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A joint lot-sizing and marketing model with reworks, scraps and imperfect products

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we establish an economic production quantity (EPQ) based inventory model by considering various types of non-perfect products. We classify products in four groups of perfect, imperfect, defective but reworkable and non-reworkable defective items. The demand is a power function of price and marketing expenditure and production unit cost is considered to be a function of lot size. The objective of this paper is to determine lot size, marketing expenditure, selling price, set up cost and inventory holding cost, simultaneously. The problem is modeled as a nonlinear posynomial geometric programming and an optimal solution is derived. The implementation of the proposed method is demonstrated using a numerical example and the sensitivity analysis is also performed to study the behavior of the model.

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1. Introduction

The main focus of inventory control problems is on optimizing order quantity or lot-sizing subject to some capacity limitations. In such problems, the objective is either to minimize the total costs associated with the inventory control system including ordering and holding costs or to maximize the benefits associated with the system. The EOQ model has been widely employed along these lines in inventory control systems to determine ordering or purchasing quantity. When the production takes place at a constant rate, the EOQ model is extended to economic production quantity (EPQ). One of the primary assumptions with most lot-sizing models is that demand is constant and is given in planning horizon. However, demand can be affected by different factors such as price and marketing expenditure (Lilian, 1992). The other issue on classical lot-sizing problems is the absence of quality characteristics. Product quality is normally affected by the state of the production process, which may shift from an "in-control" state to an "out-of-control" state and produce defective items (Lee & Rosenblatt, 1987). Hence defective items are produced when the quality characteristics are outside the specification limits and the products cannot be used until the necessary rework is applied. The production process may also produce imperfect quality products and items of imperfect quality could be used in another production/inventory situation that is less restrictive process and acceptance (Salameh & Jaber, 2000). Therefore, imperfect items can be sold to a particular purchaser at a

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discount price. However, as the production of imperfect or defective products is a natural expectation, it will be more realistic to integrate quality considerations into the classical models to cope with real life manufacturing conditions. We classify products in four groups of perfect, imperfect, defective but reworkable and non-reworkable defective items. The percentage of each type is assumed to be constant and deterministic.

The other controversial assumption with many classical EPQ model is that the unit production expenditure is assumed to be constant (Bedworth & Bailey, 1987) and all units produced are of good quality (Warets, 1994). In recent decades, researchers have tried to determine the optimal batch quantity of imperfect production systems with the consideration of different operating conditions.

Gupta and Chakraborty (1984) considered the reworking option of rejected items. They considered recycling from the last stage to the first stage and obtained an economic batch quantity model. Lee et al. (1997) developed a model of batch quantity in a multi-stage production system considering various proportions of defective items produced in every stage while they ignored the rework situation. Salameh and Jaber (2000) surveyed an EOQ model where each lot contains a certain percentage of defective items with a continuous random variable. They also considered that imperfect items could be sold as a single batch at a reduced price by the end of 100% inspection but they did not address the impact of the rejected and the reworked items and the selling price. In their paper, Salameh and Jaber did not declare the appropriate cycle time for selling the imperfect products. Hayek and Salameh (2001) assumed that all produced defective items were repairable and provided an optimal point for EPQ model under the effect of reworking the imperfect quality items. Teunter and van der Laan (2002) tried to find the solution for the non-optimal condition in an inventory model with remanufacturing. Chiu (2003) considered a finite production model with random defective rate, scrap, reworking of repairable defective items and backlogging to derive an optimal operating policy including lot size and backordering levels that minimized overall inventory costs. Chan et al. (2003) provided a framework to integrate lower pricing, rework and reject situations into a single EPQ model. They also found that the time schedule for selling imperfect items is critical, as this decision would affect the inventory cost and the batch quantities. Jamal et al. (2004) considered a single production system with rework options incorporating two cases of rework process to minimize the total system cost. In the first case, they considered that the rework executed within the same cycle and the same stage of production. In the second case, the defective items are accumulated up to N cycles to be reworked for the following cycle where all defective products could be reworked. Ben-Daya et al. (2006) developed integrated inventory inspection models with and without replacement of nonconforming items discovered during inspection where the process includes no inspection, sampling inspection, and 100% inspection. They also proposed a solution procedure to determine the operating policies for inventory and inspection which consists of order quantity, sample size, and acceptance number. Recently, Jaber et al. (2009) applied the concept of entropy cost to extend the classical EOQ model under the assumptions of perfect and imperfect quality.

There is also another types of lot sizing where the primary objective is the maximization of profit which incorporates pricing and marketing expenditure (Fathian et al., 2009; Lee & Kim, 1993; Lee et al., 1996; Sadjadi, 2005). Sadjadi et al. (2005) presented a joint production, marketing and inventory model which determines the production lot size, marketing expenditure and product's selling price when demand and production have linear relationship. Lee and Kim (1993) formulated a model incorporating production and marketing decisions in a profit maximizing firm over a planning horizon. Their integrated model simultaneously determines price, marketing expenses, demand or production volume and lot size for a single product.

Geometric Programming (GP) is a popular mathematical programming technique for engineering design purposes (Beightler & Philip, 1976; Duffin, 1967; Sadjadi et al., 2005; Jung & Klein, 2001; Abad, 1988; Kim & Lee, 1998; Lee, 1993; Lee et al., 1996). The proposed model of this study differs from the earlier ones since they assume that all items are produced by a perfect reliability. However, we classify products in four groups of perfect, imperfect, defective but reworkable and non-reworkable defective and imperfect items can be sold at the end of processing period as a single batch at a reduced price per unit. Furthermore, unlike the earlier works, the proposed model determines the holding cost per product and the set up cost per production cycle. We also consider interest and depreciation cost as part of problem formulation. The objective of this model is to maximize the total profit with decision variables of lot size (Q), marketing expenditure(M), selling price (S_P), set up cost (A) and inventory holding cost (H), where demand is a power function of selling price and marketing expenditure with constant elasticity and the cost per unit is a power function of lot size.

This paper is organized as follows: Problem definition, notations and assumptions used throughout this study are presented in Section 2. In Section 3, the proposed mathematical models are derived in order to maximize the profit per unit time. The optimal solution procedure is represented in Section 4. In Section 5, a numerical example is provided to illustrate the implementation of the proposed model. Finally, in Section 6, conclusion remarks and recommendations for possible future work are presented.

2. Problem Definition

Consider a single economic production lot-sizing where a single product in a batch size of Q is produced at a finite production rate, P units per unit time. A 100% inspection is performed in order to identify the quality of each product at negligible cost. Demand for perfect product is a power function of selling price and marketing expenditure. Each lot produced contains p_1 percent of imperfect quality items (See Fig. 1).

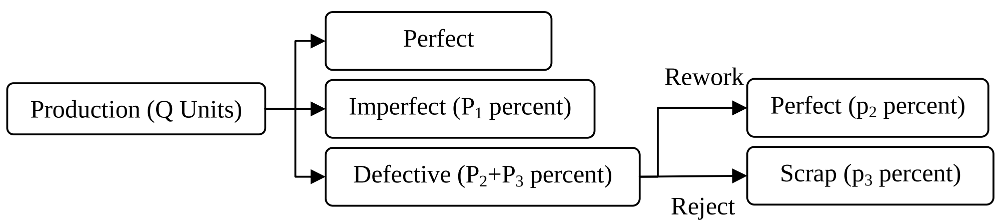

Fig .1. A schematic diagram of our model

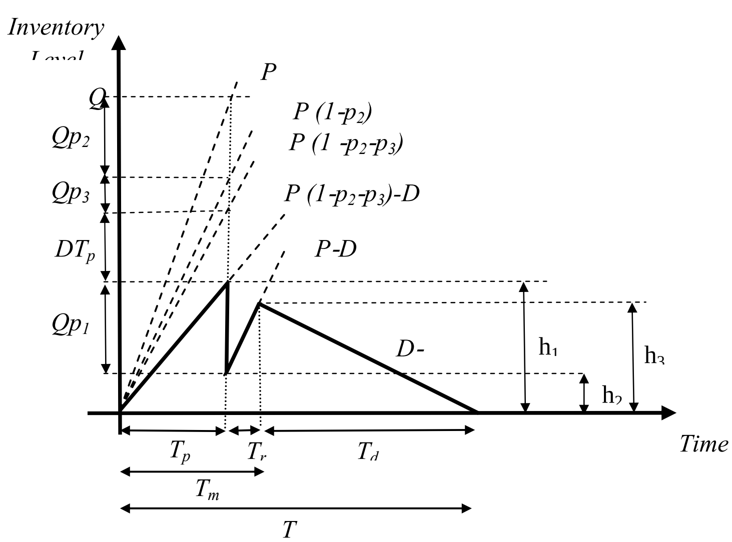

Fig. 2. Inventory level in stock during one cycle

The perfect and imperfect products are kept in stock when identified. The imperfect products are sold at the end of processing period, i.e., end of T_p in Fig. (2), as a single batch at a reduced price per unit (S_I) that is proportion of selling price of good quality products. The lot also contains a percentage of defectives, p_2 , so that these defective products can be reprocessed, or reworked, after the processing period and finally they are kept in stock. These products are assumed to be of good quality after reprocessing and they will not need any inspection. Each lot produced also contains a percentage of defectives, p_3, so that these units are rejected when identified. In other words, a defective product which could not be reworked is rejected immediately after its work operation completes. The main objective of the present study is to maximize the total profit of the inventory system.

2.1. Notations

A	Setup cost for each lot (decision variable)
C	Production cost per unit
H	Inventory holding cost per unit (decision variable)
M	Marketing expenditure (decision variable)
D	Demand per unit time
P	Production rate in units per unit time
TR	Total revenue per unit time
C_M	marketing expenditure per unit time
C_S	Setup cost per unit time
C_P	Production cost per unit time
C_H	Holding cost per unit time
C_{ID}	Interest and depreciation cost per unit time
$T(A, H)$	Total cost of interest and depreciation for a production process per cycle
R	Reliability of production (in our model R =1- p_1- p_3)
Q	Lot size in number of units per cycle (decision variable)
S_P	Unit selling price for good quality products (decision variable)
S_I	Unit selling price for imperfect quality products
p_1	Percentage of imperfect quality products
p_2	Percentage of rework products
p_3	Percentage of reject products
T	Cycle time
T_p	Processing time in each cycle
T_r	Reprocessing, reworking, time in each cycle

T_m	Total production run time, sum of the processing and reprocessing times, in each cycle
T_d	Time in each cycle when there is no production
I_1	Total quantity of inventory in stock during the processing period
I_2	Total quantity of inventory in stock during the reprocessing period
I_3	Total quantity of inventory in stock when there is no production
k	Scaling constant for demand
α	Price elasticity to demand
γ	Marketing expenditure elasticity to demand
r	Scaling constant for unit production cost
β	Lot size elasticity to production unit cost
l	Scaling constant for $T(A,H)$
x	Reliability elasticity to $T(A,H)$
y	Set up cost elasticity to $T(A,H)$
z	Inventory holding cost elasticity
i	Imperfect products price to perfect products price ratio
π	Total profit

2.2. Assumptions

The model is based on the following assumptions:

1. In this model it is assumed that demand is a function of price per unit of good quality products and marketing expenditure per unit i.e.,

$$D = kS_P{}^{-\alpha} M^{\gamma} \qquad \alpha > 1,\ 0 < \gamma < 1. \tag{1}$$

2. The production unit cost is defined as a power function of stock level and r is the scaling constant for unit production cost.

$$C = rQ^{-\beta} \qquad 0 < \beta < 1. \tag{2}$$

The exponent β represents lot size elasticity of unit production cost with $0 < \beta < 1$. This function is similar to the function considered by Sadjadi et al. (2005) and Panda and Maiti (2009).

3. Similar to the function considered by Van Beek and Putten (1987) the total cost of interest and depreciation per production cycle is affected by reliability, set up and holding cost. Therefore it can be assumed as:

$$T(A,H) = lR^{x} A^{-y} H^{-z}\ , \tag{3}$$

where $l, x, y, z \geq 0.$ Also the total reliability is $1 - p_1 - p_3$.

4.The selling price of imperfect products is proportion of the selling price of the good quality products i.e.

$$S_I = iS_p \qquad 0 < i < 1 \tag{4}$$

5. No Shortage is allowed.

6. The demand for the imperfect product with reduced price always exits.

7. Proportions of imperfect, reworked and rejected products are constant in each cycle.

8. No imperfect or defective product is produced during the rework.

9. The processing and reprocessing are accomplished using the same resources at the same speed.

10. No stop is allowed during the manufacturing operations of one lot.

11. Production rate, setup time, etc. are constant and deterministic.

12. Inspection cost is negligible.

3. Modeling

Fig. 2 presents the behavior of the inventory level in stock during one cycle. The purpose is to maximize the total profit per unit time (π). The total relevant cost per unit time includes setup, production (processing and reprocessing), inspection, rejection and inventory holding costs. According to Fig. 2, it can be easily shown that:

$$0 \le p_1 + p_2 + p_3 < 1, \tag{5}$$

$$T_p = \frac{Q}{P}, \tag{6}$$

$$T_r = \frac{p_2 Q}{P}, \tag{7}$$

$$T_m = \frac{(1+p_2)Q}{P}, \tag{8}$$

$$T = \frac{(1-p_1-p_3)Q}{D}. \tag{9}$$

To ensure that inventory level does not run into shortages, we must have:

$$\mathrm{P(1-p_2-p_3) \ge D.} \tag{10}$$

The revenue and various costs per unit time derived with respect to Eqs. 5-9 are as follows:

3.1 Revenue

The total revenue per unit time can be calculated as follows,

$$TR = \frac{S_P Q(1-p_1-p_3)}{T} + \frac{S_I Q p_1}{T} = S_P D + \frac{S_I D p_1}{(1-p_1-p_3)}. \tag{11}$$

3.2 Marketing expenditure

The marketing expenditure per unit time is as follows,

$$C_M = \frac{MQ}{T} = \frac{MD}{(1-p_1-p_3)}. \tag{12}$$

3.3 Setup cost

The setup cost for the production system during a cycle is designated as A. Using Eq. (9) yields the following for the setup cost per unit time,

$$C_S = \frac{A}{T} = \frac{D}{Q(1-p_1-p_3)}A. \tag{13}$$

3.4 Production cost

Production cost in each cycle consists of two parts: Processing cost at time T_p, and reprocessing or reworking cost at time T_r. The quantity of products reworked during each cycle is p_2Q. Therefore, according to the notation used and from Eq. (9), production cost per unit time is as follows,

$$C_P = \frac{CQ + C(p_2Q)}{T} = \frac{CD}{(1-p_1-p_3)}(1+p_2). \tag{14}$$

3.5 Inventory holding cost

The inventory holding cost per cycle is obtained as the average inventory times holding cost per product per cycle. Following Jamal et al. (2004), we do not consider any inventory holding costs for the defective items while the machines are waiting for the rework. The reason for this is the low percentage of the items as well as the low level of such costs as storage, etc. The average inventory level can be evaluated as follows,

$$\bar{\mathrm{I}} = \frac{\mathrm{I}_1 + \mathrm{I}_2 + \mathrm{I}_3}{T}. \tag{15}$$

It is evident from Fig .1 that:

$$\mathrm{I}_1 = \frac{1}{2}\mathrm{h}_1\mathrm{T}_\mathrm{p} = \frac{Q^2}{2P^2}\big(P(1-p_3-p_2)-D\big), \tag{16}$$

$$\mathrm{I}_2 = \frac{1}{2}(\mathrm{h}_2+\mathrm{h}_3)\mathrm{T}_\mathrm{r} = \frac{Q^2 p_2}{P^2}\left(P(1-p_3-p_2-p_1)-D+\frac{1}{2}(P-D)p_2\right), \tag{17}$$

$$\mathrm{I}_3 = \frac{1}{2}\mathrm{h}_3\mathrm{T}_\mathrm{d} = \frac{Q^2}{2DP^2}\big(P(1-p_1-p_3)-D(1+p_2)\big)^2. \tag{18}$$

As a result and using Eq. (5) through Eq. (8), the inventory holding cost will be expressed as:

$$C_H = H\bar{I} = \frac{HQ}{2P(1-p_1-p_3)}\Big(P(1-p_1-p_3)^2 - D(1-2p_1+p_2(1+p_2)-p_3)\Big). \tag{19}$$

3.6 Interest and depreciation cost

According to our notations and assumptions, interest and depreciation costs per unit time can be calculated as follows,

$$C_{ID} = \frac{T(A,H)}{T} = \frac{l(1-p_1-p_3)^x A^{-y} H^{-z} D}{(1-p_1-p_3)Q} = l(1-p_1-p_3)^{x-1}Q^{-1}A^{-y}H^{-z}D. \tag{20}$$

3.7 Total profit

Based on the above definitions and assumptions, we have the following formulation for the total profit per unit time (π):

$$\pi(Q, S_P, M, A, H) = TR - (C_M + C_S + C_P + C_H + C_{ID}) . \tag{21}$$

Substituting Eqs. 11-14 and Eqs.19-20 in Eq. 21 and some simplification, the total profit per unit time is as follows,

$$\begin{aligned}\pi(Q, S_P, M, A, H) &= S_P D + \frac{S_I D p_1}{(1-p_1-p_3)} - \frac{MD}{(1-p_1-p_3)} - \frac{AD}{Q(1-p_1-p_3)} - \frac{CD(1+p_2)}{(1-p_1-p_3)} \\ &- \frac{HQ}{2P(1-p_1-p_3)} * \left(P(1-p_1-p_3)^2 - D(1-2p_1+p_2(1+p_2)-p_3)\right) - l(1-p_1-p_3)^{x-1} Q^{-1} A^{-y} H^{-z} D.\end{aligned} \tag{22}$$

4. The Optimal Solution Procedure

In this section, we are interested in simultaneously determining lot size, selling price, marketing expenditure, set up and holding costs. The objective function is as follows,

$$\begin{aligned}\max \quad \pi(Q, S_P, M, A, H) &= (k + \frac{p_1 k i}{(1-p_1-p_3)}) S_P^{1-\alpha} M^{\gamma} - \frac{k}{(1-p_1-p_3)} S_P^{-\alpha} M^{1+\gamma} - \frac{k}{(1-p_1-p_3)} Q^{-1} S_P^{-\alpha} M^{\gamma} A \\ &- \frac{r\,(1+p_2)k}{(1-p_1-p_3)} Q^{-\beta} S_P^{-\alpha} M^{\gamma} - \frac{(1-p_1-p_3)}{2} QH + \frac{(1-2p_1+p_2(1+p_2)-p_3)k}{2P(1-p_1-p_3)} Q S_P^{-\alpha} M^{\gamma} H \\ &- lk(1-p_1-p_3)^{x-1} Q^{-1} S_p^{-\alpha} M^{\gamma} A^{-y} H^{-z}.\end{aligned} \tag{23}$$

The above objective function is a signomial geometric programming (GP) with 1 degree of difficulty. As the global optimality is not guaranteed for a signomial problem (Duffin et al., 1967), we modify the profit function into the posynomial GP problem with one additional variable and constraint. This technique was developed by Duffin et al. (1967) and it is assumed that there is a lower bound Z for the objective function where the maximization of Z (or minimizing Z^{-1}) is equivalent to maximization of the objective value. Also in order to simplify the objective function, we define the following,

$$\begin{aligned}a_1 &= k + \frac{p_1 k i}{(1-p_1-p_3)}, \\ a_2 &= \frac{k}{(1-p_1-p_3)} \\ a_3 &= \frac{k}{(1-p_1-p_3)}, \\ a_4 &= \frac{(1+p_2) rk}{(1-p_1-p_3)}, \\ a_5 &= \frac{(1-p_1-p_3)}{2}, \\ a_6 &= \frac{-(1-2p_1+p_2(1+p_2)-p_3)k}{2P(1-p_1-p_3)}, \\ a_7 &= lk(1-p_1-p_3).^{x-1}\end{aligned} \tag{24}$$

where a_i denotes the coefficient of ith term of Eq. (23). Therefore, the above problem is modified as follow,

$$\begin{aligned}&\min \quad Z^{-1} \\ &\text{subject to} \\ &a_1 S_P^{1-\alpha} M^{\gamma} - a_2 S_P^{-\alpha} M^{1+\gamma} - a_3 Q^{-1} S_P^{-\alpha} M^{\gamma} A - a_4 Q^{-\beta} S_P^{-\alpha} M^{\gamma} - a_5 QH - a_6 Q S_P^{-\alpha} M^{\gamma} H - a_7 Q^{-1} S_p^{-\alpha} M^{\gamma} A^{-y} H^{-z} \geq Z, \\ &Q, S_P, M, A, H, Z > 0.\end{aligned} \tag{25}$$

Since $Z > 0$, the first constraint can be transformed into the following form,

$$\min \quad Z^{-1}$$
subject to (26)

$$a_1^{-1}S_P^{\alpha-1}M^{-\gamma}Z + a_1^{-1}a_2S_p^{-1}M + a_1^{-1}a_3Q^{-1}S_P^{-1}A + a_1^{-1}a_4Q^{-\beta}S_P^{-1} + a_1^{-1}a_5QS_P^{\alpha-1}M^{-\gamma}H + a_1^{-1}a_6QS_P^{-1}H + a_1^{-1}a_7Q^{-1}S_p^{-1}A^{-y}H^{-z} \le 1,$$
$$Q, M, S_P, S_I, H, Z > 0.$$

Model (26) is a posynomial GP with 1 degree of difficulty. Hence according to Duffin et al. (1967) it can be solved globally by its dual problem which is formulated as follows,

$$\max \quad \left(\frac{1}{w_0}\right)^{w0}\left(\frac{a_1^{-1}\lambda}{w_1}\right)^{w1}\left(\frac{a_1^{-1}a_2\lambda}{w_2}\right)^{w2}\left(\frac{a_1^{-1}a_3\lambda}{w_3}\right)^{w3}\left(\frac{a_1^{-1}a_4\lambda}{w_4}\right)^{w4}\left(\frac{a_1^{-1}a_5\lambda}{w_5}\right)^{w5}\left(\frac{a_1^{-1}a_6\lambda}{w_6}\right)^{w6}\left(\frac{a_1^{-1}a_7\lambda}{w_7}\right)^{w7}$$

subject to

$$\begin{aligned}
&w_0 = 1,\\
&-w_3 + (-\beta)w_4 + w_5 + w_6 - w_7 = 0,\\
&(\alpha-1)w_1 - w_2 - w_3 - w_4 + (\alpha-1)w_5 - w_6 - w_7 = 0,\\
&-\gamma w_1 + w_2 - \gamma w_5 = 0,\\
&w_3 + (-y)w_7 = 0,\\
&w_5 + w_6 + (-z)w_7 = 0,\\
&-w_0 + w_1 = 0,\\
&\lambda = w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7,\\
&w_1, w_2, w_3, w_4, w_5, w_6, w_7 > 0.
\end{aligned} \tag{27}$$

The dual variables in Eq. (27) can be rewritten in terms of variable w_7 as follows,

$$\begin{aligned}
&w_1 = 1,\\
&w_2 = \frac{\beta\gamma + \gamma(-1+\beta - y + z + \beta y + \beta z)w_7}{\beta(\alpha-\gamma)},\\
&w_3 = yw_7,\\
&w_4 = \frac{-(1+y-z)}{\beta}w_7,\\
&w_5 = \frac{\beta(1-\alpha+\gamma) + (-1+\beta - y + z + \beta y + \beta z)w_7}{\beta(\alpha-\gamma)},\\
&w_6 = \frac{-\beta(1-\alpha+\gamma) - (-1+\beta - y + z + \beta y + \beta z - \beta\alpha z + \beta\gamma z)w_7}{\beta(\alpha-\gamma)}.
\end{aligned} \tag{28}$$

and also:

$$\lambda = 1 + \frac{\gamma}{\alpha-\gamma} + \left(\frac{\gamma(-1+\beta - y + z + \beta y + \beta z)}{\beta(\alpha-\gamma)} + y + \frac{-(1+y-z)}{\beta} - z\right)w_7. \tag{29}$$

To have feasible solution for dual problem, we need some additional assumptions to ensure that w_1 to w_6 remain positive.

$$
\begin{aligned}
&1+y-z<0,\\
&1+\gamma-\alpha<0,\\
&\beta<\frac{1+y-z}{1+y+z+\gamma z-\alpha z}.
\end{aligned}
\tag{30}
$$

Since $w_7>0$ we assume have,

$$
w_7>\frac{-\beta(1-\alpha+\lambda)}{(-1-y+z+\beta(1+y+z))}. \tag{31}
$$

Also the coefficient of 6th term of Eq. (23) must be positive $(a_6>0)$ to ensure that feasible solution exists. Therefore we assume

$$
2p_1-p_2(1+p_2)+p_3>1. \tag{32}
$$

Since the model has 1 degree of difficulty, we can eliminate 7 out of 8 dual variables by substituting Eqs. (28) and (29) into Eq. (27) and write out the dual objective function as maximizing a function in just one dual variable (w_7). By taking the logarithm of substituted dual problem we obtain a function in just one variable that is proved to be a concave function by Duffin et al. (1967) and the unique solution can be detremined using a simple line search technique such as bi-section. The procedure is similar to the method used by Sadjadi et al. (2005) and Jung and Klein (2001).

For $i=1,\ldots,7$ let

$$
\delta_i=\frac{w_i}{\lambda}. \tag{33}
$$

Note that δ_i for $i=1,\ldots,7$ are the weights of the terms in the constraints of model (26). In fact δ_1 to δ_7, represent the proportion of revenue (δ_1), marketing cost (δ_2), set up cost (δ_3), production cost (δ_4), holding cost ($\delta_5+\delta_6$) and interest and depreciation cost (δ_7) to the total profit, respectively. The following relations must hold:

$$
\begin{aligned}
&\delta_1=a_1^{-1}S_P^{\alpha-1}M^{-\gamma}Z,\\
&\delta_2=a_1^{-1}a_2S_p^{-1}M,\\
&\delta_3=a_1^{-1}a_3Q^{-1}S_P^{-1}A,\\
&\delta_4=a_1^{-1}a_4Q^{-\beta}S_P^{-1},\\
&\delta_5=a_1^{-1}a_5QS_P^{\alpha-1}M^{-\gamma}H,\\
&\delta_6=a_1^{-1}a_6QS_P^{-1}H,\\
&\delta_7=a_1^{-1}a_7Q^{-1}S_p^{-1}A^{-y}H^{-z},
\end{aligned}
\tag{34}
$$

where $\sum_{i=1}^{7}\delta_i=1$ at optimality. Using (34), the optimal solution of the problem can be summarized as follow,

$$
\begin{aligned}
&Q^* = \left(\frac{\delta_7 \delta_4^{-1-y-z} a_3^{-y} a_6^{-z}}{a_7 a_4^{-1-y-z} \delta_3^{-y} \delta_6^{-z}}\right)^{\frac{1}{-1+\beta+y-y\beta-z\beta-z}} \\
&S_P^* = \frac{a_1^{-1} a_4 Q^{-\beta}}{\delta_4}, \\
&M^* = \frac{\delta_2 a_4 Q^{-\beta}}{\delta_4 a_2}, \\
&A^* = \frac{\delta_3 a_4 Q^{1-\beta}}{\delta_4 a_3}, \\
&H^* = \frac{\delta_6 a_4 Q^{-\beta-1}}{\delta_4 a_6}, \\
&Z^* = \delta_1 a_1 \left(\frac{a_1^{-1} a_4 Q^{-\beta}}{\delta_4}\right)^{1-\alpha} \left(\frac{\delta_2 a_4 Q^{-\beta}}{\delta_4 k}\right)^{\gamma}.
\end{aligned}
\tag{35}
$$

5. A numerical example

To illustrate the implementation of the model developed in this paper, consider a production system where the parameters are as follows,

P=9000 units/year, p_1=50%, p_2=2%, p_3=3%, $\alpha = 2.5, \beta = 0.01, \gamma = 0.03, k = 10^5$, r=5, l=1, x=1.5, y=1, z=4.5, i=0.5 .

Hence the optimal dual variables are calculated to be $(w_1^*, \ldots, w_7^*) = (1,\ 0.0308,\ 0.0059,\ 1.4975,\ 0.0270,\ 0.000028,\ 0.0059)$, $\lambda^* = 2.5673$ and $(\delta_1^* \ldots \delta_7^*) = (0.3895,\ 0.0120,\ 0.0023,\ 0.5833,\ 0.0105,\ 0.000011,\ 0.0023)$. Then the optimal solution can be obtained from (35) as below:

$Q^* = 659$ units/cycle, $S_P^* = \$11.4$, $M = \$0.1$, $A = \$12.6$, $H = \$0.3$ and $\pi^* = \$1450$.

Since $S_I = iS_P$, in this example $S_I = \$5.7$.

6. Sensitivity Analysis

In this section we analyze the behavior of the proposed model when some of the parameters change in different intervals. The sensitivity analysis of pricing model could also help us extract some managerial implications.

Proposition 1. Let Q^* remain unchanged and suppose that changing p_1 does not have any effect on p_2 and p_3. As p_1 increases (decreases), S_P^* increases (decreases) (see Fig. 3).

Proof. Given $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7$ and $S_P^* = K_{S_pQ} Q^{-\beta}$ we have ($K_{S_PQ} = \frac{a_1^{-1} a_4}{\delta_4}$):

$$
\frac{\partial K_{S_PQ}}{\partial p_1} = \frac{(1+p_2) r (1-i)}{\delta_4 [(1-p_1-p_3)+p_1 i]^2} > 0 \tag{36}
$$

$$
\frac{\partial^2 K_{MQ}}{\partial p_1} = \frac{2(1+p_2) r (1-i)^2}{\delta_4 [(1-p_1-p_3)+p_1 i]^3} > 0 \tag{37}
$$

From Eq. (36) and Eq. (37), we understand that K_{S_PQ} is a concave function of p_1 when Q^* remains unchanged. Hence, as p_1 increases (decreases), the selling price for good quality products must be increased (decreased).

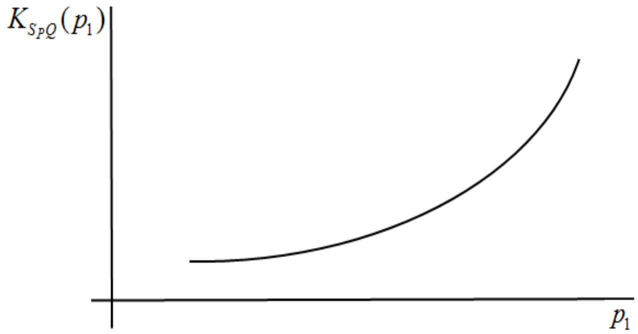

Fig. 3. The effect of p_1 on selling price

Proposition 2. Let Q^* remain unchanged. As p_2 increases (decreases), A^* increases (decreases) (see Fig. 4).

Proof. Given $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7$ and $A^* = K_{AQ}Q^{-\beta}$ we have ($K_{AQ} = \frac{\delta_3 a_4}{\delta_4 a_3}$):

$$\frac{\partial K_{AQ}}{\partial p_2} = \frac{\delta_3 r}{\delta_4} > 0 \tag{38}$$

$$\frac{\partial^2 K_{AQ}}{\partial p_2} = 0 \tag{39}$$

Note that we have:

$$\frac{\partial K_{AQ}}{\partial p_1} = 0 \tag{40}$$

$$\frac{\partial^2 K_{AQ}}{\partial p_3} = 0 \tag{41}$$

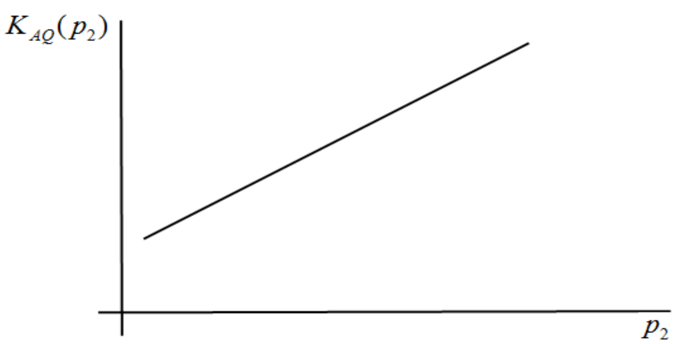

Fig. 4. The effect of p_2 on set up cost

From Eq. (38) and Eq. (39), we understand that K_{AQ} is a function of p_2 when Q^* remains unchanged. Hence, as p_2 increases (decreases), the set up cost must be increased (decreased). From Eq. (40) and Eq. (41) we realize that changing p_1 or p_3 does not affect set up cost (when Q^* remains unchanged).

7. Conclusion and future research

In this paper, a joint lot sizing and marketing problem was investigated by considering production of various types of non-perfect products. The products are classified in four groups of perfect, imperfect, defective but reworkable, and finally non-reworkable defective items. The proposed model of this paper considers demand as a power function of price and marketing expenditure and it assumes that production unit cost is a function of lot size. Furthermore, the interest and depreciation costs are also considered as part of modeling formulation. The proposed model of this paper has been solved using GP method and the implementation of the proposed method was illustrated using a numerical example. Furthermore the sensitivity analysis is presented to study the behavior of model parameters.

This research can be extended in some directions. The effect of machine breakdown on this model may be recommended for further study. Another important study is to investigate the effect of time value of money on optimal solution. Besides, it would be interesting to model the problem when various parameters are not deterministic and described in fuzzy or interval form.

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