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A multi objective geometric programming approach for electronic product pricing problem

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A R T I C L E I N F O

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A B S T R A C T

Nowadays electronic commerce plays an important role in many business activities, operations, and transaction processing. The recent advances on e-businesses have created tremendous opportunities to increase profitability. This paper presents a multi-objective marketing planning model which simultaneously determines efficient marketing expenditure, service cost and product's selling price in two competitive markets. To solve the proposed model, we discuss a multi-objective geometric programming (GP) approach based on compromise programming method. Since our proposed model is a signomial GP and global optimality is not guaranteed for the problem, we transform the model to posynomial form. Finally, the solution procedure is illustrated via a numerical example and a sensitivity analysis is presented.

1. Introduction

Making an appropriate pricing and marketing strategy is a crucial management issue in E-commerce. The most fundamental rule of e-commerce is to have internet infrastructures such as email, websites, etc to build a bridge among customers, partners and suppliers (Lee et al., 2006). Today, the digital-good providers normally use internet to perform e-commerce transactions. They also try to implement many strategies based on the consumer's preference to gain more profit creating a competitive advantage against their competitors (Lee et al., 2006; Bhargava et al., 2001). Electronic products are all types of products sold on the internet based infrastructure. Therefore, pure digital and also physical products are typical examples of electronic products (Fathian et al., 2009). Optimal pricing marketing strategy plays an important role on electronic businesses (Chen et al., 2006; Chun & Kim, 2005). Some researchers consider the effects of pricing and marketing expenditure on products. For example, Demand of many products is normally considered as a function of price, marketing, research and development, etc. Sadjadi et al. (2005) and Serel (2009) studied the joint production planning using geometric

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programming. Fathian et al. (2009) studied the effects of pricing and marketing expenditure on electronic products. Furthermore, some researches assumed that the price or cost can also be considered as independent decision variable (Sadjadi et al., 2005; Elmaghraby & Keskinocak, 2003; Jornsten & Uboe, 2009).

In this paper, pricing and marketing strategies are determined in two competitive markets. In the proposed model, the primary objective is to maximize profit in the first market. Then the objective of the second market is to remain optimal while keeping the optimality of the first objective. Geometric Programming (GP) method is used to find the optimal solution of the proposed model. GP is a mathematical programming technique which has been widely used in engineering design research (Beightler & Philip; Duffin et al., 1967; Sadjadi et al., 2005; Jung & Klein, 2001; Abad, 1988; Kim & Klein, 1998; Lee, 1993; Lee et al., 1996) to determine the optimal price and lot size. We extend the previous optimal pricing of Fathian et al. (2009) where producer faces with two competitive markets. They investigated a pricing model for electronic goods, where the resulted model considers demand as a function of price, marketing expenditure and service business in two markets. The proposed model of this paper differs from the previous works where we consider optimal pricing for two distinctive markets. Furthermore, unlike most of the earlier researches the proposed model determines the optimal value of service cost in additional to optimal price and marketing expenditure. Finally, we apply compromise programming to solve the proposed model.

This paper is organized as follows: in the next section, we represent problem statement, notations and assumptions. Furthermore our proposed model is represented in section 2. The mathematical analysis and solution procedure is discussed in section 3. The implementation of the proposed method is illustrated via a numerical example and sensitivity analysis are given in section 4. Finally, in section 5, some conclusions are drawn from the discussion.

2. Problem statement

Consider an optimal pricing, marketing and service strategy for a single electronic product in two competitive markets where demand is affected by selling price, marketing expenditure and service cost and production cost depends on demand. The basic objective of the present study is to maximize the total profit in two markets. The following summarizes the necessary notation and assumption for the proposed model.

2.1 Notations

For $i=1, 2$ consider the following notations:

D_i	Demand per unit time	α_i	Price elasticity to demand
C_i	Production cost per unit	β_i	Lot size elasticity to production unit cost
M	Marketing expenditure (decision variable)	γ_i	Marketing expenditure elasticity to demand
S	Service expenditure (decision variable)	δ_i	Service cost elasticity to demand
P_i	Unit selling price (decision variable)	r_i	Scaling constant for unit production cost
k_i	Scaling constant for demand	π_i	Manufacturer's revenue

2.2 Assumptions

The following assumption holds for the proposed model of this paper.

1. Demand is a function of price, marketing expenditure and service expenditure in two markets i.e.,

$$D_i = k_i P_i^{-\alpha_i} M^{\gamma_i} S^{\delta_i} \qquad \alpha_i > 1,\ 0 < \gamma_i < 1,\ 0 < \delta_i < 1,\ i = 1,2. \tag{1}$$

The scaling constant k_i represent other related factors and the assumption $\alpha_i > 1$ confirms that demand increases as price is reduced. Note that parameters of Eq. (1) can be easily estimated by using linear regression to the logarithm of the function (Sadjadi et al., 2005).

2. The production unit cost is defined as a power function of demand and r_i is the scaling constant for unit production cost.

$$C_i = r_i D_i^{-\beta_i} \qquad 0 < \beta_i < 1. \tag{2}$$

The exponent β_i represents demand elasticity of unit production cost with $0 < \beta_i < 1$. This function is similar to the function considered by Lee (1993) and Fathian et al., (2009).

2.3 The proposed model

The proposed model of this paper determines the price, marketing expenditure and service cost in order to maximize the profit in two competitive markets. For each market, we have following objective function,

max *Π (P, M, S)*= Total revenue- Production cost- Marketing cost- Service cost (3)

Hence, we have below two objective functions:

$$\max \Pi_1(P_1, M, S) = P_1 D_1 - C_1 D_1 - MD_1 - SD_1, \quad \max \Pi_2(P_2, M, S) = P_2 D_2 - C_2 D_2 - MD_2 - SD_2 \tag{4}$$

Substituting Eqs. (1) and (2) in Eq. (4) and simplifying the total profit per unit time in two markets yields,

$$\begin{aligned}\max \Pi_1(P_1, M, S) &= k_1 P_1^{1-\alpha_1} M^{\gamma_1} S^{\delta_1} - r_1 k_1^{1-\beta_1} P_1^{\alpha_1\beta_1-\alpha_1} M^{\gamma_1-\beta_1\gamma_1} S^{\delta_1-\beta_1\delta_1} - k_1 P_1^{-\alpha_1} M^{\gamma_1+1} S^{\delta_1} - k_1 P_1^{-\alpha_1} M^{\gamma_1} S^{\delta_1+1} \\ \max \Pi_2(P_2, M, S) &= k_2 P_2^{1-\alpha_2} M^{\gamma_2} S^{\delta_2} - r_2 k_2^{1-\beta_2} P_2^{\alpha_2\beta_2-\alpha_2} M^{\gamma_2-\beta_2\gamma_2} S^{\delta_2-\beta_2\delta_2} - k_2 P_2^{-\alpha_2} M^{\gamma_2+1} S^{\delta_2} - k_2 P_2^{-\alpha_2} M^{\gamma_2} S^{\delta_2+1}.\end{aligned} \tag{5}$$

Note that both objective functions are signomial GP problems. As the global optimality is not guaranteed for a signomial problem (Duffin et al., 1967), the above problem is modified into the posynomial GP problem with one additional variable and constraint. This technique was developed by Duffin et al. (1967). It is assumed that there are lower bounds Z_1 and Z_2 for the objective functions such that maximization of Z_1 and Z_2 (or minimizing Z_1^{-1} and Z_2^{-1}) is equivalent to maximize the objective values. Therefore, the above problems are modified as follow,

$$\begin{aligned}&\min \quad Z_1^{-1} \\ &\text{subject to} \quad k_1 P_1^{1-\alpha_1} M^{\gamma_1} S^{\delta_1} - r_1 k_1^{1-\beta_1} P_1^{\alpha_1\beta_1-\alpha_1} M^{\gamma_1-\beta_1\gamma_1} S^{\delta_1-\beta_1\delta_1} - k_1 P_1^{-\alpha_1} M^{\gamma_1+1} S^{\delta_1} - k_1 P_1^{-\alpha_1} M^{\gamma_1} S^{\delta_1+1} \geq Z_1, \\ &\qquad P_1, M, S, Z_1 > 0.\end{aligned} \tag{6}$$

$$\begin{aligned}&\min \quad Z_2^{-1} \\ &\text{subject to} \quad k_2 P_2^{1-\alpha_2} M^{\gamma_2} S^{\delta_2} - r_2 k_2^{1-\beta_2} P_2^{\alpha_2\beta_2-\alpha_2} M^{\gamma_2-\beta_2\gamma_2} S^{\delta_2-\beta_2\delta_2} - k_2 P_2^{-\alpha_2} M^{\gamma_2+1} S^{\delta_2} - k_2 P_2^{-\alpha_2} M^{\gamma_2} S^{\delta_2+1} \geq Z_2, \\ &\qquad P_2, M, S, Z_2 > 0.\end{aligned} \tag{7}$$

Since, Z_1, $Z_2>0$, the above constraints can be rearranged. Hence, problems (6) and (7) can be transformed into the following forms,

$$\begin{aligned}
&\min \quad Z_1^{-1} \\
&\text{subject to} \quad k_1^{-1}P_1^{\alpha_1-1}M^{-\gamma_1}S^{-\delta_1}Z_1 + r_1k_1^{-\beta_1}P_1^{\alpha_1\beta_1-1}M^{-\beta_1\gamma_1}S^{-\beta_1\delta_1} + P_1^{-1}M + P_1^{-1}S \le 1, \\
&\qquad P_1, M, S, Z_1 > 0.
\end{aligned} \tag{8}$$

$$\begin{aligned}
&\min \quad Z_2^{-1} \\
&\text{subject to} \quad k_2^{-1}P_2^{\alpha_2-1}M^{-\gamma_2}S^{-\delta_2}Z_2 + r_2k_2^{-\beta_2}P_2^{\alpha_2\beta_2-1}M^{-\beta_2\gamma_2}S^{-\beta_2\delta_2} + P_2^{-1}M + P_2^{-1}S \le 1, \\
&\qquad P_2, M, S, Z_2 > 0.
\end{aligned} \tag{9}$$

Models (8) and (9) are primal posynomial geometric programming problems.

3. Mathematical analysis

3.1. Multi objective optimization problem

A multi objective optimization problem (MOP) is defined to determine a vector of decision variables within a feasible region to minimize or maximize a vector of objective functions that usually conflict with each other. Such a problem can be formulated as follow,

$$\begin{aligned}
&\max \quad \{f_1(X), f_2(X), \ldots, f_m(X)\} \\
&\text{subject to} \quad g(X) \le 0,
\end{aligned} \tag{10}$$

where X is vector of decision variables; $f_i(X)$ is the ith objective function; and $g(X)$ is constraint vector. A decision vector X dominates a decision vector Y (also written as $X \succ Y$) if:

$$f_i(X) \le f_i(Y) \quad \text{for all } i \in \{1,2,\ldots,m\} \tag{11}$$

and

$$f_i(X) < f_i(Y) \quad \text{for at least one } i \in \{1,2,\ldots,m\} \tag{12}$$

Pareto optimal vectors are decision vectors that are not dominated by any other decision vector. In these solutions no objective can be improved without getting worse from, at least, another objective. There are various solution methods to solve the MOP. Some of the most widely used techniques are: sequential optimization, weighting method, goal programming, goal attainment, distance based method and direction based method. For a comprehensive study of these approaches, see Szidarovsky et al. (1986). The set of multi-objective optimization problem is convex if all the objective functions and the feasible region are convex.

3.2. Compromise programming method

In this method, the distance between some reference point and the feasible objective region is minimized. The decision maker has to select the reference point and the matrix for measuring the distances. In this way, the multiple objective functions are transferred into a single objective function. We suppose that the weighting coefficient w_r are real numbers such that $w_r \ge 0 \ \forall r = 1,2,\ldots,k$ and $\sum_{r=1}^{k} w_r = 1$. The weighted L_q-problem for minimizing distance is stated as:

$$\min \quad L_q(f(x)) = \left(\sum_{r=1}^{k} w_r \mid f_r(x) - f_r(x^*) \mid^q \right)^{\frac{1}{q}}$$
$$\text{subject to} \quad x \in X \quad \text{for } 1 \le q \le \infty, \tag{13}$$

where X is a set of constraints, such that $X = \{x \in R^n \mid g_i(x) \le b_j,\ j = 1, 2, \ldots, m\}$.

3.3 Compromise programming method to solve the proposed model

The multi objective marketing problem may be solved by several techniques including hybrid method and compromise programming method which is implemented for the proposed model of this paper. In this method the objective functions are combined to a single objective function. Let $w_r \ge 0, r = 1, 2$ are the normalized weights (i.e. $w_1 + w_2 = 1$) corresponding to the objective functions Z_1 and Z_2. Π_1 and Π_2 are the ideal objective values of Z_1 and Z_2, respectively. Ideal objective values can be obtained by using GP method (see appendix A). According to Miettinen (1999) the weighted L_q-problem is as follows,

$$\min \quad U_q(P_1, P_2, M, S, Z_1, Z_2) = \left(w_1 (Z_1^{-1} - \Pi_1^{-1})^q + w_2 (Z_2^{-1} - \Pi_2^{-1})^q \right)^{\frac{1}{q}} \tag{14}$$

For $1 \le q \le \infty$.

Case 1. For *q=1* the problem (10) is given as:

$$\min \quad U_1(P_1, P_2, M, S, Z_1, Z_2) = w_1(Z_1^{-1} - \Pi_1^{-1}) + w_2(Z_2^{-1} - \Pi_2^{-1}) \tag{15}$$

Since w_1, w_2, Π_1 and Π_2 are independent parameters, we can rearrange the problem (11) as follow,

$$\min \quad U_1(P_1, P_2, M, S, Z_1, Z_2) = [w_1 Z_1^{-1} + w_2 Z_2^{-1}] - (w_1 \Pi_1^{-1} + w_2 \Pi_2^{-1}) \tag{16}$$

Hence, it is enough to solve the following problem,

$$\begin{aligned}
\min \quad & V_1(Z_1, Z_2) = w_1 Z_1^{-1} + w_2 Z_2^{-1} \\
\text{subject to } & k_1 P_1^{1-\alpha_1} M^{-\gamma_1} S^{-\delta_1} Z_1 + r_1 k_1^{-\beta_1} P_1^{\alpha_1\beta_1 - 1} M^{-\beta_1\gamma_1} S^{-\beta_1\delta_1} + P_1^{-1} M + P_1^{-1} S \le 1, \\
& k_2 P_2^{1-\alpha_2} M^{-\gamma_2} S^{-\delta_2} Z_2 + r_2 k_2^{-\beta_2} P_2^{\alpha_2\beta_2 - 1} M^{-\beta_2\gamma_2} S^{-\beta_2\delta_2} + P_2^{-1} M + P_2^{-1} S \le 1, \\
& P_1, P_2, M, S, Z_1, Z_2 > 0.
\end{aligned} \tag{17}$$

where $U_1(P_1, P_2, M, S, Z_1, Z_2) = (V_2(Z_1, Z_2) - w_1 \Pi_1^{-1} - w_2 \Pi_2^{-1})$. Model (17) is a posynomial GP and can be solved globally by its dual problem (Duffin et al., 1967).

Case 2. For *q=2* the problem (14) is given as:

$$\min \quad U_2(P_1, P_2, M, S, Z_1, Z_2) = [w_1(Z_1^{-1} - \Pi_1^{-1})^2 + w_2(Z_2^{-1} - \Pi_2^{-1})^2]^{\frac{1}{2}} \tag{18}$$

Problem (18) cannot be transformed into posynomial form. Hence, in order to minimize this problem, we consider the following objective function that is near to (18).

$$
\begin{aligned}
\min \quad & V_2(Z_1, Z_2) = w_1 Z_1^{-2} + w_2 Z_2^{-2} \\
\text{subject to} \quad & k_1 P_1^{1-\alpha_1} M^{-\gamma_1} S^{-\delta_1} Z_1 + r_1 k_1^{-\beta_1} P_1^{\alpha_1\beta_1 - 1} M^{-\beta_1\gamma_1} S^{-\beta_1\delta_1} + P_1^{-1} M + P_1^{-1} S \le 1, \\
& k_2 P_2^{1-\alpha_2} M^{-\gamma_2} S^{-\delta_2} Z_2 + r_2 k_2^{-\beta_2} P_2^{\alpha_2\beta_2 - 1} M^{-\beta_2\gamma_2} S^{-\beta_2\delta_2} + P_2^{-1} M + P_2^{-1} S \le 1, \\
& P_1, P_2, M, S, Z_1, Z_2 > 0.
\end{aligned} \tag{19}
$$

Model (19) can be solved globally by its dual problem (Duffin et al., 1967).

4. Numerical example and sensitivity analysis

Consider the following data

Market 1	$k_1 = 10^6$	$r_1 = 4$	$\alpha_1 = 3$	$\beta_1 = 0.03$	$\delta_1 = 0.2$	$\gamma_1 = 0.4$
Market 2	$k_2 = 10^6$	$r_2 = 9$	$\alpha_2 = 2.5$	$\beta_2 = 0.05$	$\delta_2 = 0.4$	$\gamma_2 = 0.3$

For above example, the ideal value of $\Pi_1(P_1, M, S)$ and $\Pi_2(P_2, M, S)$ are $\Pi_1 = 5967.7$ and $\Pi_2 = 9376.4$. Suppose we want to decide on optimal pricing, marketing and service strategy for a single electronic product in these two competitive markets. The solutions of the proposed model by compromise programming method for *q=1* and *q=2* for $w_1 = w_2 = 0.5$ are given in Table 1.

Table1

Optimal values of decision variables for $w_1 = w_2 = 0.5$

q	p_1^*	p_2^*	M^*	S^*	Z_1^*	Z_2^*	U_q
1	7.8189	13.4219	1.1899	0.9433	5566.2	7886.5	$1.6*10^{-5}$
2	7.5788	13.1684	1.1318	0.8462	5675.6	7644.9	$2.5*10^{-5}$

Figs. 2-3, illustrate the behavior of Z_1^* and Z_2^* for different values of w_1 and w_2.

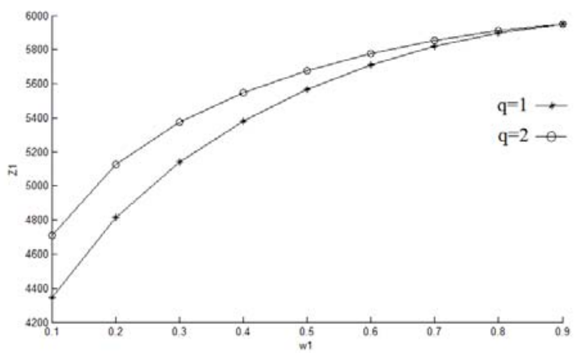

Fig. 1. Behavior of Z_1^* for different values of w_1

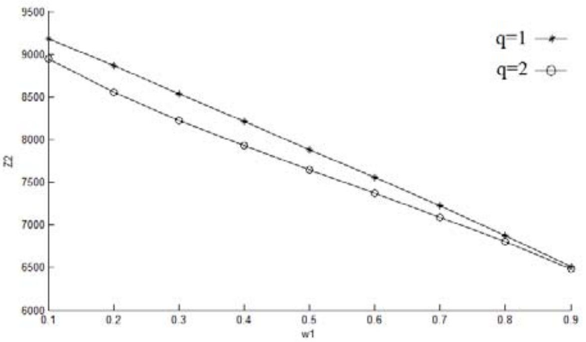

Fig. 2. Behavior of Z_2^* for different values of w_1

Fig. (1) and Fig. (2) show that by increasing w_1, Z_1^* increases as a curve and Z_2^* decreases for *q=1,2*.

5. Conclusion and future research

In this paper, we have presented a new multi-objective pricing and marketing planning. The proposed model was solved using compromise programming and the resulted model was transformed into a posynomial form to ensure that global solution is guaranteed. The implementation of the proposed model was studied using some numerical example and the results are discussed in details. As a future research, one can use multiple markets using game strategy.

Appendix A

In this section we are interested in finding the value of Π_1. This solution procedure is based on the algorithm explained by Duffin et al. (1967).Consider the following objective function:

$$\max\ \Pi_1(P_1, M, S) = k_1 P_1^{1-\alpha_1} M^{\gamma_1} S^{\delta_1} - r_1 k_1^{1-\beta_1} P_1^{\alpha_1\beta_1-\alpha_1} M^{\gamma_1-\beta_1\gamma_1} S^{\delta_1-\beta_1\delta_1} - k_1 P_1^{-\alpha_1} M^{\gamma_1+1} S^{\delta_1} - k_1 P_1^{-\alpha_1} M^{\gamma_1} S^{\delta_1+1},$$

where the model is a signomial GP problem.

$$\begin{aligned} &\min \quad T_1^{-1} \\ &\text{subject to}\ k_1 P_1^{1-\alpha_1} M^{\gamma_1} S^{\delta_1} - r_1 k_1^{1-\beta_1} P_1^{\alpha_1\beta_1-\alpha_1} M^{\gamma_1-\beta_1\gamma_1} S^{\delta_1-\beta_1\delta_1} - k_1 P_1^{-\alpha_1} M^{\gamma_1+1} S^{\delta_1} - k_1 P_1^{-\alpha_1} M^{\gamma_1} S^{\delta_1+1} \geq T_1, \\ &\qquad P_1, M, S, T_1 > 0. \end{aligned}$$

Since, $T_1 > 0$ the first constraint can be transformed into the following form,

$$\begin{aligned} &\min \quad T_1^{-1} \\ &\text{subject to}\ \ k_1^{-1} P_1^{\alpha_1-1} M^{-\gamma_1} S^{-\delta_1} T_1 + r_1 k_1^{-\beta_1} P_1^{\alpha_1\beta_1-1} M^{-\beta_1\gamma_1} S^{-\beta_1\delta_1} + P_1^{-1} M + P_1^{-1} S \leq 1, \\ &\qquad P_1, M, S, T_1 > 0. \end{aligned}$$

The above problem is a primal posynomial geometric programming with zero degree of difficulty. The corresponding dual problem is:

$$\max \quad \left(\frac{k_1^{-1}\lambda}{w_1}\right)^{w_1} \left(\frac{r_1 k_1^{-\beta_1}\lambda}{w_2}\right)^{w_2} \left(\frac{\lambda}{w_3}\right)^{w_3} \left(\frac{\lambda}{w_4}\right)^{w_4}$$

$$\begin{aligned} \text{subject to}\ \ & w_0 = 1, \\ & -w_0 + w_1 = 0, \\ & (\alpha_1 - 1)w_1 + (\alpha_1\beta_1 - 1)w_2 - w_3 - w_4 = 0, \\ & -\gamma_1 w_1 - \beta_1\gamma_1 w_2 + w_3 = 0, \\ & -\delta_1 w_1 - \beta_1\delta_1 w_2 + w_4 = 0, \\ & \lambda = w_1 + w_2 + w_3 + w_4, \\ & w_0, w_1, w_2, w_3, w_4 > 0. \end{aligned}$$

Therefore we have:

$$w_1 = 1,\quad w_2 = \frac{-(\alpha_1 - \gamma_1 - \delta_1 - 1)}{(\alpha_1\beta_1 - \beta_1\gamma_1 - \beta_1\delta_1 - 1)}, w_3 = \frac{(\beta_1 - 1)\gamma_1}{(\alpha_1\beta_1 - \beta_1\gamma_1 - \beta_1\delta_1 - 1)}, w_4 = \frac{(\beta_1 - 1)\delta_1}{(\alpha_1\beta_1 - \beta_1\gamma_1 - \beta_1\delta_1 - 1)}$$

$$\lambda = \frac{\alpha_1(\beta_1 - 1)}{(\alpha_1\beta_1 - \beta_1\gamma_1 - \beta_1\delta_1 - 1)}.$$

To have feasible solution for dual problem, we need some additional assumptions to ensure that w_1 to w_4 remain positive which are $\alpha_1 > 1 + \gamma_1 + \delta_1, \alpha_1\beta_1 - \beta_1\gamma_1 - \beta_1\delta_1 - 1 < 0.$ For $i = 1,\ldots,4$ let $\Delta_i = \frac{w_i}{\lambda}.$ Note that Δ_i, for $i = 1,\ldots,4$, are the weights of the terms in the constraints of model. In fact Δ_1 to Δ_4, represent the proportion of revenue (Δ_1), production cost (Δ_2), marketing cost (Δ_3) and service cost. The following relations must hold:

$$\Delta_1 = k_1 P_1^{1-\alpha_1} M^{-\gamma_1} S^{-\delta_1} T_1, \Delta_2 = r_1 k_1^{-\beta_1} P_1^{\alpha_1\beta_1 - 1} M^{-\beta_1\gamma_1} S^{-\beta_1\delta_1}, \quad \Delta_3 = P_1^{-1} M, \quad \Delta_4 = P_1^{-1} S.$$

where $\sum_{i=1}^{4} \Delta_i = 1$ at optimality (Lee & Kim, 1993). Using above equations, the optimal solution of the problem can be summarized as follow:

$$P_1 = \left(\frac{r}{\Delta_3^{\beta_1\gamma_1} \Delta_4^{\beta_1\delta_1} k^{\beta_1} \Delta_2} \right)^{\frac{1}{1-\alpha_1\beta_1+\beta_1\gamma_1+\beta_1\delta_1}} \quad S^* = \Delta_4 P^* \quad M^* = \Delta_3 P^*$$

Note that we can find the value of Π_2 by using a similar procedure.

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