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New procedures with new activity assumptions for solving resource constrained project scheduling problems

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ABSTRACT

The resource-constrained project scheduling problem (RCPSP) is a well-known and widely studied topic. The underlying problem assumes that non-preemptions and that constant resources are restrictions imposed on project activities, which are to be scheduled, subject to precedence relation and limited resource constraints. Project activities, in RCPSP, are classified under category A. The problem is expanded to include various other activity assumptions categories, such as B and C. In the Preemptive-RCPSP, project activities are classified under category B, which refers to the activity that can be implemented using constant resources and constant durations. In the Flexible-RCPSP, project activities are classified under category C, which refers to the activities that can be executed using flexible resources over flexible durations, and preemptions are not allowed. However, in One-of-a-Kind Production companies (OKP), such as the housing industry, plastic injection moldings, and RV manufacturing, all known as "manufactured-to-order" operations, the activities are classified under category D in addition to A, B, and C, simultaneously. Category D refers to the activities that can be executed using flexible durations and flexible resources, and preemptions are allowed. In this paper, therefore, we present a new effective model in order to deal with the projects that consist of all the previous activity assumptions simultaneously to generate feasible project schedules. Case studies are included, and the results show that the resources usage is increased and the project makespan is reduced.

1. Introduction

In the context of Project Scheduling (PS), one of the most widely studied problems is the Resource-Constrained Project Scheduling Problem (RCPSP). In an Activity-on-Node (AON) format, N is a set of nodes used to represent the n activities, and a set of pairs of activities A represents the precedence relations between activities (i.e., finish to start relations with a minimal time-lag of zero). In the RCPSP, activities are assumed to have constant durations, constant resources, and preemptions are not allowed. The decision variables are the starting of activities times when the resource availabilities are considered as given. The activities can be performed in only one possible execution mode, and the resources are assumed to be available in a constant amount for each time.

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Moreover, each activity demands a constant amount of resources during the execution. The objective of the RCPSP is to obtain a feasible schedule that meets the constraints in a way so as to minimize project makespan. The RCPSP, the subject of much attention, have been well-documented. Kolisch and Hartmann 1999, 2006; Hartmann and Kolisch 2000; Hartmann and Briskorn 2010; Zhang, Li, and Tam 2006; and Fang and Wang 2012 presented many works of literature used the exact method and heuristics method to solve the RCPSP.

In this paper, we classify the problem to four types based on the activity categories as follows. First, when project activities are classified under category A, (i.e., activities can be executed using constant resource over constant duration, and cannot be interrupted, as depicted in Fig. 1) the problem so-called "Resource-Constrained Project Scheduling Problem" (RCPSP or $RCPSP_{(A)}$). Second, when project activities under category B, (i.e., activities can be executed using the same character of resource and duration as A, but activities can be interrupted, as displayed in Fig. 2), the problem so-called "Preemptive Resource-Constrained Project Scheduling Problem" (P-RCPSP or $RCPSP_{(B)}$). Third, when project activities are classified under category C (i.e., activities can be executed using flexible resource over flexible duration and cannot be interrupted, as shown in Fig. 3) the problem so-called "Flexible Resource Constrained Project Scheduling Problem" (F-RCPSP or $RCPSP_{(C)}$). Fourth, when project activities are classified under category D (i.e., activities have the same character of resource and duration as C, but they can be interrupted, as depicted in Fig. 4) the problem so-called "Flexible Preemptive Resource -Constrained Project Scheduling Problem" (F-P-RCPSP or $RCPSP_{(D)}$).

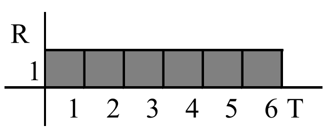

Fig. 1. Activity under category A ($RCPSP_{(A)}$).

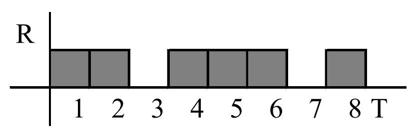

Fig. 2. Activity under category B ($RCPSP_{(B)}$)

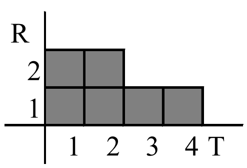

Fig. 3. activity under category C ($RCPSP_{(C)}$).

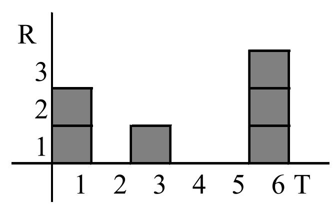

Fig. 4. Activity under category D ($RCPSP_{(D)}$)

Table 1 below summarizes the important activity assumptions implemented in different types of RCPSP. In the RCPSP, project activities are classified under category A. In the Preemptive Resource-Constrained Project Scheduling Problems (P-RCBSP), project activities are categorized under category B. In the Flexible Resource-Constrained Project Scheduling Problems (F-RCPSP) project activities under category C. In the P-F-RCPSP, when the project activities can be preemptable, and resources can be flexible, project activities are classified under category D. Equally important, in most software packages, the activities under category A has been considered as inputs. Whereas category B and C assumptions have never been considered, and if any, they can be made by the user before creating a feasible resource schedule. Activities such as welding activity, cutting activity, or assembly activity can be accelerated its execution by increasing the resources. To the best of our knowledge, these types of activities are classified under category B or C in most of the previous literature, if not all.

Table 1
The activity assumptions implemented in different types of basic RCPSPs

General	Activity Categories	Constant Duration	Constant Rresource	Flexible Resources	Interruption	State
RCPSP	A	✓	✓	x	x	Used
P-RCPSP	B	✓	✓	x	✓	Used
F-RCPSP	C	x	x	✓	x	Used
P-F-RCPSP	D	x	x	✓	✓	New

However, the problem is: researchers have classified the project activities individually under categories A, B, or C (e.g., Peteghem & Vanhoucke 2010; Kellenbrink & Helber 2015). But in practice, the F-RCPSP can be enforced in the P-F-RCPSP as a special case by allowing the activity to be preempted. Put another way, category C can be covered by category D. As a result, from a preemptive perspective, we simultaneously represent the problem related to the three types of activity assumptions, A, B, and D. To sum up, we identified the extensions of RCPSP as Resource-Constrained Project Scheduling Problem under A, B, and C activity assumptions ($RCPSP_{(ABD)}$) . The contributions of this paper are as follows: 1) Presenting a new algorithm to solve general and real cases of RCPSP when project activities are considered under (A, B, and D) simultaneously. 2) generating several schedules for various projects, which modified from PSPLIB, and measuring the impact of the activity assumptions on the project duration, resource utilization, and the percentage of the project duration improvement.

2. Literetures review

Besides no paper has handled hundreds of activities in reasonable computation time; all the literature described activities less than one hundred (Peteghem & Vanhoucke 2014). Accordingly, the previous literature has only dealt with projects having activities under category A, B, or C, each treated individually. Peteghem and Vanhoucke (2010) introduce a Genetic algorithm (GA) for solving the MRCPSP and Preemptive Multi-Resource Constrained Project Scheduling Problems (P-MRCPSPs). The MRCPSP is a generalized version of RCPSP, where each activity can be executed in one out of a set of modes, which allows the project activities to be under category B. Fundeling and Trautmann (2010) have considered a Project Scheduling Problem (PSP) in which the activities are characterized by work-content (PSPWC). That is, the resources allocated to an activity usually may vary over time subject to some restrictions. This means that the project activities are classified under category C. Ranjbar and Kianfar (2010) proposed a procedure to find all feasible work profile for each activity and used GA with a new crossover operator to schedule the project activities, the activities can not be preempted during the execution (i.e., activities are classified under category C). Bianco and Caramia (2013) proposed a new formulation for RCPSP with finish-to-start constraints, pre-emption is not allowed, scarce resources and minimum makespan objective. Project activities in this paper considered under category A. Colak et al. (2013) consider the Multi-Mode Resource Constrained Project Scheduling Problem with Renewable Resources (MRCPSP-RR), where each activity can be executed in one of the possible modes, i.e., different durations and different resources. Minimum Latest Start Time (Min-LST), Shortest Feasible Mode with Conditional Wait for the Fastest Mode (SFM-CWFM), and Shortest Feasible Mode with Conditional Wait for the Better Mode (SFM-CWBM) are heuristics, which do not use in MRCPSP-RR before for activity selection. The activities are considered under category A. Baumann and Trautmann (2013) formulated the RCPSP as a Mixed Linear Program (MLP) for small instances, and the activities have been considered under category C. Naber and Kolisch (2014) proposed four model formulations for the F-RCPSP and compared their model efficiency in terms of solution quality and computational times. Peteghem and Vanhoucke (2014) present an overview of the existing meta-heuristic for solving MRCPSP. The MRCPSP aims to find a mode and a start time for each activity to schedule the project within the minimal makespan. The research paper considers only renewable resources, and the problem has been referred to as MRCPSP/R. All the activities in this paper are under category A. Cheng et al. (2015) illustrate the difference between the preemption and activity splitting in the RCPSP as follows: P1 represents the RCPSPs without activity splitting, P2 represents the RCPSPs with non-preemptive activity splitting, and P3 represents the P-RCPSP. In this paper, project activities considered under Category B. Ma et al. (2016) address the Uncertain Resource-Constrained Project Scheduling Problem (URCPSP). The start and finish times and resource usage in most literature about the RCPSP are given in advanced for each activity. This implies the activities are under category A. Issa and Tu (2017) develop the branch and bound (B&B) heuristic to solve the RCPSP. They use the splitting activity as a way to cut down the project makespan. The activities are classified under category B. Elsayed et al. (2017) present a Consolidated Optimization algorithm (COA) which has more one optimization algorithm, each of which uses two multi-operator algorithms (MOAs) to solve the RCPSP. The activities in this paper are under category A. Oztemel and Selam (2017) use a new meta-heuristic to select an effective single mode for MRCPSP. Bee Colony Optimization (BCO) approach has been used to complete the project on time. the activities are considered under category A. Naber (2017) proposes a MIP model that uses the continuous-time system to synchronize resources and activities where each activity may start, end, or change its resource allocation at any point of time. Tritschler et al. (2017) propose a Hybrid Meta-heuristic (HM) by transferring resource quantities between selected activities as a way to improve project schedules in a variable neighborhood search. Afshar-Nadjafi (2018) extends the MRCPSP to the Preemptive Multi-mode Resource Constrained Project Scheduling Problem with permitted Mode Change (P-MRCPSP-MC) after preemption. This model is not considered in the past literature. Fixed work content is given for each project activities instead of a fixed duration and known resource requirements. Renewable and non-renewable resource types have been used in the problem. The accomplishing time of an activity can be interrupted at discrete time instances and restarted later with the same or different mode. The activities are considered under category B. Tao et al. (2018) propose an extension of MRCPSP when the project network can be selected according to specific rules. The project does not have a fixed network diagram for its execution. In real-world applications, project structure is variant and how to choose project structure is a significant decision for the scheduling problem. Project activities in this paper are under Category A. Vanhoucke and Coelho (2018) present an overview of the state-of-art algorithms for RCPSP and MRCPSP. The paper aims at demonstrating that most algorithms are still not able to solve instances much bigger in size than the ones presented between (1995-2017) or cannot solve problems with a different network and/or resource structure than usually used in the academic literature. The main goal of the paper is to provide a way to present best solutions obtained from the best performing procedure in literature and to set up a system for uploading solutions for alternative project data like PSPLIB and MMLIB uploading system. Project activities are considered under category A. Table 2 represents the glossary of symbols used in the present published papers, and Table 3 highlights the classification of project activities under different types of activity categories:

Table 2
Glossary of symbols

ACE-SP	Agarwal, Colak, and Erenguc-Single Pass.
B&B	Branch and Bound
BCO	Bee Colony Optimization
BPGA	Bi-Population Genetic Algorithm.
COA	Consolidated Optimization Algorithm
FRCPSP	Flexible-Resource-constrained Project Scheduling Problem
GA	Genetic algorithm
HM-GA-VNS	hybrid meta-heuristic with Genetic Algorithm combined with a variable neighborhood search
MILP	Mixed Integer Linear Program
MRCPSP	Multi-mode Resource-constrained Project Scheduling Problem.
MRCPSP-APS	Multi-Resource-Constrained Project Scheduling Problem with Alternative Project Structure
MRCPSP-RR	Multi-mode Resource Constrained Project Scheduling Problem with Renewable Resource.
PR	Priority Rules
P-MRCPSP	Preemptive-Multi-mode Resource-constrained Project Scheduling Problem.
P-MRCPSP-MC	Preemptive Multi-mode Resource Constrained Project Scheduling Problem with permitted Mode Change
PSPWC	Project Scheduling Problem Work-Content
RCPSP-FWP	Resource-Constrained Project Scheduling Problem- Flexible Work Profile
SA	Simulated Annealing
URCPSP	Uncertain Resource-Constrained Project Scheduling Problem
TS	Tabu Search.

The project scheduling problem addressed in this paper is extended to cover a much fuller range of engineering project requirements, and it then gives project managers more flexibility for planning and scheduling projects. However, for all these research papers, the classification of projects' activities to A, B, and D was not mentioned nor was not dealt with previously. The remainder of this paper is organized as follows. Section 3 addresses the problem description. Section 4 illustrates the proposed module. Section 5 presents a numerical example. Section 6 provides the computational results. Section 7 gives a conclusion.

Table 3
A summary about the RCPSP and RCMPSP for the papers mentioned in this paper (2010-2018)

	Author	Year	Type of the prob.	Method	Dataset	A	B	C	D
1	Peteghem and Vanhoucke	2010	MRCPSP and P-MRCPSP	Meta-heuristic BPGA	PSPLIB Boctor	√	√		
2	Fundeling and Trautmann	2010	PSPWC	Heuristic PR	Modified PSPLIB			√	
3	Ranjbar and Kianfar	2010	RCPSP-FWP	GA	PSPLIB			√	
4	Bianco and Caramia	2013	RCPSP	Exact method	PSPLIB	√			
5	Colak et al.	2013	MRCPSP-RR	Heuristic ACE-SP and meta-heuristic	PSPLIB Boctor	√			
6	Baumann and Trautmann	2013	FRCPSP	MILP	PSPLIB			√	
7	Naber and Kolisch	2014	FRCPSP	MILP	PSPLIB			√	
8	Peteghem and Vanhoucke	2014	An over view for MRCPSP	Existing Meta-heuristic	PSPLIB Boctor	√			
9	Cheng et al.	2015	(P1-P2-P3) RCPSP	Exact (B&B) meth. Heuristics-based PR	Modified PSPLIB	√	√		
10	Ma et al.	2016	URCPSP	Meta-heuristic GA	Modified PSPLIB	√			
11	Issa and Tu	2017	RCPSP	Exact-method B&B	Own		√		
12	Elsayed	2017	RCPSP	COA	PSPLIB	√			
13	Oztemel and Selam	2017	MRCPSP	Meta-heuristic BCO	Own	√			
14	Naber	2017	F-RCPSP	MILP	PSPLIB			√	
15	Tritschler e.t al.	2017	F-RCPSP	HM-GA-VNS	PSPLIB			√	
16	Nadjafi	2018	P-MRCPSP-MC	Meta-heuristic SA	ProGen/πx	√	√		
17	Tao and Dong	2018	MRCPSP-APS	Meta-heuristic TS	PSPLIB	√			
18	Vanhoucke and Coelho	2018	RCPSP-MRCPSP	-	New Datasets	√			

3. Problem description

The $\text{RCPSP}_{(\text{ABD})}$ can be described as follows: a project consists of a set of activities i = [1, 2, …., N]. The activities are subject to two types of constraints: 1) The precedence constraint, which forces each successor activity to be scheduled after all its predecessor activities are completed; and 2) The limited amount of resources is available during the activities performed. $K = (1,...,k)$, is a set of renewable resource types assigned to activities. Each activity under categories A and B, $i_{(a,b)}$, requires constant units of renewable resource, $r(i_{(a,b)}, k)$, type $k \in K$ during the non-preemptable duration, d_{ia} , or during the preemptable duration, d_{ib} . Each activity under category D, $i_{(d)}$, requires work content units, ω_d , of renewable resource type $k \in K$ during its preemptable duration, d_{i_d} . Resource type $k \in K$ has limited availability of R_k at any point along the planning horizon.

The objective of the $\text{RCPSP}_{(\text{ABD})}$ is to determine the start and finish times of the project activities, which are classified under A, B, and D categories, subject to scarce resources and precedence relationships to minimize the project makespan. A new algorithm, coded by MATLAB, employs as solving-tool to handling the problem, where many assumptions must be taken into the schedulers' account when he needs to use the model:

1- The duration of activities under category A and B must be pre-determined.
2- The activities under category A cannot be interrupted.
3- The activities under category B can be interrupted.
4- The work content of the activities under category D must be pre-determined and can be interrupted.
5- For each pre-emptive activity, no additional costs required to re-start performing them on later.
6- The resources assigned to each activity are considered as renewable resources.
7- An activity cannot start until all its predecessor activities are finished.
8- The objective is to minimize project makespan.

In practice, “manufactured-to-order” projects are generally named a one-of-a-kind project (OKP), which aims at producing highly customized projects at nearly mass production efficiency (Tu & Dean 2011). The project manager in OKP needs to deal with project activities classified under A, B, and D activity assumptions simultaneously.

4. Mathematical model

Many exact methods, heuristics, and meta-heuristics have been proposed for solving RCPSP under A, B, or C categories individually. However, the $\text{RCPSP}_{(\text{ABD})}$ has never been studied or handled previously. The mathematical model proposed in this paper employs the following assumptions and notations:

- Project activities can be classified under A, B, and D categories.
- Activities under category A can be executed using constant resources over constant durations and cannot be interrupted through the X-axis or the Time-axes.
- Activities under category B can be implemented using constant resources over constant durations and can be interrupted through the X-axis or the T-axis.
- Activities under category D can be executed using flexible resources over flexible durations and can be interrupted through the X-axes or the T-axis.
- The model is presented in the Activity-On-Node (AON) format.
- Resources are renewable and have limited capacities.
- Rescheduling activities, from time to time, is allowed due to uncertainties in activity under category D.

3.1. Inputs

i number of project activities $i \in P$

i_a activities under category A $i_a \in i \quad \forall i \in P$

i_b activities under category B $i_b \in i \quad \forall i \in P$

i_d activities under category D $i_d \in i \quad \forall i \in P$

d_{i_a} durations of activities under category A

d_{i_b} durations of activities under category B

$r_{i_{(a,b)},K}$ renewable resources type k to execute activities under A and B categories $k \in K$

R_{req} total resource required

R_{krem} resource remaining

ω_{i_d} work content

t time slots

3.2. Parameters

R_k amount of available type k resources

$r_{i_{(a,b)},K}$ resources required to execute activity under A and B categories

ω_{i_d} work-content to execute the activity under category D

$d_{i_{(a,b)}}$ duration for activity under A and B categories

$s_{i_{(a,b.d)}}$ The earliest start time for each activity i

$f_{i_{(a,b,d)}}$ The earliest finish time for each activity i

T time horizon planning

P portion of work content

3.3. Binary variables

$$s_{it} = \begin{cases} 1; \text{ if activity } i \text{ is started at time instant t} \\ 0; \textit{otherwise} \end{cases}$$

$$f_{it} = \begin{cases} 1; \text{ if activity } i \text{ is finished at time instant } t \\ 0; \text{ otherwise} \end{cases}$$

$$Y_{it} = \begin{cases} 1; \text{ if the categories A \& B are covered by protion of the } \mathrm{R_k}. \\ 0; \text{ if the categories A \& B are covered by the total of the } \mathrm{R_k}. \end{cases}$$

The objective function:

$$\text{Min } \sum_{i=1}^{n} f_{i+1} \tag{1}$$

Subject to

$$s_i + d_i \le s_j \quad \forall (i, j) \in A \tag{2}$$

$$\sum_{i_{(a)} \in s_t} r_{i_{(a)}k} \le R_k \quad \forall i_{(a)} \in A \tag{3}$$

$$\sum_{i_{(b)} \in s_t} r_{i_{(b)}k} \le R_k \quad \forall i_{(b)} \in A \tag{4}$$

$$\sum_{t=s_{i_{(d)}}}^{f_{i_{(d)}}-1} r_{i_{(d)}t} = \omega_{i_{(d)}} \quad \forall i_{(d)} \in A \tag{5}$$

$$\sum_{t=s_{i_{(d)}}}^{f_{i_{(d)}}-1} r_{i_{(d)}t} \le R_k \quad \forall i_{(d)} \in A \tag{6}$$

$$\sum_{i_{(a)} \in s_t} r_{i_{(a)}k} + \sum_{i_{(b)} \in s_t} r_{i_{(b)}k} + \sum_{t=s_{i_{(d)}}}^{f_{i_{(d)}}-1} r_{i_{(d)}t} \le R_k \tag{7}$$

$$s_i > 0 \tag{8}$$

Objective function (1) minimizes the total project's makespan. Constraint sets (2) takes the finish-start precedence relations with a minimal time lag of zero into account. Constraint set (3), (4), and (6) take care of the renewable resource limitation for activities under A, B, and D categories. Constraint (5) defines the work content for each activity under category D. Constraint set (7) ensures that the summation of the resources needed to execute activities under Categories A, B, and D simultaneously must be $\le R_k$. Constraint (8) forces the project to start at time instance zero.

In this section, we illustrate a new solution procedure for the $\mathrm{RCPSP_{(ABD)}}$ with scarce resources, finish to start constraints, and minimum makespan objective at any given time as follows:

1. If the total resource required (R_{req}) is less than the available resource (R_k), the available resource needs to be specified in order to complete the project activities.
2. If the activities under A, B, and D are brought together, and if these three need to be executed simultaneously, and if the total resource required (R_{req}) are more than the available resource (R_k) then the following two sub-loops are executed:
   2.1. Assign the available resource (R_k) to the project activities under category A, and calculate the resource remaining (R_{krem}), utilizing:

$$R_{krem} = R_k - r(i_a, k) \tag{11}$$

2.2.Assign the resource remaining (R_{krem}) to the project activities under category B, calculate the new resource remaining (R'_{krem}), and assign the new resource remaining (R'_{krem}) to cover a segment of the work content of the activities under category D. This done by:

$$R'_{krem} = R_{krem} - r(i_b, k) \tag{12}$$

$$R''_{krem} = P(\omega_{i_d}) \tag{13}$$

3. If project activities under categories A and B need to be executed simultaneously, and if ($R_{req} > R_k$), then the available resources (R_k) must first be allocated to project activities under category A, and the project activities under category B must then be delayed to (t+1).
4. If project activities that need to be executed are under categories (A and D) or (B and D) simultaneously, and if ($R_{req} > R_k$), then the available resources (R_k) must first be allocated to project activities under category A or category B. Then, secondly, a segment of project activities under category D must be covered using the resources remaining (R_{krem}). Finally, shift the rest of the work content of the activities to $(t+1)$. These two equations explained this

$$R_k < r_{i_{(a\ or\ b)},k} + \omega_{i_d} \qquad R_k = r_{i_{(a\ or\ b)},k} + P(\omega_{i_d}) \tag{15}$$

With these results in hand, we can check the resources required to perform the project activities in $(t+1)$ and repeat steps 2 through 4 until all activities in the projects are scheduled.

The concept of Project Management (PM) is the method or technique to complete the project on time. The pre-emption is a way to generate and improve a project schedule that faces the scarce resources assignment on activities over the project duration. Project activities have been assumed to be preemptive in the following papers: (Demeulemeester & Herrolen 1996; Nudtasomboon & Randhawa 1997; Valls et al. (1999), Bianco et al., 1999; Brucker & Knust 2001; Buddhakulsomsiria & Kim 2006, 2007; Damay 2007; and Peteghem & Vanhoucke 2010). Besides the difficulty of solving combinatorial optimization problems, the uncertainty, the utilization of scarce resources, and the changes in activities and time durations are the main problems with the scheduling processes. In this research, the problem becomes more much complicated because activities are classified under A, B, and D categories.

Our model-proposed handles scheduling projects, no longer through A, B or C category individually, but through the category A, B, and D simultaneously, where the problems fundamental have been extended to RCPSP problem to $\mathrm{RCPSP}_{(ABD)}$.

Three priority rules are used for activity selection when the conflicts occur; first, the Earliest Start Time (ES); second, the Latest Finish Time (LF); and the Slack Time (SL). These limits, ES, LF, and SL are determined using the traditional forward and backward pass calculations. The backward pass calculation is started from the fixed project makespan, which means that the earliest finish time of the dummy end activity, EF_n , is considered as a project makespan and must equal the LF_n. EF_n is computed using the traditional forward pass calculation. The SL can be founded from ($\mathrm{LF}-\mathrm{EF}$).

5. Numerical example

In this section, we consider a project consists of 20 activities and three renewable resources. Information of the numerical instance including predecessor activities, durations, and resource utilization are presented in Table 4.

Table 4
The properties of the project

Act	Pre.	D	ES	EF	LS	LF	SL	R1	R2	R3
1	-	0	0	0	0	0	0	0	0	0
2	1	2	0	6	0	6	0	5	6	2
3	1	3	0	3	6	9	6	3	5	2
4	2,7	4	6	10	6	10	0	2	4	4
5	1	6	0	6	7	13	7	5	4	3
6	2,3	7	6	13	9	16	3	3	5	2
7	4	5	10	15	10	15	0	4	1	4
8	5	2	6	8	13	15	7	4	1	4
9	2,3	2	6	8	13	15	7	5	5	4
10	9,8	2	8	10	15	17	7	3	2	4
11	7	6	15	21	15	21	0	1	4	5
12	4,6	1	13	14	16	17	3	3	3	2
13	6,8,9	2	13	15	17	19	4	3	2	2
14	10,12	4	14	18	17	21	3	2	2	2
15	7,13	2	15	17	19	21	4	1	4	4
16	13	3	15	18	19	22	4	5	5	4
17	11,14,15	5	21	26	21	26	0	3	2	3
18	16	8	18	26	22	30	4	4	5	4
19	5,16	2	18	20	24	26	6	5	3	3
20	17,19	6	26	32	26	32	0	2	4	6
21	18	2	26	28	30	32	4	1	6	2
22	18,21	0	32	32	32	32	0	0	0	0

For each activity in Table 4; (Act) is the activity number, (Pre.) represents the predecessor activities, and (D) is the duration of the activity. The forward-backward pass calculation can find the earliest and latest start times (ES and LS) and the earliest and latest finish time (EF and LF) times. The (SL) is the slack time (i.e., the amount of time that an activity can be delayed without causing another activity to be delayed or impacting the completion date of the project), and (R1, R2, and R3) are the resources required for each activity to be executed. When the resource limitation is not brought in, the project duration, $T_{\min}$, along the critical path can be derived. This is considered as the lower bound of the project makespan. The resource requirements to perform each activity are as indicated in Table 3, and the resource availabilities are R1 = 7, R2 = 10, and R3 = 10 units.

5.1. Case study (1)

The lower bound of project makespan, i.e., the longest period of time on the critical path, takes place when the project manager classifies all the project activities under category A and the resources are unlimited. Each activity starts based on the ES, and when only the precedence relationships constraint among project activities are considered. The lower bound makespan $T_{\min}$ equals 28 days. The non-feasible project schedule occurs due to violations of resource availabilities. As a result, the resource required (R_{req}) of (R1, R2, and R3) is = (15, 18, and 15). Table 5 shows the resource utilization and the MORR when project activities are scheduled based on the priority rules ES, LF, and SL.

Table 5
The value of the objective function obtained under ES, LF, and SL priority rules for case 1.

Project activities are classified under category A (ES, LF and SL schedule)						
Resource type	Description	Maximum resource available	Resource available in project	Resource used in project	Resource utilization %	MORR
1	R1	15	15×28=420	253	60.23	2641
2	R2	18	18×28=432	277	64.12	3430
3	R3	15	15×28=420	267	63.57	3529
Average			28 days			3200

Two facts are worth mentioning. One, the amount of resource utilization was low because of the high amount of resource requirements to carry out specific activities during certain periods and to remain idle during the rest periods. Two, project activities are not allowed to be preemptive during the execution time. However, the value of the objective function, when the project activities are classified under category A, precedence relationships and resource constraints are considered, and the resource available (R_k) of (R1, R2, and R3) is = (7, 10, and 10), is shown in Tables 6, 7, and 8 respectively:

Table 6
The value of the objective function obtained under ES priority rule for case 1

Project activities are classified under category A (ES schedule)						
Resource type	Description	Maximum resource available	Resource available in project	Resource used in project	Resource utilization %	MORR
1	R1	7	7×55=385	253	65.7	6447
2	R2	10	10×35=350	277	79.14	5069
3	R3	10	10×35=350	259	74	4674
Average			41.6 days			5393.7

Table 7
The value of the objective function obtained under LF priority rule for case 1

Project activities are classified under category A (LF schedule)						
Resource type	Description	Maximum resource available	Resource available in project	Resource used in project	Resource utilization %	MORR
1	R1	7	7×49=343	253	73.76	5985
2	R2	10	10×35=350	277	79.14	5182
3	R3	10	10×33=330	267	80.9	4653
Average			39 days			5273.3

Table 8
The value of the objective function obtained under SL priority rule for case 1

Project activities are classified under category A (SL schedule)						
Resource type	Description	Maximum resource available	Resource available in project	Resource used in project	Resource utilization %	MORR
1	R1	7	7×46=322	253	78.6	5930
2	R2	10	10×37=370	277	74.86	5440
3	R3	10	10×33=330	259	80.9	4653
Average			38.6 days			5341

The average project duration is 41.6 days under ES priority rule, 39 days under LF priority rule, and 38.6 days under SL priority rule. The upper bound of project makespan, T_{max}, is assumed to be 41.6 days.

5.2.Case study (2)

Some of the project activities are classified under category A, such as 1, 2, 4, 7, 9, 11, 14, 15, 17, 19 and 21; and some other activities are classified under category B, such as 3, 5, 6, 8, 10, 12, 13, 16, 18, and 20. The resources available to execute project activities are 7-units from R1, 10 from R2, and 10 from R3. Thus, Tables 9, 10, and 11 indicate the value of the objective function obtained under ES, LF, and SL priority rules when project activities are classified under A and B categories:

Table 9
The value of the objective function obtained under ES priority rule for case 2

Project activities are classified under category A and B (ES schedule)						
Resource type	Description	Maximum resource available	Resource available in project dur.	Resource used in project	Resource utilization %	MORR
1	R1	7	7×51=357	253	70.8	6215
2	R2	10	10×34=340	277	81.4	5023
3	R3	10	10×34=340	267	78.5	4639
Average			39.6 days			5292.3

Table 10
The value of the objective function obtained under LF priority rule for case 2

Project activities are classified under category A and B (LF schedule)						
Resource type	Description	Maximum resource available	Resource available in project dur.	Resource used in project	Resource utilization %	MORR
1	R1	7	7×49=343	253	73.76	5937
2	R2	10	10×34=340	277	81.4	5023
3	R3	10	10×34=340	267	78.5	4653
Average			39 days			15613

Table 11
The value of the objective function obtained under SL priority rule for case 2

Project activities are classified under category A and B (SL schedule)						
Resource type	Description	Maximum resource available	Resource available in project dur.	Resource used in project	Resource utilization %	MORR
1	R1	7	7×52=364	253	69.5	6391
2	R2	10	10×34=340	277	81.4	4963
3	R3	10	10×33=330	267	80.9	4653
Average			39.6 days			5335.7

Nonetheless, the average project duration is 39.6 days under ES schedule, 39 days under LF schedule, and 39.6 days under SL schedule.

5.3. Case study (3)

Project activities are classified as follows: category A includes activities, such as 1, 2, 4, 7, 9, 11, 14, 15, 17, 19, 21, and 22; category B includes activities, such as 5, 8, 10, 12, 13, 16, and 20; and category D includes activities, such as 3, 6, and 18. Tables 12, 13, and 14 show the value of the objective function obtained under ES, LF, and SL priority rules when project activities are classified under A, B, and D categories:

Table 12
The value of the objective function obtained under ES priority rule for case 3.

Project activities are classified under category A, B, and D (ES schedule)						
Resource type	Description	Maximum resource available	Resource available in project dur.	Resource used in project	Resource utilization %	MORR
1	R1	7	7×41=287	253	88.1	4930
2	R2	10	10×32=320	277	86.5	4524
3	R3	10	10×33=330	267	80.9	4706
Average			35.3 days			4720

Table 13
The value of the objective function obtained under LF priority rule for case 3

Project activities under are classified under category A, B, and D (LF schedule)						
Resource type	Description	Maximum resource available	Resource available in project dur.	Resource used in project	Resource utilization %	MORR
1	R1	7	7×44=287	253	82.1	5235
2	R2	10	10×32=320	277	86.6	4524
3	R3	10	10×33=330	267	80.9	4694
Average			36.3 days			4817.6

Table 14
The value of the objective function obtained under SL priority rule for case 3

Project activities are classified under category A, B, and D (SL schedule)						
Resource type	Description	Maximum resource available	Resource available in project dur.	Resource used in project	Resource utilization %	MORR
1	R1	7	7×41=287	253	88.1	4930
2	R2	10	10×32=320	277	86.6	4524
3	R3	10	10×33=330	267	80.9	4706
Average			35.3 days			4720

The average project makespan is reduced to 35.3 days under the ES schedule, 36.3 days under the LF schedule, and 35.3 under the SL schedule. Resources required (R_{req}) of (R1, R2, and R3) = (7, 10, and 10), and resource availability (R_k) of (R1, R2, and R3) = (7, 10, and 10). The compression between (R_{req}) and (R_k) indicates that no resource conflict occurs. Table 15, therefore, shows the best way to schedule the activities when project schedulers classify the activities under (A, B, and D) categories. The less duration and MORR (in **Bold**) are obtained under ES and SL priority rules.

Table 15
The value of the average duration and MORR for cases 1, 2, and 3.

	Activities classified under category A		Activities classified under category A and B		Activities classified under category A, B, and D	
PR	(Duration	MORR.)	(Duration	MORR.)	(Duration	MORR.)
ES	38.6	5341	39.6	5292.3	**35.3**	**4720**
LF	39	5273.3	39	5204.3	36.3	4817.6
SL	41.6	5393.7	39.6	5335.7	**35.3**	**4720**

6. Computational results

Based on the literature, test instances which classify project activities under A, B, and D categories are unavailable. Therefore, this section presents the results obtained using the proposed model with the PSPLIB modified instances. J30 and J60 activities are generated by Kolisch and Sprecher (1996). The experiments share some common characteristics, including, for example, the utilization of renewable resources.

Three parameters have been changed as follows:

1) The network complexity (NC) defines the average number of predecessors per activity.
2) The resource factor (RF) determines the average percentages of different resource types.
3) The resource strength (RS) defines the degree of the strength of resources.

Because of classified project activities to A, B, and D categories, the results obtained from the experiment provide insight into the makespan improvement. This improvement is measured and calculated as follows:

$$\%\,\text{makespan improvment} = \frac{[\text{makespan (under category A)} - \text{makespan (under category AB or AB D)}]}{\text{makespan (under category A)}}$$

As can be seen in Table A.1, the greater chance to larger the makespan improvement can be found when project activities are classified under A, B, and D categories.
Equally important, the three results from Appendix A are diagrammed in Figure 5 in graphics format: the best makespan, the best resource utilization, and the best MORR can be found when project activities are classified under A, B, and C categories.

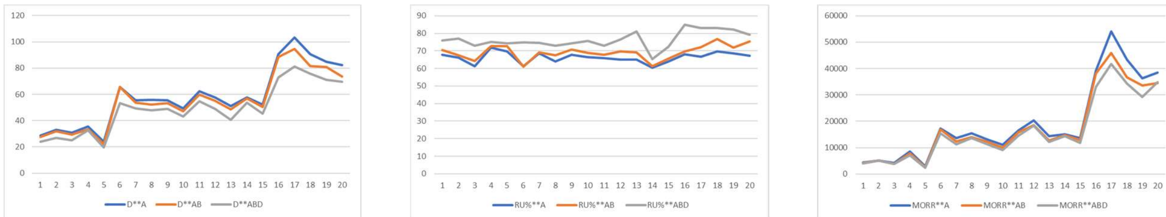

Fig. 5. Duration, resource utilization, and MORR. criterion in graphic format.

The impact of activity assumptions has been measured using the following criteria: the average of resource utilization, the average of MORR criterion, and the average of the project makespan improvement, as depicted in Table A. 1. Classify project activities, only, under category A (i.e., when the problem is considered as $RCPSP_{(A)}$ will be used as a reference to measure any improvement can occur compared with the ($RCPSP_{(AB)}$ or P-RCPSP) and with the ($RCPSP_{(ABD)}$ or F-P-RCPSP) activity assumptions.
The results can be summarized as follows: classifying project activities under "AB" can occur little improvement (4.9%) in the average of the percentage of the project-makespan-improvement whereas, classifying project activities under "ABD" increases the average of the percentage of the project-makespan-improvement (15.8%), as shown in Table A. 1 (in **Bold**).

7. Conclusion

In this paper, we present new procedures for scheduling projects under three generals of the project activities assumptions simultaneously, i.e., A, B, and D. For example, the activities under category A can be executed using constant resources over constant durations, and the pre-emptions are not allowed. The activities under category B can be executed using constant resources over constantan durations, the pre-emptions are allowed. The activities under category D can be executed using flexible resources over flexible durations, and the pre-emptions are allowed. With A, B, and D categories project schedulers can provide more flexibility in planning and scheduling projects constrained by limited multi-type of resources. In practice, many projects in construction and manufacturing-engineering include these three general categories. That is, project schedulers can interrupt (plan) activities under categories B and D. Our approach gives more flexibility to optimizing the project schedule and also offers a distinctive direction for project planning and scheduling. As seen in the three case studies, the project manager can split project activities, resulting in decreasing project duration and increasing average resource utilization.

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Appendix A

As used in this paper, the problem instants are modifications of PSPLIB created by Kolisch and Sprecher (1996). In order to investigate the impact of (A, B, and D) activity assumptions, activities for each project are classified under three assumptions: first assumption is: all the activities are classified under category A. Second assumption is: some activities are classified under category A and the rest under category B. Third assumption is: some activities are classified under category A, some under category B, and the rest under category D. The makespan (D), the resource utilization (RU), the minimum moment of resource required (MORR) criterion, the % of makespan improvement are four indicators that have been measured for each problem with ES, LF, and SL priority rules. Each project has nine feasible schedules. The first three schedules are generated when the activities are classified under category A with ES, LF, and SL priority rules. The following three schedules are generated when activities are classified under A and B categories with ES, LF, and SL. The last three schedules are generated when the activities are classified under (A, B and D) categories with ES, LF, and SL. The average of each indicator, (i.e., D, RU, MORR, and makespan improvement) has been measured for three feasible schedules under each category, as depicted in Table A.1. The procedure to get the results was programmed in MATLAB (R2018a), executed on a personal computer with an Intel(R) Core (TM) 2 Duo CPU T6500@2.10 GHz, 4GB RAM, and Windows 7.

Table A.1 The problem instances used in the paper

Project	Cat.	NC	RF	RS	R1	R2	R3	R4	PR.	D	RU%	MORR	Aver. D	Aver. RU%	Aver. MORR	%Impr.
	A	1.5	0.25	0.2					ES	30.25	66.42	4537				
									LF	27.75	68.86	4252.3	28.75	67.80	4446	0.0
1									SL	28.25	68.10	4550.8				
J30	AB	1.5	0.25	0.2	12	13	4	12	ES	27.5	70.38	4346.3				
									LF	27.5	70.45	4254.8	27.5	70.433	4285.3	4.3
									SL	27.5	70.45	4254.8				
	ABD	1.5	0.25	0.2					ES	24.25	75.25	4027				
									LF	23.75	76.37	4012.3	**23.92**	**76.00**	**4017**	**16.8**
									SL	23.75	76.37	4012.3				
	A	1.5	0.25	0.2					ES	32.75	64.28	5133.3				
									LF	33.75	63.9	5296.3	32.83	66.20	5207	0.0
2									SL	32	70.42	5193.3				
J30	AB	1.5	0.25	0.2	14	10	11	14	ES	31.75	66.43	5044.3				
									LF	31.75	67.83	5059.3	31.75	67.37	5063	3.3
									SL	31.75	67.83	5087.3				
	ABD	1.5	0.25	0.2					ES	26.75	77.03	5157.3				
									LF	26.75	77.03	5157.3	**26.75**	**77.03**	**5164**	**18.5**
									SL	26.75	77.03	5178.5				
	A	1.5	0.25	0.2					ES	29.75	63.02	3944.8				0.0
									LF	31.25	60.05	4284.8	30.67	61.19	4184	
3									SL	31	60.48	4322.5				
J30	AB	1.5	0.25	0.2	10	8	13	12	ES	29.75	63.02	3944.8				
									LF	29	65.03	3879.3	29.25	64.36	3921	4.6
									SL	29	65.03	3939.3				
	ABD	1.5	0.25	0.2					ES	25	72.84	3845				
									LF	25	72.84	3839	**25.00**	**72.85**	**3841**	**18.5**
									SL	25	72.84	3839				
	A	1.5	0.25	0.2					ES	35.5	72.01	8112				
									LF	35.25	72.48	8716.5	35.50	71.73	8515	0.0
4									SL	35.75	70.68	8719				
J30	AB	1.5	0.25	0.2	7	11	11	15	ES	33	73.42	7691.5				
									LF	33	73.42	7717	33.50	72.79	7761	5.6
									SL	34.5	71.51	7876.8				
	ABD	1.5	0.25	0.2					ES	32.5	75.04	7100.8				
									LF	32.5	75.04	7113.5	**32.50**	**75.04**	**7109**	**8.5**
									SL	32.5	75.04	7113.5				
	A	1.5	0.25	0.2					ES	21.5	74.35	2531				
									LF	25	67.33	3242	23.83	69.67	3022	0.0
5									SL	25	67.33	3294.8				
J30	AB	1.5	0.25	0.2	11	11	9	11	ES	21.5	74.35	2521.8				
									LF	22.5	71.95	2624.3	22.25	72.59	2601	6.6
									SL	22.75	71.47	2657.5				
	ABD	1.5	0.25	0.2					ES	19.75	74.32	2349.3				
									LF	19.5	75.01	2346.5	**19.75**	**74.34**	**2350**	**17.1**
									SL	20	73.67	2356.3				
	A	1.8	0.5	0.5					ES	66	60.52	17086				
									LF	64.5	61.77	17149	65.75	61.27	17322	0.0
6									SL	66.75	61.50	17732				
J30	AB	1.8	0.5	0.5	11	12	12	8	ES	65.5	60.90	16871				
									LF	64.75	61.72	16691	65.50	61.03	16839	0.4
									SL	66.25	60.46	16956				
	ABD	1.8	0.5	0.5					ES	53.25	75.09	15533				
									LF	53	75.47	15476	**53.33**	**74.97**	**15528**	**18.9**
									SL	53.75	74.32	15576				
	A	1.8	0.5	0.5					ES	56.5	65.55	13620				
									LF	56.25	68.52	13807	55.50	68.55	13716	0.0
7									SL	53.75	71.57	13721				
J30	AB	1.8	0.5	0.5	13	12	12	12	ES	54.75	69.92	12550				
									LF	53	69.06	12211	53.75	69.16	12325	3.2
									SL	53.5	68.49	12216				
	ABD	1.8	0.5	0.5					ES	51	71.96	11661				
									LF	48.25	75.69	11048	**49.17**	**74.45**	**11252**	**11.4**
									SL	48.25	75.69	11048				
	A	1.8	0.5	0.5					ES	57.5	63.72	15841				0.0
									LF	54.25	64.47	14908	55.75	63.85	15412	
8									SL	55.5	63.35	15488				
J30	AB	1.8	0.5	0.5	15	12	12	11	ES	52	67.71	13960				
									LF	52	67.71	13887	52.08	67.63	13927	6.6
									SL	52.25	67.45	13936				
	ABD	1.8	0.5	0.5					ES	48.25	72.47	13735				
									LF	47.75	73.15	13603	**47.92**	**72.93**	**13647**	**14.1**
									SL	47.75	73.15	13603				
	A	1.8	0.5	0.5					ES	55.25	68.65	12553				
									LF	54	67.95	12925	55.33	67.66	13191	0.0
9									SL	56.75	66.36	14097				
J30	AB	1.8	0.5	0.5	9	16	12	12	ES	54.25	70.14	12183				
									LF	51.5	71.96	12232	53.33	70.89	12204	3.6
									SL	54.25	70.56	12198				
	ABD	1.8	0.5	0.5					ES	49.25	73.94	11271				
									LF	49.25	73.97	11270	**48.92**	**74.42**	**11238**	**11.6**

									SL	48.25	75.33	11173				
10 J30	A	1.8	0.5	0.5					ES	50.5	64.50	10709				
									LF	48	67.94	11041	49.17	66.49	11096	0.0
									SL	49	67.02	11539				
	AB	1.8	0.5	0.5	14	15	11	11	ES	48.75	66.70	10075				
									LF	45.75	70.96	9951	47.17	68.96	10072	4.1
									SL	47	69.19	10192				
	ABD	1.8	0.5	0.5					ES	43.25	75.06	9277.3				
									LF	42.25	76.78	9104.8	**43.00**	**75.52**	**9193**	**12.5**
									SL	43.5	74.70	9199.3				
11 J60	A	1.5	0.25	0.2					ES	58.75	67.89	16174				
									LF	64.75	64.07	16955	62.33	65.91	16636	0.0
									SL	63.5	65.74	16781				
	AB	1.5	0.25	0.2	13	11	12	13	ES	58.25	68.40	15540				
									LF	58.75	69.36	15632	59.75	67.73	15832	4.1
									SL	62.25	65.43	16325				
	ABD	1.5	0.25	0.2					ES	53.5	74.44	14294				
									LF	55	72.66	14378	**54.75**	**72.91**	**14506**	**12.2**
									SL	55.75	71.61	14848				
12 J60	A	1.5	0.25	0.2					ES	56.75	66.79	18534				
									LF	54.75	67.77	20126	57.75	65.17	20363	0.0
									SL	61.75	60.96	22430				
	AB	1.5	0.25	0.2	13	15	14	14	ES	56	68.13	18725				
									LF	54.25	72.31	18374	55.25	69.55	18582	4.3
									SL	55.5	68.18	18649				
	ABD	1.5	0.25	0.2					ES	48.75	76.39	18103				
									LF	49.25	76.29	18386	**49.00**	**76.45**	**18291**	**15.2**
									SL	49	76.66	18385				
13 J60	A	1.5	0.25	0.2					ES	49.5	67.07	12931				
									LF	50.5	65.13	14630	51.08	65.03	14315	0.0
									SL	53.25	62.88	15384				
	AB	1.5	0.25	0.2	16	19	14	12	ES	48.5	68.75	12699				
									LF	48.5	69.23	12750	48.50	69.08	12733	5.1
									SL	48.5	69.23	12750				
	ABD	1.5	0.25	0.2					ES	40.75	80.57	12130				
									LF	40.5	81.2	12143	**40.58**	**80.99**	**12138**	**20.6**
									SL	40.5	81.2	12143				
14 J60	A	1.5	0.25	0.2					ES	57.25	60.89	14846				
									LF	57.75	60.37	14996	57.75	60.40	15114	0.0
									SL	58.25	59.94	15501				
	AB	1.5	0.25	0.2	15	13	13	13	ES	56.25	62.06	14679				
									LF	56.75	61.44	14767	57.00	61.22	14767	1.3
									SL	58	60.15	14855				
	ABD	1.5	0.25	0.2					ES	53.75	65.11	14523				
									LF	53.5	65.36	14416	**53.58**	**65.28**	**14451**	**7.2**
									SL	53.5	65.36	14415				
15 J60	A	1.5	0.25	0.2					ES	53.75	61.86	13232				
									LF	50.25	66.26	13520	52.17	64.03	13623	0.0
									SL	52.5	63.96	14119				
	AB	1.5	0.25	0.2	13	7	14	14	ES	51.25	64.16	13015				
									LF	49	67.57	12843	50.42	65.72	12966	3.4
									SL	51	65.41	13041				
	ABD	1.5	0.25	0.2					ES	45.5	71.86	11889				
									LF	44.75	72.84	11752	**45.17**	**72.29**	**11851**	**13.4**
									SL	45.25	72.15	11914				
16 J60	A	1.8	0.5	0.5					ES	87.25	70.44	37691				
									LF	90.5	68.27	39702	90.67	68.02	39244	0.0
									SL	94.25	65.32	40341				
	AB	1.8	0.5	0.5	13	15	11	14	ES	86.75	70.93	37609				
									LF	89.5	68.94	38607	88.33	69.74	38120	2.6
									SL	88.75	69.35	38144				
	ABD	1.8	0.5	0.5					ES	72	85.64	33104				
									LF	74.25	83.49	33523	**72.75**	**84.97**	**33123**	**19.8**
									SL	72	85.76	32744				
17 J60	A	1.8	0.5	0.5					ES	109.5	64.28	54842				
									LF	99.25	68.43	53007	103.33	66.75	54147	0.0
									SL	101.25	67.52	54593				
	AB	1.8	0.5	0.5	13	15	13	16	ES	99	71.02	47769				
									LF	92	73.14	44659	94.67	72.25	45860	8.4
									SL	93	72.58	45152				
	ABD	1.8	0.5	0.5					ES	82	82.17	42080				
									LF	80.75	83.54	41564	**81.17**	**83.09**	**41740**	**21.5**
									SL	80.75	83.54	41578				
18 J60	A	1.8	0.5	0.5					ES	91.75	68.08	42775				
									LF	82.75	75.57	39575	90.58	69.56	43432	0.0
									SL	97.25	65.02	47946				
	AB	1.8	0.5	0.5	14	18	14	14	ES	81	77.05	36644				
									LF	80	78.06	36219	81.42	76.81	36600	10.1
									SL	83.25	75.30	36939				
	ABD	1.8	0.5	0.5					ES	75	83.70	34067				
									LF	75.75	82.80	34219	**75.67**	**82.90**	**34247**	**16.5**
									SL	76.25	82.19	34457				
19 J60	A	1.8	0.5	0.5					ES	88	66.12	36010				
									LF	81.5	71.39	36544	84.92	68.72	36343	0.0
									SL	85.25	68.65	36477				
	AB	1.8	0.5	0.5	13	15	15	14	ES	84.5	68.66	33838				
									LF	78.25	74.06	33026	80.75	71.91	33530.67	4.9
									SL	79.5	73.01	33728				
	ABD	1.8	0.5	0.5					ES	72.25	80.80	29341				
									LF	69.5	83.55	29275	**71.00**	**82.06**	**29211**	**16.4**
									SL	71.25	81.83	29019				
	A	1.8	0.5	0.5					ES	79.25	69.75	36981				
									LF	83.25	66.28	38430	82.17	67.25	38453	0.0

20									SL	84	65.70	39948				
J60	AB	1.8	0.5	0.5	17	19	18	16	ES	74.5	74.38	34349				
									LF	72.25	76.53	33902	73.42	75.37	34452	10.6
									SL	73.5	75.21	35105				
	ABD	1.8	0.5	0.5					ES	69.75	79.24	34735				
									LF	69.75	79.21	34760	**69.75**	**79.22**	**34798**	**15.1**
									SL	69.75	79.21	34901				