International Journal of Industrial Engineering Computations 7 (2016) 573–584

Contents lists available at GrowingScience

International Journal of Industrial Engineering Computations

homepage: www.GrowingScience.com/ijiec

Integrated approach in solving parallel machine scheduling and location (ScheLoc) problem

Mohsen Rajabzadeh[a], Mohsen Ziaee[b] and Ali Bozorgi-Amiri[c*]

[a]*M.Sc., Industrial Engineering Group, College of Engineering, University of Bojnord, Bojnord, Iran*
[b]*Assistant Professor, Industrial Engineering Group, College of Engineering, University of Bojnord, Bojnord, Iran*
[c]*School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

C H R O N I C L E

Article history:
Received November 4 2015
Received in Revised Format
April 6 2016
Accepted April 22 2016
Available online
April 24 2016
Keywords:
Scheduling
Layout planning
Parallel machines

A B S T R A C T

Scheduling and layout planning are two important areas of operations research, which are used in the areas of production planning, logistics and supply chain management. In many industries locations of machines are not specified, previously, therefore, it is necessary to consider both location and scheduling, simultaneously. This paper presents a mathematical model to consider both scheduling and layout planning for parallel machines in discrete and continuous spaces, concurrently. The preliminary results have indicated that the integrated model is capable of handling problems more efficiently.

1. Introduction

While in many scheduling problems, the locations of machines are fixed it is possible to show how to consider location and scheduling problems simultaneously. Obviously, this integrated method enhances the modeling power of scheduling for different real-life problems (Heßler & Deghdak, 2015). Hennes and Hamacher (2002) are believed to be the first who introduced the idea of scheduling and layout planning. In their study, schedules and location are examined on a graph consisting of n nodes and m edges where, each node is considered as a storage location for a job. The primary objective was to find the optimal location and schedule of a single machine by minimizing the maximum completion time in two different scenarios. The first scenario considers location of facility could be considered only on edges while in the second scenario, locations can be considered on other parts. Elvikis et al. (2007, 2009) presented some polynomial solution methods for the planar ScheLoc makespan problem, which incorporates an special kind of a scheduling and a rather general, planar location problem, respectively.

* Corresponding author. Tel: +982182084492
E-mail: alibozorgi@ut.ac.ir (A. Bozorgi-Amiri)

doi: 10.5267/j.ijiec.2016.4.003

Kalsch and Drezner (2010) integrated both the location of the machine and the scheduling of the jobs executed by the machine by analyzing two different objectives of the makespan and the total completion time. They considered some important properties of the models and provided a lower bound for the objective functions. The single machine ScheLoc problems with Euclidean, rectilinear and general ℓq norms were solved by the "big triangle small triangle" branch-and-bound technique (Lawler, 1973; Drezner & Suzuki, 2004). According to Scholz (2012a,b), geometric branch-and-bound approaches with mixed continuous and combinatorial variables are more suitable solution methods for ScheLoc problems. Table 1 shows different single machine ScheLoc problems tackled by various authors.

Table 1
Different studies in machine scheduling and location (ScheLoc) problem

Researchers	Solution space			Objective function			Production environment		
	discrete	Network	Plane	C_{max}	TC	Other	Single machine	Parallel machine	Other
Hennes & Hamacher, 2002	✓	✓	-	✓	-	-	✓	-	-
Elvikis et al., 2007	-	-	✓	✓	-	-	✓	-	-
Elvikis et al., 2009	-	-	✓	✓	-	-	✓	-	-
Kalsch & Drezner, 2010	-	-	✓	✓	✓	-	✓	-	-
Scholz, 2011	-	-	✓	✓	-	-	✓	-	-
Present study	✓	-	✓	✓	-	-	-	✓	-

2. The proposed study

ScheLoc is associated with scheduling and layout planning of a certain jobs on some unique machines with determined processing times. All jobs are available on machines according to the following,

$$\text{The time jobs are available for processing on machine} = \text{The time jobs are available at storage} + \frac{\text{Distance between storage and machine}}{\text{Speed of transportation facilities}}$$

The primary objective is to minimize the makespan. The problem can be considered in two forms of discrete and continues.

2.1. Discrete ScheLoc

In this case, machines can be located on special places. For instance, as shown in Fig. 1, five alternative locations are considered for two machines and four jobs are stored in inventory with specified processing times.

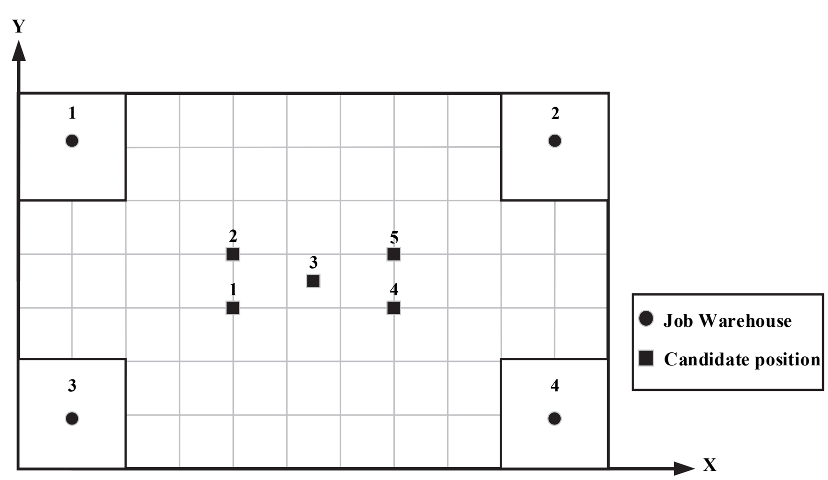

Fig. 1. The discrete parallel machine ScheLoc problem

Sets

I	Set of all jobs
J	Set of alternative locations for machines

Indices

i, i'	Jobs with ($i = 1, 2, ..., \lvert I \rvert$)
j	Locations of different machines ($j = 1, 2, ..., \lvert J \rvert$)

Parameters

ma	Number of similar machines
p_i	Processing time of job *i*
l_i	Time of availability of job *i* in storage
V_i	Velocity of moving job *i* from storage to machines
$dist_{ij}$	Distance between job *i* to candidate location *j*
r_{ij}	Time of readiness of job *i* on machine *j* $r_{ij} = l_i + \frac{dist_{ij}}{V_i}$ (1)
M	A big number calculated by $M = \max_{i,j}(r_{ij}) + \sum_{i=1}^{I} p_i$
c_i	Completion time of job *i*

Variables

c_{max}	Makespan
y_j	Binary variable, which is one if one of machines is located on alternative *j*, zero, otherwise
x_{ij}	Binary variable, which is one if job *i* is assigned to machine *j*, zero, otherwise
$z_{i'i}$	Binary variable, which is one if job *i'* is processed before job *i*, zero, otherwise

The mathematical problem is stated as follows,

$$\min \ Z = c_{max} \tag{2}$$

subject to

$$\sum_{j=1}^{|J|} y_j = ma \tag{3}$$

$$\sum_{j=1}^{|J|} x_{ij} = 1 \qquad \forall i \tag{4}$$

$$c_i - p_i \geq r_{ij} - M(1 - x_{ij}) \qquad \forall\ i, j \tag{5}$$

$$c_i - p_i \geq c_{i'} - M(1 - z_{i'i}) - M(2 - x_{ij} - x_{i'j}) \qquad \forall\ i, i'(i \neq i'), j \tag{6}$$

$$c_{i'} - p_{i'} \geq c_i - M(z_{i'i}) - M(2 - x_{ij} - x_{i'j}) \qquad \forall\ i, i'(i \neq i'), j \tag{7}$$

$$x_{ij} \leq y_j \qquad \forall\ i, j \tag{8}$$

$$c_{max} \geq c_i \qquad \forall\ i \tag{9}$$

$$c_i \geq 0,\ c_{max} \geq 0;\ y_j, x_{ij}, z_{i'i} \in \{0,1\} \tag{10}$$

As we can observe from the proposed model, the objective function given in Eq. (2) minimizes the makespan. Eq. (3) insures that all machines are assigned while Eq. (4) determines that each job has to be executed only on single machine. Eq. (5) specifies that job *i* can only be started when it reaches to machine *j*. Eq. (6) and Eq. (7) specify that job *i* can be started when the process of the previous job was already finished. According to Eq. (8), a job can be processed only on a particular machine. Eq. (9) computes the makespan and finally Eq. (10) demonstrates the type of variables. The problem can be examined under two scenarios. For the first scenario, layout planning is accomplished by minimizing sum of the times for all jobs assigned to machines and then scheduling of jobs on machines are determined. This problem is first formulated as follows,

$$\min\ Z = \sum_{j=1}^{|J|}\sum_{i=1}^{|I|}(l_i + \frac{dist_{ij}}{V_i})y_j \tag{11}$$

subject to constraint (3)

$$y_j \in \{0,1\}. \tag{12}$$

When all y_j are determined, the optimal values are denoted as $\overline{y}_j$ and the following problem is solved,

$$\min\ Z = c_{\max} \tag{13}$$

subject to constraints (4)(5)(6)(7)(9)

$$x_{ij} \le \overline{y}_j \qquad \forall\ i, j \tag{14}$$

$$c_i \ge 0,\ c_{\max} \ge 0;\ x_{ij}, z_{i'i} \in \{0,1\} \tag{15}$$

For the second scenario, we first solve the following problem

$$\min\ Z = \sum_{j=1}^{|J|}\sum_{i=1}^{|I|}(l_i + \frac{dist_{ij}}{V_i})x_{ij} \tag{16}$$

subject to constraints (3)(4)(8)

$$y_j, x_{ij} \in \{0,1\} \tag{17}$$

Now, the jobs are sorted according to non-decreasing order of availability times. In case, there are two jobs with the same availability times, the one with less processing time is considered and then the jobs are assigned to machines.

2.2. Continous ScheLoc

In this case, machines can be assigned anywhere in job floor as shown in Fig. 2 as follows,

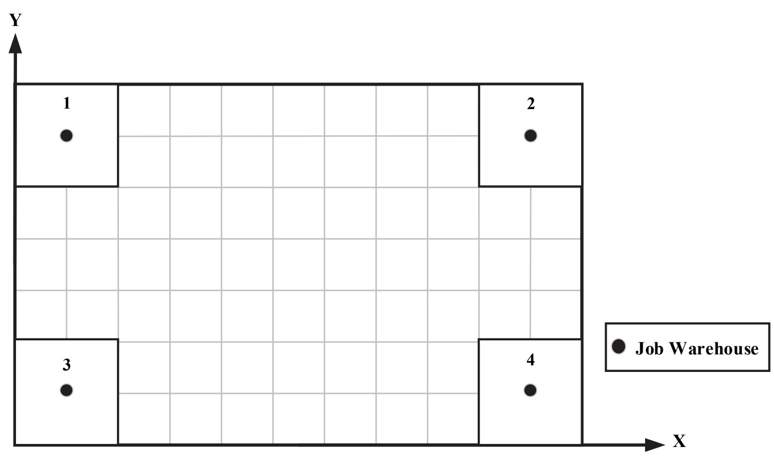

Fig. 2. Continuos parallel machine ScheLoc problem

For this problem, we define the following assumptions

Sets

I	Set of all jobs
K	Set of all machines

Indices

i	Jobs with ($i=1,2,...,\lvert I\rvert$)
k	Locations of different machines ($j=1,2,...,\lvert J\rvert$)

Parameters

n	Number of jobs
ma	Number of similar machines
p_i	Processing time of job *i*
l_i	Time of availability of job *i* in storage
V_i	Velocity of moving job *i* from storage to machines
(a_i, b_i)	Location of storage for job *i*
M	A big number

Variables

$dist_{ik}$	Distance between location of machin *k* and storage of job *i*
r_{ik}	Time of readiness of job *i* on machine *j* $r_{ij}=l_i+\dfrac{dist_{ij}}{V_i}$
$c_{\max}$	Makespan
(x_k, y_k)	Location of machine *k*
w_{ik}	If job *i* is assigned to machine *k,* zero, otherwise
$z_{i'i}$	Binary variable, which is one if job *i'* is processed before job *i*, zero, otherwise

The proposed study is formulated as follows,

$$\min\ Z=c_{\max} \tag{20}$$

subject to

$$\sum_{k=1}^{|K|} w_{ik}=1 \qquad \forall i \tag{21}$$

$$c_i-p_i\geq l_i+\frac{dist_{ik}}{V_i}-M(1-w_{ik}) \qquad \forall\ i,k \tag{22}$$

$$c_i-p_i\geq c_{i'}-M(1-z_{i'i})-M(2-w_{ik}-w_{i'k}) \qquad \forall\ i,i'(i\neq i'),k \tag{23}$$

$$c_{i'}-p_{i'}\geq c_i-M(z_{i'i})-M(2-w_{ik}-w_{i'k}) \qquad \forall\ i,i'(i\neq i'),k \tag{24}$$

$$c_{\max}\geq c_i \qquad \forall\ i \tag{25}$$

$$c_i\geq 0,\ c_{\max}\geq 0;\ w_{ik},\mathrm{z}_{i'i}\in\ \{0,1\} \tag{26}$$

The objective function minimizes the makespan, Eq. (21) ensures that each job is processed only on one machine, Eq. (22) specifies that job *i* can only be started when it reaches to machine *j*. Eq. (23) and Eq. (24) specify that job *i* can be started when the process of the previous job was already finished. Eq. (25) computes the makespan and finally Eq. (26) demonstrates the type of variables. In this study, distances are computed as follows,

$$dist_{ik}=|x_k-\mathrm{a}_i|+|\mathrm{y}_k-\mathrm{b}_i|, \tag{27}$$

and Eq. (27) is linearalized as follows,

$$|x_k-\mathrm{a}_i|=u_{ki}+v_{ki}, \tag{28}$$

$$|y_k - b_i| = g_{ki} + h_{ki}. \tag{29}$$

Therefore, the following equations are added to the problem statement,

$$x_k - \mathrm{a}_i = u_{ki} - v_{ki}, \tag{30}$$

$$y_k - b_i = g_{ki} - h_{ki}, \tag{31}$$

$$u_{ki} \cdot v_{ki} = 0, \tag{32}$$

$$g_{ki} \cdot h_{ki} = 0, \tag{33}$$

$$u_{ki}, v_{ki} \geq 0, \tag{34}$$

$$g_{ki}, h_{ki} \geq 0. \tag{35}$$

The resulted problem formulation is still nonlinear because of nonlinear terms given in Eq. (32) and Eq. (33). However, since only u_{ki} or v_{ki} appears nonzero in the final solution and g_{ki} or h_{ki} appears in nonzero form in the objective function, we may disregard them in the problem formulation (Lawler, 1973) and the problem is formulated as follows,

$$\min\ Z = c_{\max} \tag{36}$$

subject to constraints (23)(24)(25)

$$\sum_{k=1}^{|K|} w_{ik} = 1 \qquad \forall i \tag{37}$$

$$c_i - p_i \geq l_i + \frac{(u_{ki} + v_{ki}) + (g_{ki} + h_{ki})}{V_i} - M(1 - w_{ik}) \qquad \forall\ i,k \tag{38}$$

$$x_k - \mathrm{a}_i = u_{ki} - v_{ki} \qquad \forall\ i,k \tag{39}$$

$$y_k - b_i = g_{ki} - h_{ki} \qquad \forall\ i,k \tag{40}$$

$$x_k \geq 0, y_k \geq 0, u_{ki} \geq 0, v_{ki} \geq 0, g_{ki} \geq 0, h_{ki} \geq 0, c_i \geq 0,\ c_{\max} \geq 0;\ w_{ik}, \mathrm{z}_{i'i} \in \{0,1\} \tag{41}$$

To solve the resulted model in continuous form, we consider two scenarios. For the first scenario, the following problem formulation has to be solved.

$$\min\ Z = \sum_{i=1}^{|I|}\sum_{k=1}^{|K|}(l_i + \frac{dist_{ik}}{V_i}) = \sum_{i=1}^{|I|}\sum_{k=1}^{|K|}(l_i + \frac{|\mathrm{x}_k - \mathrm{a}_i| + |\mathrm{y}_k - \mathrm{b}_i|}{V_i}) \tag{42}$$

$$(x_k, \mathrm{y}_k) \in R^2. \tag{43}$$

Using Eqs. (28-35) yields the following problem statement,

$$\min\ Z = \sum_{i=1}^{|I|}\sum_{k=1}^{|K|}(l_i + \frac{(u_{ki} + v_{ki}) + (g_{ki} + h_{ki})}{V_i}) \tag{44}$$

subject to constraints (39)(40)

$$x_k \geq 0, y_k \geq 0, u_{ki} \geq 0, v_{ki} \geq 0, g_{ki} \geq 0, h_{ki} \geq 0 \tag{45}$$

Once the problem is solved for optimality, $\overline{u}_{ki}$ ، $\overline{v}_{ki}$ ، $\overline{g}_{ki}$ و $\overline{h}_{ki}$ are optimal values of u_{ki} ، v_{ki} ، g_{ki} و h_{ki}, respectively. Therefore we have,

$$\min\ Z = c_{\max} \tag{46}$$

subject to constraints (21)(23)(24)(25)

$$c_i - p_i \geq l_i + \frac{(\overline{u}_{ki} + \overline{v}_{ki}) + (\overline{g}_{ki} + \overline{h}_{ki})}{V_i} - M(1 - w_{ik}) \qquad \forall\ i,k \tag{47}$$

$$c_i \geq 0,\ c_{\max} \geq 0;\ w_{ik}, \mathrm{z}_{i'i} \in \{0,1\} \tag{48}$$

For the second scenario, we first solve the following

$$\min\ Z=\sum_{i=1}^{|I|}\sum_{k=1}^{|K|} AT_{ik} \tag{49}$$

subject to constraints (21)(39)(40)

$$AT_{ik} \geq l_i + \frac{(u_{ki}+v_{ki})+(g_{ki}+h_{ki})}{V_i} - M(1-w_{ik}) \qquad \forall\ i,k \tag{50}$$

$$AT_{ik} \geq 0, u_{ki} \geq 0, v_{ki} \geq 0, g_{ki} \geq 0, h_{ki} \geq 0, \mathrm{w}_{ik} \in \{0,1\} \tag{51}$$

Here if AT_{ik} receives a value one, it means that job *i* is assigned to machine *k* and the assignment of jobs to machines are the same as discrete form.

3. The results

In this section, we present the results of the implementation of the proposed study using some randomly generated numbers. All problems are coded in GAMS using personal computer with 2.2GHz core i7 CPU and 6 GB RAM. Table 2 shows the input parameters.

Table 2

Input data used for testing different instances

Sample problem	Coordination of storage	Coordination of candidate locations	Processing time	Velocity of vehicles	Time of availability of jobs
$r \simeq p$	$u[0,25]$	$u[0,25]$	$normrnd[15,5]$	$u\{1.5,2,2.5,3\}$	$normrnd[8,2]$
$r \simeq 0.1p$	$u[0,15]$	$u[0,15]$	$normrnd[25,5]$	$u\{3,3.5,...,6\}$	$normrnd[4,1]$
$r \simeq 10p$	$u[0,200]$	$u[0,200]$	$normrnd[10,3]$	$u\{0.5,1,1.5\}$	$normrnd[25,5]$

3.1. Discrete ScheLoc

In this case, we consider additional input data for the proposed study as given in Table 3 as follows,

Table 3

Input data for discrete ScheLoc

Job number	Processin time (S)	Time of availability in storage	Velocity of vehicle (m/s)
1	6	2	2
2	4	5	1
3	10	3	2
4	8	4	1

Table 4 shows the results of the implementation of the proposed study. In addition, Fig. 3 shows the results.

Table 4

The summary of the results of the optimal solution

Problem	Objective function	Locations of machines	CPU time
Integrated	19.5	1, 3	0.027
Scenario one	21	4, 5	0.026
Scenario two	21	1, 4	0.027

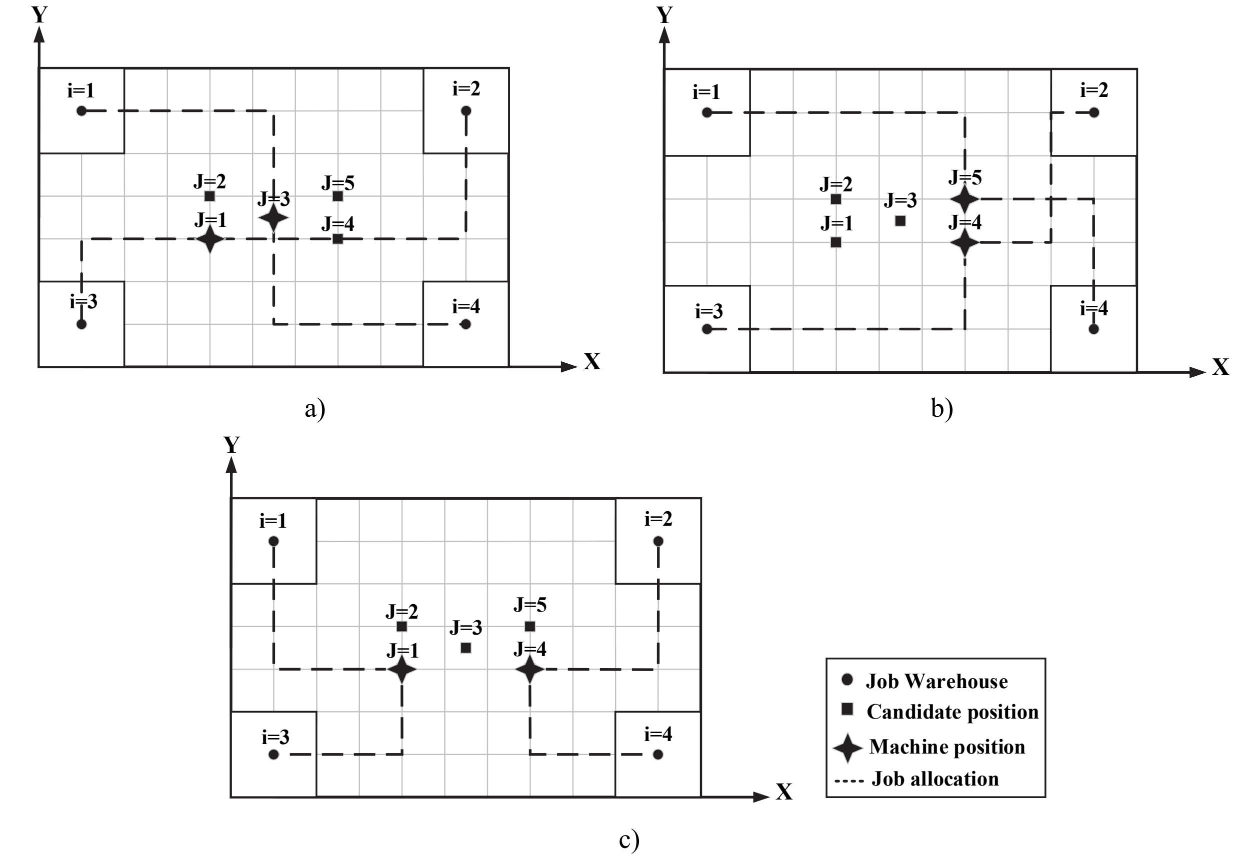

Fig. 3. The locations of the machines and job schedule for discrete case

In order to have a better understanding on the performance of the proposed method we have generated 10 sample test problems, solved with GAMS software package and in case GAMS software could not reach optimal solution, we have reported the best solution after 1000 seconds shown in (*). Table 5 shows the results of the problem. For all problems relative gaps are computed as follows,

$$Gap_{\cdot a\&b} = \left(\frac{sol_a - sol_b}{sol_b}\right) \times 100. \tag{52}$$

Table 5
The results of the optimal solution for 10 sample test problems

Problem #	Input data			Integrated model		Scenario one			Scenario two		
	# of jobs	Candidate locations	# of machines	Cmax	Time	Cmax	Time	Gap	Cmax	Time	Gap
1	5	5	2	23	0.352	23	0.395	0	25	0.098	8.69
2	4	5	3	17	0.107	19	0.256	11.76	21	0.1	23.52
3	5	5	3	29	0.813	31	0.497	6.89	31	0.101	6.89
4	6	4	2	54	3.107	54	0.524	0	72	0.269	33.33
5	6	5	2	39	0.709	40	0.464	2.56	42	0.267	7.69
6	6	5	3	42	1.878	42	0.665	0	58	0.278	38.09
7	7	5	2	67	3.264	70.5	0.836	5.22	79.33	0.26	18.4
8	8	5	2	83.66	14.667	85	2.172	1.6	106	0.269	26.7
9	9	5	3	60	152.198	63	6.542	5	63.5	0.233	5.83
10	10	6	3	68.33*	1000	71	50.577	3.9	78	0.245	14.15
Mean		-		-	117.71	-	6.292	3.7	-	0.212	18.34

Table 6
The results of the optimal solution for 10 sample test problems

Problem #	Input data			Integrated model		Scenario one			Scenario two		
	# of jobs	Candidate locations	# of machines	Cmax	Time	Cmax	Time	Gap	Cmax	Time	Gap
1	5	5	2	46	1.391	46	0.746	0	48.66	0.536	5.78
2	4	5	3	30.75	0.643	30.75	0.415	0	39.09	0.305	27.12
3	5	5	3	46.83	2.285	46.83	0.741	0	66.25	0.539	41.47
4	6	4	2	60.9	1.43	61	0.708	0.16	67.9	0.426	11.49
5	6	5	2	73	1.664	73.57	0.593	0.78	88.25	0.486	20.89
6	6	5	3	51.6	3.734	52.2	1.064	1.16	68.64	0.473	33.02
7	7	5	2	78.85	58.186	79.11	1.726	0.33	103.11	0.396	30.77
8	8	5	2	114.85	564.65	116	11.99	0.99	118	0.253	2.74
9	9	5	3	81.12*	1000	81.71*	1000	0.73	102.12	0.323	25.893
10	10	6	3	83.44*	1000	83.77*	1000	0.39	103.55	0.477	24.101
Mean		-		-	263.398	-	113.109	0.46	-	0.421	22.32

Table 7
The results of the optimal solution for 10 sample test problems

Problem #	Input data			Integrated model		Scenario one			Scenario two		
	# of jobs	Candidate locations	# of machines	Cmax	Time	Cmax	Time	Gap	Cmax	Time	Gap
1	5	5	2	80	0.146	80	0.234	0	89	0.249	11.25
2	4	5	3	77	0.103	77	0.249	0	77	0.24	0
3	5	5	3	193	0.383	195	0.268	1.04	193	0.188	0
4	6	4	2	120	0.112	120	0.252	0	134	0.256	11.67
5	6	5	2	65	0.339	71	0.378	9.23	66.67	0.253	2.56
6	6	5	3	61.5	0.416	85	0.384	38.21	79.66	0.263	29.52
7	7	5	2	167	0.13	167	0.367	0	167	0.277	0
8	8	5	2	159	0.338	159	1.57	0	159	0.228	11.38
9	9	5	3	136	0.389	152	0.346	11.76	152	0.231	11.76
10	10	6	3	137	0.236	137	0.267	0	146	0.443	6.57
Mean		-		-	0.301	-	0.344	6.02	-	0.262	7.33

As we can observe from the results of Tables (5-7), the proposed integrated model has relatively performed better than alternative methods.

3.2. Continous model

For the case of continuous model, we have solved the problem and Table 8 and Fig. 4 show the results of the proposed study.

Table 8
The summary of the results of the optimal solution

Problem	Objective function	Locations of machines	CPU time
Integrated	18	(10, 6), (1, 6)	0.114
Scenario one	21.5	(10, 1), (10, 1)	0.261
Scenario two	18	(10, 6), (1, 6)	0.384

Again, we have generated 10 addional test problems and Table 9, Table 10 and Table 11 present details of our findings.

Table 9
The results of the optimal solution for 10 sample test problems

Prob.	Input data		Integrated model		Scenario one			Scenario two		
	# of jobs	locations	Cmax	time	Cmax	time	Gap	Cmax	time	Gap
1	5	2	19	0.36	22	0.26	15.78	21	0.258	10.52
2	4	3	16	0.115	22	0.26	37.5	16	0.235	0
3	5	3	28	0.111	31	0.257	10.71	33	0.307	17.85
4	6	2	51	0.234	53	0.487	3.92	72.5	0.277	42.15
5	6	2	39	0.387	40	0.508	2.56	42	0.249	7.69
6	6	3	50	0.379	52.33	0.38	4.66	60.33	0.4	20.66
7	7	2	48	0.592	52	0.569	8.33	65	0.372	35.41
8	8	2	82	1.344	83.66	1.122	2.02	104.3	0.362	27.19
9	9	3	57	1.514	64.5	1.789	13.15	96	1.483	68.42
10	10	3	67	11.866	74.5	27.55	11.19	87	1.675	29.85
Mean	-		-	1.69	-	3.318	10.98	-	0.561	25.98

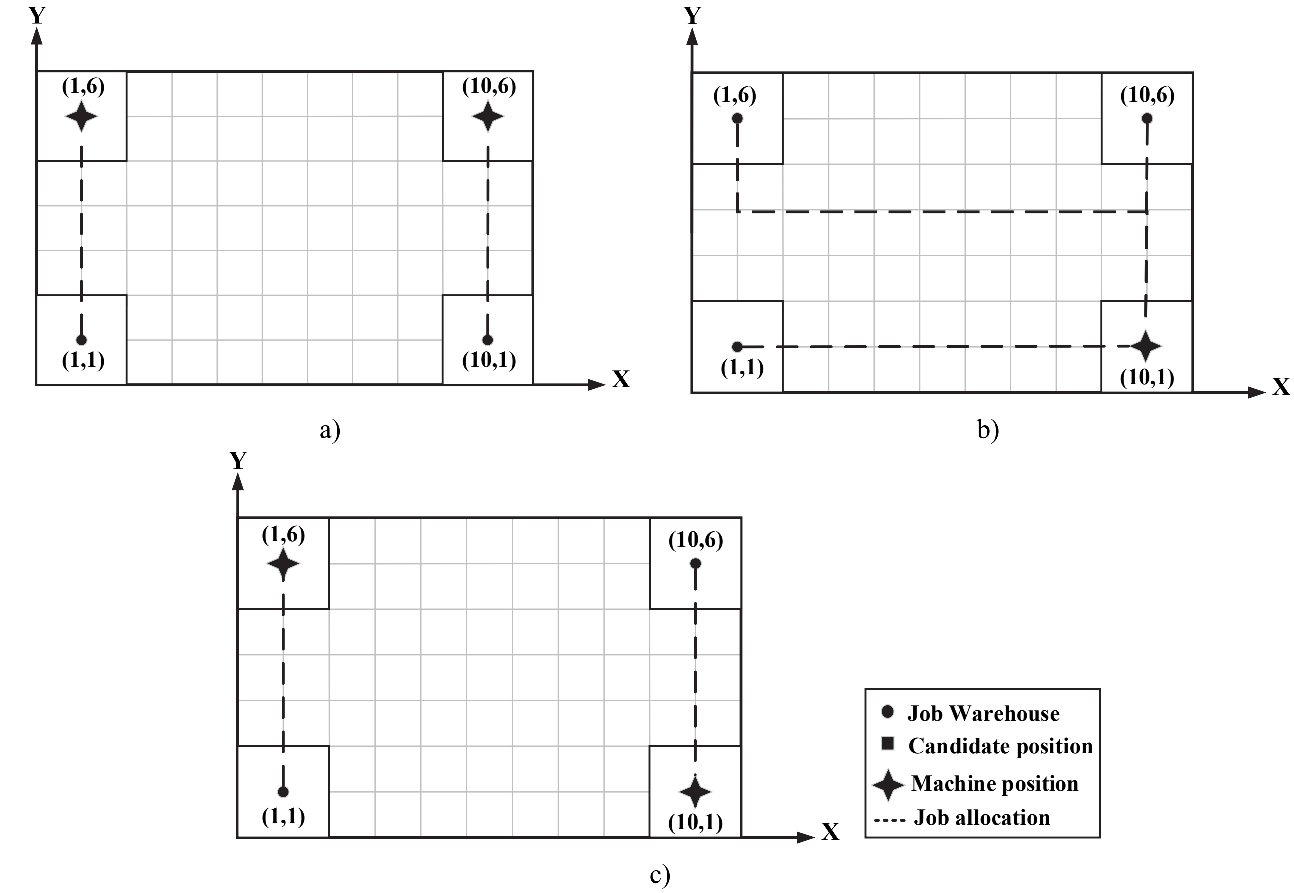

Fig. 4. The locations of the machines and job schedule for continous case

Table 10
The results of the optimal solution for 10 sample test problems

Problem #	Input data		Integrated model		Scenario one			Scenario two		
	# of jobs	Candidate locations	Cmax	time	Cmax	time	Gap	Cmax	time	Gap
1	5	2	63	0.271	63	0.247	0	68	0.232	7.93
2	4	3	18	0.214	21	0.301	16.66	26	0.247	44.44
3	5	3	39	0.187	40	0.354	2.56	40	0.305	2.56
4	6	2	65.66	0.456	73.33	0.576	11.68	71.33	0.458	8.63
5	6	2	51	0.481	51	0.498	0	51	0.384	0
6	6	3	42	0.654	45.5	0.573	8.33	58.5	0.356	39.28
7	7	2	48.33	0.756	51.33	1.103	6.2	59	0.268	22.07
8	8	2	87.25	1.853	89.25	1.785	2.29	104.25	0.787	19.48
9	9	3	79	1.968	80.5	1.565	1.89	86	1.457	8.86
10	10	3	96.48	13.457	97.76	6.312	1.32	102.54	1.965	6.28
Mean	-		-	2.029	-	1.331	5.09	-	0.646	6.28

Again, as we can observe from the results of Table 9, Table 10 and Table 11, the proposed integrated model has been able to solve the randomly generated problems in less amount of time. In most cases, the proposed model has provided better objective function values. In summary, in both cases, there have been some improvement on the performance of the ScheLoc problem using both discrete and continuous spaces.

Table 11
The results of the optimal solution for 10 sample test problems

Problem #	Input data		Integrated model		Scenario one			Scenario two		
	# of jobs	Candidate locations	cmax	time	cmax	Time	Gap	cmax	Time	Gap
1	5	2	24	0.269	24	0.187	0	24	0.247	0
2	4	3	17	0.245	17	0.201	0	17	0.256	0
3	5	3	24.33	0.158	25.66	0.216	5.46	29	0.235	19.19
4	6	2	45	0.354	48	0.354	6.67	51	0.263	13.33
5	6	2	39	0.387	42.33	0.587	8.54	42	0.265	7.69
6	6	3	25	0.745	25.33	0.685	1.32	25.33	0.304	1.32
7	7	2	56.66	0.572	56.66	0.406	0	61.33	0.256	8.24
8	8	2	57	0.987	61	1.485	7.01	68	0.435	19.3
9	9	3	88	1.541	94.5	1.987	7.38	95	1.895	7.95
10	10	3	83.33	10.565	86.66	2.356	3.99	90.66	1.987	8.79
Mean	-		-	1.58	-	0.846	4.03	-	0.614	8.58

4. Conclusion

In this paper, we have presented a mathematical model for ScheLoc problem in discrete and continuous spaces. The proposed study has been formulated under different conditions and the implementations were examined using various randomly generated numbers. The preliminary results have indicated that the proposed study of this paper could provide promising results. As future study, one may consider the problem under uncertain conditions using fuzzy numbers and we leave it as a future study for interested researchers.

Aknowledgement

The authors would like to thank the anonymous referees for constructive comments on earlier version of this paper.

References

Drezner, Z., & Suzuki, A. (2004). The big triangle small triangle method for the solution of nonconvex facility location problems. *Operations Research*,*52*(1), 128-135.

Elvikis, D., Hamacher, H. W., & Kalsch, M. T. (2009). Simultaneous scheduling and location (ScheLoc): the planar ScheLoc makespan problem.*Journal of Scheduling*, *12*(4), 361-374.

Elvikis, D., Hamacher, H. W., & Kalsch, M. T. (2007). *Scheduling and location (ScheLoc): makespan problem with variable release dates*. Technische Universität Kaiserslautern, Fachbereich Mathematik.

Hennes, H., & Hamacher, H. W. (2002). *Integrated scheduling and location models: single machine makespan problems*. Technische Universität Kaiserslautern, Fachbereich Mathematik.

Heßler, C., & Deghdak, K. (2015). *Discrete Parallel Machine Makespan ScheLoc Problem*. Technische Universität Kaiserslautern, Fachbereich Mathematik.

Kalsch, M. T., & Drezner, Z. (2010). Solving scheduling and location problems in the plane simultaneously. *Computers & Operations Research*,*37*(2), 256-264.

Lawler, E. L. (1973). Optimal sequencing of a single machine subject to precedence constraints. *Management science*, *19*(5), 544-546.

Nickel, S., & Puerto, J. (1999). A unified approach to network location problems. *Networks*, *34*(4), 283-290.

Pinedo, M. L. (2012). *Scheduling: theory, algorithms, and systems*. Springer Science & Business Media.

Scholz, D. (2011). *Deterministic global optimization: geometric branch-and-bound methods and their applications* (Vol. 63). Springer Science & Business Media.

Scholz, D. (2012a). Integrated scheduling and location problems. In*Deterministic Global Optimization* (pp. 109-116). Springer New York.

Scholz, D. (2012b). Summary and discussion. In *Deterministic Global Optimization* (pp. 129-132). Springer New York.