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An imperfect quality items with learning and inflation under two limited storage capacity

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In this paper, we develop a two-warehouse imperfect production model under two cases: (i) model starts with shortages (ii) model ends with shortages. Most of the researchers proposed the models for perfect items but we develop for imperfect quality items, which is very realistic. Demand is taken as time dependent and dependent on the production. Holding cost in rented warehouse (RW) is greater than own warehouse (OW). Deterioration is taken as Weibull distribution in both OW and RW. Shortages are allowed and partially backlogged. The effect of learning on production cost is also considered. Learning from one cycle to other cycle, improve the efficiency of the organization. A numerical example including the sensitivity analysis is given to validate the results of the production-inventory model.

1. Introduction

The classical inventory models usually assume the available warehouse has unlimited capacity. In many practical situations, there exist many factors like temporary price discounts making retailers buy a capacity of goods exceeding their own warehouse (OW). In this case, retailers will either rent other warehouses or rebuild a new warehouse. However, from economical point of views, they usually choose to rent other warehouses. Hence, an additional storages space known as rented warehouses (RW) is often required due to limited capacity of showroom facility. In recent years, various researchers have discussed a two-warehouse inventory system. Therefore, due to the limited capacity of the available showroom facility (existing storage, own warehouse (OW)), an additional storage which is assumed to be available with abundant space is required to hold a large stock. This additional storage facility may be a rented warehouse (RW) with better preserving facility. This is first proposed by Hartely (1976). In this system, it is assumed that the holding cost in RW is greater than that in OW.

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Hence, items in RW are first transferred to OW to meet the demand until the stock level in RW drops to zero and then items in OW are released.

By assuming constant demand rate, Sarma (1987) developed a deterministic inventory model for a single deteriorating item with shortages and two levels of storage. Pakkala and Achary (1992) extended the two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages. Besides, the ideas of time-varying demand for deteriorating items with two storage facilities were considered by Benkherouf (1997) and Bhunia and Maiti (1998). Singh et al. (2008) provided a two-ware inventory model for deteriorating items. In that model shortages are allowed and partially backlogged. Singh et al.(2009) offered a two-warehouse inventory model for deteriorating items with shortages under inflation and time-value of money. Recently, Jaggi and Verma (2010) developed a two-warehouse inventory model with linear trend in demand under the inflationary conditions. Shortage was allowed and completely backlogged.

Most of the existing EOQ models unrealistically ignored the presence of the imperfect production process and equipment. Porteus (1986) and Rosenblatt and Lee (1986) was the first who developed a model with imperfect quality items. Furthermore, various researchers have discussed a two-warehouse inventory system. Kimand Hong (1999) determined the optimal production run length in deteriorating production process. Salameh and jaber (2000) developed an economic production/ inventory quantity model for items with imperfect quality. They assumed that poor-quality items are sold as a single batch by the end of the 100% screening process. Goyal et al. (2002) extended the model of Salameh and Jaber (2000) to develop a practical approach to determine the EPQ for items with imperfect quality. Chung and Hou (2003) developed a model to determine an optimal run time for a deteriorating production system with shortages.

Papachristos and Konstantaras (2006) developed economic ordering quantity models for items with imperfect quality and discussed many of the assumption of Salameh and Jaber (2000). Huang (2004) and Chung and Huang (2006) investigated the model of Salameh and Jaber (2000) in a two-level supply chain (vendor-buyer), while Wee et al. (2007) and Eroglu and Ozdemir (2007) independently extended it by allowing for shortages. In addition, Chan et al. (2003) develop an economic production model using similar assumptions as Salameh and Jaber (2000). Jaber et al. (2008) develop the model by using the assumption of Salameh and Jaber (2000) and discussed the effect of learning effects. In the classical economic production/order quantity models, the items produced/ received are implicitly assumed to be with perfect quality. However, it may not always be the case. Due to imperfect production process, natural disasters, damage or breakage in transit, or for many other reasons, the lot sizes produced/ received may contain some defective items.

Goyal and Giri (2003) considered the production-inventory problem with time varying demand, production and deterioration rate. Salameh and Jaber (2000) developed an economic production/ inventory quantity model for items with imperfect quality. Goyal et al. (2002) extended the model of Salameh and Jaber (2000) to develop a practical approach to determine the EPQ for items with imperfect quality. Chun et al. (2009) developed a two warehouse model with imperfect quality. Recently Singh et al. (2012) proposed a warehouse imperfect fuzzified production model with shortages and inflation.

There are lots of real life problem where the defective rate, ordering cost are decreases from one cycle to other. Such as automotive manufacturing for shipments of raw material where the percentage of defective items per lot decreases with cumulative number of shipments conforming to a learning curve and the demand of raw material is highly uncertain due to inflation and market complexities. We developed the models where percentage of defective items in each lot, production cost are follows learning effects. Most of the papers are develop for perfect quality items. But In this paper, we developed a two warehouse model with imperfect quality items with learning effect which is more realistic. We consider the two models (i) shortages at the end and, (ii) starts with the shortages. We assume that demand is time- dependent and deterioration is taken as Weibull for both OW and RW. Shortages are also allowed in this model.

2. Assumptions and Notations

2.1 Assumptions

In developing the mathematical models of the inventory system the following assumptions are used:

1. The demand rate $D(t)$ is deterministic and is a known function of time; the function $D(t)$ is given by:

   $D(t) = ae^{bt}$, where a and b> 0.

2. Production rate is dependent on the demand rate i.e. $P = kd = kae^{bt}$
3. Shortages are allowed and partially backlogged where $B = e^{-\delta t}$, δ is a backlogging parameter, $\delta > 0$.
4. Salvage value is associated to deteriorated units during the cycle time.
5. The time horizon of the inventory system is infinite.
6. Replenishment rate is infinite, and lead-time is zero.
7. The owned warehouse (OW) has a fixed capacity of W units, the rented warehouse (RW) has unlimited capacity.
8. The goods of OW are consumed only after consuming the goods kept in RW.
9. The unit inventory costs (including holding cost and deterioration cost) per unit time in RW are higher than those in OW.
10. The deterioration rate is taken as weibull in both OW and RW.

In addition, the following notations are used throughout this study:

2.2 Notations

W Fixed capacity level of OW

α Scale parameter of the deterioration rate in OW

β Shape parameter of the deterioration rate in OW

a, b Parameters of the demand rate

$(C_p + \frac{C_0}{n^{\delta}})$ Production cost with learning effect

C_{RW} Present worth of Holding cost in RW

C_{OW} Present worth of Holding cost in OW

C_3 Present worth of Deterioration cost

C_4 Present worth of Opportunity cost

C_s Present worth of Shortage cost

C_5 Present worth of Rework cost

f(X) Probability density function of X

I_{i1} Inventory level in OW at time t with $t \in [0, t_1]$

I_{i2} Inventory level in RW at time t with $t \in [t_1, t_2]$

I_{i3} Inventory level in RW at time t with $t \in [t_2, t_3]$

I_{i4} Inventory level in OW at time t with $t \in [t_1, t_3]$

I_{i5} Inventory level in OW at time t with $t \in [t_3, t_4]$

I_{i6} Inventory level in OW at time t with $t \in [t_4, t_5]$

I_{i7} Inventory level in OW at time t with $t \in [t_5, T]$

3. Formulation of the model

In Fig.1, the inventory level during a production cycle in which both OW and RW are used. Initially, the inventory level is zero. The production starts at time t = 0 and items accumulate from 0 up to W units in OW in t_1 units of time. After time t_1 any production quantity exceeding W will be stored in RW. After this production stopped and the inventory level in RW begins to decrease at t_2 and will reach 0 units at t_3 because of demand and deterioration. The inventory level in OW comes to decrease at $t_1$and then falls below W at t_3 due to deterioration. The remaining stocks in OW will be fully exhausted at t_4 owing to demand and deterioration, the inventory becomes zero. At this time shortage starts developing and at time t_5 it reaches to maximum shortage level, at this time fresh production starts to clear the backlog by the time T.

3.1. Model I: When shortages at the end

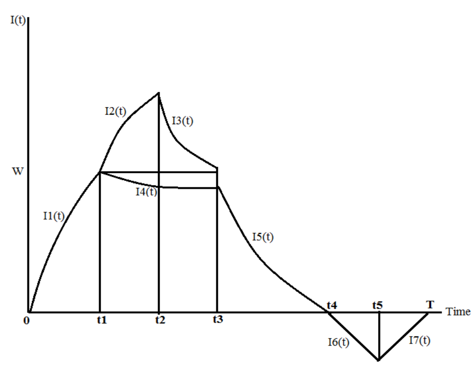

Fig. 1. Two warehouse model with the shortages at the end

$$I_1'(t) + \alpha\beta t^{\beta-1} I_1(t) = P - D \qquad 0 \le t \le t_1 \qquad (1)$$

$$I_2'(t) + \alpha\beta t^{\beta-1} I_2(t) = P - D \qquad t_1 \le t \le t_2 \qquad (2)$$

$$I_3'(t) + \alpha\beta t^{\beta-1} I_3(t) = -D \qquad t_2 \le t \le t_3 \qquad (3)$$

$$I_4'(t) + \alpha\beta t^{\beta-1} I_5(t) = 0 \qquad t_1 \le t \le t_3 \qquad (4)$$

$$I_5'(t) + \alpha\beta t^{\beta-1} I_4(t) = -D \qquad t_3 \le t \le t_4 \qquad (5)$$

$$I_6'(t) = -BD \qquad t_4 \le t \le t_5 \qquad (6)$$

$$I_7'(t) = P - D \qquad t_5 \le t \le T \qquad (7)$$

with these boundary conditions

$I_1(0)=0, I_2(t_1)=0, I_3(t_3)=0, I_4(t_1)=W, I_5(t_4)=W, I_6(t_4)=0,$ and $I_7(T)=0,$

$$I_1(t)=(k-1)a\left[t+\frac{bt^2}{2}+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha bt^{\beta+2}}{\beta+2}\right](1-\alpha t^\beta) \tag{8}$$

$$I_2(t)=(k-1)a\left[(t-t_1)+\frac{b}{2}(t^2-t_1^2)+\frac{\alpha}{\beta+1}(t^{\beta+1}-t_1^{\beta+1})+\frac{b\alpha}{\beta+2}(t^{\beta+2}-t_1^{\beta+2})\right](1-\alpha t^\beta) \tag{9}$$

$$I_3(t)=a\left[(t_3-t)+\frac{b}{2}(t_3{}^2-t^2)-\frac{\alpha}{\beta+1}(t_3^{\beta+1}-t^{\beta+1})-\frac{b\alpha}{\beta+2}(t_3{}^{\beta+2}-t^{\beta+2})\right](1-\alpha t^\beta) \tag{10}$$

$$I_4(t)=We^{\alpha(t_1^\beta-t^\beta)} \tag{11}$$

$$I_5(t)=a\left[(t_4-t)+\frac{b}{2}(t_4{}^2-t^2)-\frac{\alpha}{\beta+1}(t_4^{\beta+1}-t^{\beta+1})+\frac{b\alpha}{\beta+2}(t_4{}^{\beta+2}-t^{\beta+2})\right](1-\alpha t^\beta) \tag{12}$$

$$I_6(t)=a\left[t_4+\frac{(b-\delta)t_4^2}{2}-\frac{b\delta t_4^3}{3}\right] \tag{13}$$

$$I_7(t)=(k-1)a\left[(t-T)+\frac{b}{2}(t^2-T^2)\right] \tag{14}$$

Production cost

$$PC=(C_p+\frac{C_0}{n^\gamma})\int_0^{t_2} kae^{bt}e^{-rt}dt=(C_p+\frac{C_0}{n^\gamma})ka\left(t_2+\frac{(b-r)t_2^2}{2}-\frac{brt_2^2}{2}\right) \tag{15}$$

Present worth of Holding cost in RW

$$H_{RW}=C_{RW}\left\{\int_{t_1}^{t_2} I_2(t)e^{-rt}dt+\int_{t_2}^{t_3} I_3(t)e^{-rt}dt\right\}$$

$$=aC_{RW}\left[(k-1)\left\{\left(\frac{t_2^2}{2}+\frac{t_1^2}{2}\right)-r\left(\frac{t_2^3}{3}+\frac{t_1^3}{6}\right)+\frac{b}{2}\left(\frac{t_2^3}{3}+\frac{2t_1^3}{3}\right)\right\}+\left\{\left(\frac{t_2^2}{2}+\frac{t_3^2}{2}\right)-r\left(\frac{t_2^3}{3}+\frac{t_3^3}{6}\right)+\frac{b}{2}\left(\frac{t_2^3}{3}+\frac{2t_3^3}{3}\right)\right\}\right] \tag{16}$$

Present worth of Holding cost in OW

$$H_{OW}=C_{OW}\left\{\int_0^{t_1} I_1(t)e^{-rt}dt+\int_{t_1}^{t_3} I_5(t)e^{-rt}dt+\int_{t_3}^{t_4} I_4(t)e^{-rt}dt\right\} \tag{17}$$

$$=C_{OW}\left[\begin{array}{l}(k-1)a\left\{\frac{t_1^2}{2}-\frac{rt_1^3}{3}+\frac{bt_1^3}{6}-\frac{brt_1^4}{8}\right\}+a\left\{\left(\frac{t_4^2}{2}+\frac{t_3^2}{2}\right)-r\left(\frac{t_4^3}{6}+\frac{t_3^3}{3}\right)+\frac{b}{2}\left(\frac{2t_4^3}{3}+\frac{t_3^3}{3}\right)-\frac{br}{2}\left(\frac{t_4^4}{2}+\frac{t_3^4}{4}\right)\right\}\\ +W\left\{(t_3-t_1)-\frac{\alpha t_3^{\beta+1}}{\beta+1}-\frac{\alpha t_1^{\beta+1}}{\beta+1})\right\}\end{array}\right]$$

Present worth of Deteriorated items

$$D=C_3\left\{\int_0^{t_1}\alpha\beta t^{\beta-1}I_1(t)e^{-rt}dt+\int_{t_1}^{t_2}\alpha\beta t^{\beta-1}I_2(t)e^{-rt}dt+\int_{t_2}^{t_3}\alpha\beta t^{\beta-1}I_3(t)e^{-rt}dt+\int_{t_1}^{t_3}\alpha\beta t^{\beta-1}I_4(t)e^{-rt}dt\int_{t_3}^{t_4}\alpha\beta t^{\beta-1}I_5(t)e^{-rt}dt\right\}$$

$$=\alpha\beta C_3\left[\begin{array}{l}(k-1)a\left\{\frac{t_1^{\beta+1}}{(\beta+1)}+\frac{t_2^{\beta+1}}{(\beta+1)}-\frac{rt_1^{\beta+2}}{(\beta+2)}-\frac{rt_2^{\beta+2}}{(\beta+2)}+\frac{bt_1^{\beta+2}}{2(\beta+2)}+\frac{bt_2^{\beta+2}}{2(\beta+2)}-\frac{brt_1^{\beta+3}}{2(\beta+3)}-\frac{brt_2^{\beta+3}}{2(\beta+3)}\right\}\\ +a\left\{\begin{array}{l}\frac{t_3^{\beta+1}}{\beta(\beta+1)}+\frac{t_4^{\beta+1}}{\beta(\beta+1)}+\frac{t_3^{\beta+1}}{(\beta+1)}+\frac{t_2^{\beta+1}}{(\beta+1)}-\frac{rt_4^{\beta+2}}{(\beta+1)(\beta+2)}\frac{rt_3^{\beta+2}}{(\beta+2)}-\frac{rt_3^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_2^{\beta+2}}{(\beta+2)}\\ \frac{bt_3^{\beta+2}}{\beta(\beta+2)}+\frac{bt_4^{\beta+2}}{\beta(\beta+2)}+\frac{bt_3^{\beta+2}}{2(\beta+2)}+\frac{bt_2^{\beta+2}}{2(\beta+2)}-\frac{brt_1^{\beta+3}}{2(\beta+3)}-\frac{brt_2^{\beta+3}}{2(\beta+3)}\end{array}\right\}+\\ W\left\{\frac{1}{\beta}\left(t_3^{\beta}-t_1^{\beta}\right)-\frac{r}{\beta+1}(t_3^{\beta+1}-t_1^{\beta+1})\right\}\end{array}\right] \quad (18)$$

Salvage value for Deteriorated Items

$$SV=\pi\left\{\int_0^{t_1}\alpha\beta t^{\beta-1}I_1(t)e^{-rt}dt+\int_{t_1}^{t_2}\alpha\beta t^{\beta-1}I_2(t)e^{-rt}dt+\int_{t_2}^{t_3}\alpha\beta t^{\beta-1}I_3(t)e^{-rt}dt+\int_{t_1}^{t_3}\alpha\beta t^{\beta-1}I_4(t)e^{-rt}dt\int_{t_3}^{t_4}\alpha\beta t^{\beta-1}I_5(t)e^{-rt}\right.$$

$$=\alpha\beta\pi\left[\begin{array}{l}(k-1)a\left\{\frac{t_1^{\beta+1}}{(\beta+1)}+\frac{t_2^{\beta+1}}{(\beta+1)}-\frac{rt_1^{\beta+2}}{(\beta+2)}-\frac{rt_2^{\beta+2}}{(\beta+2)}+\frac{bt_1^{\beta+2}}{2(\beta+2)}+\frac{bt_2^{\beta+2}}{2(\beta+2)}-\frac{brt_1^{\beta+3}}{2(\beta+3)}-\frac{brt_2^{\beta+3}}{2(\beta+3)}\right\}\\ +a\left\{\begin{array}{l}\frac{t_3^{\beta+1}}{\beta(\beta+1)}+\frac{t_4^{\beta+1}}{\beta(\beta+1)}+\frac{t_3^{\beta+1}}{(\beta+1)}+\frac{t_2^{\beta+1}}{(\beta+1)}-\frac{rt_4^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_3^{\beta+2}}{(\beta+2)}-\frac{rt_3^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_2^{\beta+2}}{(\beta+2)}\\ \frac{bt_3^{\beta+2}}{\beta(\beta+2)}+\frac{bt_4^{\beta+2}}{\beta(\beta+2)}+\frac{bt_3^{\beta+2}}{2(\beta+2)}+\frac{bt_2^{\beta+2}}{2(\beta+2)}-\frac{brt_1^{\beta+3}}{2(\beta+3)}-\frac{brt_2^{\beta+3}}{2(\beta+3)}\end{array}\right\}+\\ W\left\{\frac{1}{\beta}\left(t_3^{\beta}-t_1^{\beta}\right)-\frac{r}{\beta+1}(t_3^{\beta+1}-t_1^{\beta+1})\right\}\end{array}\right] \quad (19)$$

Present worth of Shortage cost

$$I_s=C_s\left[-\int_{t_4}^{t_5}I_6(t)e^{-rt}dt-\int_{t_5}^{T}I_7(t)e^{-rt}dt\right]$$

$$=-aC_s\left[\left\{-\frac{t_4^2}{2}-\frac{t_5^2}{2}+\frac{rt_5^3}{3}+\frac{rt_4^3}{6}+\frac{b\delta t_5^4}{12}+\frac{b\delta t_4^4}{4}\right\}+(k-1)\left\{-\frac{T^2}{2}-\frac{t_5^2}{2}+\frac{rT^3}{6}-\frac{rt_5^3}{3}-\frac{bT^3}{3}-\frac{bt_5^3}{6}\right\}\right] \quad (20)$$

Present worth of lost sales quantity

$$I_L=C_L\int_{t_4}^{t_5}[1-e^{-\delta t}]ae^{bt}e^{-rt}dt=C_La\left[\frac{\delta}{2}(t_5^2-t_4^2)+\frac{(b-r)\delta}{3}(t_5^3-t_4^3)-\frac{br\delta}{4}(t_5^4-t_4^4)\right] \quad (21)$$

The number of defective items N in a production cycle is

$$N=\begin{cases}0 & X\geq t_2\\ \int_0^{t_2}kae^{bt}dt & X\leq t_2\end{cases}=t_2\begin{cases}0 & X\geq t_2\\ ka\left(t_2+\frac{bt_2^2}{2}\right) & X\leq t_2\end{cases}$$

The expected number of defective items is

$$E(N)=\int_0^{t_2}ka\left(t_2+\frac{bt_2^2}{2}\right)(1-rt_2)f(X)dX$$

where $f(X)=\mu e^{-\mu X}$

$$E(N)=ka\mu\left[t_2^2-rt_2^3-\frac{\mu t_2^2}{2}+\frac{bt_2^3}{2}\right] \quad (23)$$

Present worth of rework cost

$$RC= C_5 E(N) \tag{24}$$

Present worth of Total cost

$$TC = \frac{1}{T}\left[PC + H_{RW} + H_{OW} + D + I_s + I_L + RC - SV\right] \tag{25}$$

3.2. Model 2: When model starts with the shortages

In Fig.2, the inventory level during a production cycle in which both OW and RW are used. Initially, the inventory level is zero. At this time shortages starts developing and at time t_1 it reaches to maximum shortage level, at this time fresh production starts to clear the backlog by the time t_2.The production starts at time $t = t_2$ and items accumulate from 0 up to W units in OW in t_3 units of time. After time t_3 any production quantity exceeding W will be stored in RW. After this production stopped and the inventory level in RW begins to decrease at t_4 and will reach 0 units at t_5 because of demand and deterioration. The inventory level in OW comes to decrease at t_3 and then falls below W at $t_5$due to deterioration. The remaining stocks in OW will be fully exhausted at T owing to demand and deterioration, the inventory becomes zero.

$$I_1'(t) = -Bd \qquad 0 \le t \le t_1 \tag{26}$$

$$I_2'(t) = P - d \qquad t_1 \le t \le t_2 \tag{27}$$

$$I_3'(t) + \alpha\beta t^{\beta-1} I_3(t) = P - d \qquad t_2 \le t \le t_3 \tag{28}$$

$$I_4'(t) + \alpha\beta t^{\beta-1} I_4(t) = P - d \qquad t_3 \le t \le t_4 \tag{29}$$

$$I_5'(t) + \alpha\beta t^{\beta-1} I_5(t) = -d \qquad t_4 \le t \le t_5 \tag{30}$$

$$I_6'(t) + \alpha\beta t^{\beta-1} I_6(t) = -d \qquad t_5 \le t \le T \tag{31}$$

$$I_7'(t) + \alpha\beta t^{\beta-1} I_7(t) = 0 \qquad t_3 \le t \le t_5 \tag{32}$$

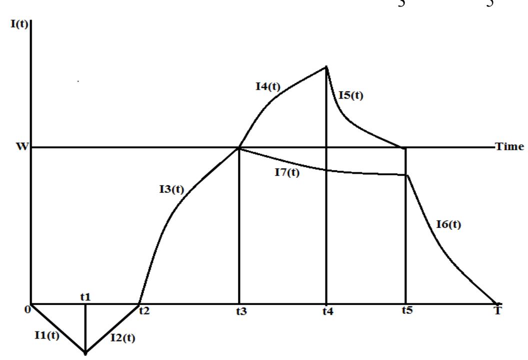

Fig. 2. Two warehouse model which is starts from shortages.

With these boundary conditions

$I_1(0) = 0, I_2(t_2) = 0, I_3(t_2) = 0, I_4(t_3) = 0, I_5(t_5) = 0, I_6(T) = 0,$ and $I_7(t_3) = W$

$$I_1(t) = -a\left[t + \frac{(b-\delta)t^2}{2} - \frac{b\delta t^3}{3}\right] \tag{33}$$

$$I_2(t) = (k-1)a\left[\left(t - t_2\right) + \frac{b}{2}\left(t^2 - t_2^{\,2}\right)\right] \tag{34}$$

$$I_3(t) = (k-1)a\left[\left(t - t_2\right) + \frac{b}{2}\left(t^2 - t_2^{\,2}\right) + \frac{a}{\beta+1}(t^{\beta+1} - t_2^{\beta+1}) - \frac{\alpha b}{\beta+2}(t^{\beta+2} - t_2^{\beta+2})\right](1 - \alpha t^{\beta}) \tag{35}$$

$$I_4(t)=(k-1)a\left[\left(t-t_3\right)+\frac{b}{2}\left(t^2-t_3^{\,2}\right)+\frac{\alpha}{\beta+1}(t^{\beta+1}-t_3^{\beta+1})+\frac{\alpha b}{\beta+2}(t^{\beta+2}-t_3^{\beta+2})\right](1-\alpha t^{\beta}) \tag{36}$$

$$I_5(t)=a\left[\left(t_5-t\right)+\frac{b}{2}\left(t_5^{\,2}-t^2\right)+\frac{\alpha}{\beta+1}\left(t_5^{\,\beta+1}-t^{\beta+1}\right)+\frac{\alpha b}{\beta+2}\left(t_5^{\,\beta+2}-t^{\beta+2}\right)\right](1-\alpha t^{\beta}) \tag{37}$$

$$I_6(t)=a\left[\left(T-t\right)+\frac{b}{2}\left(T^2-t^2\right)+\frac{\alpha}{\beta+1}\left(T^{\beta+1}-t^{\beta+1}\right)+\frac{\alpha b}{\beta+2}\left(T^{\beta+2}-t^{\beta+2}\right)\right](1-\alpha t^{\beta}) \tag{38}$$

$$I_7(t)=We^{\alpha(t_3^{\beta}-t^{\beta})} \tag{39}$$

Production cost

$$PC_1=\left(C_p+\frac{C_0}{n^{\phi}}\right)\int_{t_2}^{t_4}kae^{bt}e^{-rt}dt\ \left(C_p+\frac{C_0}{n^{\phi}}\right)k\left[a(t_4-t_2)+\frac{(b-r)}{2}(t_4^2-t_2^2)\right] \tag{40}$$

Present worth of Holding cost in RW

$$H_{1RW}=C_{RW}\left\{\int_{t_3}^{t_4}I_4(t)e^{-rt}dt+\int_{t_4}^{t_5}I_5(t)e^{-rt}dt\right\}$$

$$=aC_{RW}\left[(k-1)\left\{\left(\frac{t_4^2}{2}+\frac{t_3^2}{2}\right)-r\left(\frac{t_4^3}{3}+\frac{t_3^3}{6}\right)+\frac{b}{2}\left(\frac{t_4^3}{3}+\frac{2t_3^3}{3}\right)\right\}+\left\{\left(\frac{t_5^2}{2}+\frac{t_4^2}{2}\right)-r\left(\frac{t_4^3}{3}+\frac{t_5^3}{6}\right)+\frac{b}{2}\left(\frac{t_4^3}{3}+\frac{2t_5^3}{3}\right)\right\}\right] \tag{41}$$

Present worth of Holding cost in OW

$$H_{1OW}=C_{OW}\left\{\int_{t_2}^{t_3}I_3(t)dt+\int_{t_3}^{t_5}I_7(t)dt+\int_{t_5}^{T}I_6(t)dt\right\}$$

$$=C_{OW}\left[\begin{array}{l}(k-1)a\left\{\left(\frac{t_2^2}{2}+\frac{t_3^2}{2}\right)-r\left(\frac{t_2^3}{6}+\frac{t_3^3}{3}\right)+\frac{b}{2}\left(\frac{2t_2^3}{3}+\frac{t_3^3}{3}\right)-\frac{br}{2}\left(\frac{t_2^4}{2}+\frac{t_3^4}{4}\right)\right\}+a\left\{\begin{array}{l}\left(\frac{T^2}{2}+\frac{t_5^2}{2}\right)-r\left(\frac{T^3}{6}+\frac{t_5^3}{3}\right)\\+\frac{b}{2}\left(\frac{2T^3}{3}+\frac{t_5^3}{3}\right)-\frac{br}{2}\left(\frac{T^4}{2}+\frac{t_5^4}{4}\right)\end{array}\right\}\\+W\left\{\left(t_5-t_3\right)-\frac{\alpha t_5^{\beta+1}}{\beta+1}\right\}\end{array}\right] \tag{42}$$

Present worth of deteriorated items

$$D_1=C_3\left[\int_{t_2}^{t_3}\alpha\beta t^{\beta-1}I_3(t)dt+\int_{t_3}^{t_4}\alpha\beta t^{\beta-1}I_4(t)dt+\int_{t_4}^{t_5}\alpha\beta t^{\beta-1}I_5(t)dt+\int_{t_5}^{T}\alpha\beta t^{\beta-1}I_6(t)dt+\int_{t_3}^{t_5}\alpha\beta t^{\beta-1}I_7(t)dt\right] \tag{43}$$

$$=\alpha\beta C_3\left[\begin{array}{l}(k-1)a\left\{\begin{array}{l}\frac{t_3^{\beta+1}}{(\beta+1)}+\frac{t_2^{\beta+1}}{\beta(\beta+1)}-\frac{rt_3^{\beta+2}}{(\beta+2)}-\frac{rt_2^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{bt_3^{\beta+2}}{2(\beta+2)}+\frac{bt_2^{\beta+2}}{\beta(\beta+2)}-\frac{brt_3^{\beta+3}}{2(\beta+3)}-\frac{brt_2^{\beta+3}}{(\beta+1)(\beta+3)}\\+\frac{t_4^{\beta+1}}{(\beta+1)}+\frac{t_3^{\beta+1}}{\beta(\beta+1)}-\frac{rt_4^{\beta+2}}{(\beta+2)}-\frac{rt_3^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{bt_4^{\beta+2}}{2(\beta+2)}+\frac{bt_3^{\beta+2}}{\beta(\beta+2)}-\frac{brt_4^{\beta+3}}{2(\beta+3)}-\frac{brt_3^{\beta+3}}{(\beta+1)(\beta+3)}\end{array}\right\}\\+a\left\{\begin{array}{l}\frac{t_5^{\beta+1}}{\beta(\beta+1)}+\frac{t_4^{\beta+1}}{(\beta+1)}+\frac{t_5^{\beta+1}}{(\beta+1)}+\frac{T^{\beta+1}}{\beta(\beta+1)}-\frac{rt_5^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_4^{\beta+2}}{(\beta+2)}-\frac{rT^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_5^{\beta+2}}{(\beta+2)}\\\frac{bt_5^{\beta+2}}{\beta(\beta+2)}+\frac{bt_4^{\beta+2}}{(\beta+2)}+\frac{bt_5^{\beta+2}}{2(\beta+2)}+\frac{bT^{\beta+2}}{\beta(\beta+2)}-\frac{brt_4^{\beta+3}}{2(\beta+3)}-\frac{brt_5^{\beta+3}}{2(\beta+3)}\end{array}\right\}+\\W\left\{\frac{1}{\beta}\left(t_5^{\beta}-t_3^{\beta}\right)-\frac{r}{\beta+1}(t_5^{\beta+1}-t_3^{\beta+1})\right\}\end{array}\right]$$

Salvage value for Deteriorated Items

$$SV=\pi\left\{\int_0^{t_1}\alpha\beta t^{\beta-1}I_1(t)e^{-rt}dt+\int_{t_1}^{t_2}\alpha\beta t^{\beta-1}I_2(t)e^{-rt}dt+\int_{t_2}^{t_3}\alpha\beta t^{\beta-1}I_3(t)e^{-rt}dt+\int_{t_1}^{t_3}\alpha\beta t^{\beta-1}I_4(t)e^{-rt}dt\int_{t_3}^{t_4}\alpha\beta t^{\beta-1}I_5(t)e^{-rt}dt\right\}$$

$$=\alpha\beta\pi\left[\begin{array}{l}(k-1)a\left\{\begin{array}{l}\frac{t_3^{\beta+1}}{(\beta+1)}+\frac{t_2^{\beta+1}}{\beta(\beta+1)}-\frac{rt_3^{\beta+2}}{(\beta+2)}-\frac{rt_2^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{bt_3^{\beta+2}}{2(\beta+2)}+\frac{bt_2^{\beta+2}}{\beta(\beta+2)}-\frac{brt_3^{\beta+3}}{2(\beta+3)}-\frac{brt_2^{\beta+3}}{(\beta+1)(\beta+3)}\\+\frac{t_4^{\beta+1}}{(\beta+1)}+\frac{t_3^{\beta+1}}{\beta(\beta+1)}-\frac{rt_4^{\beta+2}}{(\beta+2)}-\frac{rt_3^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{bt_4^{\beta+2}}{2(\beta+2)}+\frac{bt_3^{\beta+2}}{\beta(\beta+2)}-\frac{brt_4^{\beta+3}}{2(\beta+3)}-\frac{brt_3^{\beta+3}}{(\beta+1)(\beta+3)}\end{array}\right\}\\+a\left\{\begin{array}{l}\frac{t_5^{\beta+1}}{\beta(\beta+1)}+\frac{t_4^{\beta+1}}{(\beta+1)}+\frac{t_5^{\beta+1}}{(\beta+1)}+\frac{T^{\beta+1}}{\beta(\beta+1)}-\frac{rt_5^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_4^{\beta+2}}{(\beta+2)}-\frac{rT^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_5^{\beta+2}}{(\beta+2)}\\\frac{bt_5^{\beta+2}}{\beta(\beta+2)}+\frac{bt_4^{\beta+2}}{(\beta+2)}+\frac{bt_5^{\beta+2}}{2(\beta+2)}+\frac{bT^{\beta+2}}{\beta(\beta+2)}-\frac{brt_4^{\beta+3}}{2(\beta+3)}-\frac{brt_5^{\beta+3}}{2(\beta+3)}\end{array}\right\}+\\W\left\{\frac{1}{\beta}\left(t_5^{\beta}-t_3^{\beta}\right)-\frac{r}{\beta+1}(t_5^{\beta+1}-t_3^{\beta+1})\right\}\end{array}\right] \tag{44}$$

Present worth of shortage cost

$$I_s=C_s\left[-\int_0^{t_1}I_1(t)e^{-rt}dt-\int_{t_1}^{t_2}I_2(t)e^{-rt}dt\right] \tag{45}$$

$$=-aC_s\left[\left\{-\frac{t_1^2}{2}+\frac{rt_1^3}{3}+\frac{(b-\delta)rt_1^4}{8}+\frac{b\delta t_1^4}{12}+\frac{b\delta rt_1^5}{15}\right\}+(k-1)\left\{-\frac{t_2{}^2}{2}-\frac{t_1^2}{2}+\frac{rt_2{}^3}{6}-\frac{rt_1^3}{3}-\frac{bt_2{}^3}{3}-\frac{bt_1^3}{6}\right\}\right]$$

Present worth of lost sales

$$I_L=C_L\int_0^{t_1}[1-e^{-\delta t}]ae^{bt}e^{-rt}dt=C_4a\left[\frac{\delta}{2}t_1^2+\frac{(b-r)\delta}{3}t_1^3-\frac{br\delta}{4}t_1^4\right] \tag{46}$$

Present worth of total cost

$$\text{TC}=\frac{1}{T}[OC_1+H_{1RW}+H_{1OW}+D_1+I_s+I_L-SV] \tag{47}$$

Numerical Example

For Model 1:

$$a=250,b=2.2,C_p=2.2,k=2,\gamma=0.1,\alpha=0.05,\beta=0.03,W=50,\delta=0.1,C_{RW}=1.9,$$
$$C_{OW}=1.6,C_3=5,\pi=50,C_S=5,C_L=3,\mu=0.001,C_5=0.7,r=0.02$$

$$t_1^*=0.183197,t_2^*=0.260739,t_3^*=1.5391,t_4^*=2.92868,t_5^*=9.18837,T^*=11.2367,TC=13387.1$$

For Model 2:

$$a=100,b=1.8,C_p=2.2,k=2,\alpha=0.05,\beta=0.03,W=50,\delta=0.1,C_{RW}=1.9,C_{OW}=1.6,$$
$$C_3=5,\pi=50,C_s=5,C_L=3,\mu=0.001,C_5=0.7,r=0.02$$

Output results

$$t_1{*}=1.57369,t_2{*}=2.39014,t_3{*}=2.53839,t_4{*}=6.65712,t_5{*}=8.82671,T{*}=10.635,$$
$$TC=1767.38$$

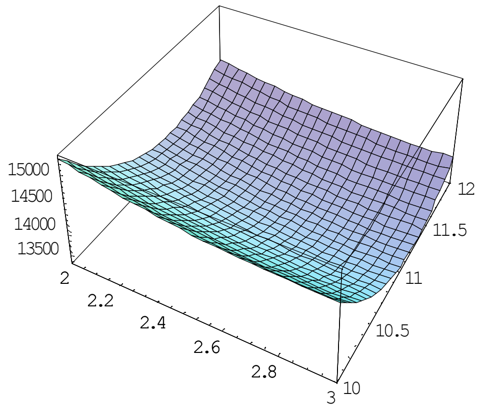

Fig. 3. Convexity of t_4* and T w.r.t TC

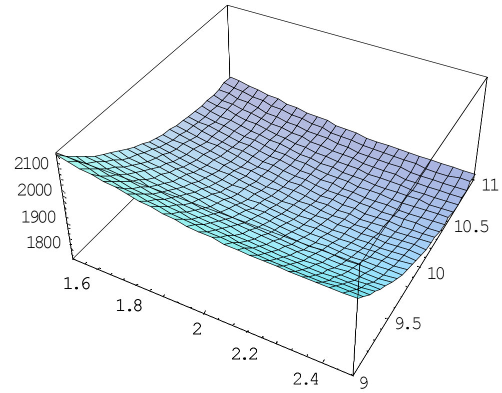

Fig. 4. Convexity of t_2* and T* w.r.t. TC

Table 1
Sensitivity analysis of model 1

Parameter	Change in Parameter	t_1*	t_2*	t_3*	t_4*	t_5*	T*	TC
a	260	0.17714	0.261657	1.54058	2.92889	9.1884	11.2367	13922.2
	270	0.171478	0.262508	1.54194	2.92908	9.18842	11.2367	14457.2
	280	0.166172	0.263299	1.5432	2.92925	9.18844	11.2367	14992.3
	290	0.161191	0.264034	1.54437	2.92942	9.18844	11.2367	15527.4
	300	0.156504	0.264721	1.54547	2.92957	9.18848	11.2367	16062.4
b	2.3	0.182018	0.26215	1.52991	2.90771	9.18374	11.2321	13939.4
	2.4	0.180868	0.263546	1.52142	2.88707	9.17945	11.2278	14491.1
	2.5	0.179743	0.264915	1.51354	2.86839	9.17548	11.2238	15042.4
	2.6	0.178644	0.266253	1.50622	2.85095	9.17178	11.2201	15593.4
	2.7	0.17757	0.267555	1.49939	2.83461	9.16834	11.2166	16144
C_p	2.3	0.183053	0.224436	1.51869	2.92593	9.1886	11.237	13388.5
	2.4	0.182922	0.191502	1.50015	2.92345	9.18882	11.2374	13389.6
	2.5	0.182902	0.161598	1.48331	2.92122	9.18903	11.2377	13390.6
	2.6	0.182694	0.134457	1.46801	2.91921	9.18923	11.2381	13391.3
	2.7	0.182595	0.109881	1.45412	2.91739	9.18941	11.2382	13391.9
α	0.06	0.186096	0.247408	1.52553	2.9298	9.18853	11.2368	13384.7
	0.07	0.189041	0.234358	1.51212	2.93101	9.18869	11.2368	13382.2
	0.08	0.192033	0.221596	1.49886	2.93229	9.18884	11.2369	13379.7
	0.09	0.195074	0.209128	1.48575	2.93365	9.18899	11.237	13377.1
	0.1	0.198165	0.196963	1.47279	2.93508	9.18914	11.237	13374.4
β	0.04	0.182885	0.262414	1.53973	2.92885	9.18834	11.2366	13387.2
	0.05	0.182583	0.264029	1.54033	2.92901	9.18831	11.2366	13387.2
	0.06	0.182292	0.265586	1.5409	2.92917	9.18828	11.2365	13387.3
	0.07	0.18201	0.267089	1.54144	2.92932	9.18825	11.2365	13387.3
	0.08	0.181738	0.268539	1.54196	2.92948	9.18823	11.2365	13387.3
W	52	0.189424	0.259787	1.53758	2.92847	9.18835	11.2366	13387.5
	54	0.195591	0.258834	1.53604	2.92826	9.18832	11.2366	13888
	56	0.201701	0.257881	1.53451	2.92805	9.1883	11.2366	13388.4
	58	0.207755	0.256928	1.53298	2.92784	9.18827	11.2366	13388.8
	60	0.213754	0.255973	1.53144	2.92762	9.18825	11.2366	13389
C_{RW}	2.0	0.183356	0.297961	1.55288	2.91725	9.18781	11.2368	13403.9
	2.1	0.183513	0.334343	1.5667	2.90626	9.18725	11.2368	13420.3
	2.2	0.183667	0.37064	1.5805	2.8957	9.18669	11.2369	13436.2
	2.3	0.183818	0.406504	1.59427	2.88554	9.18614	11.237	13451.6
	2.4	0.183967	0.441881	1.60795	2.87578	9.18559	11.237	13466.6
C_3	6	0.183075	0.262842	1.53997	2.92868	9.18834	11.2366	13387.4
	7	0.182953	0.264914	1.54083	2.92867	9.1883	11.2366	13387.7
	8	0.182833	0.266954	1.54167	2.92866	9.18827	11.2366	13387.9
	9	0.182713	0.268964	1.5425	2.92865	9.18824	11.2366	13388.2
	10	0.182595	0.270946	1.54332	2.92863	9.1882	11.2366	13388.5
C_s	6	0.18322	0.266389	1.54812	2.94099	9.07451	11.0474	15323.5
	7	0.183236	0.270407	1.55451	2.94971	8.99171	10.9114	17267.8
	8	0.183248	0.273408	1.55928	2.95621	8.92881	10.8088	19217.3
	9	0.183257	0.275735	1.56297	2.96123	8.87941	10.7287	21170.1
	10	0.183264	0.277591	1.56591	2.96523	8.83958	10.6645	23125.4
C_L	4	0.183326	0.293247	1.59061	2.99876	9.41126	11.6076	14532
	5	0.183457	0.326279	1.64211	3.06825	9.62541	11.9737	15714.7
	6	0.183591	0.359894	1.69372	3.13734	9.8309	12.3347	16933.3
	7	0.183726	0.394166	1.7456	3.20623	10.0278	12.6907	18185.5
	8	0.183865	0.429186	1.7979	3.27512	10.2162	13.0413	19469.3
r	0.022	0.183036	0.218916	1.47151	2.83615	8.69963	10.7537	12146.1
	0.024	0.182887	0.180084	1.40714	2.74741	8.24577	10.2924	11050.6
	0.026	0.182749	0.144287	1.34611	2.66259	7.82609	9.85503	10083.4
	0.028	0.182624	0.111599	1.28851	2.58172	7.43902	9.44255	9228.73
	0.03	0.182511	0.0822342	1.23445	2.50479	7.08244	9.05498	8472.11

Table 2
Sensitivity analysis of model 2

Parameter	Change in Parameter	t_1*	t_2*	t_3*	t_4*	t_5*	T*	TC
a	102	1.57369	2.39014	2.53839	6.65712	8.82671	10.635	1801.75
	104	1.5679	2.38209	2.5252	6.6554	8.82409	10.6306	1836.13
	106	1.56515	2.37828	2.51895	6.65458	8.82285	10.6286	1870.5
	108	1.56251	2.3746	2.51292	6.6538	8.82166	10.6266	1904.86
	110	1.55996	2.37105	2.5071	6.65304	8.82051	10.6247	1939.23
C_3	5.2	1.57406	2.39065	2.53923	6.65723	8.82688	10.6352	1767.45
	5.4	1.57443	2.39117	2.54006	6.65727	8.82705	10.6354	1767.55
	5.6	1.5748	2.39169	2.5409	6.65773	8.82721	10.6355	1767.65
	5.8	1.57518	2.3922	2.54173	6.65788	8.82738	10.6357	1767.75
	6.0	1.57555	2.39272	2.54257	6.65789	8.82754	10.6359	1767.85
C_s	5.2	1.45436	2.22143	2.37725	6.64029	8.8097	10.6181	1776.85
	5.4	1.3453	2.0666	2.23002	6.62487	8.79407	10.6024	1785.09
	5.6	1.24482	1.92335	2.09444	6.61057	8.77957	10.5876	1792.28
	5.8	1.15156	1.78983	1.96874	6.59721	8.76958	10.5735	1798.58
	6.0	1.06439	1.6645	1.85141	6.58458	8.75313	10.56	1804.11
C_L	4	1.50156	2.31125	2.46294	6.64747	8.81691	10.624	1770.47
	5	1.43724	2.24092	2.39579	6.63885	8.80813	10.6142	1773.22
	6	1.37934	2.17762	2.33547	6.63108	8.80023	10.6054	1775.7
	7	1.32684	2.12022	2.28086	6.62403	8.79304	10.5974	1777.7
	8	1.27892	2.06782	2.23109	6.6176	8.78648	10.59	1779.92
r	0.022	1.30904	2.09852	2.25131	6.32867	8.28486	10.25	1627.36
	0.024	1.32511	2.13369	2.28271	5.6715	7.79891	9.59038	1492.31
	0.026	1.34888	2.16748	2.3105	5.29331	7.40159	9.18875	1381.16
	0.028	1.37346	2.20234	2.33988	4.962	7.05203	8.83589	1283.53
	0.03	1.39911	2.23866	2.37114	4.66914	6.74171	8.52317	1196.83

Observations

1. With increase in demand parameter a, t_1^* decrease and $a, t_2^*, t_3^*, t_4^*, t_5^*, T^*$ and total cost increases.
2. With increase in demand parameter b, t_2^* decrease and $t_1^*, t_3^*, t_4^*, t_5^*, T^*$ and total cost increases.
3. With increase in production cost $C_p, t_1^*, t_2^*, t_3^*, t_4^*, t_5^*$ decreases and t_5^*, T^* and total cost slightly increases.
4. With increase in warehouse capacity W, t_3^*, t_4^*, t_5^* decreases and t_1^*, t_2^* increases and T^* and total cost slightly increases.
5. With increase in holding cost of RW, t_4^*, t_5^* decreases and t_1^*, t_2^*, t_3^*, T^* and total cost increases.
6. With increase in deterioration cost C_3, t_1^*, t_4^*, t_5^* decreases and t_2^*, t_3^*, T^* and total cost increases.
7. With increase in shortage cost of C_s, t_5^*, T^* decreases and $t_1^*, t_2^*, t_3^*, t_4^*$, and total cost increases.
8. With increase in lost sale cost of $C_L, t_1^*, t_3^*, t_4^*, t_5^*, T^*$ and total cost increases.

4. Conclusion

In this paper, we developed an imperfect quality items with learning and inflation under two storage capacity. We assumed two cases in this paper (i) model ends with shortages (ii) model starts with shortages. Demand is taken as time dependent and dependent on the production. Deterioration is taken as Weibull distribution in both OW and RW. Shortages are allowed and partially backlogged. The effect of learning on production cost is also considered. Learning from one cycle to other cycle, improve the efficiency of the organization. Fig 3 shows the convexity of total cost function for model I. Fig 4 shows the convexity of total cost function for model II. Table 1 and Table 2 show the sensitivity analysis for model 1 and 2, respectively. This paper can be further extended in so many ways: permissible delay, fuzzy environment etc.

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