{1,i}) P_{2,i}\right]} + \frac{1}{\left[(1+\alpha_{1,i}) P_{1,i}\right]} \right];\ E_{3i} = \left[\frac{E[x_i](1 - E[x_i])}{(1+\alpha_{1,i}) P_{2,i}} + \frac{1}{\lambda_i} \right]\ for\ \ i = 1,...,L. \tag{B-3}$$

Substitute Eqs. (B-2) and (B-3) in Eq. (B-1), $E[TCU(T_Z)]$ becomes as follows:

$$E\left[TCU(T_Z,n)\right] = \left\{ \begin{array}{l} C_0(1-\pi_0)\lambda_0 + \dfrac{K_0(1+\beta_{1,0})}{T_Z} + C_0(1+\beta_{2,0})\pi_0\lambda_0 + \dfrac{h_{1,0}\lambda_0^{\ 2}T_Z}{2}(1-\pi_0)^2 E_{0P} \\ + \dfrac{K_0}{T_Z} + C_{R,0}(1-\pi_0)\lambda_0 E[x_0] + \dfrac{h_{2,0}\lambda_0^{\ 2}(1-\pi_0)^2}{2P_{2,0}} E[x_0]^2 T_Z \\ + h_{1,0}\sum_{i=1}^{L}\left\{ \dfrac{\lambda_i^{\ 2}T_Z}{2\left[(1+\alpha_{1,i})P_{1,i}\right]} + \left(\sum_{i=1}^{L}[\lambda_i T_Z] - \sum_{j=1}^{i}[\lambda_j T_Z] \right)\lambda_i E_{2i} \right\} \end{array} \right\}$$
$$+ \sum_{i=1}^{L} \left\{ \begin{array}{l} C_i(1+\alpha_{3,i})\lambda_i + \dfrac{nK_{D,i}}{T_Z} + \dfrac{K_i(1+\alpha_{2,i})}{T_Z} + C_{D,i}\lambda_i + C_{R,i}(1+\alpha_{3,i})\lambda_i E[x_i] + h_{1,i}T_Z\lambda_i^2 E_{3i} \\ + h_{2,i}\dfrac{T_Z}{2\left[(1+\alpha_{1,i})P_{2,i}\right]}(\lambda_i E[x_i])^2 + \left(\dfrac{\lambda_i^{\ 2}T_Z}{2n}\right)\left[\dfrac{1}{\lambda_i} - E_{2i}\right](h_{3,i} - h_{1,i}) + \dfrac{h_{3,i}}{2}(\lambda_i^2 T_Z)E_{2i} \end{array} \right\} \tag{32}$$

Appendix - C

Table C
Assumption of the equivalent variable values in a single-phase fabricating system

Product i	i_i	$h_{3,i}$	$P_{1,i}$	$C_{D,i}$	$h_{2,i}$	C_i	$K_{D,i}$	λ_i	$h_{1,i}$	x_i	$P_{2,i}$	$C_{R,i}$	K_i
1	0.2	\$70	58000	\$0.1	\$16	\$80	\$1800	3000	\$16	5%	46400	\$50	\$17000
2	0.2	\$75	59000	\$0.2	\$18	\$90	\$1900	3200	\$18	10%	47200	\$55	\$17500
3	0.2	\$80	60000	\$0.3	\$20	\$100	\$2000	3400	\$20	15%	48000	\$60	\$18000
4	0.2	\$85	61000	\$0.4	\$22	\$110	\$2100	3600	\$22	20%	48800	\$65	\$18500
5	0.2	\$90	62000	\$0.5	\$24	\$120	\$2200	3800	\$24	25%	49600	\$70	\$19000

Appendix – D

Table D-1
Different critical system features influenced by overtime factor $\alpha_{1,0}$

$\alpha_{1,0}$	E[$TCU(T_Z*,n*)$] (A)	Added cost due to Overtime	Utilization (B)	(A) %surge	(B) %drop	Sum of $t_{1,i}$* (C)	(C) %drop	n*	End product's shipping cost	Rework cost stage 2	T_Z*
0.0	\$2,244,654	\$0	0.2389	-	-	0.0743	-	3	\$66,885	\$43,831	0.4871
0.1	\$2,299,604	\$56,124	0.2250	2.45%	-5.82%	0.0681	-8.34%	3	\$66,358	\$43,830	0.4913
0.2	\$2,354,727	\$112,217	0.2134	4.90%	-10.67%	0.0629	-15.34%	3	\$65,872	\$43,828	0.4953
0.3	\$2,409,982	\$168,283	0.2037	7.37%	-14.73%	0.0585	-21.27%	3	\$65,417	\$43,827	0.4990
0.4	\$2,465,338	\$224,323	0.1953	9.83%	-18.25%	0.0547	-26.38%	3	\$64,987	\$43,826	0.5026
0.5	**\$2,517,055**	**\$279,345**	**0.1880**	**12.14%**	**-21.31%**	**0.0576**	**-22.48%**	**4**	**\$75,968**	**\$43,827**	**0.5660**
0.6	\$2,572,258	\$335,150	0.1817	14.59%	-23.94%	0.0543	-26.92%	4	\$75,519	\$43,826	0.5696
0.7	\$2,627,525	\$390,937	0.1761	17.06%	-26.29%	0.0514	-30.82%	4	\$75,089	\$43,826	0.5732
0.8	\$2,682,844	\$446,707	0.1711	19.52%	-28.38%	0.0488	-34.32%	4	\$74,677	\$43,825	0.5766
0.9	\$2,738,206	\$502,460	0.1666	21.99%	-30.26%	0.0465	-37.42%	4	\$74,281	\$43,824	0.5799
1.0	\$2,793,604	\$558,198	0.1626	24.46%	-31.94%	0.0445	-40.11%	4	\$73,897	\$43,824	0.5831
1.1	\$2,849,031	\$613,921	0.1590	26.93%	-33.44%	0.0425	-42.80%	4	\$73,526	\$43,823	0.5863
1.2	\$2,904,483	\$669,629	0.1557	29.40%	-34.83%	0.0408	-45.09%	4	\$73,166	\$43,823	0.5894
1.3	\$2,959,955	\$725,324	0.1526	31.87%	-36.12%	0.0393	-47.11%	4	\$72,816	\$43,822	0.5925
1.4	\$3,015,445	\$781,004	0.1499	34.34%	-37.25%	0.0378	-49.13%	4	\$72,475	\$43,822	0.5955
1.5	\$3,070,949	\$836,672	0.1473	36.81%	-38.34%	0.0365	-50.87%	4	\$72,142	\$43,822	0.5984
1.6	\$3,126,466	\$892,327	0.1450	39.28%	-39.31%	0.0353	-52.49%	4	\$71,817	\$43,821	0.6013
1.7	\$3,181,993	\$947,969	0.1428	41.76%	-40.23%	0.0341	-54.10%	4	\$71,499	\$43,821	0.6042
1.8	\$3,237,529	\$1,003,598	0.1408	44.23%	-41.06%	0.033	-55.59%	4	\$71,188	\$43,821	0.6071
1.9	\$3,293,072	\$1,059,216	0.1389	46.71%	-41.86%	0.0321	-56.80%	4	\$70,883	\$43,821	0.6099
2.0	\$3,348,620	\$1,114,822	0.1372	49.18%	-42.57%	0.0312	-58.01%	4	\$70,584	\$43,820	0.6127

Table D-2

Different critical system-features influenced by outsourcing factor π_0

π_0	$E[TCU(T_Z*,n*)]$ (A)	Added cost due to outsourcing	Utilization (B)	(A) %surge	(B) decline %	t_0*	t_0* %drop	T_Z*	Rework expense stage 1	$n*$
0.00	$2,412,185	$0	0.2456	-	-	0.0712	-	0.4948	$5,313	4
0.05	$2,430,151	$52,676	0.2384	0.74%	-2.93%	0.0687	-3.51%	0.5024	$5,048	4
0.10	$2,443,023	$100,270	0.2312	1.28%	-5.86%	0.0651	-8.57%	0.5030	$4,782	4
0.15	$2,455,920	$147,864	0.2240	1.81%	-8.79%	0.0615	-13.62%	0.5036	$4,516	4
0.20	$2,468,842	$195,458	0.2168	2.35%	-11.73%	0.058	-18.54%	0.5042	$4,251	4
0.25	$2,481,788	$243,053	0.2096	2.89%	-14.66%	0.0544	-23.60%	0.5047	$3,985	4
0.30	$2,491,116	$290,114	0.2024	3.27%	-17.59%	0.0569	-20.08%	0.5649	$3,719	4
0.35	$2,504,071	$337,709	0.1952	3.81%	-20.52%	0.0529	-25.70%	0.5655	$3,454	4
0.40	**$2,517,055**	**$385,305**	**0.1880**	**4.35%**	**-23.45%**	**0.0489**	**-31.32%**	**0.5660**	**$3,188**	**4**
0.45	$2,530,065	$432,901	0.1808	4.89%	-26.38%	0.0448	-37.08%	0.5665	$2,922	4
0.50	$2,543,104	$480,497	0.1736	5.43%	-29.32%	0.0408	-42.70%	0.5670	$2,657	4
0.55	$2,556,171	$528,094	0.1664	5.97%	-32.25%	0.0368	-48.31%	0.5674	$2,391	4
0.60	$2,569,266	$575,691	0.1592	6.51%	-35.18%	0.0327	-54.07%	0.5678	$2,125	4
0.65	$2,582,389	$623,289	0.1520	7.06%	-38.11%	0.0286	-59.83%	0.5681	$1,860	4
0.70	$2,595,539	$670,886	0.1449	7.60%	-41.00%	0.0246	-65.45%	0.5684	$1,594	4
0.75	$2,608,718	$718,484	0.1377	8.15%	-43.93%	0.0204	-71.35%	0.5686	$1,328	4
0.80	$2,621,926	$766,083	0.1305	8.70%	-46.86%	0.0164	-76.97%	0.5688	$1,063	4
0.85	$2,635,161	$813,682	0.1233	9.24%	-49.80%	0.0123	-82.72%	0.5690	$797	4
0.90	$2,648,425	$861,281	0.1161	9.79%	-52.73%	0.0082	-88.48%	0.5691	$531	4
0.95	$2,661,717	$908,880	0.1089	10.34%	-55.66%	0.0041	-94.24%	0.5692	$266	4
1.00	$2,659,783	$956,677	0.1017	10.26%	-2.93%	0.0000	-100.00%	0.5453	$0	4