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A mathematical model for optimization of an integrated network logistics design

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A B S T R A C T

In this study, the integrated forward/reverse logistics network is investigated, and a capacitated multi-stage, multi-product logistics network design is proposed by formulating a generalized logistics network problem into a mixed-integer nonlinear programming model (MINLP) for minimizing the total cost of the closed-loop supply chain network. Moreover, the proposed model is solved by using optimization solver, which provides the decisions related to the facility location problem, optimum quantity of shipped product, and facility capacity. Numerical results show the power of the proposed MINLP model to avoid th sub-optimality caused by separate design of forward and reverse logistics networks and to handle various transportation modes and periodic demand.

1. Introduction

Logistics network design is the important strategic issue in supply chain management. In general, logistics network design decisions include determining the numbers, locations, and capacities of facilities and the quantity of the flow between them (Amiri, 2006). Since opening and closing a facility is fabulously expensive and time-consuming, making changes in facility location decisions is impossible in the short run. Because tactical and operational decisions are determined after the strategic decisions, the configuration of logistics network will become a constraint for tactical and operational level decisions (Meepetchdee & Shah, 2007). In the last decade, because of legal requirements, environmental protection and also related economic benefits, many companies such as Dell, General Motors, Kodak, and Xerox focused on remanufacturing and recovery activities and have met with notable successes in this area (Meade et al., 2007; Üster et al., 2007).

In the recent years, some researches (Meade et al., 2007; Bei & Linyan, 2005) classify driving forces led to increased interest and investment in reverse supply chain into two groups: environmental

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factors and business factors. Reverse logistics network design includes determining the numbers, locations, and capacities of collection, recovery and disposal centers, buffer inventories in each site, and the quantity of flow between each pair of facilities. Reverse logistics networks have special characteristics differentiating them from forward logistics networks. One of these characteristics is the important role of collection/inspection centers. After testing returned products in collection/inspection centers, returned products are divided into recoverable and scrapped products to prevent excessive transportation and to ship the returned products directly to proper facilities (Fleischmann et al., 2001). In most of the past researches the design of forward and reverse logistics networks is considered separately that may lead to sub-optimal design, but due to the fact that the configuration of the reverse logistics network has a strong influence on the forward logistics network and vice versa; designing the forward and reverse logistics should be integrated (Lee & Dong, 2007).

Previous research in the area of reverse and integrated logistics network design often limited itself to proposing a single capacity level for each facility and often did not address how facility capacity for reverse and forward activity can be determined (see Table 2 and Amiri, 2006). Nevertheless, capacity levels are important decision variables in real-life applications due to their significant effect on logistics network efficiency (Amiri, 2006).

In addition, as shown in Table 2, a significant part of literature in logistics network design is associated with single-period problem, a smaller part is associated with multi-periods and in recent years a few papers have dealt with multi-periods integrated forward/reverse logistics network design. Based on the aforementioned considerations, this paper addresses the issue of integrated multi-periods, multi-product, multi-stage forward/reverse logistics network design including production, distribution, collection/inspection, recovery and disposal facilities with multiple capacity levels.

The rest of this paper is organized as follows. After offering the literature review to assess the state-of-the-art in forward/reverse logistics network design, a generalized mixed integer non-linear programming (MINLP) formulation model is developed. The application of the model is shown with a numerical example. Finally, concluding remarks and some possible future works are given.

2. Literature review

Most of the literature about logistics network design considers various facility location models based on the MILP.

These models include a range of models from simple uncapacitated facility location models (e.g. Sung & Song, 2003) to more complex models such as capacitated multistage or multi-commodity models (e.g. Tsiakis & Papageorgiou, 2008) and they are usually aimed at determining the cost minimizing or profit maximizing system design.

In this paper, specific network design problems for forward, reverse and integrated supply chain design problems are surveyed. To structure the related literature review, logistics network design problems have been classified according to four general specifications: problem definition, modeling, outputs and objectives (see Table 1), and some of the available models in the literature in the last decade are reviewed in Table 2.

During the last decade, many models were developed for supply chain reverse logistics network design. Jayaraman et al. (1999) developed an MILP model for reverse logistics network design under a pull system based on customers' demand for recovered products. Fleischmann et al. (2001) illustrated that an integrated approach, optimizing the forward and return network, simultaneously, offers considerable cost savings compared to a sequential design of both networks. Salema et al. (2007) extended the model of Fleischmann et al. (2001) to multi-product networks under demand uncertainty.

Table 1
State-of-the-art of classification of reverse and integrated logistics network design problem

Problem specifications			Code
Problem definition	*Periods*	Single Period	SPr
		Multi Period	MPr
	Number of facilities to be opened	Exogenous (determined)	Ex
		Endogenous (undetermined)	En
	Product	Single product	SP
		Multi product	MP
	Flow capacity	Uncapacitated flow	UCF
		Capacitated flow	CF
	Demand	Deterministic	D
		Stochastic	S
	Facility capacity	Uncapacitated	UC
		Capacitated	Ca
Modeling	Mixed integer linear programming		MILP
	Mixed integer non-linear programming		MINLP
	Stochastic mixed integer programming		SMIP
	Mixed integer goal programming		MIGP
Outputs	Location/allocation		L
	Facility capacity		FC
	Service region		SR
	Transportation amount		TA
	Price of product		P
	Demand satisfaction quantity		DS
	Number of vehicles		NV
	Inventory		I
Objectives	Min cost/Max profit		C
	Max responsiveness		Res

Table 2
Review of some existing models in logistics network design

Reference articles	Problem definition	Modeling	Outputs	Objectives
Reverse logistics				
Marin and Pelegrin (1998)	UC, D, UCF, SP, En, SPr	MILP	L, TA	C
Jayaraman et al. (1999)	Ca, D, UCF, MP, En, SPr	MILP	L, TA	C
Krikke et al. (1999)	UC, D, UCF, SP, Ex, SPr	MILP	L, TA, I	C
Jayaraman et al. (2003)	Ca, D, UCF, SP, En, SPr	MILP	L, TA	C
Listes and Dekker (2005)	Ca, S, UCF, MP, En, SPr	SMIP	L, TA	C
Min et al. (2006)	UC, D, UCF, SP, En, MPr	MILP	L, TA	C
Üster et al. (2007)	UC, D, UCF, MP, En, SPr	MILP	L, TA	C
Demirel and Gokcen (2008)	Ca, D, UCF, MP, En, SPr	MILP	L, TA	C
Aras et al. (2008)	UC, D, UCF, MP, Ex, SPr	MINLP	L, TA, NV, P	C
Du and Evans (2008)	Ca, D, UCF, MP, En, SPr	MILP	L, FC, TA	C, Res
Pati et al. (2008)	Ca, D, CF, MP, En, SPr	MIGP	L, TA	C
Integrated logistics				
Fleischmann et al. (2001)	UC, D, UCF, SP, En, SPr	MILP	L, TA, DS	C
Lu and Bostel (2007)	UC, D, UCF, SP, En, SPr	MILP	L, TA	C
Salema et al. (2006)	Ca, D, UCF, SP, En, SPr	MILP	L, TA, DS	C
Ko and Evans (2007)	Ca, D, UCF, MP, En, MPr	MINLP	L, TA	C
Salema et al. (2007)	Ca, S, UCF, MP, En, SPr	SMIP	L, TA, DS	C
Min and Ko (2008)	Ca, D, UCF, MP, En, MPr	MINLP	L, TA	C
Lee and Dong (2008)	Ca, D, CF, SP, Ex, SPr	MILP	L, TA	C
Pishvaee et al. (2010)	Ca, D, UCF, SP, En, SPr	MILP	L, FC, TA	C, Res

As cost pressures continue, a growing number of firms have begun to explore the possibility of managing product returns in a more cost-efficient and timely manner. However, few studies have addressed the problem of determining the number and location of collection points in a multiple time horizon, while determining the desirable holding time for consolidation of returned products into a large shipment. To fill the void in such a line of research, Min et al. (2006) proposed a MINLP model and a genetic algorithm that can solve the reverse logistics problem involving both spatial and temporal consolidation of returned products. Miranda and Garrido (2004) proposed a simultaneous approach to incorporate inventory control decisions into typical facility location models under stochastic demand. They presented a MINLP model and a heuristic solution approach, based on Lagrangian relaxation and the sub-gradient method. Pati et al. (2008) formulated a mixed integer goal programming (MIGP) model which studies the inter-relationship between multiple objectives to assist in proper management of the paper recycling logistics system.

Ko and Evans (2007) considered the model for dynamic supply chain management by third party logistics providers (3PLs). The model belongs to a class of the multi-period, two-echelon, multi-commodity, capacitated location models. The main differences of this model as compared with existing location models lie in handing forward flow simultaneous with reverse one. Thus, the paper presented a mixed integer nonlinear programming model for the design of a dynamic integrated distribution network to account for the integrated aspect of optimizing the forward and return network, simultaneously. In addition, Min and Ko (2008) developed a mixed-integer programming model and a genetic algorithm that can solve the reverse logistics problem involving the location and allocation of repair facilities for 3PLs.

In the area of logistics network design, many models have been developed for various kinds of networks. Most of research in logistics network design was often limited to considering a single capacity level for each facility and often did not address how capacity levels can be determined (Miranda & Garrido, 2004). Amiri (2006) developed a MILP model for a multi-stage forward network and also considered multiple capacity levels for each facility. The model not only determines the number and location of facilities, but also it is able to find the optimal capacity level for each facility. Tsiakis and Papageorgiou (2006) determined the optimal configuration of complex capacitated multi-product, multi-echelon production and distribution network subject to operational and financial constraints. In addition, the production capacity of each manufacturing site is modeled and distribution centers are described by upper and lower bounds on their material handling capacity. Du and Evans (2008) minimized tardiness and total costs for location and capacity decisions in a closed-loop logistics network operated by 3PL providers.

As summarized above, a majority of existing logistics networks design models have, so far, focused on forward and reverse logistics network separately and neglected integrated logistics network design. In addition, a few of recent studies considered the coordination of integrated logistics activities in multiple periods (Ko & Evans, 2007; Min & Ko, 2008). More importantly none of these prior studies addressed the integrated logistics network design in multiple time periods that also considers multiple capacity levels for each facility and various modes of transportation. The proposed model in this study will aim to design a multi-periods and multi-product integrated logistics network for capacitated supply chain. The model not only determines the number and location of facilities, but also it is able to find the optimal capacity level for each facility and optimal operating capacity for production/recovery and distribution/collection facilities over different periods.

3. Problem definition

The integrated logistics network (ILN) discussed in this paper is a multi-stage logistics network including production, distribution, customer zones, collection/inspection, recovery and disposal centers with multi-level capacities.

As illustrated in Fig. 1, in the forward network, new products are shipped by various transportation modes from production centers to customer zones through distribution centers to meet the demand of each customer in different periods. Customer zones are assumed to be predetermined and fixed. In the reverse network, returned products are collected in collection/inspection centers and, after testing, the recoverable products are shipped to recovery facilities, and scrapped products are shipped to disposal centers.

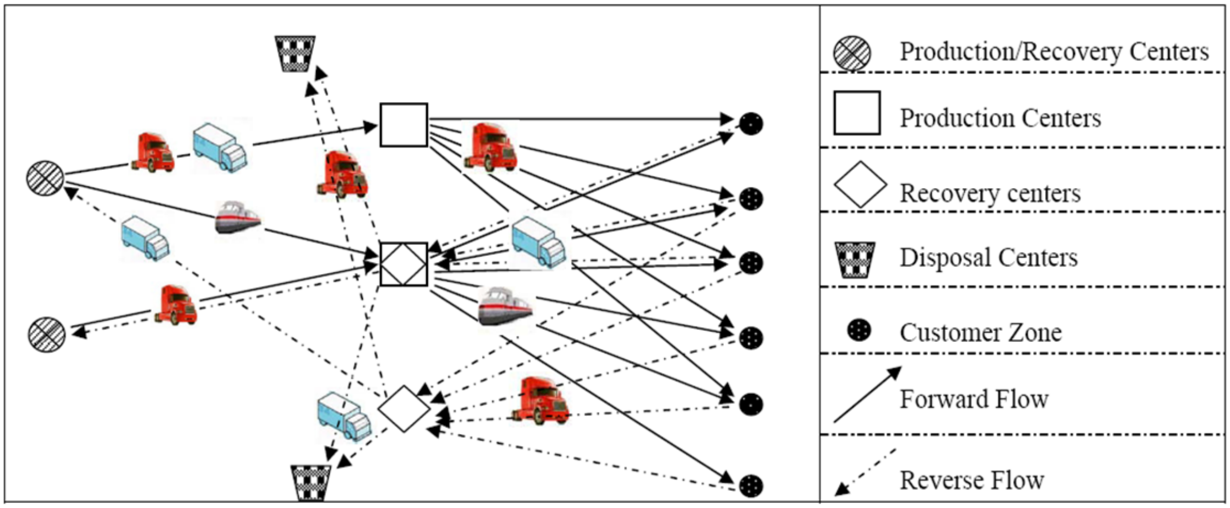

Fig. 1. An integrated forward/reverse logistics network

Lee and Dong (2008) discussed that in an integrated logistics network, hybrid processing facilities offer potential cost savings compared with separate distribution or collection centers. Thus the ILN considers a distribution-collection facility whereby both distribution and collection centers are established at the same location. The resulting cost saving is reflected in the objective function, which considers the tradeoff of fixed opening costs of facilities, transportation costs, operation costs, and warehousing costs of distribution/collection centers over time. Thus, unlike previous models with hybrid distribution/collection facilities (e.g. Lee & Dong, 2008), the use of hybrid-collection facilities is a decision variable in the proposed model and also the capacity of facilities for production, recovery, distribution, and collection activity are decision variables in different time periods in the ILN model.

In the forward flow, products are pulled through a divergent network and in the reverse flow, returned products are shipped through a semi-convergent network according to push principles. A pre-defined percentage of demand from each customer zone in each period after satisfying demands is returned products and a pre-defined value is determined as an average disposal fraction.

With the above situations in mind, the main issues to be addressed by this study are to determine the location, the number and the capacity of production/recovery, distribution, collection/ inspection and disposal centers, and also to determine the product flow between the facilities. ILN is a generic network, so it can support a variety of industries such as electronic and digital equipment industries (e.g. Lee & Dong, 2008; Krikke et al. 1999) and vehicle industries (e.g. Üster et al., 2007). According to Table 1, the problem in question can be coded as shown in Table 3.

Table 3
Coding of the problem in question

Problem definition	Modeling	Outputs	Objectives
Ca, D, UCF, MP, En, MPr	MINLP	L, FC, TA	C

4. Model formulation

To support the presentation of the proposed mathematical model, we first provide a verbal description of the model as follows.

Minimize cost
= Fixed Opening Costs + Fixed and Variable Transportation Costs + Operation Costs + Warehousing Costs
Subject to:

- Satisfying all forward and reverse demands,
- Balancing of flows between nodes,
- Facility and transportation capacity constraints,
- Logical constraints related to the different capacity levels and inventories,
- Non-negativity and binary constraints.

The following notations are used in the formulation of the integrated logistic network (ILN) model:

Sets and Indices

I : Set of potential production/recovery center locations $i \in I$
J : Set of potential distribution, collection/inspection and hybrid center locations $j \in J$
M : Set of potential disposal center locations $m \in M$
K : Fixed locations of customer zones $k \in K$
P : Index for time periods $p \in P$
N : Set of capacity levels available for facilities $n \in N$
V : Set of potential modes of transportation $v \in V$
W : Set of various products $w \in W$

Parameters

d_{kpw} : Demand of w^{th} product related to k^{th} customer zone occurred in p^{th} period.
r_{kaw} : Rate of return of w^{th} used products from k^{th} customer zone occurred after a period.
s_w : Average disposal fraction of w^{th} used products
$|p|$: Number of time periods
WC_{jpw} : Warehousing cost w^{th} product at j^{th} distribution center in p^{th} period.
f_i^n : Fixed cost of opening i^{th} production/recovery center with n^{th} capacity level.
o_j^n : Fixed cost of opening j^{th} distribution center with n^{th} capacity level.
h_j^n : Fixed cost of opening j^{th} collection/inspection center with n^{th} capacity level.
g_j^n : Fixed cost of opening j^{th} hybrid center with n^{th} capacity level.
a_m^n : Fixed cost of opening m^{th} disposal center with n^{th} capacity level.
FCX_{ijvp} : Fixed cost of transportation related to v^{th} transportation mode from i^{th} production/ recovery center to j^{th} distribution center at p^{th} period.
FCU_{jkvp} : Fixed cost of transportation related to v^{th} transportation mode from j^{th} distribution center to k^{th} customer zone at p^{th} period.
FCQ_{kjvp} : Fixed cost of transportation related to v^{th} transportation mode from k^{th} customer zone to j^{th} collection/inspection center at p^{th} period.
FCP_{jivp} : Fixed cost of transportation related to v^{th} transportation mode from j^{th} collection/inspection center to i^{th} production/recovery center at p^{th} period.
FCT_{jmvp} : Fixed cost of transportation related to v^{th} transportation mode from j^{th} collection/inspection center to m^{th} disposal center at p^{th} period.
VCX_{ijvp} : Unit variable cost of transportation related to v^{th} transportation mode from i^{th} production/ recovery center to j^{th} distribution center at p^{th} period.
VCU_{jkvp} : Unit variable cost of transportation related to v^{th} transportation mode from j^{th} distribution center to k^{th} customer zone at p^{th} period.
VCQ_{kjvp} : Unit variable cost of transportation related to v^{th} transportation mode from k^{th} customer zone to j^{th} collection/inspection center at p^{th} period.
VCP_{jivp} : Unit variable cost of transportation related to v^{th} transportation mode from j^{th} collection/inspection center to i^{th} production/recovery center at p^{th} period.

VCT_{jmvp} : Unit variable cost of transportation related to v^{th} transportation mode from j^{th} collection/inspection center to m^{th} disposal center at p^{th} period.
CAX_{ijvp} : Capacity of v^{th} transportation mode for carrying various product from i^{th} production/ recovery center to j^{th} distribution center at p^{th} period.
CAU_{jkvp} : Capacity of v^{th} transportation mode for carrying various product from j^{th} distribution center to k^{th} customer zone at p^{th} period.
CAQ_{kjvp} : Capacity of v^{th} transportation mode for carrying various product from k^{th} customer zone to j^{th} collection/inspection center at p^{th} period.
CAP_{jivp} : Capacity of v^{th} transportation mode for carrying various product from j^{th} collection/inspection center to i^{th} production/recovery center at p^{th} period.
CAT_{jmvp} : Capacity of v^{th} transportation mode for carrying various product from j^{th} collection/inspection center to m^{th} disposal center at p^{th} period.
cap_i^n : Total capacity in n^{th} level related to i^{th} production/recovery center.
cap_j^n : Total capacity in n^{th} level related to j^{th} hybrid center.
cav_m^n : Capacity in n^{th} level related to m^{th} disposal center.
cw_{iw} : Manufacturing cost of w^{th} product at i^{th} production/recovery center.
cr_{iw} : Recovery cost of w^{th} product at i^{th} production/recovery center.
cy_{jw} : Processing cost of w^{th} product at j^{th} distribution center.
cz_{jw} : Processing cost of w^{th} product at j^{th} collection/inspection center.
cv_{mw}: Disposal cost of w^{th} product at m^{th} disposal center.

Decision variables
WI_{jpw} : Inventory of w^{th} product related to j^{th} distribution center at the beginning of p^{th} period.
caw_{ip}: Capacity of production related to i^{th} production/recovery center at p^{th} period.
car_{ip}: Capacity of recovery related to i^{th} production/recovery center at p^{th} period.
cay_{jp}: Capacity of distribution related to j^{th} distribution center at p^{th} period.
caz_{jp}: Capacity of collection/inspection related to j^{th} collection/inspection center at p^{th} period.
X_{ijwvp}: Quantity of w^{th} product shipped from i^{th} production/recovery center to j^{th} distribution center by v^{th} transportation mode in p^{th} period.
U_{jkwvp}: Quantity of w^{th} product shipped from j^{th} distribution center to k^{th} customer zone by v^{th} transportation mode in p^{th} period.
Q_{kjwvp}: Quantity of w^{th} product shipped from k^{th} customer zone to j^{th} collection/inspection center by v^{th} transportation mode in p^{th} period.
P_{jiwvp}: Quantity of w^{th} product shipped from j^{th} collection/inspection center to i^{th} production/recovery center by v^{th} transportation mode in p^{th} period.
T_{jmwvp}: Quantity of w^{th} product shipped from j^{th} collection/inspection center to m^{th} disposal center by v^{th} transportation mode in p^{th} period.
CX_{ijvp} : 1, if v^{th} transportation mode is utilized for carrying products from i^{th} production/ recovery center to j^{th} distribution center at p^{th} period; 0, otherwise.
CU_{jkvp} : 1, if v^{th} transportation mode is utilized for carrying products from j^{th} distribution center to k^{th} customer zone at p^{th} period; 0, otherwise.
CQ_{kjvp} : 1, if v^{th} transportation mode is utilized for carrying products from k^{th} customer zone to j^{th} collection/inspection center at p^{th} period; 0, otherwise.
CP_{jivp} : 1, if v^{th} transportation mode is utilized for carrying products from j^{th} collection/inspection center to i^{th} production/recovery center at p^{th} period; 0, otherwise.
CT_{jmvp} : 1, if v^{th} transportation mode is utilized for carrying products from j^{th} collection/inspection center to m^{th} disposal center at p^{th} period; 0, otherwise.
W_i^n= 1, if a production/recovery center with n^{th} capacity level is opened at i^{th} location; 0, otherwise.

Y_j^n = 1, if a distribution center with n^{th} capacity level is opened at j^{th} location; 0, otherwise.
Z_j^n= 1, if a collection/inspection center with capacity level is opened at j^{th} location; 0, otherwise.
S_j^n= 1, if a hybrid center with n^{th} capacity level is opened at j^{th} location; 0, otherwise.
V_m^n = 1, if a disposal center with n^{th} capacity level is opened at m^{th} location; 0, otherwise.

The transportation costs between facilities include fixed and variable costs. Variable transportation costs are calculated by multiplying the transportation cost of one unit of product per unit of distance (e.g. one kilometer) by the corresponding shipping distance. In term of the above notation, the ILN design problem can be formulated as follows:

Minimum Total Costs:

$$\min\ TC = \mathrm{TC}_1 + \mathrm{TC}_2 + \mathrm{TC}_3 + \mathrm{TC}_4 \tag{1}$$

Subject to:

$$\mathrm{TC}_1 = \sum_i \sum_n f_i^n W_i^n + \sum_j \sum_n o_j^n Y_j^n + \sum_j \sum_n h_j^n Z_j^n + \sum_m \sum_n a_m^n V_m^n + \sum_j \sum_n g_j^n S_j^n \tag{2}$$

$$\mathrm{TC}_2 = \sum_w \sum_v \sum_p \sum_i \sum_j (FCX_{ijvp}) CX_{ijwvp} + \sum_w \sum_v \sum_p \sum_j \sum_k (FCU_{jkvp}) CU_{jkwvp} + \sum_w \sum_v \sum_p \sum_k \sum_j (FCQ_{kjvp}) CQ_{kjwvp} + \sum_w \sum_v \sum_p \sum_j \sum_i (FCP_{jivp}) CP_{jiwvp} + \sum_w \sum_v \sum_p \sum_j \sum_m (FCT_{jmvp}) CT_{jmwvp} \tag{3}$$

$$\mathrm{TC}_3 = \sum_w \sum_v \sum_p \sum_i \sum_j (cw_{iw} + VCX_{ijvp}) X_{ijwvp} + \sum_w \sum_v \sum_p \sum_j \sum_k (cy_{jw} + VCU_{jkvp}) U_{jkwvp} + \sum_w \sum_v \sum_p \sum_k \sum_j (cz_{jw} + VCQ_{kjvp}) Q_{kjwvp} + \sum_w \sum_v \sum_p \sum_j \sum_i (cr_{iw} + VCP_{jivp}) P_{jiwvp} + \sum_w \sum_v \sum_p \sum_j \sum_m (cv_{mw} + VCT_{jmvp}) T_{jmwvp} \tag{4}$$

$$\mathrm{TC}_4 = \sum_w \sum_j \sum_p WI_{jpw} WC_{jpw} \tag{5}$$

$$\sum_v \sum_j U_{jkwvp} = d_{kpw}; \qquad \forall k \in K, \forall p \in P, \forall w \in W \tag{6}$$

$$\sum_v \sum_j Q_{kjwvp} = \sum_{a=1}^{p} r_{kaw} d_{k(p-a+1)w} + \sum_{a=p+1}^{|p|} r_{kaw} d_{k(p-a+1+|p|)w}; \qquad \forall k \in K, \forall p \in P, \forall w \in W \tag{7}$$

$$WI_{jpw} + \sum_v \sum_i X_{ijwvp} = WI_{j(p+1)w} + \sum_v \sum_k U_{jkwvp}; \qquad \forall j \in J, \forall p \in P, \forall w \in W \tag{8}$$

$$\sum_v \sum_m T_{jmwvp} = s_w \sum_v \sum_k Q_{kjwvp}; \qquad \forall j \in J, \forall p \in P, \forall w \in W \tag{9}$$

$$\sum_v \sum_i P_{jiwvp} = (1 - s_w) \sum_v \sum_k Q_{kjwvp}; \qquad \forall j \in J, \forall p \in P, \forall w \in W \tag{10}$$

$$\sum_w \sum_v \sum_j X_{ijwvp} \le caw_{ip}; \qquad \forall i \in I, \forall p \in P \tag{11}$$

$$\sum_w WI_{jpw} + \sum_w \sum_v \sum_i X_{ijwvp} \le cay_{jp}; \qquad \forall j \in J, \forall p \in P \tag{12}$$

$$\sum_w \sum_v \sum_k Q_{kjwvp} \le caz_{jp}; \qquad \forall j \in J, \forall p \in P \tag{13}$$

$$\sum_w \sum_v \sum_j T_{jmwvp} \le \sum_n V_m^n\, cav_m^n; \qquad \forall m \in M, \forall p \in P \tag{14}$$

$$\sum_w \sum_v \sum_j P_{jiwvp} \le car_{ip}; \qquad \forall i \in I, \forall p \in P \tag{15}$$

$$caw_{ip} + car_{ip} \le \sum_n cap_i^n W_i^n; \qquad \forall i \in I, \forall p \in P \tag{16}$$

$$cay_{jp} + caz_{jp} \sum_n S_j^n \le \sum_n Y_j^n\, cap_j^n + \sum_n S_j^n\, cap_j^n ; \qquad \forall j \in J,\ \forall p \in P \tag{17}$$

$$caz_{jp} + cay_{jp} \sum_n S_j^n \le \sum_n Z_j^n\, cap_j^n + \sum_n S_j^n\, cap_j^n ; \qquad \forall j \in J,\ \forall p \in P \tag{18}$$

$$\sum_n W_i^n \le 1 ; \qquad \forall i \in I \tag{19}$$

$$\sum_n Y_j^n \le 1 ; \qquad \forall j \in J \tag{20}$$

$$\sum_n Z_j^n \le 1 ; \qquad \forall j \in J \tag{21}$$

$$\sum_n S_j^n \le 1 ; \qquad \forall j \in J \tag{22}$$

$$\sum_n V_m^n \le 1 ; \qquad \forall m \in M \tag{23}$$

$$\sum_n Y_j^n + Z_j^n + S_j^n \le 1; \qquad \forall j \in J \tag{24}$$

$$WI_{j1W} = WI_{j(|p|+1)W} = 0; \qquad \forall j \in J, \forall w \in W \tag{25}$$

$$\sum_w X_{ijwvp} \le CAX_{ijvp} \times CX_{ijvp} ; \qquad \forall i \in I, \forall j \in J, \forall v \in V, \forall p \in P \tag{26}$$

$$\sum_w U_{jkwvp} \le CAU_{jkvp} \times CU_{jkvp} ; \qquad \forall j \in J, \forall k \in K, \forall v \in V, \forall p \in P \tag{27}$$

$$\sum_w Q_{kjwvp} \le CAQ_{kjvp} \times CQ_{kjvp} ; \qquad \forall k \in K, \forall j \in J, \forall v \in V, \forall p \in P \tag{28}$$

$$\sum_w T_{jmwvp} \le CAT_{jmvp} \times CT_{jmvp} ; \qquad \forall i \in I, \forall j \in J, \forall v \in V, \forall p \in P \tag{29}$$

$$\sum_w P_{jiwvp} \le CAP_{jivp} \times CP_{jivp} ; \qquad \forall i \in I, \forall j \in J, \forall v \in V, \forall p \in P \tag{30}$$

$$P_{jiwvp}, X_{ijwvp}, U_{jkwvp}, Q_{kjwvp}, T_{jmwvp} \ge 0 \qquad \forall i \in I, \forall j \in J, \forall k \in K, \forall m \in M, \forall p \in P, \forall w \in W, \forall v \in V \tag{31}$$

$$W_i^n, Y_j^n, Z_j^n, S_j^n, V_m^n \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall m \in M, \forall n \in N \tag{32}$$

$$CX_{ijvp}, CU_{jkvp}, CQ_{kjvp}, CT_{jmvp}, CP_{jivp} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall m \in M, \forall v \in V, \forall p \in P \tag{33}$$

The objective function given in Eq. (1) minimizes sum of the fixed opening, transportation, operation, and warehousing costs through the whole logistics network. Term TC_1 in Eq. (2) is the fixed opening costs of production/recovery, distribution, collection, distribution/collection and disposal centers. Term TC_2 in Eq. (3) denotes fixed cost of transporting products in forward and reverse networks. Term TC_3 in Eq. (4) is the variable transportation and operation costs. Term TC_4 in Eq. (5) stands for warehousing cost in distribution and distribution/collection centers. Eq. (6) and Eq. (7) ensure that the demands of all customers are satisfied in each period for each product and returned products from all customers are collected in different periods by various transportation modes. Eqs. (8)-(10) are flow balance constraints at production/recovery, distribution, and collection/inspection centers in forward and reverse flows. Constraints (11)-(15) refer to capacity constraints on facilities. Constraints (16)-(18) represent total capacity constraints in production/recovery and distribution/collection center. Constrains (17) and (18) assure that locating distribution and collection centers at the same place, results in establishing hybrid (distribution/collection) centers. Constraints (19)-(23) are logical constraints associated with different capacity levels, these constraints certify that, at most, one capacity level can be assigned to a facility. Constraint (24) assures that only one of the distribution, collection or hybrid center is located at the same place. Eq. (25) is a constraint refers to warehousing amount in the initial and last periods. Constraints (26)-(30) refer to transportation capacity. Constraints (31)-(33) enforce the binary and non-negativity restrictions on corresponding decision variables.

5. An illustrative example

To illustrate the properties of the problem and the model, the proposed model has been applied to a fictitious, but practical problem. The example contains 3 potential production/recovery centers, 5 potential distribution, collection/inspection, and hybrid centers, 3 potential disposal centers and 4 customer zones. It is assumed that each facility have 4 possible capacity levels but the production, recovery, distribution, and collection capacity are the continuous decision variables. There are 4 periods, 3 transportation mode, and 3 various products in this example. Other parameters are generated randomly using uniform distributions (Pishvaee et al., 2010) specified in Tables 4 and 5.

Table 4
d_{kpw}, r_{kpw}, WC_{jpw} used in the example

Periods (p)	Demand (d_{kpw})	Quantity rate of return (r_{kpw})	Warehousing cost (WC_{jpw})
1	$\sim Unif[120,250]$	$\sim Unif[0.45,0.65]$	$\sim Unif[4,10]$
2	$\sim Unif[220,300]$	$\sim Unif[0.45,0.7]$	$\sim Unif[4,9]$
3	$\sim Unif[140,300]$	$\sim Unif[0.65,0.75]$	$\sim Unif[3,9]$
4	$\sim Unif[110,350]$	$\sim Unif[0.5,0.8]$	$\sim Unif[2,9]$

Table 5
The value of the parameters used in the example

Parameter	Range	Parameter	Range
f_i^n	$\sim Unif[40000,80000]$	cap_i^n	$\sim Unif[500,1800]$
o_j^n	$\sim Unif[15000,40000]$	cw_{iw}	$\sim Unif[10,13]$
h_j^n	$\sim Unif[12000,20000]$	cr_{iw}	$\sim Unif[6,7]$
g_j^n	$\sim Unif[20000,40000]$	cy_{jw}	$\sim Unif[2,5]$
a_m^n	$\sim Unif[14000,25000]$	cz_{jw}	$\sim Unif[2,9]$
cav_m^n	$\sim Unif[300,600]$	cv_{mw}	$\sim Unif[2.5,4.5]$
cap_j^n	$\sim Unif[400,2000]$	s_w	$\sim Unif[0.5,0.7]$
$VCX_{ijvp}, VCU_{jkvp}, VCQ_{kjvp}, VCP_{jivp}, VCT_{jmvp}$	$\sim Unif[1,10]$		
FCX_{ijvp}, FCU_{jkvp}, FCQ_{kjvp}, FCP_{jivp}, FCT_{jmvp}	$\sim Unif[30,90]$		
CAX_{ijvp}, CAU_{jkvp}, CAQ_{kjvp}, CAP_{jivp}, CAT_{jmvp}	$\sim Unif[100,500]$		

The test is carried out on a Pentium dual-core 2.50 GHz computer with 2 GB RAM. Using LINGO 8.0 with at most 25(min) elapsed time, the optimal solution is obtained as shown in Fig. 2 and Fig. 3. The optimal solution is W_i^n: $W_1^3 = W_3^4 = 1$; $Y_j^n : Y_5^2 = 1$; S_j^n: $S_1^4 = S_4^3 = 1$; $V_m^n : V_2^3 = 1$ yields 51804592.3 for the objective function.

According to results shown on Fig. 3, because of cost saving associated with opening hybrid distribution-collection centers the model considers the number of opening hybrid distribution-collection facilities is more than distribution and collection/inspection facilities. In addition, the optimal capacities for distribution, collection, production, and recovery operation in time periods are shown in Fig. 2 and 3.

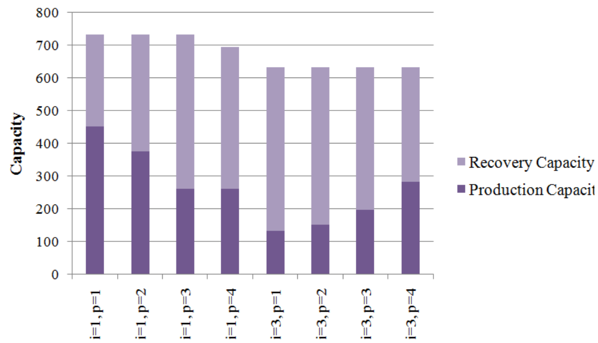

Fig. 2. Allocated production and recovery capacity in time periods

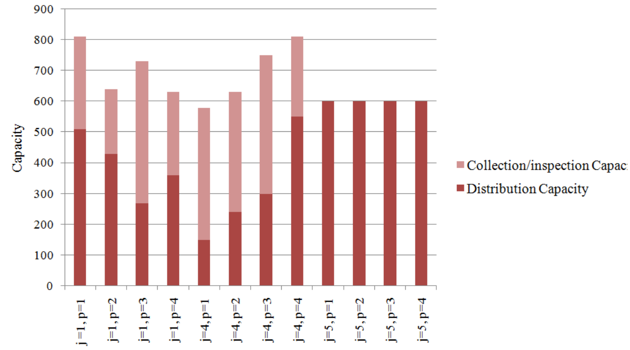

Fig. 3. Allocated distribution and collection/inspection capacity in time periods

6. Conclusions and future research

In this paper, we have presented a mixed integer non-linear programming (MINLP) model for forward/reverse logistics network design.

The logistics network considered in this paper is a closed-loop integrated forward/reverse logistics network including production/recovery, distribution, collection/inspection, customer, and disposal centers. The proposed model is able to integrate the forward and reverse network design decisions to avoid the sub-optimality leads from separated and sequential designs. In the proposed model demands, quantities of returned products, and warehousing costs are assumed to be periodic. Moreover, the model supports multiple capacity levels for each facility, various transportation modes and various products. In addition the model considers cost savings associated with combined distribution centers and collection/inspection centers by means of opening hybrid centers. To cope with the issue of time periods in integrated logistics network design, the proposed model determined the optimal production, recovery, distribution, and collection capacity in time periods. Computational results show that the capacitated model could handle data over time periods and therefore it can be concluded that the proposed MINLP model can be used as a powerful tool in practical cases.

For future research the model can be expanded to include the element of risk and uncertainty involved in the reverse logistics network design problem. For future development, addressing the multi-objective treatments of the reverse logistics network design which explicitly analyze the tradeoffs among cost, response time, market potential, and speedy returns in a multi-product integrated logistics network is a promising research avenue. Although exact solution for small incidents of the proposed model can be obtained by optimization software such as LINGO, meta-heuristic methods e.g. genetic algorithm (GA), are applicable for fast exploration in large scale problems and can be considered as an efficient research in future.

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