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Uncertain portfolio optimization based on Dempster-Shafer theory

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CHRONICLE

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ABSTRACT

Nowadays, the selection and management of the optimal portfolio are the most primary fields of financial decision-making. Thereby, selecting a portfolio capable of providing the highest efficiency and, at the same time, the lowest investment risk has been turned into one of the most critical concerns among financial activists. However, in this selection, the two factors above are not the only determining ones. Various factors are affecting financial markets' behavior under different possible scenarios, which should be identified. In this paper, we examine the high sensitivity of the Iranian capital market to the exchange rate fluctuations in the different scenarios due to the lack of a unified view of the value of that rate among experts as one of the mentioned factors and obtain its value using Dempster–Shafer theory (DST). Then, a portfolio selection model that prefers stocks with higher ranks is proposed. Representative results of the real-life case study reveal that the submitted approach is productive and practically applicable.

1. Introduction

Nowadays, numerous factors (directly or indirectly) affect the capital market, which makes assets' value estimation uncertain or, in some cases, impossible. One of the crucial problems Iran's economy is suffering from in the last years is the US sanctions and their effects on the currency fluctuations and imbalance. Increasing flexibility and involving such a condition in analyses and financial decision-making has become an inevitable necessity in Iran's economy. In addition, according to the historical experience between 2012 and 2013, currency exchange rate fluctuations, particularly in US dollars in the free market, will affect the revenue gained in export-oriented and import-orient business groups. One of the most important subjects in financial issues is portfolio optimization. Regarding the most important and influential investigations in this area, we can point out Markowitz (1952) and Sharpe's models (Sharpe, 1963). Markowitz provided the fundamental portfolio model, a foundation for modern portfolio theory (MPT). He proposed that besides the returns of assets, the risk criteria should be considered in asset selection for investment.

One of the fundamental drawbacks of the Markowitz model is that he assumes returns and variances are accurate and can be calculated. However, the provided model's nature indicates its significant sensitivity to the changes made in the mentioned parameters. In this study, we have used DST in a new way in portfolio optimization issues to bring optimization issues closer to the real world. Therefore, prioritizing influential factors and acquiring behavioral scenarios in terms of practical aspects to calculate the behavior of those factors are fundamental problems in investigating and analyzing the changes in the stock market. In addition, there are situations where there is not enough historical data or the historical data are not stable in the real world. Available historical data should be used, but it is less critical when different scenarios are at the forefront. The stock

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market is no exception, affected by political, social, and economic factors. In this case, utilizing the viewpoint of experts in decisions can be very useful.

Plenty of researches have been done in portfolio selection, making an effort to ameliorate the efficiency of different nominal models of portfolio optimization under different conditions (Sharpe, 1963; Grossman & Stiglitz, 1980; Yunusoglu & Selim, 2013; Konno & Yamazaki, 1991; Pavlou et al., 2019; Xidonas et al., 2011). Portfolio selection is authenticated to be a multi-dimensional problem. A multi-criteria decision-making (MCDM) approach has been adopted to determine the constitutional multi-criteria nature of this problem by many (Thakur et al., 2018; Abdollahzadeh, 2002; Xidonas et al., 2011; Siskosa et al., 1999). Although all these researches made an effort to create efficiency in portfolio construction models, it is utterly difficult to engender an effective portfolio, especially in an uncertain dynamic atmosphere. In addition, when an incident or an unexpected event changes an investor's environmental conditions, the current strategy in the investment portfolio might alter. Such condition change requires a reasonable and regulated evaluation of the portfolio for striking a balance (Markowitz, 1952). Recent developments in the discipline of portfolio theory imply that the knowledge of future returns and variances, provided by classic point-estimation techniques, is not thoroughly trustful. It should be considered that problem data could be defined by a set of scenarios as risk and return are specified by randomness. Bradley and Crane (1972) first recognized the applicability of these techniques for financial purposes and, by Mulvey and Vladimirou (1992) for asset allocation. Mulvey et al. (1995) was the first to work on models of mathematical optimization where data values come in sets of scenarios while explaining the concept of robust solutions and introducing the robust model formulation. Guastaroba et al. (2009) surveyed different techniques and also compared the techniques by providing in-sample and out-of-sample analysis of the portfolios obtained by using these techniques to generate the rates of return. Barro and Canestrelli (2005) studied a dynamic portfolio management problem over a finite horizon with transaction costs and a risk objective function. They presumed that the uncertainty faced by the investor could be estimated using discrete probability distributions via a scenario approach. As a consequence, a scenario decomposition approach was used to solve the problem. Liesiö and Salo (2012) used a scenario-based approach to model uncertainty involved in the selection of a portfolio. The two key features of their approach include the use of a set inclusion technique to model incomplete information associated with planning scenarios and an integer programming technique to determine non-dominance relations between portfolios. Şakar and Köksalan (2013) evaluated the return through a regression equation for the single-index model and generated scenarios of index returns from a random walk model. Fulton and Bastian (2019) utilized Monte Carlo simulation to generate scenarios based on the assessment of sample means and covariance matrices from a multivariate normal distribution, omitted the outlier data based on percentiles, and resampled the remaining data to obtain three types of scenarios: Positive, negative or neutral outlook. Thakur et al. (2018) used the Fuzzy Delphi method in the first stage for critical factors identification. In this regard, they hierarchically organized the stocks via critical factors and historical data using Dempster–Shafer theory. Ultimately, the mentioned researchers used Ant Colony Simulation for portfolio optimization. The performance of obtained results was satisfactory, compared to the recent assets' efficiency. However, justified information about the scenario probabilities or the decision-makers (DMs) risk preferences may be effortful to evoke: for instance, in group settings, the DMs may have differing views about the scenario probabilities, and they may also illustrate different risk attitudes. DST is famous for its capability of dealing with uncertain and incomplete information, but its use remained unnoticed in-stock selection and portfolio recommendation. In this research, DST is applied for the first time to estimate the US dollar rate and its effect on the selection of stocks based on the sharp multi-factor model in the Tehran Stock Exchange.

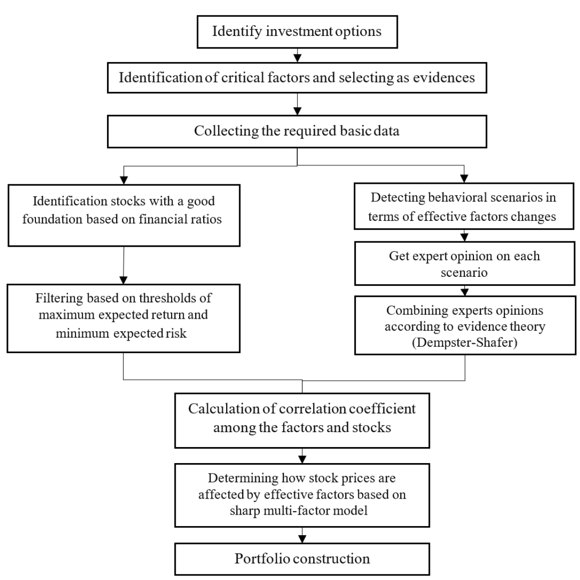

Fig. 1. Conceptual model of this research

After a general review of the study subject, it is worth mentioning that we will introduce DST to receive experts' viewpoints under ambiguous conditions. In section three, we will further introduce the used model in this study for portfolio optimization. In section four, the Symbiotic Organisms Search (SOS) algorithm will be investigated to solve the problem numerically. Section five will investigate how to select a portfolio in the form of a numerical instance and by taking advantage of actual datasets extracted from Tehran Stock Exchange. In section six, we will provide the obtained results and suggestions for further studies. The conceptual flow chart is depicted in Fig 1.

2. Dempster–Shafer theory

Dempster (1967) proposed a multivalued mapping from one space to another space. It has been used for statistical inference when we have multiple sample information, and we need to identify a single hypothesis. The evidence theory is one of the important instruments in defining uncertainty, providing the opportunity for a decision-maker (DM) to understand new probabilities. This theory copes with the discussion regarding existing beliefs of a situation or a system of situations. Individuals' beliefs are not the same when facing a single type of incidence, though they can be examined and combined by a particular method. Indeed, Dempster–Shafer theory has been formed based on a number of beliefs caused by observation and perception of evidence. The DST is successfully applied in various kinds of problems under uncertainty. However, the DST has not played a pivotal role in the portfolio selection problem. We will shortly explain DST for multi-criteria decision-making analysis under uncertainty conditions (Mohammadi & Makui, 2017). Presume $Y = \{y_1, y_2, \dots, y_m\}$ is a set of options, $E = \{e_1, e_2, \dots, e_n\}$ a set of criteria, $W = \{w_1, w_2, \dots, w_n\}$ a set of weights, such that $0 \le w_j \le 1, \quad 1 \le j \le n, \sum_{j=1}^{n} w_j = 1$. Presume that p is the assessment rank of $H_1, H_2, \dots, H_p$ for options' multi-criteria assessment. Presume that $\beta_{q,j}(y_i)$ shows a belief degree of the fact that e_j gauge has been evaluated for y_i with H_q degree. Where $0 \le \beta_{q,j}(y_i) \le 1$ and $\sum_{q=1}^{p} \beta_{q,j}(y_i) \le 1$. Presume that $S\big(e_j(y_i)\big)$ shows criterion evaluation value e_j for y_i option, defined as follows.

$$S(\mathrm{e}(y_i)) = \{H_q, \beta_{q,j}(y_i)\} \tag{1}$$

where H_q is an assessment degree, such that $1 \le q \le p$, $1 \le i \le m$, $1 \le j \le n$.

Firstly, we commute belief degree $\beta_{q,j}(y_i)$ about appraisal degree H_q concerning e_j gauge of y_i option to basic probable mass $m_{q,j}(y_i)$. Such that:

$$m_{q,j}(y_i) = w_j \beta_{q,j}(y_i) \tag{2}$$

where, $1 \le q \le p$, $1 \le i \le m$, $1 \le j \le n$.

Now, persume that $m_{H,j}(y_i)$ demonstrates the probable residual mass of e_j criteria concerning the appraisal of y_i option, which is construed as follows:

$$m_{H,j}(y_i) = \overline{m}_{H,j}(y_i) + \widetilde{m}_{H,j}(y_i)$$

$$\overline{m}_{H,j}(y_i) = 1 - w_j$$

$$\widetilde{m}_{H,j}(y_i) = w_j \left(1 - \sum_{q=1}^{p} \beta_{q,j}(y_i) \right)$$

where, $1 \le q \le p$,$1 \le i \le m$, and $1 \le j \le n$. Probable residual mass that is not allocated to each appraisal degree divided into two sections: The part pertinent to relative weights of criteria and the section pertaining to the violation in the assessment process.

$\overline{m}_{H,j}(y_i)$ is the first part of probable hesitancy mass, which is not yet allocated to appraisal degrees. Based on the fact that e_j gauge contributes to the appraisal process according to its weight, that is w_j, $\overline{m}_{H,j}(y_i)$ is a descending function of w_j. $\overline{m}_{H,j}(y_i)$ will be equal to 1, if the weight of a_j is $w_j = 0$. $\overline{m}_{H,j}(y_i)$ will be zero if a_j prevails the appraisal or $w_j = 1$. In other words, $\overline{m}_{H,j}(y_i)$ illustrates the degree to which other criteria could make a contribution to the appraisal process.

$\widetilde{m}_{H,j}(y_i)$ is the second part of the probable hesitancy mass, which is not yet allocated to an appraisal degree, and because of violation in the appraisal process of $S\big(e_j(y_i)\big)$ will ensue. $\widetilde{m}_{H,j}(y_i)$ will be zero if $S\big(e_j(y_i)\big)$ is impeccable or $\sum_{q=1}^{p} \beta_{q,j}(y_i) = 1$, otherwise $\widetilde{m}_{H,j}(y_i)$ will be a positive value. $\overline{m}_{H,j}(y_i)$ will be corresponding to w_j, whose positive values will lead to the next constraint's violation.

$G_{I(l)}$ as a subset of l number of first criteria will be defined as follows:

$G_{I(l)} = \{e_1, e_2, \dots, e_l\}$

Assume that $m_{q,I(l)}(y_i)$ is a probable mass that illustrates the support degree of a belief that all criteria prevailing in $G_{I(l)}$ subset emphasize that y_i option with H_q degree is evaluated. $m_{H,I(l)}(y_i)$ illustrates the probable hesitancy mass that is not allocated to appraisal degrees after all criteria in $G_{I(l)}$ subset are evaluated. $m_{q,I(l)}(y_i)$ and $m_{H,I(l)}(y_i)$ can be acquire amalgamating basic probable mass of $m_{q,j}(y_i)$ and $m_{H,j}(y_i)$ for all $q = 1,2,\dots,p$ and $j = 1,2,\dots,l$.

Evidence-based reasoning recursive algorithm can be summarized as follows:

$$\{H_q\}: m_{q,I(l+1)}(y_i) = K_{I(l+1)}\left[m_{q,I(l)}(y_i)m_{q,l+1}(y_i) + m_{H,I(l)}(y_i)m_{q,l+1}(y_i) + m_{q,I(l)}(y_i)m_{H,l+1}(y_i)\right],$$

$$m_{H,I(l+1)}(y_i) = \bar{m}_{H,I(l+1)}(y_i) + \tilde{m}_{H,I(l+1)}(y_i),$$

$$\{H\}: \bar{m}_{H,I(l+1)}(y_i) = K_{I(l+1)}\left[\bar{m}_{H,I(l)}(y_i)\bar{m}_{H,l+1}(y_i)\right],$$

$$\{H\}: \tilde{m}_{H,I(l+1)}(y_i) = K_{I(l+1)}\left[\tilde{m}_{H,I(l)}(y_i)\tilde{m}_{H,l+1}(y_i) + \bar{m}_{H,I(l)}(y_i)\tilde{m}_{H,l+1}(y_i) + \tilde{m}_{H,I(l)}(y_i)\bar{m}_{H,l+1}(y_i)\right],$$

$$K_{I(l+1)} = \left[1 - \sum_{u=1}^{p}\sum_{\substack{f=1 \\ f\neq u}}^{p} m_{u,I(l)}(y_i)m_{f,l+1}(y_i)\right]^{-1}$$

$K_{I(l+1)}$ is a normalization factor through which $\sum_{q=1}^{p} m_{q,I(l+1)}(y_i) + m_{H,I(l+1)}(y_i) = 1$. Remember that $m_{q,I(1)}(y_i) = m_{q,1}(y_i)$ $(q = 1,2,\dots,p)$ and $m_{H,I(1)}(y_i) = m_{H,1}(y_i)$. Besides, the criteria existing in G are numbered randomly. It means that the results of $m_{q,I(l)}(y_i)$, $(q = 1,2,\dots,p)$, and $m_{H,I(l)}(y_i)$ do not depend on the sum order of criteria.Besides, the criteria existing in G are numbered randomly. It means that $m_{q,I(l)}(y_i)$, $(q = 1,2,\dots,p)$and $m_{H,I(l)}(y_i)$ results do not depend on the sum order of the requirements. In the DST, after that entire n criteria are composed, the amalgamated belief degree β_q is directly computed from the following equation:

$$\{H_q\}: \beta_q(y_i) = \frac{m_{q,I(n)}(y_i)}{1 - \bar{m}_{H,I(n)}(y_i)}$$

$$\{H\}: \beta_H(y_i) = \frac{\tilde{m}_{H,I(n)}(y_i)}{1 - \bar{m}_{H,I(n)}(y_i)}$$

β_H indicates the violation degree existing in the assessment process. Thus, we will have:

$$\sum_{q=1}^{p} \beta_q(y_i) + \beta_H(y_i) = 1$$

Note that $\mathrm{B_H} = 0$ if the main assessment of $S\left(a_j(x_i)\right)$ is complete

3. Markowitz portfolio optimization model

The essential parameters in deciding on investment are the risk level and the return of invested assets. Appointing optimum investment ratios for the assets is the premier objective of constructing a portfolio such that the total return is maximized under an acceptable risk or minimized risk for a specific level of return for a given period of investment. People invest based on their expected utility and disregard today's consumption in anticipation of more advantages in the future. Optimal portfolio selection is often fulfilled by the exchange between return and risk so that the more risk of a portfolio, the more investors' expected efficiency. Concerning the goal of this paper, which is considering currency fluctuation and vagueness related to this subject in portfolio optimization, we developed our approach based on the model proposed by Markowitz, which will be explicated in the following. In this case, it is presumed that short sales are not allowed, and the weights of the assets in the portfolios are positive. Markowitz (1952) has developed a basilar model of MPT based on an issue relevant to rational investor behavior. Markowitz utilizes profit fluctuation as an investment risk. MPT is a quadratic model, where the variance of each stock or its square root, i.e., Standard Deviation (SD) is adjusted to measure the risk. He conveyed the issue as a Quadratic Programming (QP) aiming to minimize portfolio risk, provided that the expected efficiency is an invariable value. The standard form of the mean-variance model is as follows:

$$max\mu_p = \sum_{i=1}^{n} \mu_i x_i$$

subject to:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} = \sigma_p^2$$

$$\sum_{i=1}^{n} x_i = 1,$$

$$x_i \geq 0, \quad (i = 1, \dots, n)$$

$$x_j \geq 0. \quad (j = 1, \dots, n)$$

where the sum of stock weights must be equal to 1, and also the weight of each stock in the selected portfolio must be a real and non-negative number. This mathematical model that is the basic model delved in this paper obtains an efficient investment frontier after solving the portfolio optimization problem, considering different efficiencies, and determining optimal weights. In this regard, it is impossible to select a portfolio higher than the efficient investment frontier. Also, selecting a portfolio lower than the efficient frontier of investment is not suggested because a higher efficiency can be obtained at the same risk level in portfolio selection.

$$R = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2]\,[n\sum y^2 - (\sum y)^2]}} \tag{19}$$

4. The proposed model

Several assumptions regarding the system's behavior must be considered in order to analyze any real problem with a mathematical modeling tool. To start with, the efficiency parameter of the model fluctuates within a symmetric range, and all investors have an identical single-period time horizon. Besides, a trade in the market is costless, and personal incomes are tax-free such that investors do not differentiate between capital profit and dividend profit. In addition, inflation does not affect this problem. Moreover, no capital can solely affect the stock price according to sell and buy decisions. Finally, at a certain level of risk, investors prefer a higher efficiency level and seek a minimum level of risk for a certain level of efficiency. The portfolio analysis problem is as follows. Given such a set of predictions, determine the set of efficient portfolios; a portfolio is efficient if none other gives either (a) a higher expected return and the same variance of return or (b) a lower variance of return and the same expected return. We can see this analytically: suppose there are N securities; Let r_i be the discounted return of the i^{th} security; Let x_i be the relative amount invested in security i. Short sales are excluded, thus $x_i \geq 0$ for all i. The x_i are not random variables, but are fixed by the investor. Since the x_i are percentages we have $\sum x_i = 1$. Let σ_{ij} be the covariance between $r_i$and r_j (thus σ_{ii} is the variance of r_i).

Indexes:	
i, j	Indexes for stocks; i, j={1, 2, …, n}
G_m	Indexes of specified stocks; M={1, 2, …, M}
Parameters:	
r_i	Return of security
R_p	Return of portfolio
n	The number of stocks
M	A large amount
σ_i^2	Expected risk
σ_{ij}	Covariance between shares *i* and *j*
β_i	Measurement unit of systematic risk
p	The number of stocks should be selected
k	A unique upper bound for all stocks
Variables:	
x_i	The weight value of the stock *i* if selected
z_i	$\begin{cases} 1 & if\ the\ i^{th}\ security\ is\ selected; \\ 0 & if\ the\ i^{th}\ security\ is\ not\ selected; \end{cases}$

Thereby, we have:

$$MaxR_p = \sum_{i=1}^{n} r_i x_i \tag{20}$$

Eq. (20) is the final objective function, where r_i is independent of x_i. Since $x_i \geq 0$ for all i and $\sum x_i = 1$, R_p is a weighted average of r_i, with the x_i as non-negative weights.

Subjected to:

$$\sum_i \sum_j \sigma_{ij} x_i x_j = \sigma_i^2 \tag{21}$$

The set constraint (21) represents the acceptable risk in order to maximize the portfolio return.

$$L_m \leq \sum_{i \in G_m} x_i \leq U_m \tag{22}$$

The set constraint (22) put lower and upper bounds on each specified stock class.

$$Z_i \leq M x_i \tag{23}$$

$$\sum_i z_i \geq p \tag{24}$$

The set constraint (23) and (24) determine the selected stocks and put a lower bound on the number of invested stocks.

$$\sum_{i=1}^{n} x_i = 1 \tag{25}$$

The set constraint (25) indicates that the total budget must be allocated to different assets.

$$x_i \leq \mathrm{k} \tag{26}$$

The set constraint (26) considers a unique upper bound for all stocks.

$$x_i \geq 0 \tag{27}$$

The set constraint (27) defines no security may be held in negative quantities.

5. Illustrative example

In this research, the gathered data belong to 20 companies authorized by Tehran Stock Exchange, authorized in the time interval between 2019, April to 2021, July. The following constraint is applied for the selection of stock sets:

A) Their financial year ends on 20 March B) in the study period, they do not experience trading halt for more than six months C) their financial statements and information are complete and available D) Their monthly return is more than 10%.

As mentioned earlier, the correlation coefficient between each stock's price and dollar has captured more interest than before, which plays a pivotal role in the success or failure of the investment, especially in Iran's financial markets. In a considerable number of conducted investigations, researchers employed the Beta coefficient to assess the place of stocks existing in the market. In other words, they analyzed the correlation coefficient between stock price and stock exchange index. In the present study, the dollar exchange rate is employed as a pivotal factor in the stock exchange and other financial markets in Iran, according to the importance of exchange rate and its impact on the economy and domestic markets and also inappropriate efficiency of stock exchange index. In this section, the value of the dollar exchange rate, an uncertain parameter, will first be calculated by considering Dempster-Schaefer's theory. Afterward, the results of solving the model and the optimal values of the stock portfolio, including x_i and R_p, based on the Sharpe ratio and PPMC, will be presented. Accordingly, the belief degree $\beta_{n,i}$ about the dollar exchange rate B_i is ascertained by the DM_i. The results are shown in the Table1.

Table 1
Belief degree based on evidence theory

DM	Weight	Scenario A	Scenario B	Scenario C	Scenario D	B_H
		0.117	0.186	0.299	0.359	0.037
DM1	0.300	0.050	0.150	0.250	0.500	0.050
DM2	0.300	0.000	0.100	0.450	0.400	0.050
DM3	0.200	0.200	0.400	0.200	0.200	0.000
DM4	0.200	0.400	0.200	0.200	0.150	0.050

As mentioned above, the dollar exchange rate is predicted using Dempster-Schaefer's theory in the 285600 rial. Therefore, its percentage of deviation from the current amount (257000 rial) is %10. In this regard, using this information and considering the value of β between stock returns and dollar returns, is calculated. It should be noted that the correlation coefficient between changes in the dollar exchange rate and changes in the total index of the Iranian Stock Exchange, based on PPMC, is equal to 0.340. Finally, due to the Mixed Integer Non-linear Programming (MINLP) of the obtained model, the model is solved by GAMS 25.0.3 on a personal computer based on 2.8 GHz at 45 seconds since the computer with the ANTIGONE solver. The results are represented in Table 2.

Table 2
The optimal answer based on Markowitz Model

Portfolio variance	**0.0025**	**0.0050**	**0.0075**	**0.0100**	**0.0125**	**0.0150**	**0.0175**	**0.0200**	**0.0225**	**0.0250**
Portfolio returns	**0.1710**	**0.1770**	**0.1820**	**0.1850**	**0.1860**	**0.1890**	**0.1910**	**0.1980**	**0.2070**	**0.2090**
Stock1	0.0500	0.0500	0.0500	0.0500	0.0500	0.0490	0.0490	0.0490	0.0480	0.0480
Stock2	0.0010	0.0010	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock3	0.1890	0.1100	0.0500	0.0430	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000
Stock4	0.0010	0.0370	0.0970	0.3480	0.5000	0.5000	0.5000	0.3470	0.0010	0.0010
Stock5	0.0010	0.0010	0.0010	0.0490	0.0490	0.0490	0.0490	0.2990	0.3000	0.3000
Stock6	0.0010	0.0010	0.0000	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.3470
Stock7	0.0000	0.0000	0.0000	0.0000	0.0010	0.0010	0.0010	0.0010	0.3470	0.0010
Stock8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock9	0.2990	0.2990	0.2990	0.0010	0.0010	0.0010	0.0010	0.0000	0.0000	0.0000
Stock10	0.0000	0.0000	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock11	0.0000	0.0000	0.5000	0.0000	0.0000	0.0010	0.0010	0.0010	0.0010	0.0001
Stock12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock16	0.0010	0.0010	0.0010	0.0010	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock17	0.4570	0.5000	0.5000	0.5000	0.3480	0.3480	0.0980	0.0010	0.0010	0.0010
Stock18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0010	0.0010
Stock19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Stock20	0.0000	0.0000	0.0000	0.0070	0.0490	0.0500	0.3000	0.3000	0.3000	0.3000

After solving the model with the software Gams, we formed an optimal stock portfolio. As shown in Fig 2, by increasing the value of σ_i^2 due to the increase in the degree of risk-taking, the return of the optimal portfolio boost.

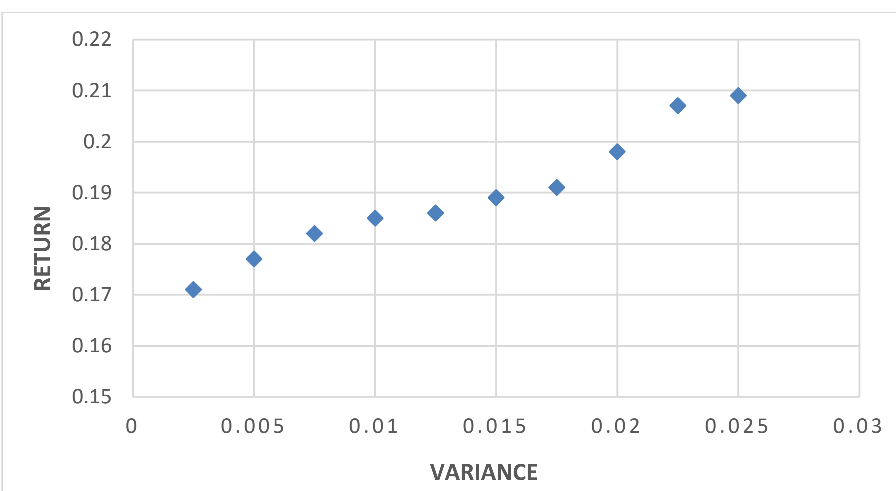

Fig. 2. Portfolio Returns Based on Different Levels of Expected risk

6. Conclusion

Uncertainty is an inherent feature in human mental judgments that should be given special attention to in decisions. The present study shows a stock portfolio optimization model considering the dollar exchange rate, which seeks to consider information deficiencies to improve performance using the logic based on Dempster-Schaefer's theory. It is also formulated in a given atmosphere, and then the effectiveness of the submitted model is assessed by the case study.

References

Barro, D., & Canestrelli, E. (2005). Dynamic portfolio optimization: Time decomposition using the maximum principle with a scenario approach. *European Journal of Operational Research*, *163*(1), 217-229.

Bradley, S. P., & Crane, D. B. (1972). A dynamic model for bond portfolio management. *Management science*, *19*(2), 139-151.

Dempster, A. P. (1967). Upper and lower probability inferences based on a sample from a finite univariate population. *Biometrika*, *54*(3-4), 515-528.

Fulton, L. V., & Bastian, N. D. (2019). Multiperiod stochastic programming portfolio optimization for diversified funds. *International Journal of Finance & Economics*, *24*(1), 313-327.

Guastaroba, G., Mansini, R., & Speranza, M. G. (2009). On the effectiveness of scenario generation techniques in single-period portfolio optimization. *European Journal of Operational Research*, *192*(2), 500-511.

Grossman, S. J., & Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. *The American Economic Review*, *70*(3), 393-408.

Konno, H., & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. *Management Science*, *37*(5), 519-531.

Liesiö, J., & Salo, A. (2012). Scenario-based portfolio selection of investment projects with incomplete probability and utility information. *European Journal of Operational Research*, *217*(1), 162-172.

Markowitz, H. M. (1952). *Portfolio selection*. Yale university press.

Mohammadi, S. E., & Makui, A. (2017). Multi-attribute group decision making approach based on interval-valued intuitionistic fuzzy sets and evidential reasoning methodology. *Soft Computing*, *21*(17), 5061-5080.

Mulvey, J. M., & Vladimirou, H. (1992). Stochastic network programming for financial planning problems. *Management science*, *38*(11), 1642-1664.

Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations research*, *43*(2), 264-281.

Pavlou, A., Doumpos, M., & Zopounidis, C. (2018). The robustness of portfolio efficient frontiers: A comparative analysis of bi-objective and multi-objective approaches. *Management Decision*, *57*(2), 300-313.

Şakar, C. T., & Köksalan, M. (2013). A stochastic programming approach to multicriteria portfolio optimization. *Journal of Global Optimization*, *57*(2), 299-314.

Sharpe, W. F. (1963). A simplified model for portfolio analysis. *Management science*, *9*(2), 277-293.

Siskos, Y., Spyridakos, A., & Yannacopoulos, D. (1999). Using artificial intelligence and visual techniques into preference disaggregation analysis: The MUDAS system. *European Journal of Operational Research*, *113*(2), 281-299.

Thakur, G. S. M., Bhattacharyya, R., & Sarkar, S. (2018). Stock portfolio selection using Dempster–Shafer evidence theory. *Journal of King Saud University-Computer and Information Sciences*, *30*(2), 223-235.

Yunusoglu, M. G., & Selim, H. (2013). A fuzzy rule based expert system for stock evaluation and portfolio construction: An application to Istanbul Stock Exchange. *Expert Systems with Applications*, *40*(3), 908-920.

Xidonas, P., Mavrotas, G., Zopounidis, C., & Psarras, J. (2011). IPSSIS: An integrated multicriteria decision support system for equity portfolio construction and selection. *European Journal of Operational Research*, *210*(2), 398-409.