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A precedence constrained flow shop scheduling problem with transportation time, breakdown times, and weighted jobs

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A B S T R A C T

Job precedence can often be seen in various manufacturing process scenarios. For instance, in the context of flow shop scheduling, certain jobs must be processed before a specific job may be executed. Formally, this scenario is known as precedence constraint, which influences the optimal job sequence. Because of this practical significance, in this study, a two-machine flow shop scheduling problem in which transportation times, breakdown time, and weighted jobs are considered. In addition to that, an ordered precedence constraint is considered that ensures a successor job cannot start on any machine before its predecessor job has been done on all machines. This is the first study that deals with flow shop scheduling problems with transportation times, breakdown time, job weights, and precedence constraints altogether, to the best of the author's knowledge. To solve this problem, a simple and efficient solution methodology is developed that assures optimal or near-optimal solutions effectively. The developed algorithm is tested on various test instances and results are reported, which will be useful for future comparative studies.

1. Introduction

Several manufacturing companies organize their industrial practices in a progressive order. The progressive standard adheres to the idea that each process has suppliers and clients, which are represented by the preceding and succeeding processes, respectively. This scenario often could be seen as Flow Shop Scheduling Problem (FSSP) (Pinedo and Hadavi, 1992). The scheduling process was regarded as the most essential topic of operational research and it is critical for any company's survival in today's competitive market. Concerning the developments of FSSP, at first, Johnson (1954) and Bellman (1956) addressed the scheduling problem with two-machine n jobs in which the transportation time is negligible i.e. the time necessary to transfer jobs from one machine to the other was insignificant. Essentially, the machines in the flow-shop process, the jobs can be placed at different locations, so that a job completed on the first machine can take some amount of time such as loading time, shipping time, and unloading time to process on the next machine (Maggu & Das, 1980). In the FSSP, each process can be considered as a single or a group of machines. Each machine is accountable for performing a particular task. Besides, all jobs should be completed in the same order as the machines. As a result, after a job finishes a task in one machine, it requires joining the line in the next machine. In addition, the jobs must be completed within certain parameters, such as release dates and resource availability. However, the majority of FSSP research assumes that machines are accessible during the scheduling horizon, but more practically, machines are not constantly available throughout the planning horizon (e.g., due to breakdown or preventive maintenance). Considering the FSSP model studied

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by both Johnson and Bellman as a base, several practical FSSP models have been discussed and a wide variety of solution methodologies is presented in the literature.

Before 1980, in most the scheduling-related studies, all jobs are supposed to have the same priority for handling them on the machines in a flow shop. However, from a practical perspective, this assumption is to be somewhat restrictive due to varying inventory costs involved with the jobs. Miyazaki and Nishiyama, (1980) addressed the FSSP model with job weights and presented an efficient methodology for optimizing the makespan time. Maggu et al., (1984) studied the idea of job weights and transportation time of a job and proposed an efficient solution methodology that minimizes the total makespan. In addition to the classical makespan objective, several researchers addressed other key objectives such as total flow time, lateness, and idle time of machines. Rajendran and Ziegler (1997) studied FSSP intending to minimize the overall weighted job flow times. Chou and Lee (1999) tried to solve bi-objective FSSP with work release dates, where the aim is to optimize the makespan and weighted sum of total flow time. Chandramouli (2005) offered a heuristic solution for the n-job, three-machine FSSP, which included transportation time, breakdown time, and job weights. Later, Pandian and Rajendran, (2010) have proposed an improved version of this algorithm that assures the best solutions. Khodadadi (2011) studied the constrained three-machine FSSP with transportation times and suggested a heuristic algorithm. Gupta et al., (2013) studied a two-stage FSSP model in which transportation times and job weights are considered. To solve this problem optimally, a branch and bound algorithm have been developed. Ren et al., (2015) considered FSSP that aims to optimize the makespan with release dates and proposed a local search heuristic algorithm. Thangaraj and Rajendran, (2016) studied a multi-stage FSSP model with job weights in a fuzzy environment and suggested a simple and efficient heuristic algorithm that assures giving optimal or near-optimal solutions. Fabri et al., (2019) addressed FSSP with release dates, shipping times, and precedence constraints which is solved using the efficient Lagrangean relaxation method. Janaki and Mohamed Ismail, (2020) attempted *m* machines *n* jobs FSSP with probabilistic processing times along with job delays and developed a heuristic method. An extensive review of intelligent scheduling problems can be found in Fazel Zarandi et al., (2020). More recently, Ren et al., (2021) have studied FSSP with release dates and proposed an exact branch and bound algorithm and a hybrid discrete differential evolution algorithm for obtaining optimal solutions. Branda et al., (2021) addressed the FSSP model including maintenance activities to minimize the makespan and the earliness-delay penalty. To solve this problem effectively, two novel metaheuristics namely genetic algorithm and Harmony search have been developed.

In industries, the prime objective of makespan condition intends to minimize machine load and save energy. Reviewing the literature, to the best of the author's knowledge, no study that deals with FSSP with transportation times, breakdown time, job weights and precedence constraints altogether, which is more realistic in industry. This motivates us to consider a practical scenario that is analogous to the flow shop scenario in terms of transportation times, breakdown times, job weights, and precedence constraints. Given that the present problem is NP-hard and cannot be solved in polynomial time. A simple and efficient heuristic method is developed to solve the present model.

The article is structured as follows: Section 2 describes the present problem. Section 3 presents the proposed algorithm. A numerical illustration is given in Section 4. Computational results and a discussion are presented in Section 5. Section 6 concludes the study.

2. Problem Statement

The present FSSP model can be described as follows: Let $J = \{1, 2, .., n\}$ be a set of n jobs to be performed in the sequence $1-2$ (i.e. Machine 1 followed by Machine 2) by the two machines $I = \{1, 2\}$. The processing time that represents the time when job j lapses on machine i is denoted by P_{ij}^{i}, which often assumes a positive number. Transportation time g_i, a non-negative number that denotes the delivery time of job i from machine 1 to machine 2. Let w_j be a job weight that says its relative significance in the particular sequence. Furthermore, we consider an ordered precedence constraint (j_p, j_s) that ensures the successor job (j_s) cannot start on any machine before its predecessor job (j_p) has done on all machines irrespective of their processing times. Finally, breakdown time interval (a, b) has been considered that often occurs due to power failure or no supply of raw material, or other technical interruptions. The length of the breakdown time interval is $b-a$. It is assumed that initially, all jobs are ready for processing and each job is allowed to go through the same production stage i.e. in the specified sequence only. The problem aims to determine an optimal schedule that minimizes the total elapsed time and mean weighted flow time for jobs. It is noted that the mean weighted flow time is denoted by F and it is computed by using the formula $F = \dfrac{\sum_{j=1}^{n} w_j \times f_j}{\sum_{j=1}^{n} w_j}$, where f_j is flow time of the j^{th} job. The structure of the stated problem can be represented in the following Table 1. The schematic representation of the present problem is demonstrated in Fig. 1.

Table 1
Structure of 2-machine flow shop scheduling problem with transportation times and job weights

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time $\left(g_{j}\right)$	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job (w_{j})
1	P_{11}^{1}	g_1	P_{21}^{1}	w_1
2	P_{12}^{1}	g_2	P_{22}^{1}	w_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	P_{1n}^{1}	g_n	P_{2n}^{2}	w_n

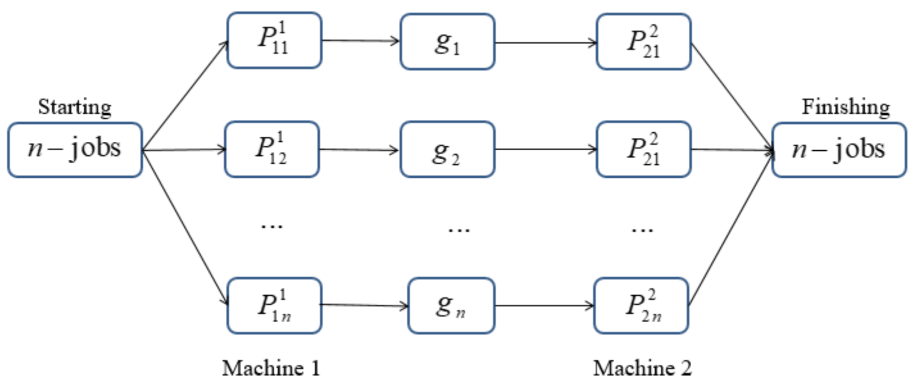

Fig 1. Schematic representation of the present model

3. Proposed Algorithm

This section presents a simple and efficient solution methodology for determining the optimal sequence for a 2-machine flow-shop scheduling problem with transportation and breakdown times, job weights, and precedence constraints.
The systematic procedure of the proposed algorithm is described below:

Step 1: Convert the given problem into a regular two-machine flow-shop problem by using $K_{ij} = P_{ij}^{1} + g_i$ and $L_{ij} = P_{ij}^{2} + g_i$

Step 2: Calculate the $Min(K_{ij}, L_{ij})$

a. If $Min(K_{ij}, L_{ij}) = K_{ij}$, then $K'_{ij} = K_{ij} - w_i$, $L'_{ij} = L_{ij}$

b. If $Min(K_{ij}, L_{ij}) = L_{ij}$, then $K'_{ij} = K_{ij}$, $L'_{ij} = L_{ij} + w_i$

Step 3: Construct a revised scheduling problem as shown below:

Job (j)	$K''_{ij} = \dfrac{K'_{ij}}{w_i}$	$L''_{ij} = \dfrac{L'_{ij}}{w_i}$
1	K''_{11}	L''_{12}
2	K''_{12}	L''_{22}
$\vdots$	$\vdots$	$\vdots$
n	K''_{1n}	L''_{n2}

Step 4: Determine the optimal sequence by applying Johnson's method to the revised scheduling problem (obtained in Step 3) by considering job precedence constraints into account.

Step 5: Determine the total elapsed time to the given problem using the optimal sequence found in Step 4.

Step 6: Determine the impact of breakdown time interval (a,b) on each job and revise the original problem by considering new processing times $P_{ij}^{1'}$ & $P_{ij}^{2'}$, which are computed as follows:

(a) If the breakdown time interval has an impact on job (i), then $P_{ij}^{1\prime} = P_{ij}^{1} + (b-a)$ and $P_{ij}^{2\prime} = P_{ij}^{2} + (b-a)$.

(b) If the breakdown time interval has no impact on job (i),

then $P_{ij}^{1\prime} = P_{ij}^{1}$ and $P_{ij}^{2\prime} = P_{ij}^{2}$.

Step 7: Using the revised problem (obtained in Step 6) and the optimal sequence (obtained in Step 4), calculate the total elapsed time, flow time of each job, machine idle time, and mean weighted flow time.

4. Numerical Example

This section presents a numerical example to validate the proposed algorithm.

Example 1: Let us consider a two-machine flow shop scheduling problem on five jobs with transportation time and job weights. Let the breakdown interval time and precedence constraint be $(a,b)=(19,23)$ and $(l,m)=(3,5)$, respectively. The problem's objective is to minimize the total elapsed time and mean weighted flow time.

Table 2
Numerical instance with transportation times and job weights

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time $\left(g_i\right)$	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job $\left(w_j\right)$
1	5	5	8	4
2	8	3	9	3
3	10	1	4	2
4	9	4	7	1
5	7	5	6	5

The proposed algorithm is described for the considered instance in the following steps:

Step 1: Initially, the given problem is converted to the two-machine flow-shop problem and is reported in Table 3 shown below:

Table 3
Converted two-machine flow-shop problem

Job (j)	$K_{ij} = P_{ij}^{1} + g_i$	$L_{ij} = P_{ij}^{2} + g_i$
1	10	13
2	11	12
3	11	5
4	13	11
5	12	11

Step 2-3: Compute $K_{ij}{}'$ & $L_{ij}{}'$ based on the minimum value of K_{ij} & L_{ij}, respectively and the revised scheduling problem is shown in Table 4 as follows:

Table 4
Revised two-machine flow-shop problem

Job (j)	$K_{ij}'' = \dfrac{K_{ij}'}{w_i}$	$L_i'' = \dfrac{L_i'}{w_i}$
1	1.5	3.25
2	2.67	4
3	5.5	3.5
4	13	12
5	2.4	3.2

Step 4: As per the considered precedence constraint $(3,5)$, the predecessor job (3) has to process on all the machines before starting the successor job (5) on any machine irrespective of their processing times. Therefore, this constraint affects the optimal sequence and it has been taken into account while determining the optimal sequence using the Johnson method for the revised scheduling problem (obtained in Step 3). The optimal sequence is given as follows:

1	3	5	2	4

Step 5: Using the above optimal sequence, the total elapsed time is calculated and is shown in Table 5 as follows:

Table 5
Total elapsed time in the absence of breakdown interval

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time(g_i)	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job (w_i)
1	0-5	5	10-18	4
3	5-15	1	**18-22**	2
5	**15-22**	5	27-33	5
2	**22-30**	3	33-42	3
4	30-39	4	43-50	1

The total elapsed time is 50 hours. The breakdown interval $(a,b)=(19,\ 23)$ affected jobs are Job 2, Job 3, and Job 5. The breakdown interval length is $b-a=23-19=4$, which is to be added to those job-processing times.

Step 6: The original job processing times have been affected due to the breakdown interval. The resultant flow-shop scheduling problem after implementing the breakdown interval is shown in Table 6 as follows:

Table 6: Effect of original processing times due to breakdown interval

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1\prime}\right)$	Transportation time(g_i)	Processing time on Machine 2 $\left(P_{ij}^{2\prime}\right)$	Weights of job (w_i)
1	5	5	8	4
2	**12**	3	9	3
3	10	1	**8**	2
4	9	4	7	1
5	**11**	5	6	5

Step 7: The total elapsed time, flow time of each job, idle time of the machines and mean weighted flow time are determined using the revised job processing times with respective obtained optimal sequence and reported in Tables 7-8. Finally, the end solution is represented through Gantt chart shown in Fig.2.

Table 7
Total elapsed time due to effect of breakdown interval

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1\prime}\right)$	Transportation time(g_i)	Processing time on Machine 2 $\left(P_{ij}^{2\prime}\right)$	Weights of job (w_i)
1	0-5	5	10-18	4
3	5-15	1	18-26	2
5	15-26	5	31-37	5
2	26-38	3	41-50	3
4	38-47	4	51-58	1

Table 8
Summary of final results

Break down Interval time	Precedence constraint	Optimal Sequence	Breakdown affected jobs	Overall flow time of each job (in hrs)	Idle time of each machine (in hrs)	Mean weighted flow time (in hrs)	Total elapsed time due to breakdown (in hrs)
(19, 23)	(3, 5)	1-3-5-2-4	2, 3, & 5	$f_1 = 18$ $f_3 = 19$ $f_5 = 22$ $f_2 = 24$ $f_4 = 20$	$M_1 = 0$ $M_2 = 20$	20.80	58

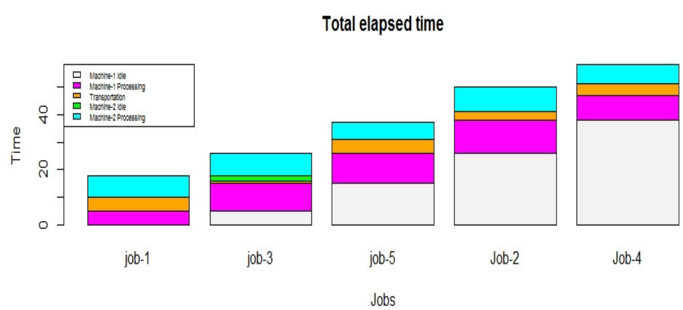

Fig. 2 Gantt chart of solution

5. Computational Results

This section presents computational results. It is noted that there are no existing studies on the present model, thus this study has not attempted any comparative studies to test the algorithm's performance. However, a set of six numerical instances (those are reported in the appendix) has been created for computational experiments and these instances are represented with A1, A2, A3, A4, A5, and A6, respectively. The proposed algorithm was implemented in MATLAB 2021a and all the experiments were tested on an Intel Core i5 with 2.10 GHz CPU B950 and 4 GB of RAM PC running Microsoft Windows 2010 Operating System. Fifteen test cases with distinct breakdown and precedence constraint values were tested and results are reported in Table 9. It is noted that the CPU runtime of the proposed algorithm for all the test cases is ranging from 10 seconds to 34 seconds. It shows that the present algorithm provides optimal or near-optimal solutions within considerable time.

Table 9: Computational results

Name of the Instance & its size	Break down Interval time	Precedence constraint	Optimal Sequence	Breakdown affected jobs	Overall flow time of each job (in hrs)	Idle time of each machine (in hrs)	Mean weighted flow time (in hrs)	Total elapsed time due to breakdown (in hrs)
A1 (2×7)	(501,550)	(2, 6)	5-1-3-7-2-6-4	2 & 3	$f_5 = 208$ $f_1 = 341$ $f_3 = 406$ $f_7 = 426$ $f_2 = 450$ $f_6 = 498$ $f_4 = 408$	$M_1 = 0$ $M_2 = 65$	383.4902	1622

A1 (2×7)	(600,700)	(3, 6) & (2, 4)	5-1-7-3-6-2-4	2 & 6	$f_5=208$ $f_1=341$ $f_7=526$ $f_3=357$ $f_6=598$ $f_2=501$ $f_4=408$	$M_1=0$ $M_2=65$	404.4118	1673
A2 (2×4)	(3000,31000)	(1, 4)	3-2-1-4	1 & 2	$f_3=1989$ $f_2=3579$ $f_1=2508$ $f_4=2801$	$M_1=0$ $M_2=1293$	2769.10	7233
A3 (2×10)	(101,115)	(5, 6)	1-3-2-8-9-4-10-5-6-7	4, 8, 9, & 1	$f_1=37$ $f_3=42$ $f_2=53$ $f_8=64$ $f_9=66$ $f_4=79$ $f_{10}=54$ $f_5=42$ $f_6=41$ $f_7=22$	$M_1=0$ $M_2=16$	46.0930	244
A3 (2×10)	(101,115)	(2, 5) & (3,9)	1-8-3-9-4-10-2-5-6-7	2, 9, & 10	$f_1=37$ $f_8=50$ $f_3=42$ $f_9=66$ $f_4=65$ $f_{10}=54$ $f_2=67$ $f_5=42$ $f_6=41$ $f_7=22$	$M_1=0$ $M_2=20$	46.7442	234
A3 (2×10)	(100,120)	(2, 7)	1-3-8-9-4-10-5-6-2-7	4, 5, 9 , & 10	$f_1=37$ $f_3=42$ $f_8=50$ $f_9=72$ $f_4=85$ $f_{10}=60$ $f_5=62$ $f_6=41$ $f_2=53$ $f_7=22$	$M_1=0$ $M_2=21$	47.9535	261
A3 (2×10)	(100,120)	(3, 10)	1-2-8-9-4-3-10-5-6-7	3, 4, 9, & 10	$f_1=37$ $f_2=53$ $f_8=50$ $f_9=72$ $f_4=105$ $f_3=62$ $f_{10}=60$	$M_1=0$ $M_2=26$	49.3488	266

					$f_5 = 42$ $f_6 = 41$ $f_7 = 22$			
A3 (2×10)	(90,100)	(2, 4) & (7, 10)	1-3-8-9- 2-4-7- 10-5-6	4 & 9	$f_1 = 37$ $f_3 = 42$ $f_8 = 50$ $f_9 = 62$ $f_2 = 53$ $f_4 = 75$ $f_7 = 22$ $f_{10} = 40$ $f_5 = 42$ $f_6 = 41$	$M_1 = 0$ $M_2 = 16$	43.7674	226
A4 (2×6)	(200, 250)	(2, 5)	1-4-6-2- 5-3	2, 4, & 6	$f_1 = 166$ $f_4 = 195$ $f_6 = 278$ $f_2 = 218$ $f_5 = 160$ $f_3 = 145$	$M_1 = 0$ $M_2 = 112$	188.68	643
A5 (2×15)	(101, 110)	(5, 10)	9-11-8- 13-7-15- 5-10-14- 1-6-2-3- 4-12	5, 7, 13, & 15	$f_9 = 49$ $f_{11} = 43$ $f_8 = 35$ $f_{13} = 47$ $f_7 = 65$ $f_{15} = 42$ $f_5 = 29$ $f_{10} = 34$ $f_{14} = 32$ $f_1 = 30$ $f_6 = 51$ $f_2 = 31$ $f_3 = 41$ $f_4 = 27$ $f_{12} = 37$	$M_1 = 0$ $M_2 = 56$	40.549	306
A5 (2×15)	(101, 110)	(6, 13) & (7, 11)	5-9-7- 11-8-6- 13-15- 10-14-1- 2-3-4-12	6, 8, & 11	$f_5 = 20$ $f_9 = 49$ $f_7 = 56$ $f_{11} = 52$ $f_8 = 44$ $f_6 = 60$ $f_{13} = 38$ $f_{15} = 33$ $f_{10} = 34$ $f_{14} = 32$ $f_1 = 30$ $f_2 = 31$ $f_3 = 41$	$M_1 = 0$ $M_2 = 47$	41.2549	297

					$f_4 = 27$ $f_{12} = 37$			
A5 (2×15)	(100, 150)	(3, 8)	5-9-11-3-8-13-7-15-10-14-1-6-2-4-12	7, 8, 10, 13, & 15	$f_5 = 20$ $f_9 = 49$ $f_{11} = 43$ $f_3 = 41$ $f_8 = 85$ $f_{13} = 138$ $f_7 = 156$ $f_{15} = 83$ $f_{10} = 84$ $f_{14} = 32$ $f_1 = 30$ $f_6 = 51$ $f_2 = 31$ $f_4 = 27$ $f_{12} = 37$	$M_1 = 0$ $M_2 = 106$	58.5686	488
A5 (2×15)	(150, 165)	(2, 14)	5-9-11-8-13-7-15-10-2-14-1-6-3-4-12	1, 2, 10, 14, & 15	$f_5 = 20$ $f_9 = 49$ $f_{11} = 43$ $f_8 = 35$ $f_{13} = 38$ $f_7 = 56$ $f_{15} = 48$ $f_{10} = 49$ $f_2 = 46$ $f_{14} = 47$ $f_1 = 45$ $f_6 = 51$ $f_3 = 41$ $f_4 = 27$ $f_{12} = 37$	$M_1 = 0$ $M_2 = 41$	42.1961	318
A5 (2×15)	(100, 150)	(5, 11) & (2, 13)	9-5-11-8-2-13-7-15-10-14-1-6-3-4-12	2, 7, 10, 13, 14, & 15	$f_9 = 49$ $f_5 = 20$ $f_{11} = 43$ $f_8 = 35$ $f_2 = 81$ $f_{13} = 88$ $f_7 = 156$ $f_{15} = 83$ $f_{10} = 84$ $f_{14} = 82$ $f_1 = 30$ $f_6 = 51$ $f_3 = 41$ $f_4 = 27$ $f_{12} = 37$	$M_1 = 0$ $M_2 = 106$	56.6078	488

A6 (2×6)	(18, 22)	(3, 1)	3-1-2-5-4	1, 2, & 3	$f_3 = 22$ $f_1 = 21$ $f_2 = 24$ $f_5 = 18$ $f_4 = 17$	$M_1 = 0$ $M_2 = 40$	19.7333	67

6. Conclusion

This study investigates an FSSP model with transportation times, breakdown time, job weights, and precedence constraints intending to minimize the makespan. As the present problem is NP-hard and cannot be solved in polynomial time, a simple and efficient heuristic algorithm is developed to get optimal or near-optimal solutions effectively. Since no studies on the present model are available, a comparative study is not carried out to test the effectiveness of the algorithm. However, the performance of the proposed algorithm is tested with various numerical instances of distinct sizes. Numerical results show the effectiveness of the proposed algorithm. Furthermore, the reported results will fetch for future comparative studies.

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Appendix

Name	Instance

A1

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time $\left(g_i\right)$	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job $\left(w_j\right)$
1	113	12	216	10
2	128	21	252	8
3	150	15	192	9
4	216	17	175	5
5	56	9	143	8
6	233	22	243	7
7	128	11	287	4

A2

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time $\left(g_i\right)$	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job $\left(w_j\right)$
1	1123	13	1342	6
2	1541	17	1921	8
3	856	12	1121	7
4	1324	21	1456	4

A3

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time $\left(g_i\right)$	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job $\left(w_j\right)$
1	12	4	21	8
2	21	7	25	7
3	11	3	28	5
4	31	2	32	2
5	23	5	14	4
6	25	4	12	5
7	14	3	5	4
8	18	8	24	3
9	19	6	27	2
10	24	4	12	3

A4

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time $\left(g_i\right)$	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job $\left(w_j\right)$
1	56	12	98	5
2	78	8	82	7
3	84	6	55	4
4	62	10	73	3
5	94	8	58	4
6	108	5	65	2

Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time $\left(g_i\right)$	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job $\left(w_j\right)$
1	15	3	12	2
2	17	4	10	4
3	25	1	15	5
4	18	2	7	3
5	5	3	12	2
6	31	2	18	4
7	23	5	28	3
8	18	4	13	5

A5	9	21	3	25	6
	10	15	7	12	2
	11	18	3	22	4
	12	23	2	12	5
	13	15	4	19	3
	14	17	2	13	2
	15	13	6	14	1
	Job (j)	Processing time on Machine 1 $\left(P_{ij}^{1}\right)$	Transportation time $\left(g_i\right)$	Processing time on Machine 2 $\left(P_{ij}^{2}\right)$	Weights of job $\left(w_j\right)$
A6	1	9	1	7	3
	2	11	3	6	1
	3	13	2	3	4
	4	10	5	2	5
	5	9	4	5	2