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An integer linear programming approach for a location-allocation problem in online stores industry: A real world case study

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A B S T R A C T

As the population grows and demand increases, cities have seen a rise in the number of chain stores. To remain competitive, these companies must reduce costs and attract more customers. A key factor in achieving this is the strategic placement of store branches, which reduces the distance between stores and customers, instilling trust and increasing their appeal while also cutting costs by reducing the need for employees to navigate longer distances. In this study, an integer linear programming model is presented with the goal of dividing a zone in Ahvaz city into several scenarios to determine the optimal number of stores while maintaining control over the distance between active stores. This research is the first to include this specific limitation in the mathematical model of the problem. The results of the study demonstrate a significant reduction in the distance between customers and stores.

1. Introduction

Chain stores are a group of stores with centralized management that offer similar services or products to consumers. Currently, there is a significant increase in the number of chain stores in our country. With rising competition in the market, stores are continuously striving to increase their profit margins by cutting costs and ensuring customer satisfaction (Fooeik et al., 2022; Ghanbari et al., 2022). One of the primary challenges that chain stores face is the strategic placement of their branches to reduce the distance between stores and customers. This approach fosters trust in customers and lowers costs by minimizing the need for staff to navigate longer distances. Since Cooper (1963) introduced the location-allocation problem, it has been widely applied in many fields, such as supply chain design, traffic network design, facility layout problems, telecommunication networks, emergency services, healthcare systems, and the construction of various public facilities. Location–allocation problems that make use of hubs for shipping goods from origins to destinations is a crucial issue in logistics systems design due to its wide applications in real-life problems and it has been studied many times in the literature. Logendran and Terrell (1988) developed an uncapacitated plant location-allocation problem with price sensitive stochastic demands to maximize the expected net profits. Sherali and Rizzo (1991) studied the capacitated location-allocation problem with a continuum of demand on a chain graph. Recherche et al. (1995) modeled the location-allocation problem where the locations of both customers and facilities may be regions with some probability distributions inside each region. Zhou (2000) proposed the expected value model, chance-constrained programming, and dependent-chance programming for uncapacitated location-allocation problem with stochastic demands. Wang et al. (2002) studied a facility location problem with stochastic customer demand and immobile servers using three heuristic algorithms. Akella et al. (2005) formulated a mixed-integer programming technique to decide base station location and the allocation of channels. Four different greedy heuristics were proposed to obtain high quality solutions efficiently and a Lagrangean heuristic is built to improve the optimality gap. Adler et al. (2014) formulated the location allocation problem using four integer linear programs to jointly solve the location and allocation of routine traffic police patrol vehicles. Ren and Awasthi (2015) addressed the multi-objective problem of capacitated location allocation and developed solution approaches based on four metaheuristics,

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namely genetic algorithms, simulated annealing, tabu search, and ant colony optimization. Hajipour et al. (2016) introduced queuing theory to the study of location allocation for congested facilities, which can be used to support public health. Fathollahi Fard and Hajaghaei-Keshteli (2018) developed a tri-level location allocation model and proposed a nested metaheuristic approach to design the supply chain network that includes distribution centers, customer zones, and recovery centers. Praneetpholkrang et al. (2021) proposed a multi-objective optimization model for shelter location-allocation in response to humanitarian relief logistics.

This research aims to use a real case study with assumptions that closely resemble reality. These assumptions include considering the maximum distance between the customer and the store, assigning each customer to only one store, and limiting the distance between the built stores, which is a new approach proposed in this research. The study uses an integer linear programming model for location and divides a zone in Ahvaz city into different scenarios for the appropriate number of stores. The research is structured in a reverse manner, starting with a detailed description of the problem in the second part, followed by the presentation of the model and mathematical relationships in the third part. The case study is described in the fourth section, while the fifth section presents the numerical results. Finally, the conclusion and future suggestions are presented in the sixth section.

2. Problem Statement

Chain stores have a specific strategy and purpose of providing goods with high circulation in the city. Thanks to their strong relationships with suppliers and well-equipped warehouses, they can offer unique discounts to customers. The optimization of logistics between suppliers, store branches, and end customers plays a crucial role in this industry. Without effective management to reduce costs, companies risk bankruptcy and leaving the market.

One effective way to reduce costs is to employ a "dispersion" strategy in the distribution of branches. By minimizing the distance between customers and stores, and potentially utilizing a third-party logistics (3PL) company for order fulfillment, delivery times can be decreased. In this study, the supply chain network illustrated in Figure (1) is composed of two main components: store branches and customers.

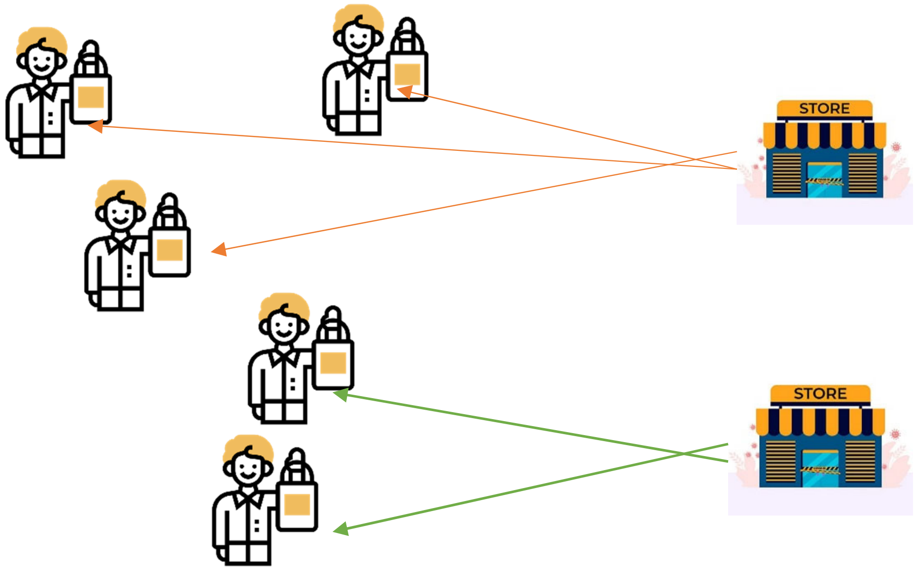


Fig. 1. General scheme of the desired supply chain network

The first step of this study is to determine the optimal candidate points for store locations, followed by assigning customers to the stores based on the following assumptions:

- Each customer can only be assigned to a store located within a maximum radius of 5000 meters.
- Each customer can only make purchases from a single store.
- The distance between any two stores must be at least 1000 meters.
- To streamline the calculations, only customers with at least 5 purchase records have been included in the analysis.

3. Mathematical Model

Sets

S	Set of candidate locations for stores.
S'	Set of existing stores.
C	Set of customers.

Parameters

d_{cs}	Distance between candidate location $s \in S$ and customer $c \in C$
$p_{ss'}$	Binary parameter that equals zero if the distance between candidate location $s \in S$ and existing location $s \in S'$ is less than 2 kilometers, and one otherwise
α	Minimum number of stores required to be built
β	Maximum number of stores that can be built
θ	Percentage of customer coverage

Variables

Y_s	A binary variable that equals one if a store is built at candidate point $\in S$, and zero otherwise.
X_{cs}	Binary variable that equals one if customer $c \in C$ is assigned to store $s \in S$, and zero otherwise.

Mathematical Equations

$$min\ Z = \sum_{s} \sum_{c} d_{cs}\, X_{cs} \tag{1}$$

$$d_{cs}\, X_{cs} \ \leq 5000\, Y_s \qquad \forall c \in C, s \in S \tag{2}$$

$$\sum_{s} X_{cs} \leq 1 \qquad \forall c \in C \tag{3}$$

$$\sum_{s} Y_s \ \geq \alpha \tag{4}$$

$$\sum_{s} Y_s \ \leq \beta \tag{5}$$

$$\sum_{c} \sum_{s} X_{cs} \ \geq \theta * |C| \tag{6}$$

$$\sum_{c} X_{cs} \ \geq \ Y_s \qquad \forall\, s \in S \tag{7}$$

$$Y_{s'} \leq \ p_{ss'}\, Y_s + (1 - Y_s) \ \forall s, s' , s \neq s', s' > s \tag{8}$$

$$Y_s \in \{0,1\} \qquad \forall s, s' \tag{9}$$

$$X_{cs} \in \{0,1\} \qquad \forall c \in C, s \in S \tag{10}$$

The proposed model aims to minimize the cost of assigning customers to stores, subject to a set of constraints. These constraints are defined by a set of equations as follows: The objective function (1) tends to minimize the total cost of assigning customers to stores. constraints (2) guarantee that A customer $c \in C$ can be assigned to a store $s \in S$ only if the distance between them is no more than 5000 meters. Constraints (3) guarantee that ensures that each customer $c \in C$ can be assigned to at most one store. Constraints (4) and (5) define the minimum and maximum number of stores that can be built. Constraints (6) guarantee that ensures that at least θ percent of customers must be covered. Constraints (7) guarantee that ensures that if a store $s \in S$ is built, at least one customer must be covered. Constraints (8) guarantee that Ensures that a store $s \in S$ must be at least 2 kilometers away from other stores if it is built. Constraints (9) and (10) guarantee that indicate the type of variables used.

4. Case Study

"Kala Resane Hasti" is a startup operating in Iran and a subsidiary of the Golrang Industrial Group. Their aim is to become one of the leading logistics companies in the country by creating a balanced flow of value for their customers and stakeholders. With over 3000 employees, the company provides support for the logistics operations of major retail chains such as "Ofoq Kourosh Chain Stores", "Akala", "Hyper Family", and others in Iran. As Ahvaz is one of the most populous cities in Iran, the company is following a distribution strategy for its stores in the region. In line with this, they are seeking to construct appropriate stores in the designated zone. Fig. 2 illustrates the customer base of this zone, while Fig. 3 shows potential locations for building the stores.

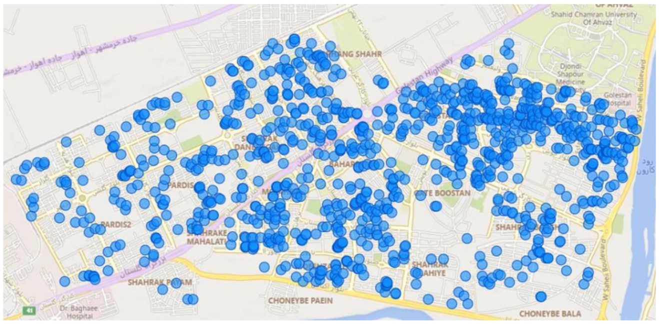


Fig. 2. Distribution of customers with at least 5 orders

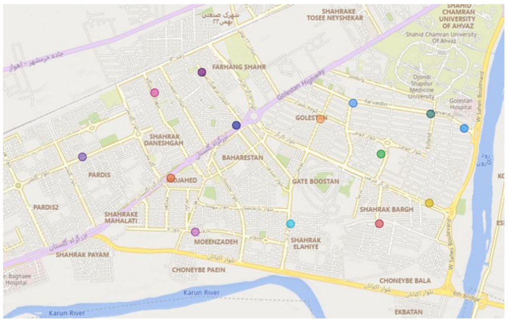


Fig. 3. Candidate points for building a store

5. Numerical Results

As determining the ideal number of stores within the decision-making zone involves considering multiple factors, four different scenarios have been outlined for the number of stores, and the outcomes of each are elaborated upon below.

Scenario 1: Choosing 3 stores

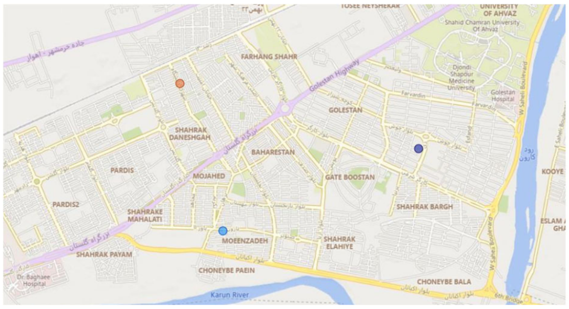


Fig. 4. Location of the 3 selected stores for building

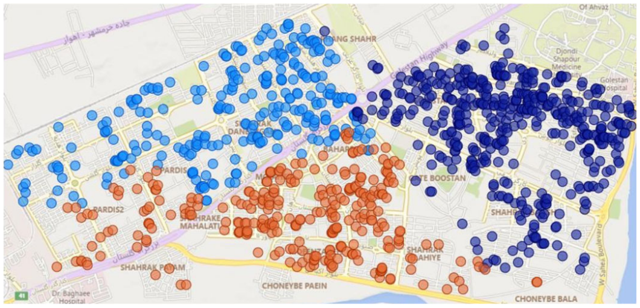
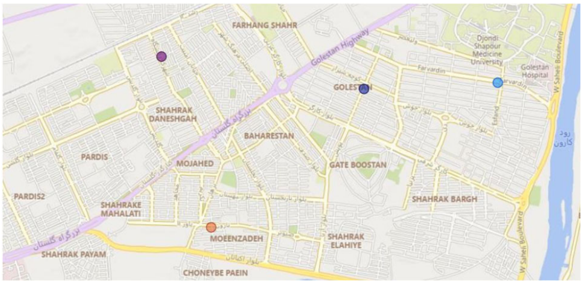


Fig. 5. Coverage of customers by the 3 built stores

Scenario 2: Choosing 4 stores



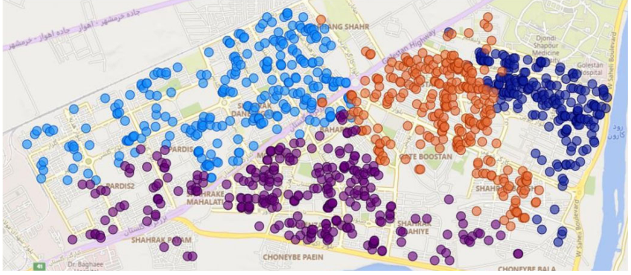
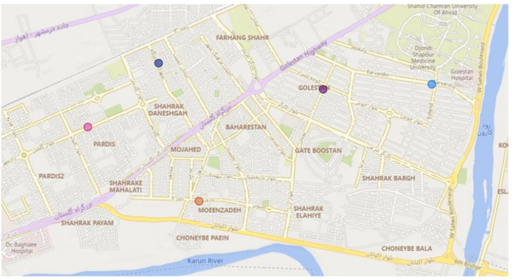


Fig. 6. Location of the 4 selected stores for building

Fig. 7. Coverage of customers by the 4 built stores

Scenario 3: Choosing 5 stores



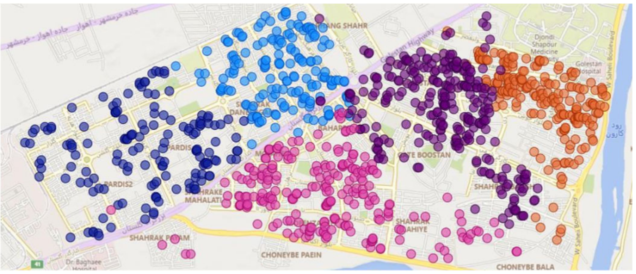
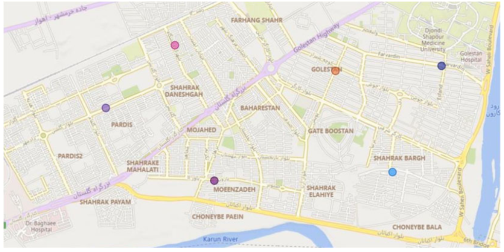


Fig. 8. Location of the 5 selected stores for building

Fig. 9. Coverage of customers by the 5 built stores

Scenario 4: Choosing 6 stores



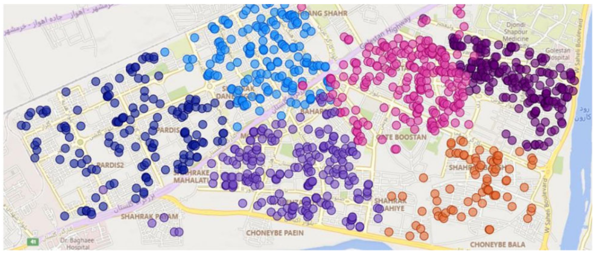


Fig. 10. Location of the 6 selected stores for building

Fig. 11. Coverage of customers by the 6 built stores

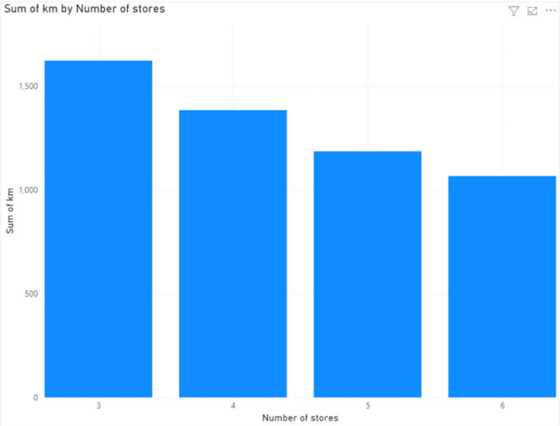


Fig. 12. Total traveled distance in kilometers for each store in different scenarios

The figure below displays the locations of the three stores currently in use within this zone:

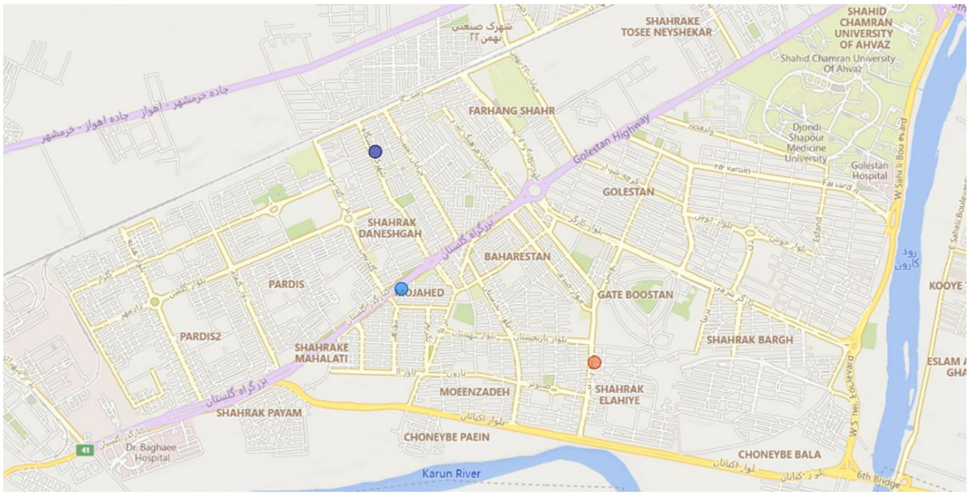


Fig. 13. Current status of stores used in the city

Based on the given relationships, the current average distance between stores and customers within the zone is 2400 meters. If any of the scenarios are implemented, the corresponding reduction in the average distance between stores and customers can be seen in Fig. 14:

Fig. 14. The value of the average distance reduction between stores and customers for each scenario.

6. Conclusion

In the current competitive market, chain stores are constantly seeking ways to reduce costs and enhance customer satisfaction to increase their profit margins and sustain their business. One of the most pressing challenges for these stores is determining the optimal locations for their branches to minimize the distance between stores and customers. In this study, an integer linear programming model was developed to allocate store locations in a zone of Ahvaz city. To make the model more realistic, assumptions were added, such as the maximum allowable distance between a customer and a store and the allocation of each customer to a single store. The model also included a constraint on the minimum distance between activated stores, which has not been previously explored in this type of research. Different scenarios were defined in this study for varying numbers of store branches, and the results were analyzed and categorized based on the mathematical model. The results of the model indicate an improvement in the current situation and provide useful insights for decision-making regarding the optimal placement of stores. As a recommendation for future research, incorporating the demand level in a non-deterministic manner and considering the number of required delivery personnel for each activated store could enhance the stability and accuracy of the presented results.

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