A study on the ranking performance of some MCDM methods for industrial robot selection problems

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CHRONICLE	ABSTRACT
	In this paper, the ranking performance of six most popular and easily comprehensive multi-criteria decision-making (MCDM) methods, i.e. weighted sum method (WSM), weighted product method (WPM), weighted aggregated sum product assessment (WASPAS) method, multi-objective optimization on the basis of ratio analysis and reference point approach (MOORA) method, and multiplicative form of MOORA method (MULTIMOORA) is investigated using two real time industrial robot selection problems. Both single dimensional and high dimensional weight sensitivity analyses are performed to study the effects of weight variations of the most important as well as the most critical criterion on the ranking stability of all the six considered MCDM methods. The identified local weight stability interval indicates the range of weights within which the rank of the best alternative remains unaltered, whereas, the global weight stability interval determines the range of weights within which the overall rank order of all the alternatives remains unaffected. It is observed that for both the problems, multiplicative form of MOORA is the most robust method being least affected by the changing weights of the most important and the most critical criteria.

1. Introduction

Multi-criteria decision-making (MCDM) refers to making decisions in the presence of multiple, usually conflicting criteria. MCDM problems are quite common in everyday life. A typical MCDM problem involves a number of alternatives to be assessed and a number of criteria to evaluate the alternatives. Each alternative has a performance value for each criterion and based on these values, the alternatives can be assessed and ranked. Although MCDM problems are widespread all the time, MCDM as a discipline only has a relatively short history of about 30 years. The development of MCDM discipline is closely related to the advancement of computer technology. In one hand, the rapid development of computer technology in recent years has made it possible to conduct systematic analysis of complex

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MCDM problems. On the other hand, the widespread use of computers and information technology has generated a huge amount of information, which makes MCDM increasingly important and useful in supporting efficient decision-making. MCDM methods are gaining importance as potential tools for analyzing complex real time problems due to their inherent ability to judge different alternatives (choice, option, strategy, policy, scenario can also be used synonymously) on various criteria for possible selection of the best/suitable alternative(s). The MCDM analysis has some unique characteristics, such as the presence of multiple non-commensurable and conflicting criteria, different units of measurement among the criteria, and the presence of quite different alternatives.

There are three primary steps in utilizing any MCDM method involving numerical analysis of alternatives:

  (a) Determination of the relevant criteria and alternatives.
  (b) Attach numerical measures to the relative importance/priority to criteria and impact of the alternatives on these criteria.
  (c) Process the numerical values to determine a ranking of each alternative.

MCDM methods can help to improve the quality of decisions by making the decision-making process more explicit, rational and efficient (Lourenzutti & Krohling, 2013; Podvezko & Sivilevičius, 2013; Tavana et al., 2013; Chai et al., 2013; Ruzgys et al., 2014). An intriguing problem with MCDM methods which rank a set of alternatives in terms of a number of competing criteria is that oftentimes different MCDM methods may yield different rank orders when they are fed with exactly the same numerical data. Thus, the issue of evaluating the relative ranking performance of such methods is naturally raised. Since it is practically impossible to know which one is the best alternative for a given decision-making problem, some kind of testing procedures to identify the most efficient MCDM method need to be explored (Zavadskas et al., 2006, 2013[b]; Peldschus, 2009; Podvezko, 2011).

Salminen et al. (1998) compared the performance of ELECTRE III (ELimination and Et Choice Translating REality), PROMETHEE I, II (Preference Ranking Organization METHod for Enrichment Evaluation) and SMART (Simple Multi-Attribute Rating Technique) methods on application problems concerning environmental decision-making situations, and concluded that it would be better to use several methods for the same problem when possible; when not, the application of ELECTRE III method would be recommended. Zanakis (1998) investigated the performance of eight methods, e.g. ELECTRE, TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), multiplicative exponential weighting (MEW), simple additive weighting (SAW), and four versions of analytic hierarchy process (AHP) (original vs. geometric scale and right eigenvector vs. mean transformation solution). It was observed that in general, all AHP versions would behave similarly and closer to SAW than the other methods. ELECTRE would be the least similar to SAW, followed by MEW. TOPSIS would behave closer to AHP and differently from ELECTRE and MEW, except for problems with few criteria. Raju and Pillai (1999) compared the performance of five MCDM methods, e.g. ELECTRE-2, PROMETHEE-2, AHP, compromise programming and EXPROM-2 (extended PROMETHEE) while selecting the best reservoir configuration. It was shown that although the same preference strategy had been reached by all the methods, compromise programming had a slight advantage over the others. Parkan and Wu (2000) compared the performance of operational competitiveness rating, AHP and data envelopment analysis methods to understand their similarities and differences. Opricovic and Tzeng (2004) performed a comparative analysis of VIKOR (VIse kriterijumsko KOmpromisno Rangiranje) and TOPSIS methods with a numerical example, showing their similarities and differences. Caterino et al. (2009) compared the performance of some of the most widely adopted MCDM methods, and concluded that TOPSIS and VIKOR methods would seem to be more appropriate for solving the retrofit selection problem because of their capability to deal with each kind of judgment criteria, clarity of their results, and reduced difficulty to deal with parameters and choices. Chatterjee et al. (2011) compared the applicability and capability of two MCDM methods, i.e. complex proportional assessment (COPRAS) and evaluation of

mixed data (EVAMIX) methods for materials selection. Athawale and Chakraborty (2012) studied the ranking performance of ten most commonly used MCDM methods for materials selection and observed that VIKOR method had a relatively better performance than the others due to its computational simplicity. It was concluded that for a given material selection problem, more attention had to be paid on the proper selection of relevant criteria and alternatives, not on choosing the most appropriate MCDM method to be adopted. Mela et al. (2012) selected six methods, i.e. weighted sum method (WSM), weighted product method (WPM), VIKOR, TOPSIS, PROMETHEE II and a procedure based on the PEG (PEG = Pareto-Edgeworth-Grierson) theorem for comparing their performance. It was observed that the PEG-procedure would tend to provide a well-balanced solution. Chatterjee and Chakraborty (2012) applied four preference ranking-based MCDM methods, i.e. PROMETHEE II (EXPROM2), COPRAS with gray relations (COPRAS-G), ORESTE (Organization, Rangement Et Synthese De Donnes Relationnelles) and operational competitiveness rating analysis (OCRA) for solving a gear material selection problem, and compared their relative ranking performance. Anojkumar et al. (2014) considered FAHP-TOPSIS, FAHP-VIKOR, FAHP-ELECTRE and FAHP-PROMTHEE methods for pipe material selection in sugar industry, and explored the effectiveness and flexibility of VIKOR method.

In this paper, six most popular MCDM methods, e.g. WSM, WPM, weighted aggregated sum product assessment (WASPAS) method, multi-objective optimization on the basis of ratio analysis and reference point approach (MOORA) method, and multiplicative form of MOORA method (MULTIMOORA) are considered in order to investigate their robustness with respect to changing weights of the most important as well as the most critical criterion while solving two real time robot selection problems. Applying single dimensional and high dimensional weight sensitivity analyses, the local and the global weight stability intervals for these six MCDM methods are determined in order to identify the most robust and the most sensitive MCDM methods.

2. MCDM methods

Each of the MCDM problems starts with a decision/evaluation matrix exhibiting the performance of different alternatives with respect to various criteria.

$$X = \begin{bmatrix} x_{11} & x_{12} & ... & x_{1n} \\ x_{21} & x_{22} & ... & x_{2n} \\ ... & ... & ... & ... \\ x_{m1} & x_{m2} & ... & x_{mn} \end{bmatrix} \tag{1}$$

where x_{ij} is the performance measure of ith alternative on j^{th} criterion, m is the number of alternatives and n is the number of criteria.

This decision matrix first requires to be normalized so that it becomes dimensionless and all its elements are comparable. This normalization procedure is a ratio system in which the performance of an alternative on a criterion is compared to a denominator which is a representative for all the alternatives concerning that criterion. Although, there are various normalization procedures, the following simple normalization procedure is adopted here.
For beneficial criteria,

$$\bar{x}_{ij} = \frac{x_{ij}}{\max_i x_{ij}} \tag{2}$$

For non-beneficial criteria,

$$\bar{x}_{ij} = \frac{\min_i x_{ij}}{x_{ij}} \tag{3}$$

where $\bar{x}_{ij}$ is the normalized value of x_{ij} and it is a dimensionless number in the [0, 1] interval.

2.1 Weighted sum method

The weighted sum method (WSM) (Fishburn, 1967; MacCrimon, 1968; Triantaphyllou & Mann, 1989), also known as SAW method, is the simplest and most widely used MCDM method. In this method, each criterion is assigned a weight in such a way that all the criteria weights must add up to one. Each alternative is now assessed with respect to every criterion. The overall performance score of an alternative is computed using the following equation:

$$Q_i^{(1)} = \sum_{j=1}^{n} \bar{x}_{ij} w_j \, , \tag{4}$$

where w_j is weight (relative importance or significance) of jth criterion. For a given MCDM problem, the criteria weights are determined applying AHP or entropy method. In WSM method, the $Q_i^{(1)}$ values are arranged in descending order and the alternative having the highest $Q_i^{(1)}$ value becomes the best choice.

2.2 Weighted product method

The weighted product method (WPM) (Miller & Starr, 1969; Triantaphyllou & Mann, 1989) is quite similar to WSM method. The main difference in that instead of addition in this method, there is multiplication. The overall performance score of an alternative is given by the following expression:

$$Q_i^{(2)} = \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} \, . \tag{5}$$

In this method, each normalized value of an alternative with respect to a criterion is raised to the power of the relative weight of the corresponding criterion. The alternative with the maximum overall performance score $Q_i^{(2)}$ is considered as the most preferred option.

2.3 Weighted aggregated sum product assessment method

The weighted aggregated sum product assessment (WASPAS) method is a unique combination of WSM and WPM methods. In this method, a joint generalized criterion of weighted aggregation of additive and multiplicative methods is proposed as follows (Zavadskas et al., 2013[a,b]):

$$Q_i = 0.5 Q_i^{(1)} + 0.5 Q_i^{(2)} = 0.5 \sum_{j=1}^{n} \bar{x}_{ij} w_j + 0.5 \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} \, . \tag{6}$$

In order to have increased ranking accuracy and effectiveness of the decision-making process, in WASPAS method, a more generalized equation for determining the total relative importance of ith alternative is developed as below (Zavadskas et al., 2012):

$$Q_i = \lambda Q_i^{(1)} + (1 - \lambda) Q_i^{(2)} = \lambda \sum_{j=1}^{n} \bar{x}_{ij} w_j + (1 - \lambda) \prod_{j=1}^{n} (\bar{x}_{ij})^{w_j} \quad (\lambda = 0, 0.1, \ldots, 1) \tag{7}$$

The candidate alternatives are now ranked based on the Q values and the best alternative has the highest Q value. In Eq. (7), when the value of λ is 0, WASPAS method is transformed to WPM, and when λ is 1, it becomes WSM method. It has now been applied for solving MCDM problems for increasing ranking

accuracy and it has the capability to reach the highest accuracy of estimation. Till date, WASPAS method has limited successful applications in location selection (Hashemkhani Zolfani et al., 2013), civil engineering domain (Dėjus & Antuchevičienė, 2013; Staniūnas et al., 2013; Šiožinyte & Antuchevičiene, 2013), port site selection (Bagočius et al., 2013) and manufacturing decision-making (Chakraborty & Zavadskas, 2014).

2.4 Multi-objective optimization on the basis of ratio analysis method

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting criteria (objectives) subject to certain constraints (Brauers, 2004). In a real time decision-making scenario, different decision makers with varying interests and values, make a decision-making process much more difficult. In a decision-making problem, the objectives (criteria) must be measureable and their outcomes can be measured for every candidate alternative. Among the conflicting criteria (objectives), some are beneficial (where maximum values are desired) and some are non-beneficial (where minimum criteria values are always preferred). The multi-objective optimization on the basis of ratio analysis (MOORA) method (Brauers & Zavadskas, 2006; Chakraborty, 2011; Karande & Chakraborty, 2012) considers both beneficial and non-beneficial objectives (criteria) for ranking or selecting one or more alternatives from a set of available options. MOORA method mainly consists of two components, i.e. ratio system and reference point approach. It can be described in the other way that a combination of ratio system and reference point approach results in the development of MOORA method. In ratio system of MOORA method, the elements of the decision matrix are first normalized by comparing the performance (x_{ij}) of ith alternative with respect to j^{th} objective for all objective values. Each response of an alternative on an attribute is compared with the denominator representing all alternatives, which is square root of the sum of squares of all alternatives concerning that attribute. The following expression is applied for normalization of the decision matrix:

$$\bar{x}_{ij} = x_{ij} \Big/ \left[\sum_{i=1}^{m} x_{ij}^2 \right]^{1/2} \ (j = 1,2,\ldots,n) \tag{8}$$

For ratio system of MOORA method, the normalized performances are added for beneficial criteria and subtracted for non-beneficial criteria, as given in the following expression:

$$y_i = \sum_{j=1}^{g} \bar{x}_{ij} - \sum_{j=g+1}^{n} \bar{x}_{ij} \ , \tag{9}$$

where g is the number of criteria to be maximized, (n - g) is the number of criteria to be minimized and y_i is the assessment value of ith alternative with respect to all the criteria. The y_i value can be positive or negative depending on the totals of its maxima (beneficial attributes) and minima (non-beneficial attributes) in the decision matrix. When sorted in descending order, the best alternative is that which has the highest assessment value. It is recommended to have an ordinal ranking of y_i values to derive the final preference of the candidate alternatives (Brauers & Zavadskas, 2009). Brauers and Zavadskas (2009) proved that this method is quite robust with respect to stakeholders (decision makers), objectives, and interrelations between objectives and alternatives. As this method is non-subjective, based on cardinal and most recent data, it is more robust than the existing MCDM methods (Karande & Chakraborty, 2012).

In some cases, it is often observed that some attributes are more important than the others. In order to give more importance to an attribute, it can be multiplied with its corresponding weight (significance coefficient) (Brauers & Zavadskas, 2009). When these attribute weights are taken into consideration, Eq. (9) becomes as follows:

$$y_i = \sum_{j=1}^{g} w_j \bar{x}_{ij} - \sum_{j=g+1}^{n} w_j \bar{x}_{ij} \tag{10}$$

2.5 Reference point approach

Reference point approach is the second part of MOORA method, used in succession to ratio system and is based on $\bar{x}_{ij}$ values obtained from the ratio system. It is an MCDM method used for interactive analysis of multi-criteria optimization problems. This approach, being capable of handling general preference information, can be used to rank and choose an appropriate alternative for a desired application. It is developed for dealing with linear, non-linear and discrete decision-making problems. Normally, while proceeding towards a decision with multiple criteria, the decision maker is often interested in achieving a certain level of accomplishment or target with respect to each criterion, which is generally the most preferred performance measure. That level of achievement is conventionally recognized as a reference point. In actual practice, the performance of alternatives with respect to different criteria will deviate from those reference points by different margins. In this approach, measurement of such deviations from the corresponding reference points with respect to all the considered criteria, combined into a unique overall deviation, is the basis of comparison for favoring an option from a discrete set of alternatives. The notion of reference point was originally used in the earlier development of multi-objective programming as part of the work done with regards to goal programming by Charnes and Cooper (1963). Further, Wierzbicki (1982) defined the idea of achievement scalarizing function to overcome some of the drawbacks of goal programming. Therefore, the concept of reference point relied on the idea of achievement scalarizing function thereafter. Scalarizing functions used in the reference point approach are the so-called achievement functions and this approach relies on their properties.

In fact, reference point approach is a family of methods that considers different reference points charted on different hypotheses. The one considered here is a maximal objective reference point, used in MOORA method (Brauers & Zavadskas, 2010). The maximal objective reference point approach is more realistic and non-subjective because the coordinates (r_j), which are selected for the reference point, are realized in one of the candidate alternatives. It is based on ratio system and is one of the most commonly used methods for obtaining an order of preferences between the non-dominated alternatives.

Here, the weighted normalized values (r_{ij}) are obtained by multiplying $\bar{x}_{ij}$ values by the respective criteria weights w_j as follows:

$$r_{ij} = w_j \bar{x}_{ij} \tag{11}$$

In this approach, a maximal objective reference point (r_j) is considered. Therefore, after obtaining the weighted normalized data, the maximal objective reference point (r_j) is identified as the optimal weighted normalized value for each criterion. Therefore, in case of beneficial criteria, the highest performance value, being optimal is identified as the reference point, whereas, in case of non-beneficial criteria, it is the lowest performance value. Consequently, the reference points for beneficial and non-beneficial criteria are obtained as follows:

For beneficial criteria:

$$r_j = \text{Max} \ (r_{ij}) \tag{12}$$

For non-beneficial criteria:

$$r_j = \text{Min} \ (r_{ij}) \tag{13}$$

Further, the deviation of a criterion value from the set reference point (r_j) can be obtained as ($r_j - r_{ij}$). For a decision-making problem, an alternative would be the best when all of its beneficial criteria would attain the maximum values and all of its non-beneficial criteria would have the minimum values. But it

is not possible to have such an alternative satisfying both these requirements. There would be deviations for beneficial criteria for not attaining the maximum values and also for non-beneficial criteria for not having the minimum values. In this approach, the performance index (P_i) captures this total deviation for all the considered criteria for i^{th} alternative, which can be expressed using Min-Max metric of Tchebycheff as follows:

$$P_i = \underset{(i)}{\text{Min}}(\underset{(j)}{\text{Max}}|r_j - r_{ij}|) \tag{14}$$

The best alternative is that which has the minimum total deviation when all the beneficial and non-beneficial criteria are considered. It means that the best alternative would have the lowest P_i value.

2.6 Full multiplicative form

Brauers and Zavadskas (2010, 2011) extended MOORA method with the full multiplicative form embodying maximization as well as minimization of purely multiplicative utility function. Full multiplicative form of MOORA (MULTIMOORA) uses multiplication of criteria for beneficial and non-beneficial attributes separately and their ratio, providing dimensionless number for comparison. MOORA method, when combined with the full multiplicative form is identified as MULTIMOORA approach. Besides additive utility, its utility function also includes multiplication of the attributes. MULTIMOORA becomes the most robust system of multiple optimizations under condition of support from the ameliorated nominal group technique and Delphi (Brauers & Zavadskas 2010). The procedural diagram of MULTIMOORA is shown in Fig. 1. Brauers and Zavadskas (2010,2011) developed the following equation for the full multiplicative form of MOORA method to distinguish it from the mixed forms.

$$U_i = \frac{A_i}{B_i}, \tag{15}$$

where $A_i = \prod_{j=1}^{g} \bar{x}_{ij}, B_i = \prod_{j=g+1}^{n} \bar{x}_{ij}$ and U_i is the degree of utility for i^{th} alternative. In Eq. (15), the criteria to be maximized (beneficial attributes) are taken as the numerator and the criteria to be minimized (non-beneficial attributes) are taken as denominator (Baležentis et al., 2010). Brauers and Zavadskas (2010, 2011) suggested that if any of the x_{ij} value is 0, which signifies the absence of a particular criterion in the decision matrix, a foregoing filtering stage or withdrawal of that criterion from the decision matrix can be considered. MULTIMOORA method has been applied for personnel selection (Balezentis et al., 2012; Balezentis and Zeng, 2013), energy management (Streimikiene et al., 2012; Balezentiene et al., 2013) and property selection (Brauers & Zavadskas, 2011).

It is worthwhile to mention here that for ratio system, reference point and full multiplicative form of MOORA methods, the vector normalization procedure as given in Eq. (8) is adopted.
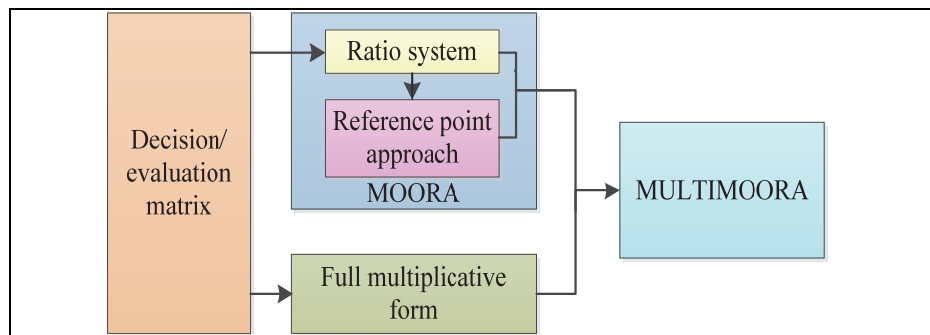

Fig. 1. Procedural diagram of MULTIMOORA

3. Sensitivity analysis

Sensitivity analysis is the study to show the impacts of variations in input data on the recommendations generated by the multi-criteria models, and is performed to establish a sense of strength of the model. Ustinovichius and Simanaviciene (2010) defined sensitivity analysis as "the study of how uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input". Thus, it is an effective tool for testing the robustness of the results generated by the decision-making models in presence of uncertainty. The sensitivity analysis thus helps to a) validate the robustness of the results generated by the decision models, b) identify the most uncertain input parameters that cause significant variation on the output, and c) find out the range of values of input parameters for which the model shows a stable output (Butler et al., 1997). In MCDM methods, the generated outputs are affected by two input parameters, i.e. criteria weights and performance data (Zavadskas et al., 2006). The criteria weights can be measured using different approaches based on dissimilar concepts. It is a general observation that a set of weights obtained using one approach differs from another, and those sets of weights do not show any relationship amongst them. Often, the criteria weights in MCDM problems are challenged because of assortment and uncertainty involved in their calculations. Also, the weights obtained by some methods, like AHP are subjective (judgmental) and biased with the decision makers' perceptions. While developing a decision-making problem to be solved by any of the MCDM methods, utmost care is usually taken in accumulating realistic information from reliable sources. Therefore, performing sensitivity analysis on performance data would be impractical, and hence, not justified at all. As a result, sensitivity analysis is performed to study the effects of changes in criteria weights on the final rankings of the alternatives. Sensitivity analysis on weights shows the stability of the derived solutions with respect to changes in criteria weights. It helps the decision makers to investigate the capability of MCDM methods in trading-off ostensible performance, address uncertainty in the selection problems and find out the least sensitive approach. Therefore, in this paper, two rigorous weight sensitivity analyses are performed to investigate the effects of varying criteria weights on the final rankings of the alternatives as derived employing the six considered MCDM methods.

3.1 Single dimensional weight sensitivity analysis

In this approach, weight of the most important criterion in a decision-making problem is varied (increased as well as decreased in steps) within a feasible range and all other criteria weights are adjusted equally to satisfy the weight additivity constraint, i.e. $\sum_{j=1}^{n} w_j = 1$. Since the difference in weight for the most important criterion is apportioned equally among the remaining criteria, it is also called non-proportional sensitivity analysis. Due to this non-proportional variation in criteria weights, the ratio of weights does not remain fixed and hence, a separate combination of new weights is obtained. Usually, a criterion with the highest weight assumes to have the major influence on the preference ranking of the alternatives, therefore, it can be considered as the most important criterion. As this approach is based on additivity concept of weights, and because the criteria weights cannot be negative, the highest possible weight of the selected criterion is thus bounded. Therefore, it is necessary to find out the possible range within which the weight of the selected criterion can be varied. In this case, the weight of the most important criterion can be reduced up to 0 and increased up to w'_j. The value of w'_j can be obtained applying the following equation:

$$w'_j = [(w_{j\,max} + (n - 1) \times (w_{j\,min})], \tag{16}$$

where $w_{j\,max}$ is the highest criterion weight and $w_{j\,min}$ is the lowest criterion weight. Increasing the weight of the most important criterion beyond w'_j will make the lowest criterion weight to be negative. Now, the new performance scores of all the alternatives and the corresponding ranking orders are computed for these changing criteria weights using any of the MCDM methods. Sensitivity (robustness) of MCDM

methods can be investigated while exploring the weight stability intervals for these methods. In this paper, both local and global weight stability intervals for each MCDM method are obtained. The local weight stability interval indicates the range of weights within which the rank of the best alternative remains unaltered, whereas, the global weight stability interval identifies the range of weights within which the overall rank order of all the alternatives remains unaffected.

3.2 High dimensional weight sensitivity analysis

Traditionally, weight sensitivity analysis is performed on the intuitive belief that the criterion with the highest weight is the most important or critical one. However, the sensitivity of a criterion in a decision-making process may not be necessarily related only to its weight. In some instances, the criterion with the lowest weight may be the most critical one. Therefore, determination of the most critical criterion will help the decision maker to take better decisions. Although, the single dimensional weight sensitivity analysis approach provides some insight about the robustness of MCDM methods, it can be sometimes misleading as the potential interactions due to simultaneous manipulation of multiple criteria weights in a wider range are ignored as well as the relative preference order among the attributes are not considered. In order to overcome these shortcomings of single dimensional weight sensitivity analysis approach, high dimensional weight sensitivity analysis approach is considered. In this approach, at first, the most critical criterion is identified by determining the criticality degree of each criterion (Triantaphyllou & Sánchez, 1997) and then, high dimensional weight sensitivity analysis is performed to find out the most robust and the most sensitive MCDM methods.

The mathematical procedure to identify the most critical criterion is given as follows:
Initially, all the alternatives are ranked according to their preference scores P_i ($i = 1,2,3,…,m$) as

$$P_1 \geq P_2 \geq P_3 \geq … \geq P_m .\tag{17}$$

Now, the value of $\delta_{k,i,j}$ ($1 \leq i \leq j \leq m$; $1 \leq k \leq n$), i.e. minimum change in the current weight (w_k) for a criterion C_k which will alter the preference of alternatives A_i and A_j is determined. The value of $\delta_{k,i,j}$ may be feasible for the limited conditions as:

$$\delta_{k,i,j} < \frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \leq w_k, \quad \text{if}\,(a_{jk} > a_{ik})\tag{18}$$

or

$$\delta_{k,i,j} > \frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \leq w_k, \quad \text{if}\,(a_{jk} < a_{ik})\tag{19}$$

Further, to express the changes in relative terms, $\delta_{k,i,j}$ values are converted to $\delta'_{k,i,j}$ scores as follows:

$$\delta'_{k,i,j} = (\delta_{k,i,j} \times 100)/w_k \tag{20}$$

In the next step, the criticality degree of criterion C_k, denoted as D'_k is computed. It is the smallest percent amount by which the current value of w_k must change, so that the existing ranking of the alternatives will be reversed.

$$D'_k = \min\,\{|\,\delta'_{k,i,j}\,|\} \text{ for all } 1 \leq k \leq n \tag{21}$$

The reciprocal of D'_k for criterion C_k is identified as sensitivity coefficient and is denoted as $\text{Sens}(C_k)$.

$$\text{Sens}(C_k) = \frac{1}{D'_k} \text{ for all } 1 \leq k \leq n \tag{22}$$

Higher the criticality degree or lower the sensitivity coefficient, more critical is the criterion. Therefore, a criterion having the maximum criticality degree is identified as the most critical criterion. If the criticality degree of any criterion is found to be infeasible (NF), then the value of $\text{Sens}(C_k)$ is set to zero.

This infeasibility implies that change in weight of that criterion by any magnitude cannot change the alternative's rank ordering. Also, a criterion is robust if all its $\delta'_{k,i,j}$ values are infeasible.

The next step after determining the most critical criterion is to perform the high dimensional weight sensitivity analysis. In this step, the weight of the most critical criterion is varied within the feasible range while maintaining weight proportionality, and its effect on the original rank ordering is observed for each of the considered MCDM methods. The weight proportionality is maintained by employing the following equation:

$$w_c = (1 - w_s) \times [w_c^o/W_c^o] = w_c^o - \Delta x \, \alpha_c, \tag{23}$$

where w_c is the change of weight for a criterion C, w_s is the weight of the most critical (sensitive) criterion, w_c^o is the original weight of criterion C and W_c^o represents the sum of the original weights that are being changed. The parameter α_c is defined as the weight coefficient of elasticity. The value of α_c represents the relative trade-off of other criteria weights in relation to the given changes in the weight of the most critical criterion during the sensitivity analysis.

$$\alpha_c = w_c^o/W_c^o. \tag{24}$$

The value of α_c for the most critical criterion is always defined to be one. The Δx is the amount of change implemented to the set of weights according to their associated α_c values. The values of Δx should be bounded on both sides within a feasible limit such that none of the criteria weights can take negative values. The bound for Δx can be calculated as follows:

$$-w_s^o \le \Delta x \le \min \{w_c^o/\alpha_c\} \tag{25}$$

Now, the new sets of criteria weights are calculated using the following expressions:

$$w_s = w_s^o + \alpha_s\Delta x \tag{26}$$
$$w_c = w_c^o - \alpha_c\Delta x \tag{27}$$

where w_s^o is the original weight of the criterion undergoing sensitivity analysis.

4. Illustrative Examples

In order to visualize the effects of changing criteria weights on the ranking performance of the considered MCDM methods and identify the most robust MCDM method, the following two real time examples are cited.

4.1 Example 1

Bhangale et al. (2004) considered a problem for selection of the most appropriate industrial robot for some pick-n-place operations where it had to avoid certain obstacles. In this example (Bhangale et al., 2004), five different robot selection attributes are considered as load capacity (LC), maximum tip speed (MTS), repeatability (RE), memory capacity (MC) and manipulator reach (MR), among which load capacity, maximum tip speed, memory capacity and manipulator reach are the beneficial attributes (where higher values are desirable), whereas repeatability is a non-beneficial attribute (where lower value is preferable). Thus, the industrial robot selection problem consists of five criteria and seven alternative robots, as given in Table 1. Using AHP method, Rao (2007) determined the weights (relative importance) of the criteria, as shown in Table 2, which are subsequently adopted for comparison of the ranking performance of six MCDM methods. The decision matrix of Table 1 is now linear normalized employing Eq. (2) and Eq. (3), and the corresponding normalized decision matrix is exhibited in Table 3 which is subsequently used for WSM, WPM and WASPAS method-based analyses. On the other hand, Table 4 shows the vector normalized decision matrix for ratio system of MOORA method, reference point approach and full multiplicative form of MOORA method-based analyses. Table 5 shows the related weighted normalized decision matrix which is also required for ratio system of MOORA method, reference point approach and full multiplicative form of MOORA method-based analyses. Now, this

industrial robot selection problem is solved applying all the six MCDM methods and the corresponding ranking orders of all the seven robots are provided in Table 6. This table also exhibits the original ranking of the robots as obtained by Bhangale et al. (2004). From this table, it is observed that Cybotech V15 Electric Robot (R_3) emerges out as the best choice in all the MCDM methods, except in ratio system of MOORA and the method adopted by Bhangale et al. (2004). On the other hand, Unimation PUMA 500/600 (R_5) is the worst preferred robot alternative in all the methods, except in reference point approach.

Table 1
Decision matrix for Example 1 (Bhangale et al., 2004)

Robot	LC	RE	MTS	MC	MR
ASEA-IRB 60/2 (R_1)	60	0.4	2540	500	990
Cincinnati Milacrone T3-726 (R_2)	6.35	0.15	1016	3000	1041
Cybotech V15 Electric Robot (R_3)	6.8	0.1	1727.2	1500	1676
Hitachi America Process Robot (R_4)	10	0.2	1000	2000	965
Unimation PUMA 500/600 (R_5)	2.5	0.1	560	500	915
United States Robots Maker 110 (R_6)	4.5	0.08	1016	350	508
Yaskawa Electric Motoman L3C (R_7)	3	0.1	1778	1000	920

Table 2
Criteria weights for Example 1

Criteria	LC	RE	MTS	MC	MR
Weight	0.036	0.192	0.326	0.326	0.12

Table 3
Normalized decision matrix for WAS, WPM and WASPAS methods

Robot	LC	RE	MTS	MC	MR
R_1	1.0000	0.2000	1.0000	0.1667	0.5907
R_2	0.1058	0.5333	0.4000	1.0000	0.6211
R_3	0.1133	0.8000	0.6800	0.5000	1.0000
R_4	0.1667	0.4000	0.3937	0.6667	0.5758
R_5	0.0417	0.8000	0.2205	0.1667	0.5459
R_6	0.0750	1.0000	0.4000	0.1167	0.3031
R_7	0.0500	0.8000	0.7000	0.3333	0.5489

Table 4
Normalized decision matrix for ratio system of MOORA, reference point and full multiplicative form of MOORA methods

Robot	LC	RE	MTS	MC	MR
R_1	0.9705	0.7861	0.6355	0.1217	0.3557
R_2	0.1027	0.2948	0.2542	0.7304	0.3740
R_3	0.1100	0.1965	0.4321	0.3652	0.6022
R_4	0.1618	0.3931	0.2502	0.4869	0.3467
R_5	0.0404	0.1965	0.1401	0.1217	0.3288
R_6	0.0728	0.1572	0.2542	0.0852	0.1825
R_7	0.0485	0.1965	0.4449	0.2435	0.3306

Table 5
Weighted normalized decision matrix

Robot	LC	RE	MTS	MC	MR
R_1	0.0349	0.1509	0.2072	0.0397	0.0427
R_2	0.0037	0.0566	0.0829	0.2381	0.0449
R_3	0.0040	0.0377	0.1409	0.1190	0.0723
R_4	0.0058	0.0755	0.0816	0.1587	0.0416
R_5	0.0015	0.0377	0.0457	0.0397	0.0395
R_6	0.0026	0.0302	0.0829	0.0278	0.0219
R_7	0.0017	0.0377	0.1450	0.0794	0.0397

Table 6
Comparison of rankings for Example 1

Robot	Bhangale et al. (2004)	WSM	WPM	WASPAS	Ratio system	Reference point	Full multiplicative form
R_1	3	4	5	5	5	5	2
R_2	1	2	2	2	1	2	3
R_3	2	1	1	1	2	1	1
R_4	6	5	4	4	4	3	4
R_5	7	7	7	7	7	5	7
R_6	4	6	6	6	6	7	6
R_7	5	3	3	3	3	4	5

Single dimensional weight sensitivity analysis

Now, single dimensional weight sensitivity analysis is carried out to study the ranking stability of the considered MCDM methods for possible changes in the criteria weights. In this example, both MTS and MC criteria have the maximum priority weight of 0.326. Between them, MTS is arbitrarily chosen as the most important criterion for this sensitivity analysis. Its weight is now varied within a feasible range and all other criteria weights are equally adjusted such that the weight additivity constraint is maintained. In this sensitivity analysis, the lower limit up to which the weight of the most important criterion can be reduced always equals to zero, whereas, the upper limit can be obtained using Eq. (16). But in this case, the lower limit of the weight of the most important criterion cannot be set as zero, because it would then cause a distortion in the calculated value of the performance score for WPM method. Therefore, in this example, the sensitivity analysis is performed while varying the weight of MTS criterion in a range of $0.0001 \leq w_{MTS} \leq 0.47$, and six new sets of criteria weights, as shown in Table 7, are thus obtained. The weight of MTS criterion cannot be increased beyond 0.47, because then the weight of LC criterion becomes negative. The problem is again solved for these new sets of criteria weights using the six MCDM methods and the derived rank orders for the alternative robots are exhibited in Figures 2(a)-(f).

These figures reveal that except in full multiplicative form of MOORA method, all the alternative robots are to some extent rank affected due to the weight variations in the considered MCDM methods. It is also observed that alternative R_6 is the best choice in the lower weight region, however, alternative R_3 becomes the best choice in the upper weight region of criterion MTS for all the methods. For attaining more clarity on the performance of MCDM methods, the local as well as global weight stability intervals are determined for each method, as given in Table 8. From this table, it is evident that the full multiplicative form of MOORA method appears to be the most robust (least sensitive) MCDM method with the highest local weight stability interval of $0.0001 \leq w_{MTS} \leq 0.47$, followed by ratio system of MOORA method having a weight stability interval of $0.0001 \leq w_{MTS} \leq 0.4$. However, reference point approach is the most sensitive technique without any steady weight stability interval. Now, when the global weight stability intervals for these MCDM methods are considered, the full multiplicative form of MOORA method with the highest weight stability interval of $0.0001 \leq w_{MTS} \leq 0.47$ again emerges out as the most robust technique, followed by WPM method with an interval of $0.3 \leq w_{MTS} \leq 0.47$. Reference point approach is observed to be the most sensitive method according to global weight stability interval obtained in this single dimensional weight sensitivity analysis.

Table 7
New sets of criteria weights for Example 1

Set	LC	RE	MTS	MC	MR
1	0.1175	0.2735	0.0001	0.4075	0.2015
2	0.0925	0.2485	0.10	0.3825	0.1765
3	0.0675	0.2235	0.20	0.3575	0.1515
4	0.0425	0.1985	0.30	0.3325	0.1265
5	0.0175	0.1735	0.40	0.3075	0.1015
6	0.0001	0.1560	0.47	0.2900	0.0840

(a) WSM

(b) WPM

(c) WASPAS

(d) Ratio system

(e) Reference point approach
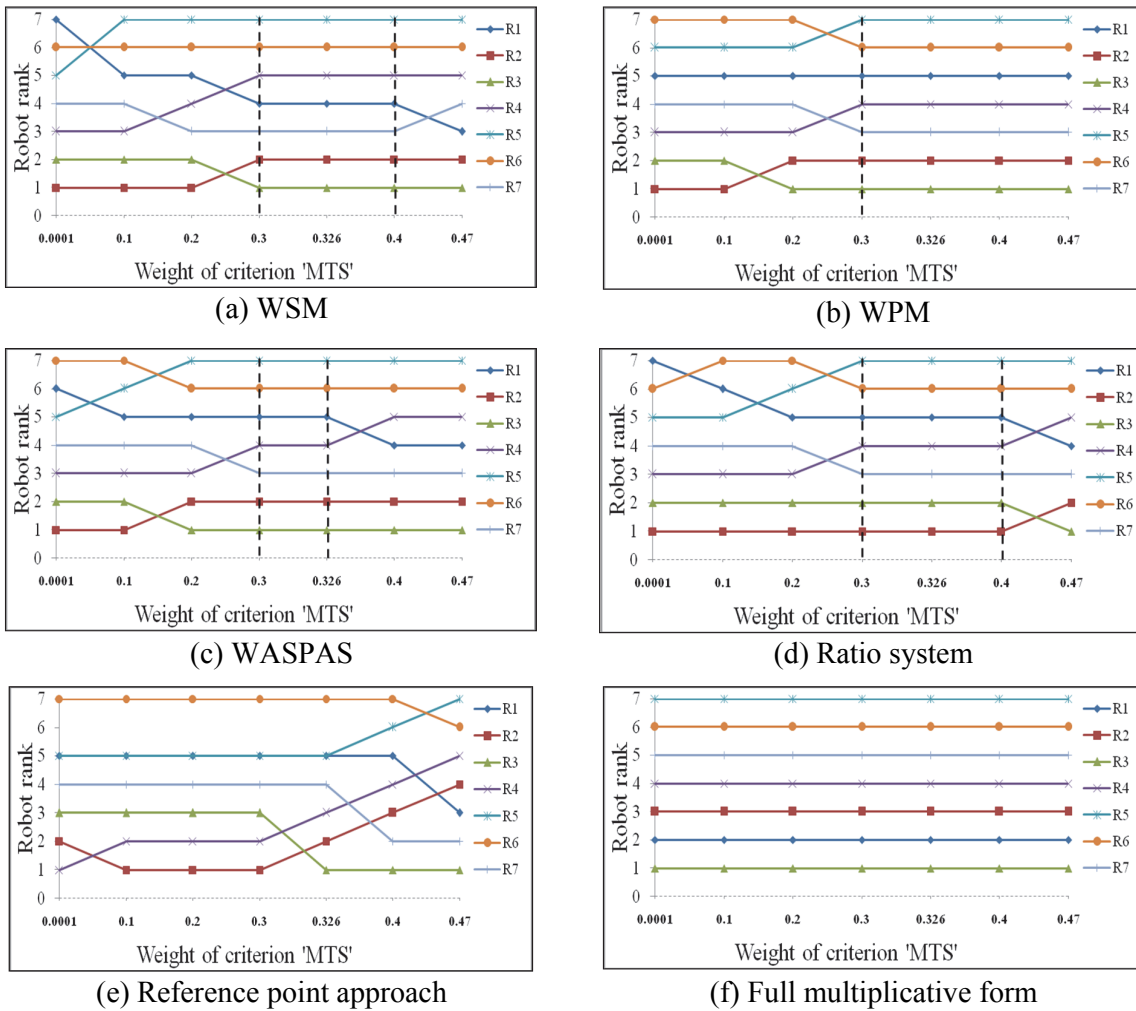
(f) Full multiplicative form

Fig. 2. Results of single dimensional weight sensitivity analysis for Example 1

Table 8
Single dimensional weight stability intervals for Example 1

Method	Weight stability interval	
	Local	Global
WSM	$0.3 \leq w_{MTS} \leq 0.47$	$0.3 \leq w_{MTS} \leq 0.4$
WPM	$0.2 \leq w_{MTS} \leq 0.47$	$0.3 \leq w_{MTS} \leq 0.47$
WASPAS	$0.2 \leq w_{MTS} \leq 0.47$	$0.3 \leq w_{MTS} \leq 0.326$
Ratio system	$0.0001 \leq w_{MTS} \leq 0.4$	$0.3 \leq w_{MTS} \leq 0.4$
Reference point approach	NIL	NIL
Full multiplicative form	$0.0001 \leq w_{MTS} \leq 0.47$	$0.0001 \leq w_{MTS} \leq 0.47$

High dimensional weight sensitivity analysis

To perform high dimensional weight sensitivity analysis and determine the most robust MCDM method for this example, the most critical criterion is first searched out. For identifying the most critical criterion, the alternative robots are arranged as R_3-R_2-R_7-R_4-R_1-R_6-R_5 in order of their performance scores obtained using WPM approach. The $\delta_{k,i,j}$ and $\delta'_{k,i,j}$ values are then calculated for all the criteria, as shown in Tables 9 and 10 respectively. Now, to find out the criticality ranks of the robot selection criteria, their criticality degree and sensitivity coefficient values are calculated in Table 11. From this table, it is observed that

MC with the highest Sens(C_k) value of 0.0514 evolves out as the most critical criterion. Therefore, for this problem, high dimensional weight sensitivity analysis is performed with respect to weight of criterion MC while maintaining the weight additivity and proportionality constraints. Now, the weight coefficient of elasticity for the most critical criterion is considered as one and the same for the other criteria are calculated in Table 12.

In high dimensional weight sensitivity analysis, the weight of the most critical criterion is reduced by 0.326 as well as increased by 0.674 in steps, and the respective sets of new criteria weights are thus achieved. In other words, the Δx range is attained as $-0.326 \le \Delta x \le 0.674$. Here, if Δx value is reduced below -0.326, weight of criterion MC becomes negative, and if it is increased above 0.674, all other criteria weights become negative. Therefore, the weight of criterion MC is varied within the obtained Δx range, and 11 new sets of criteria weights are developed, as given in Table 13. When the Δx value is set to zero, it provides the original weights of the criteria. The performance scores of the seven alternative robots and their respective rank orders are obtained for these 11 new sets of criteria weights using the six MCDM methods. The rank orders thus obtained for these methods against the changing weight of criterion MC are shown in Figures 3(a)-(f). From these figures, it is revealed that in WSM, WPM, WASPAS, ratio system of MOORA and reference point approaches, all the alternative robots are rank affected due to this weight variation. It is also observed that the best robot choice as given by all these five methods is shifted from robot R_3 to robot R_2 for higher criterion weight.

Table 9
$\delta_{k,i,j}$ values for robot selection Example 1

Alternative pair	LC	RE	MTS	MC	MR
(R_3,R_2)	5.0466	0.1419	0.1352	NF	0.0999
(R_3,R_7)	1.8108	NF	-5.7342	0.6881	0.2542
(R_3,R_4)	NF	0.3902	0.5452	-0.9365	0.3679
(R_3,R_1)	NF	0.3130	-0.5869	0.5634	0.4588
(R_3,R_6)	7.7657	NF	1.0632	0.7766	0.4272
(R_3,R_5)	4.6609	NF	0.7269	1.0021	0.7357
(R_2,R_7)	1.3761	NF	-0.2561	0.1153	1.0642
(R_2,R_4)	NF	0.8867	18.7689	0.3547	2.6073
(R_2,R_1)	NF	0.4499	NF	0.1799	4.9279
(R_2,R_6)	8.4271	NF	NF	0.2942	0.8170
(R_2,R_5)	4.6158	NF	1.6498	0.3554	3.9397
(R_7,R_4)	NF	0.1035	0.1351	NF	-1.5418
(R_7,R_1)	NF	0.1219	-0.2437	0.4387	-1.7507
(R_7,R_6)	-7.3201	NF	0.6100	0.8446	0.7444
(R_7,R_5)	26.3216	NF	0.4574	1.3161	73.5250
(R_4,R_1)	NF	0.1586	NF	0.0635	-2.1269
(R_4,R_6)	1.5448	NF	-22.4799	0.2575	0.5193
(R_4,R_5)	1.4236	NF	1.0273	0.3559	5.9649
(R_1,R_6)	0.1188	NF	0.1831	2.1976	0.3821
(R_1,R_5)	0.1526	NF	0.1876	NF	3.2676
(R_6,R_5)	1.0904	0.1817	0.2024	-0.7269	NF

Table 10

$\delta'_{k,i,j}$ values for robot selection Example 1

Alternative pair	LC	RE	MTS	MC	MR
(R_3,R_2)	14018.3787	73.9250	41.4654	NF	83.2493
(R_3,R_7)	5029.9838	NF	-1758.9514	211.0742	211.8714
(R_3,R_4)	NF	203.2280	167.2273	-287.2622	306.5971
(R_3,R_1)	NF	163.0252	-180.0278	172.8267	382.3629
(R_3,R_6)	21571.3759	NF	326.1229	238.2115	355.9646
(R_3,R_5)	12946.9109	NF	222.9753	307.3898	613.0473
(R_2,R_7)	3822.5874	NF	-78.5624	35.3531	886.8719
(R_2,R_4)	NF	461.8340	5757.3418	108.8002	2172.7265
(R_2,R_1)	NF	234.3053	NF	55.1983	4106.6209
(R_2,R_6)	23408.5914	NF	NF	90.2311	680.8694
(R_2,R_5)	12821.6745	NF	506.0670	109.0236	3283.0610
(R_7,R_4)	NF	53.9004	41.4562	NF	-1284.7937
(R_7,R_1)	NF	63.4734	-74.7662	134.5792	-1458.9453
(R_7,R_6)	-20333.4841	NF	187.1180	259.0864	620.3687
(R_7,R_5)	73115.5574	NF	140.3136	403.7055	61270.8371
(R_4,R_1)	NF	82.6194	NF	19.4637	-1772.4182
(R_4,R_6)	4291.0866	NF	-6895.6840	78.9771	432.7706
(R_4,R_5)	3954.4705	NF	315.1115	109.1725	4970.7694
(R_1,R_6)	329.9700	NF	56.1759	674.1104	318.3936
(R_1,R_5)	423.8415	NF	57.5405	NF	2723.0402
(R_6,R_5)	3028.7763	94.6493	62.1012	-222.9774	NF

Table 11

D'_k and Sens(C_k) values for robot selection Example 1

Criteria	LC	RE	MTS	MC	MR
D_k'	329.9700	53.9004	41.4562	19.4637	83.2493
Sens(C_k)	0.0030	0.0186	0.0241	0.0514	0.0120
Criticality rank	5	3	2	1	4

Table 12

Weight coefficient of elasticity for criteria

Criteria	LC	RE	MTS	MR
α_c value	0.0534	0.2849	0.4837	0.1780

Table 13

Changing criteria weights for high dimensional weight sensitivity analysis for Example 1

Δx value	LC	RE	MTS	MC	MR
-0.326	0.0534	0.2849	0.4837	0.0001	0.1780
-0.226	0.0481	0.2564	0.4353	0.1000	0.1602
-0.126	0.0427	0.2279	0.3869	0.2000	0.1424
-0.026	0.0374	0.1994	0.3386	0.3000	0.1246
0.074	0.0320	0.1709	0.2902	0.4000	0.1068
0.174	0.0267	0.1424	0.2418	0.5000	0.0890
0.274	0.0214	0.1139	0.1935	0.6000	0.0712
0.374	0.0160	0.0855	0.1451	0.7000	0.0534
0.474	0.0107	0.0570	0.0967	0.8000	0.0356
0.574	0.0053	0.0285	0.0484	0.9000	0.0178
0.674	0.0001	0.0001	0.0001	1.0000	0.0001

(a) WSM

(b) WPM

(c) WASPAS

(d) Ratio system

(e) Reference point approach

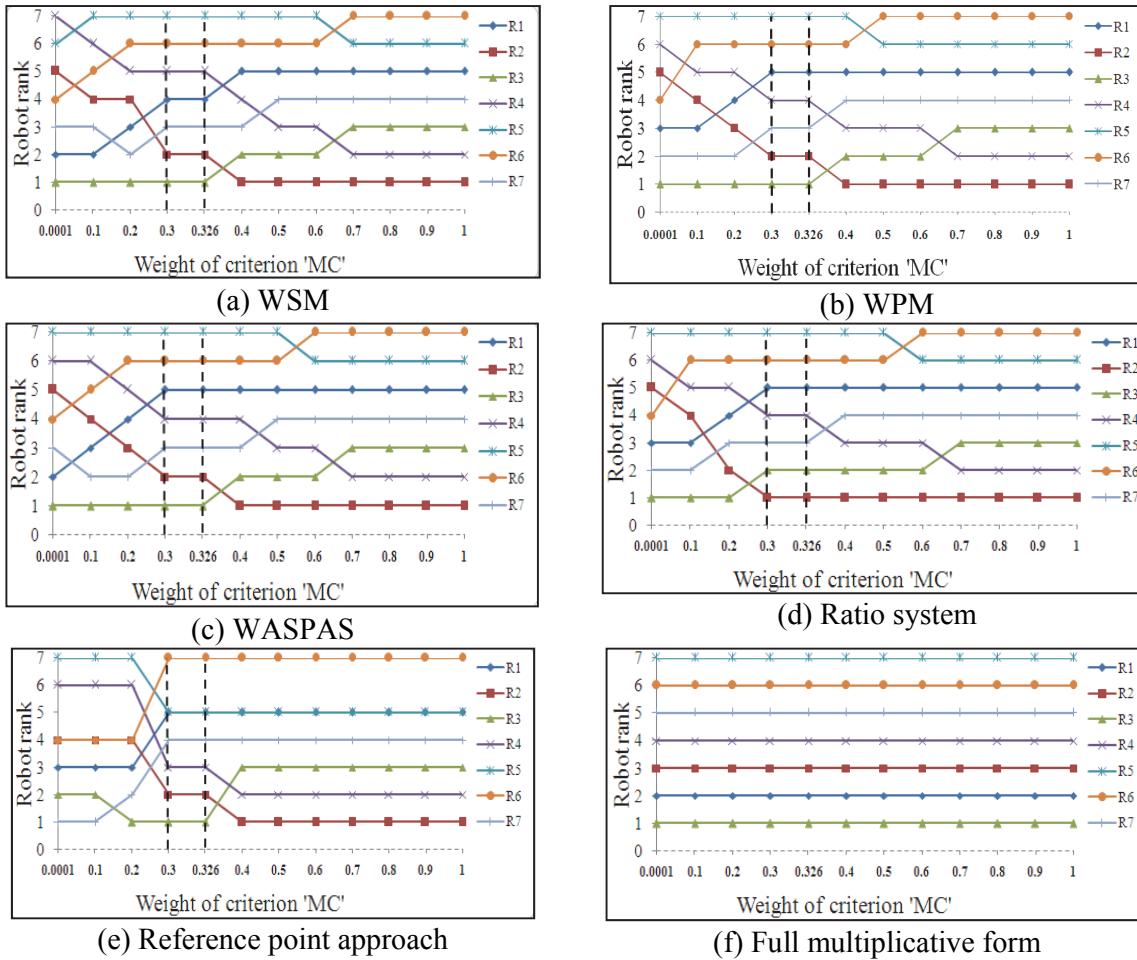(f) Full multiplicative form

Fig. 3. Results of high dimensional weight sensitivity analysis for Example 1

Table 14

High dimensional weight stability intervals for Example 1

Method	Weight stability interval	
	Local	Global
WSM	$0.0001 \le w_{MC} \le 0.326$	$0.3 \le w_{MC} \le 0.326$
WPM	$0.0001 \le w_{MC} \le 0.326$	$0.3 \le w_{MC} \le 0.326$
WASPAS	$0.0001 \le w_{MC} \le 0.326$	$0.3 \le w_{MC} \le 0.326$
Ratio system	$0.3 \le w_{MC} \le 1$	$0.3 \le w_{MC} \le 0.326$
Reference point approach	$0.2 \le w_{MC} \le 0.326$	$0.3 \le w_{MC} \le 0.326$
Full multiplicative form	$0.0001 \le w_{MC} \le 1$	$0.0001 \le w_{MC} \le 1$

From Figs. 3(a)-(c), it is observed that the ranking performance of WSM, WPM and WASPAS methods is the same with respect to local and global weight stability intervals in this high dimensional sensitivity analysis. In case of ratio system of MOORA method, the local weight stability interval is set as $0.3 \le w_{MC} \le 1$, whereas, the global weight stability interval is $0.3 \le w_{MC} \le 0.326$. The local as well as global weight stability intervals for reference point approach, as exhibited in Figure 3(e), are found to be $0.2 \le w_{MC} \le 0.326$ and $0.3 \le w_{MC} \le 0.326$ respectively. On the other hand, it is observed from Figure 3(f) that for full multiplicative form of MOORA method, the position of the top ranked robot alternative remains entirely unaffected with respect to weight variation of the most critical criterion. All the weight stability intervals attained for the six MCDM methods are summarized in Table 14. From this table, it is thus evident that the full multiplicative form of MOORA method has the highest local as well as global weight stability intervals, and hence, it is the most robust approach among the six considered MCDM methods. On the other hand, reference point approach is the most sensitive method being significantly affected by the variation in weight of the most critical criterion.

4.2 Example 2

In order to validate the ranking performance of the six MCDM methods, another industrial robot selection problem is considered here. Karsak et al. (2012) applied the cross efficiency analysis model of data envelopment analysis (DEA) to solve this robot selection problem in a given industrial scenario while considering 12 competitive robot alternatives and five criteria, i.e. cost (C) (in US$), handling coefficient (HC), load capacity (LC) (in kg), repeatability (R) (in (mm^{-1}) and velocity (V) (in m/s). Among these, cost is the sole non-beneficial attribute and the remaining four attributes are beneficial in nature. In this example, the reciprocal of repeatability is considered as a beneficial attribute. Cost is the catalogue price of a robot. The value of handling coefficient can be determined from various features, like diameter (in mm), elevation (in mm), basic rotation (in degree), roll (in degree), pitch (in degree) and yaw (in degree). The diameter, elevation and basic rotation are related to the work area to a robot arm, whereas, roll, pitch and yaw are related to rotational angles of the robot wrist about the three principal axes. The original decision matrix is shown in Table 15. Employing AHP method, Mondal and Chakraborty (2013) determined the criteria weights as $w_C = 0.1071$, $w_{HC} = 0.1071$, $w_{LC} = 0.1624$, $w_R = 0.3910$ and $w_V = 0.2323$, which are used here for the subsequent analyses. The original decision matrix is subsequently normalized according to the requirements and the corresponding ranking orders of 12 robot alternatives as derived employing the considered MCDM methods are given in Table 16. Karsak et al. (2012) identified robot 12 as the best choice, which is also validated by Mondal and Chakraborty (2013) while applying different variants of DEA.

Table 15
Decision matrix for Example 2 (Karsak et al., 2012)

Robot	C	HC	LC	R	V
1	100000	0.995	85	1.7	3
2	75000	0.933	45	2.5	3.6
3	56250	0.875	18	5	2.2
4	28125	0.409	16	1.7	1.5
5	46875	0.818	20	5	1.1
6	78125	0.664	60	2.5	1.35
7	87500	0.88	90	2	1.4
8	56250	0.633	10	8	2.5
9	56250	0.653	25	4	2.5
10	87500	0.747	100	2	2.5
11	68750	0.88	100	4	1.5
12	43750	0.633	70	5	3

Table 16
Comparison of rankings for Example 2

Robot	WSM	WPM	WASPAS	Ratio system	Reference point	Full multiplicative form
R_1	6	9	7	9	11	4
R_2	4	5	5	5	7	3
R_3	5	4	4	4	2	6
R_4	12	12	12	12	11	12
R_5	9	8	9	7	2	10
R_6	11	11	11	11	7	11
R_7	10	10	10	10	9	8
R_8	1	2	2	1	1	9
R_9	8	6	6	6	5	7
R_{10}	7	7	8	8	9	5
R_{11}	3	3	3	3	5	2
R_{12}	2	1	1	2	2	1

Single dimensional weight sensitivity analysis

For performing single dimensional weight sensitivity analysis, the most important criterion is first identified as R having the highest priority weight of 0.3910. Its weight is now varied within a feasible range of $0.0001 \leq w_R \leq 0.8194$ while maintaining the weight additivity constraint. The weight of criterion

R cannot be increased beyond 0.8194, because the weights of C and HC criteria then become negative. Ten new sets of criteria weights, as given in Table 17, are thus generated and this problem is again solved with those new sets of criteria weights for obtaining the rankings of the robot alternatives using the six MCDM methods. The effects of this weight variation on the ranking orders of robots are exhibited in Figs. 4(a)-(f).

Table 17
New sets of criteria weights for Example 2

Set	C	HC	LC	R	V
1	0.2049	0.2049	0.2602	0.0001	0.3301
2	0.1799	0.1799	0.2352	0.1000	0.3051
3	0.1549	0.1549	0.2102	0.2000	0.2801
4	0.1299	0.1299	0.1852	0.3000	0.2551
5	0.1049	0.1049	0.1602	0.4000	0.2301
6	0.0799	0.0799	0.1352	0.5000	0.2051
7	0.0549	0.0549	0.1102	0.6000	0.1801
8	0.0299	0.0299	0.0852	0.7000	0.1551
9	0.0048	0.0048	0.0602	0.8000	0.1301
10	0.0001	0.0001	0.0553	0.8194	0.1252

(a) WSM

(b) WPM

(c) WASPAS

(d) Ratio system

(e) Reference point approach
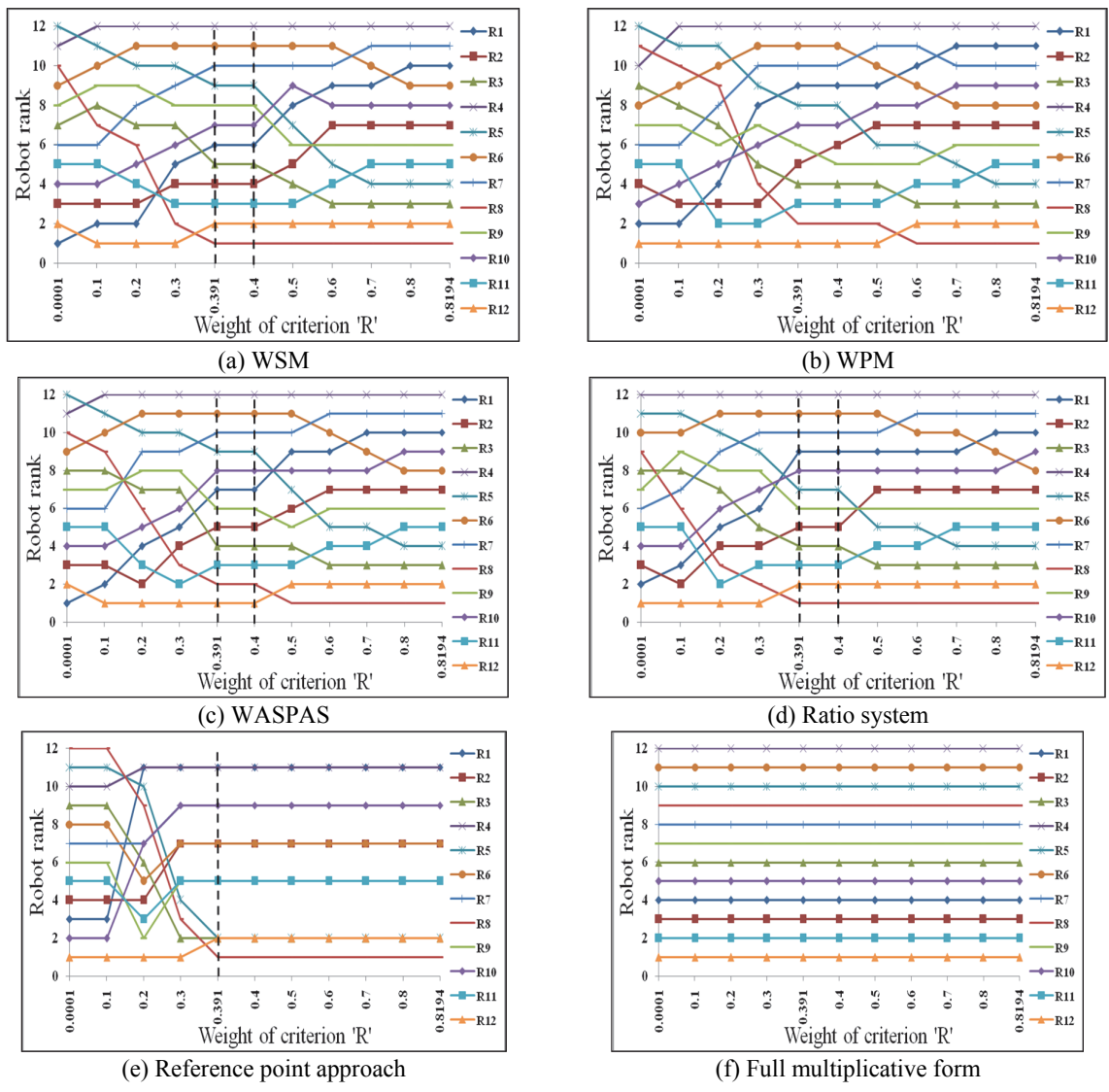
(f) Full multiplicative form

Fig. 4. Results of single dimensional weight sensitivity analysis for Example 2

Table 18
Single dimensional weight stability intervals for Example 2

Method	Weight stability interval	
	Local	Global
WSM	$0.391 \leq w_R \leq 0.8194$	$0.391 \leq w_R \leq 0.4$
WPM	$0.0001 \leq w_R \leq 0.5$	NIL
WASPAS	$0.1 \leq w_R \leq 0.4$	$0.391 \leq w_R \leq 0.4$
Ratio system	$0.391 \leq w_R \leq 0.8194$	$0.391 \leq w_R \leq 0.4$
Reference point approach	$0.391 \leq w_R \leq 0.8194$	$0.391 \leq w_R \leq 0.8194$
Full multiplicative form	$0.0001 \leq w_R \leq 0.8194$	$0.0001 \leq w_R \leq 0.8194$

These figures again indicate that the full multiplicative form of MOORA method appears to be the most robust MCDM method with the highest local as well as global weight stability interval of $0.0001 \leq w_{MTS} \leq 0.8194$, followed by reference point approach with a global weight stability interval of $0.391 \leq w_R \leq 0.8194$. It is quite interesting to observe that for WSM, WASPAS and ratio system of MOORA methods, the global weight stability interval is the same as $0.391 \leq w_R \leq 0.4$. These three MCDM methods are found to be quite similar in their ranking performance. For WPM method, no global weight stability interval is obtained and it can be concluded that its ranking performance is seriously affected due to weight variation of the most important criterion in single dimensional weight sensitivity analysis. All the weight stability intervals achieved using the adopted MCDM methods are provided in Table 18.

High dimensional weight sensitivity analysis

In this example, criterion V is identified as the most critical one and high dimensional weight sensitivity analysis is now performed while varying its weight within the feasible range of $-0.2323 \leq \Delta x \leq 0.7677$ after maintaining the weight additivity and proportionality constraints. In this sensitivity analysis, the weight of criterion V is reduced by 0.2323 as well as increased by 0.7677 in steps, and 11 sets of new criteria weights are derived, as given in Table 19.

Table 19
Changing criteria weights for high dimensional weight sensitivity analysis for Example 2

Δx value	C	HC	LC	R	V
-0.2323	0.1395	0.1395	0.2115	0.5093	0.0001
-0.1323	0.1256	0.1256	0.1904	0.4584	0.1000
-0.0323	0.1116	0.1116	0.1692	0.4075	0.2000
0.0677	0.0977	0.0977	0.1481	0.3565	0.3000
0.1677	0.0837	0.0837	0.1269	0.3056	0.4000
0.2677	0.0697	0.0697	0.1058	0.2546	0.5000
0.3677	0.0558	0.0558	0.0846	0.2037	0.6000
0.4677	0.0418	0.0418	0.0634	0.1528	0.7000
0.5677	0.0279	0.0279	0.0423	0.1018	0.8000
0.6677	0.0139	0.0139	0.0211	0.0509	0.9000
0.7677	0.0001	0.0001	0.0001	0.0001	1.0000

If Δx value is reduced below -0.2323, the weight of criterion V becomes negative, and if it is increased above 0.7677, all other criteria weights become negative. Now, based on the changing weight of criterion V, the rank orders for the alternative robots are determined employing the considered methods, as displayed in Figures 5(a)-(f). From these figures, it is observed that the performance of all the MCDM methods, except the full multiplicative form of MOORA method is seriously affected by the varying weight of the most critical criterion. Table 20 provides the local as well as global weight stability intervals for all the six MCDM methods. It is quite interesting to note that the full multiplicative form of MOORA method has the highest local and global weight stability intervals of $0.0001 \leq w_V \leq 1$. It signifies that its ranking performance is quite stable over the entire range of weight variation of criterion V. The performance of reference point approach is moderate with a global weight stability interval of $0.0001 \leq$

$w_V \leq 0.2323$. On the other hand, no global weight stability interval exists for WSM, WPM, WASPAS and ratio system of MOORA methods. In all these four methods, the position of the top ranked robot alternative cannot be maintained over the considered range of weight variation for criterion V.
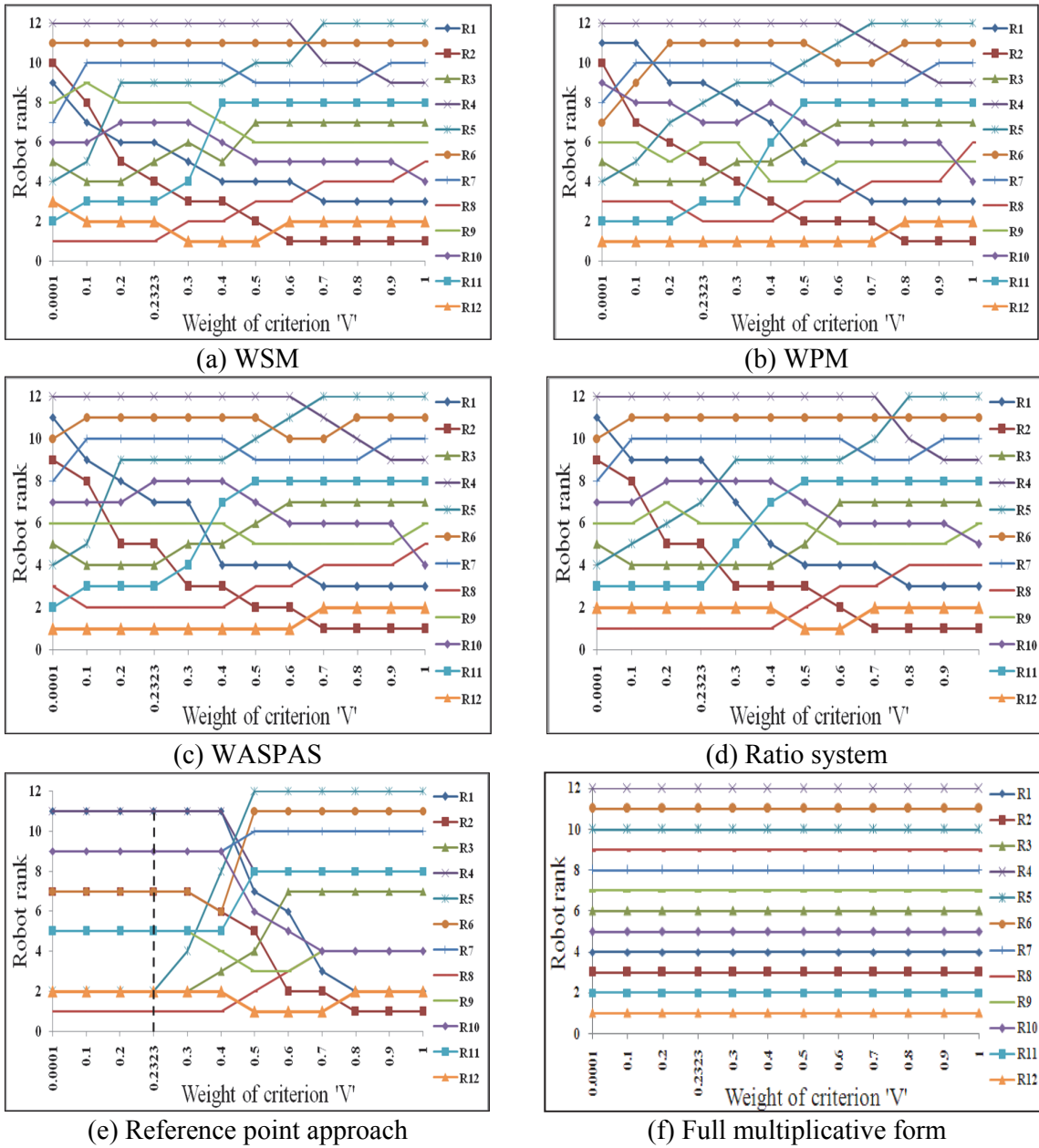

(a) WSM

(b) WPM

(c) WASPAS

(d) Ratio system

(e) Reference point approach

(f) Full multiplicative form

Fig. 5. Results of high dimensional weight sensitivity analysis for Example 2

Table 20
High dimensional weight stability intervals for Example 2

Method	Weight stability interval	
	Local	Global
WSM	$0.0001 \leq w_V \leq 0.2323$	NIL
WPM	$0.0001 \leq w_V \leq 0.7$	NIL
WASPAS	$0.0001 \leq w_V \leq 0.6$	NIL
Ratio system	$0.0001 \leq w_V \leq 0.4$	NIL
Reference point approach	$0.0001 \leq w_V \leq 0.4$	$0.0001 \leq w_V \leq 0.2323$
Full multiplicative form	$0.0001 \leq w_V \leq 1$	$0.0001 \leq w_V \leq 1$

5. Conclusions

Although MCDM plays a critical role in many real time problems, it is hard to accept an MCDM method as being accurate all the time. When the alternatives are very distinctive from each other, it is less likely that the ranking irregularities among the MCDM methods will take place. However, the decision maker needs a more powerful MCDM method when alternatives are closely related with each other. Although it is doubtful that the 'perfect' MCDM method can ever be found, it is always a prudent idea for the decision maker to be aware of the main controversies related to the ranking performance of MCDM methods. Although the search for finding out the best MCDM method may never end, research in this area of decision-making is still critical and valuable. In this paper, using two real time robot selection problems, the ranking performance of six popular MCDM methods is compared in order to identify the most robust MCDM method with respect to varying weights of the most important as well as the most critical criterion. Based on single dimensional and high dimensional weight sensitivity analysis results, the full multiplicative form of MOORA method is identified as the most robust performer being least affected by the varying weights of the evaluation criteria. The other MCDM methods are observed to be seriously affected by the changing criteria weights.

MULTIMOORA methods (ratio system, reference point approach and full multiplicative form) treat beneficial and non-beneficial objectives separately. However, in WSM, WPM and WASPAS methods, beneficial and non-beneficial criteria are treated equally, but their normalization is done separately. The normalization approach employed in WSM, WPM and WASPAS methods takes into consideration only two performance values, i.e. minimum (for non-beneficial attributes) and maximum (for beneficial attributes), and does not consider all the performance values as in the case of MULTIMOORA methods. Therefore, the normalized scores obtained in MULTIMOORA methods are more realistic. MULTIMOORA is thus the most robust system of multiple objectives optimization with respect to stakeholders (decision makers), objectives, and interrelations between objectives and alternatives as it is non-subjective and based on cardinal data. In full multiplicative form of MOORA method, maximization as well as minimization is obtained by purely multiplicative utility function. It uses multiplication of criteria values for beneficial and non-beneficial attributes separately and their ratio, providing dimensionless number for comparison. Purely multiplicative utility function adopted in this approach helps to obtain robust ranking results.

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