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Sourcing and pricing strategies for two retailers in a decentralized supply chain system under supply disruption

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A B S T R A C T

This paper presents the decentralized supply chain with two suppliers and two competing retailers. It also investigates the sourcing and pricing strategies of two retailers in a decentralized supply chain system under a supply disruption environment. These retailers face their individual stochastic demand markets; however, they compete with each other through a two-stage price and service operation. The interactive dynamics among retailers is characterized, including the existence and uniqueness of the Nash equilibrium in service and price games demonstrated.

1. Introduction

It is important to consider an appropriate response against different risks in business since the intensification of global competition in the world, which is constantly changing. One of the important issues in supply chain management identifying competitors is risk and disruption management. The environment is often disrupted by some haphazard events, such as the promotion of sale, the raw material shortage, the new tax or tariff policy, machine breakdown, and so on. It is important to know how the supply chain can benefit under different disruptions. For example, in 2002, the US government raised imported steel tariffs by up to 30%, causing a big raw material cost disruption to automotive parts producers in the US (Stundza, 2002) or in the late 1990s, both Nokia and Ericsson depended on the supply of computer chips for their mobile phones from Philips Electronics Inc. in New Mexico. In March 2000, a fire caused by lightning rendered Philips unable to supply the chips for several weeks. Adopting different disruption management approaches resulted in vastly different destinies for the two communication companies. Nokia gained 3% for its mobile phones global market share from 27 to 30% in the year of 2000. However, Ericsson retreated from the phone

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handset production market in January 2001 with a loss of 1.68 billion dollars for its mobile phone division (Latour, 2001).

In this paper, we study the decentralized of a supply chain including two suppliers and two retailers. In a decentralized supply chain with long-term competition between independent retailers facing random demands and buying from two suppliers, we investigate how should retail prices and service levels be specified in such a way that the profits of each retailer is maximized. We assume that a retailer faces stochastic demand functions, which depend on his/her own retail price and service level as well as all other retailers. We first compute the optimal order quantity for suppliers then obtain the optimal equilibrium service levels and price for retailers. The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes our basic model. Numerical examples are provided in Section 4. Conclusions and suggestions for future research are given in Section 5.

2. Literature review

Competitive strategies and risk management have been important issues in the study of supply chain management. The supply chain risk management has been extensively studied over the past decade. We can divide articles in the field of competition into two categories, namely competition inside and competition between chains. There are many articles in the field of competition inside chain.

In the normal operation, our competition schedule is closely related to Tsay and Agrawal (2000) who studied a distribution system, in which a manufacturer supplies a common product to two retailers who use price and service quality to directly compete for end customers in a deterministic setting. Bernstein and Federgruen (2003) studied a two-echelon supply chain with one supplier, multiple competing retailers, fixed-ordering costs, and deterministic demand, which depend on the firm's prices. They also used a non-cooperative game theory to study coordination mechanisms. Moreover, Bernstein and Federgruen (2007) considered coordination mechanisms for supply chains under price and service competition. They treat service competition in the same way as price competition, with both affecting the deterministic portion of the demand and found that perfect coordination can be achieved if a whole sale price is combined with a backlogging penalty, and the sign of the penalty can be either positive or negative. There are a few articles in the field of competition outside chain. McGuire and Staelin (1983) considered two single-item manufacturers each selling through exclusive retailers, with partial substitutability between the products (i.e., the manufacturers compete in the retail markets). They explored additional channel management strategies and characterized coordination mechanisms.

Wu et al. (2009) considered a problem with a single buyer and *n* sellers. The buyer enters into option contracts with the sellers, and each contract fixes a price and a capacity. Then, the buyer observes the market price and demand and decides to exercise some of the contracts or to make purchases at a market price. The paper shows that option contracts coordinate this supply chain. Zhao and Shi (2011) considered two competing supply chains, each with multiple upstream suppliers producing complementary products and selling to a single buyer (e.g., assembler or retailer), who then sells the finished assembled product to a market that involves both demand uncertainty and competition. Its main research questions focus on what supply chain structure (integration vs. decentralization) and which contracting strategy a business should choose. Xiao and Yang (2008) developed a price–service competition model of two supply chains to investigate the optimal decisions of players under demand uncertainty. Each supply chain consists of one risk-neutral supplier and one risk-averse retailer who make decisions on both price and its service level.

For ease of reference, we shall provide a summary of the articles reviewed in the field of the chain competition in Table 1.

Table 1
Summary of the articles reviewed in the field of chain competition

Reference Name	competition		Demand		Competitive basis			Type of competition	
	inside chain	between chains	certain	uncertain	price	service	other	Horizontal	Vertical
Tsay & Agrawal , 2000	√		√		√	√			√
Bernstein & Federgruen, 2004	√			√	√	√		√	
Bernstein & Federgruen, 2003	√		√		√				√
Gurnani et al., 2005	√			√	√		√	√	
Bernstein, & Federgruen, Wu et al., 2009	√			√	√	√		√	
Bernstein & Federgruen, 2005	√			√	√			√	
Zhao, 2008	√			√	√		√		√
Leng & Parlar, 2009	√			√			√		√
Bernstein & Federgruen, 2004	√			√	√		√	√	
Xiao et al., 2010	√			√	√		√		√
McGuire & Staelin, 1983		√	√		√			√	
Boyaci & Gallego, 2004		√	√			√			√
Wu et al., 2009		√		√	√		√	√	
Zhao & Shi, 2011		√		√	√			√	
Xiao & Yang, 2008		√		√	√	√		√	
Baron et al., 2007		√	√				√	√	
Anderson & Bao, 2010		√		√	√				√

We now turn our attention to the literature on disruption in supply chains. Although the field of competition inside and outside chain have been addressed by many researchers, there are few studies in the field of disruption. Xiao and Qi (2006) studied the coordination of a supply chain with one manufacturer and two competing retailers after the production cost of the manufacturer was disrupted. They considered two coordination mechanisms, namely an all-unit quantity discount and an incremental quantity discount. For each mechanism, they developed the conditions under which supply chain was coordinated and discussed how the cost disruption might affect the coordination mechanisms. Chen and Xiao (2009) developed two coordination models of a supply chain consisting of one manufacturer, one dominant retailer and multiple fringe retailers to investigate how to coordinate the supply chain after demand disruption. They considered two coordination schedules, linear quantity discount schedule and Groves wholesale price schedule. Li et al. (2010) investigated the sourcing strategy of a retailer and the pricing strategies of two suppliers in a supply chain under an environment of supply disruption. They characterized the sourcing strategies of the retailer in a centralized and a decentralized system and derived a sufficient condition for the existence of an equilibrium price in the decentralized system when the suppliers were competitive. We investigate the pricing strategies of two suppliers in a supply chain under an environment of supply disruption, and we study the sourcing strategy of two retailers that compete on the price and service levels in a supply chain.

3. New mathematical model

We consider a decentralized supply chain system in which two suppliers sells a single product to two competing retailers. All four firms are assumed to be risk neutral and pursue expected profit maximization. The retailers buy the same product from two suppliers and sell the product to its customers in a single selling season or incur stock-out penalty cost. Each retailer faces a random demand, the distribution of which depends on not only his own retail price and service level but also all other retailer's. The uncertain source of supply is a state of the suppliers, which are subject to random failures. We assume that there are two types of failure: common-cause affects both suppliers. For example, a war may affect all the suppliers in a region, and supplier-specific failures, a supplier may still fail for some supplier-specific reason even if there is no common-cause failure. For example, equipment failure might affect one supplier but not the other supplier. If a supplier is in the failure state, no orders can be supplied. We also assume that supplier 1 is affected only by the common-cause failure but supplier 2 is affected by both types of failure. Wholesale price for supplier 1 is larger than wholesale price of supplier 2. We investigate the case where the retailers' stochastic demands have multiplicative form $D_i(p,s) = d_i(p,s).\ \varepsilon_i, i = 1,2$. Here, ε_i is a non-negative random variable with continuous, differentiable, reversible distribution function $F_i(.)$, which is independent of the price vector and service level vector , and its density function$\mathrm{f_i}(.)$ and $d_i(p,s) = a_1 - mp_1 + \rho p_2 + \gamma s_1 - \theta s_2$. Thus, the corresponding demand distribution function is $F_i(./d_i(p,s))$. Without loss of generality, it is assumed that $E\varepsilon_i = 1$; therefore, the expected demand is $ED_i(p,s) = d_i(p,s)$.

Each retailer's demand is decreasing in its own price and increasing in its own service, and one retailer's price increase can only increase its own rival's demand, and likewise one retailer's service increase can only decrease its rival's demand. In this article retailers depend on wholesale price of the two suppliers provided by two suppliers. The retailers choose their own order quantity and use the Nash equilibrium for determining prices and service levels for retailers. Service cost $\frac{\vartheta s_1^2}{2}$ (ϑ is a service factor) of a quadratic form implying diminishing returns. The following notations are used in our proposed model.

w_i	Whole sale price of a unit of the product offered by supplier *i*, (*i*=1, 2),
q_{il}	Order quantity of retailers *i* placed with suppliers *l*, (*i*=1,,2 and *l*=1, 2),
p_i	Retail price of retailers *i* (*i*=1, 2),
s_i	Service level of retailers *i* (*i*=1, 2),
k	Stock-out penalty cost for retailers,
a_i	Retailer *i* base market,
m_i	Sensitivity of a retailer's demand to its own retail price,
ρ_i	Sensitivity of a retailer's demand to its rival retail price,
γ_i	Sensitivity of a retailer's demand to its own service,
θ_i	Sensitivity of a retailer's demand to its rival service,
π_i	Retailer *i* profit,
α	Probability of a common-cause failure not occurring, where $0 < \alpha < 1$,
β	Probability that supplier 1does not fail conditional on a common-cause failure not occurring, where $0 < \beta < 1$,
w_i	Wholesale price of a unit of the product offered by a supplier.

We first obtain order quantity of retailers *i* placed with suppliers. For this purpose, we define the profit function for retailers.

$$\begin{aligned}\pi_i = \alpha\beta &\left[p_i\left[q_{i1} + q_{i2} - \left(q_{i1} + q_{i2} - D_i(p,s)\right)^+\right] - w_1 q_{i1} - w_2 q_{i2} - k[D_i(p,s) - q_{i1} - q_{i2}]^+ - \frac{\vartheta_i s_{i1}^2}{2}\right] \\ &+ \alpha(1-\beta)\left[p_i\left[q_{i2} - \left(q_{i2} - D_i(p,s)\right)^+\right] - w_2 q_{i2} - k[D_i(p,s) - q_2]^+ - \frac{\vartheta_i s_1^2}{2}\right] \\ &+ (1-\alpha)\left[-k[D_i(p,s) - 0]^+ - \frac{\vartheta_i s_{i1}^2}{2}\right]\end{aligned} \quad (1)$$

Since the order quantities of each retailer only influences the profit function of that retailer, so we can get the optimal order quantities as follows,

$$\begin{aligned}\frac{\partial \pi_i}{\partial q_{i1}} = \alpha\beta[p_i - p_i F(q_{i1} + q_{i2}) - w_1 - kF(q_{i1} + q_{i2}) + k = 0 \rightarrow (q_{i1} + q_{i2})^* = F^{-1}\left(\frac{p_i - w_1 + k}{p_i + k}\right) d_i \\ = F^{-1}(\nabla) d_i\end{aligned} \quad (2)$$

$$\frac{\partial^2 \pi_i}{\partial {q_{i1}}^2} < 0$$

$$\frac{\partial \pi_i}{\partial q_{i2}} = \alpha\beta[p_i - w_1 - p_i F(q_{i1} + q_{i2}) - kF(q_{i1} + q_{i2}) + k] + \alpha(1-\beta)[p_i - w_2 - p_i F(q_{i2}) - kF(q_{i2}) + k] + (1-\alpha)[0] = 0 \rightarrow q_2^* = F^{-1}\left(\frac{\alpha(p_1 - w_2 + k) - \alpha\beta(p_1 - w_1 + k)}{\alpha(1-\beta)(p_1 + k)}\right) d_i = F^{-1}(\Delta) d_i \quad (3)$$

$$\frac{\partial^2 \pi_i}{\partial {q_{i2}}^2} < 0$$

Thus, π_i is strictly concave in q_2^* and $(q_{i1} + q_{i2})^*$. Then, replacing the optimal order quantities into Eq. (1) yields the following,

$$\begin{aligned} \pi_r(p,s) = \alpha\beta &\left[(p_1 + k)\left[F^{-1}(\nabla)d_i - \int_0^{F^{-1}(\nabla)d_i} F\left(\frac{t}{d_i}\right)\right] - kd_i - w_1(F^{-1}(\nabla) - F^{-1}(\Delta))d_i - w_2 F^{-1}(\Delta)d_i - \frac{\vartheta s_1^2}{2}\right] \\ &+ \alpha(1-\beta)\left[(p_1 + k)\left[F^{-1}(\Delta)d_i - \int_0^{F^{-1}(\Delta)d_i} F\left(\frac{t}{d_i}\right)\right] - kd_i - w_2 F^{-1}(\Delta)d_i - \frac{\vartheta s_1^2}{2}\right] + (1-\alpha) \\ &= \left[\alpha\beta(p_1 + k)\int_0^{F^{-1}(\nabla)} tf\left(\frac{t}{d_i}\right) + \alpha(1-\beta)(p_1 + k)\int_0^{F^{-1}(\Delta)} tf\left(\frac{t}{d_i}\right) - k\right] d_i - \frac{\vartheta s_1^2}{2} \\ &= Q(p_i)d_i(p_i, s) - \frac{\vartheta s_i^2}{2} \end{aligned} \quad (4)$$

In this paper, we assume that all retailers simultaneously determine their private retail prices and service levels. We first get the equilibrium service level based on equilibrium price, and then determine equilibrium price. The equilibrium service level based on the Nash equilibrium is equal with the following formula,

$$\frac{\partial \pi_r}{\partial s_i} = \gamma Q(p_i) - \vartheta s_1 \rightarrow s_i^* = \frac{\gamma Q(p_i)}{\vartheta} \quad (5)$$

$$\frac{\partial^2 \pi_r}{\partial {s_i}^2} = -\vartheta$$

It implies that profit function is strictly concave for any given retail price vector p and all other retailers' service level. Therefore, we have a unique equilibrium service level. After obtaining the equilibrium of service level the retailer *i* expected demand function can be reduced as:

$$d_i(p_i) = a_i - m_i p_i + \rho_i p_j + \frac{{\gamma_i}^2 Q(p_i)}{\vartheta_i} - \theta_i \frac{\gamma_i Q(p_j)}{\vartheta_i} \quad (6)$$

and, we can replace the profit function as:

$$\pi_i(p) = Q(p_i)d_i(p) - \frac{\gamma^2 Q^2(p_i)}{2\vartheta} \quad (7)$$

To obtain the equilibrium price and service level, we must first solve the below system based on the Nash equilibrium.

$$\frac{\partial \pi_{1r}}{\partial p_1} = 0$$
$$\frac{\partial \pi_{2r}}{\partial p_2} = 0$$

Then based on Eq. (6), the equilibrium level of service is obtained.

However, because of the difficulty of solving the exact value of the derivative to obtain the equilibrium wholesale price and then to examine the price equilibrium behavior between retailers, we need the following assumptions.

Assumption 1: For all i =1, 2, ..., *N*, the retail price p_i is defined on $[w_i - k, \overline{p_i}]$, where $\overline{p_i}$ is the maximum price such $d_i(p)|_{p_i = \overline{p_i}} = 0$

Assumption 2: $\frac{\partial d_i(p_i)}{\partial p_i} \le 0, \frac{\partial d_i(p_i)}{\partial p_j} \ge 0, i \ne j$

Consequently, the following theorems are based on the above assumptions.

Proposition 1: There is the unique Nash equilibrium strategy of the game for the retail price.

Proof: From Eq. (7), first we take the derivative of the function of the wholesale price, based on the Nash equilibrium.

$$\frac{\partial \pi_i}{\partial p_i} = \alpha\beta \left[\left[F^{-1}(\nabla)d_i - \left(F^{-1}(\nabla)d_i - D(p,s)\right)^+\right] + (p_1 + k) \right.$$

$$* \left[-mF^{-1}(\nabla) + \frac{1}{\frac{\partial F\left(F^{-1}(\nabla)\right)}{\partial p_1}} d_i - \left[F\left(F^{-1}(\nabla)\right) * \left[\frac{1}{\frac{\partial F\left(F^{-1}(\nabla)\right)}{\partial p_1}} * d_i - mF^{-1}(\nabla) \right] \right] \right] + mk$$

$$\left. - w_1\left(\frac{1}{F'\left(F^{-1}(\nabla)\right)} - \frac{1}{F'\left(F^{-1}(\Delta)\right)}\right)d_i + w_1\left(F^{-1}(\nabla) - F^{-1}(\Delta)\right)m + mw_2F^{-1}(\Delta) - w_2 d_i \frac{1}{\frac{\partial F\left(F^{-1}(\Delta)\right)}{\partial p_1}} \right]$$

$$+ \alpha(1-\beta) \left[\left[F^{-1}(\Delta)d_i - \left(F^{-1}(\Delta)d_i - D(p,s)\right)^+\right] + (p_1 + k) \right.$$

$$* \left[-mF^{-1}(\Delta) + \frac{1}{\frac{\partial F\left(F^{-1}(\Delta)\right)}{\partial p_1}} d_i - \left[F\left(F^{-1}(\Delta)\right) * \left[\frac{1}{\frac{\partial F\left(F^{-1}(\Delta)\right)}{\partial p_1}} * d_i - mF^{-1}(\Delta) \right] \right] \right] + mk$$

$$\left. + mw_2F^{-1}(\Delta) - w_2 d_i \frac{1}{\frac{\partial F\left(F^{-1}(\Delta)\right)}{\partial p_1}} \right] + (1-\alpha)[mk]$$

Based on Assumption 1, we have:

$$\min p_i = w_1 - k$$

$$\lim_{p \to w_1 - k \,\&\, p = w_2 - k} \frac{\partial \pi_r}{\partial p_1}$$

$$= \alpha\beta \left[(p_1 + k - w_2) \left[\frac{1}{\frac{\partial F\left(F^{-1}(0)\right)}{\partial p_1}} d_i \right] + mk \right] + \alpha(1-\beta) \left[(p_1 + k - w_2) * \left[\frac{1}{\frac{\partial F\left(F^{-1}(0)\right)}{\partial p_1}} d_i \right] + mk \right]$$

$$+ (1-\alpha)[mk] = \alpha \left[(p_1 + k - w_2) * \left[\frac{1}{\frac{\partial F\left(F^{-1}(0)\right)}{\partial p_1}} d_i \right] + mk \right] + (1-\alpha)[mk] = mk \ge 0$$

$$max\ \ d_i(\overline{p}) = 0$$

$$\frac{\partial \pi_r}{\partial p_i} = Q'_i(p) d_i(p,s) - mQ_i(p), \lim_{p \to \bar{p}} \frac{\partial \pi_r}{\partial p_i} = -mQ(p) \leq 0$$

Since $\frac{\partial \pi_r}{\partial p_i}$ is continuous and based on the above relationships, there is a retail equilibrium price that is $\frac{\partial \pi_r}{\partial p_i} = 0$. After a supply disruption has occurred, in different conditions, the optimal strategy of the decentralized supply chain is as follows.

1-If $\alpha(p_1 - w_2) + \alpha k - \alpha\beta(k + p_1)\left(\frac{p_1 - w_2 + k}{k + p_1}\right) - \alpha(1-\beta)(k + p_1)F(q_2) + k \leq 0$ و $\alpha\beta[p_1 - p_1 F(q_1 + q_2) - w_1 - kF(q_1 + q_2) + k \leq 0$ both supplier 1 and supplier 2 are placed with zero order quantity and retailers pay penalty cost.

2-If $\alpha(p_1 - w_2) + \alpha k - \alpha\beta(k + p_1)\left(\frac{p_1 - w_2 + k}{k + p_1}\right) - \alpha(1-\beta)(k + p_1)F(q_2) + k \geq 0$ و $\alpha\beta[p_1 - p_1 F(q_1 + q_2) - w_1 - kF(q_1 + q_2) + k \leq 0$, the total optimal quantity order from retailer 1 is equal the optimal quantity ordered from supplier 2.

3- If $w_2 \leq w_1$ and $\alpha(p_1 - w_2) + \alpha k - \alpha\beta(k + p_1)\left(\frac{p_1 - w_2 + k}{k + p_1}\right) - \alpha(1-\beta)(k + p_1)F(q_2) + k \geq 0$ the optimal quantity ordered from supplier 1 is zero and the total optimal quantity order from retailer 1 is equal the optimal quantity ordered from supplier 2.

4- If $a_1 = a_2, m_1 = m_2, \gamma_1 = \gamma_2, \vartheta_1 = \vartheta_2, \theta_1 = \theta_2, \rho_1 = \rho_2$, retailers have the same price and service level.

5- If $a_1 \neq a_2, m_1 = m_2, \gamma_1 = \gamma_2, \vartheta_1 = \vartheta_2, \theta_1 = \theta_2, \rho_1 = \rho_2$, each of the retailers that have a greater base market, that retailer has a greater price and service levels.

4. Computational analysis

In this section, we present numerical examples to illustrate the theoretical results and analyze the changes of variables in different scenarios. We study example created by all possible combinations of the following parameters: α=0.8, β=0.7, k=3, w_1=7, w_2=8, a_1=a_2=30, m_1=m_2=0.7, γ_1=γ_2=0.4, ϑ_1= ϑ_2=4, θ_1=θ_2=0.4 and ρ_1=ρ_2=0.6. First, we determine the optimum amount of order quantity, but to avoid the complicated formulas and solve them, we approximate the exact equation with the following linear equation by using linear regression method:

$$q_{i2} = (0.806 + 0.0035 p_i) d_i$$

$$q_{i1} = (0.326 - 0.0009 p_i) d_i$$

Accordingly, the amount of profit for retailers based on the following formula will be as follows,

$$\pi_r(p) = Q(p_i) d_i(p) - \frac{\gamma_i^2 Q^2(p_i)}{2\,\vartheta_i}$$

$$\frac{\partial \pi_{r1}}{\partial p_1} = Q'(p_1) d_i(p_1) - m_1 Q(p_1)$$

$$\frac{\partial \pi_{r2}}{\partial p_2} = Q'(p_2) d_i(p_2) - m_2 Q(p_2)$$

$$d_i(p_i) = a_i - m_i p_i + \rho_i p_j + \frac{\gamma_i^2 Q(p_i)}{\vartheta_i} - \theta_i \frac{\gamma_i Q(p_j)}{\vartheta_i}$$

$$Q(p_i) = \alpha\beta(p_i + k)\left[\int_0^{F^{-1}(\nabla)} tf(t)\right] + \alpha(1-\beta)(p_i + k)\left[\int_0^{F^{-1}(\Delta)} tf(t)\right] - k$$

$$Q'(p_i) = \alpha\beta\left[\int_0^{F^{-1}(\nabla)} tf(t) + (1-\nabla)F^{-1}(\nabla)\right] + \alpha(1-\beta)\left[\int_0^{F^{-1}(\Delta)} tf(t) + (1-\Delta)F^{-1}(\Delta)\right]$$

$0 \le d_i(p_i) \le a$

$$\begin{aligned}\pi_i = 0.56\Bigg[p_i\left[1.1320 + 0.0026p_i - \left(1.1320 + 0.0026p_i - D_i(p,s)\right)^+\right] - w_1(0.326 - 0.0009p_i) - w_2(0.8060 \\ + 0.0035p_i) - 3[D_i(p,s) - 1.1320 - 0.0026p]^+ - \frac{\vartheta s_1^2}{2}\Bigg] \\ + 0.24\Bigg[p_1\left[0.8060 + 0.0035p_i - \left(0.8060 + 0.0035p_i - D_i(p,s)\right)^+\right] - 8(0.8060 + 0.0035p_i) \\ - 3[D(p,s) - 0.8060 - 0.0035p_i]^+ - \frac{\vartheta s_1^2}{2}\Bigg] + 0.2\left[-3[D_i(p,s) - 0]^+ - \frac{\vartheta s_1^2}{2}\right]\end{aligned}$$

After the profit function, we determine the equilibrium price and level of service. Thus, first to obtain the equilibrium retail prices, base on the Nash equilibrium, we derive the profit function with respect to retail price:

$$\frac{\partial \pi_{r1}}{\partial p_1} = Q'(p_1)d_i(p_1) - mQ(p_1)$$

$$\begin{aligned}Q(p_i) &= 0.56(p_i + 3)\left[\frac{1}{\sqrt{2\pi}}\int_0^{(1.1320+0.0026p_i)} te^{-\frac{1}{2}t^2}\right] + 0.24(p_i + 3)\left[\frac{1}{\sqrt{2\pi}}\int_0^{(0.8060+0.0035p_i)} te^{-\frac{1}{2}t^2}\right] - 3 \\ &= \frac{1}{\sqrt{2\pi}}\left[0.56(p_i + 3)\left[1 - e^{-\frac{1}{2}((1.1320+0.0026p_i)^2}\right] + 0.24(p_i + 3)\left[1 - e^{-\frac{1}{2}(0.8060+0.0035p_i)^2}\right]\right] - 3\end{aligned}$$

$$\begin{aligned}Q'(p_i) = 0.56\left[\frac{1}{\sqrt{2\pi}}\int_0^{((1.1320+0.0026p_i)} te^{-\frac{1}{2}t^2} + \left(1 - \frac{p_i - 4}{p_i + 3}\right) * ((1.1320 + 0.0026p_i)\right] \\ + 0.24\left[\frac{1}{\sqrt{2\pi}}\int_0^{(08060+0.0035p_i)} te^{-\frac{1}{2}t^2} + (1 - \frac{0.8(p_i - 5) - 0.56(p_i - 4)}{0.24(p_i + 3)})(0.8060 + 0.0035p_i)\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi_{ri}}{\partial p_i} = \Bigg[0.56\left[\frac{1}{\sqrt{2\pi}}\int_0^{((1.1320+0.0026p_i)} te^{-\frac{1}{2}t^2} + \left(1 - \frac{p_i - 4}{p_i + 3}\right) * ((1.1320 + 0.0026p_i)\right] \\ + 0.24\Bigg[\frac{1}{\sqrt{2\pi}}\int_0^{(08060+0.0035p_i)} te^{-\frac{1}{2}t^2} \\ + \left(1 - \frac{0.8(p_i - 5) - 0.56(p_i - 4)}{0.24(p_i + 3)}\right)(0.8060 + 0.0035p_i)\Bigg]\Bigg] d_i(p_1) \\ - m_i\Bigg[0.56(p_i + 3)\left[\frac{1}{\sqrt{2\pi}}\int_0^{(1.1320+0.0026p_i)} te^{-\frac{1}{2}t^2}\right] + 0.24(p_i + 3)\left[\frac{1}{\sqrt{2\pi}}\int_0^{(0.8060+0.0035p_i)} te^{-\frac{1}{2}t^2}\right] \\ - 3 = \frac{1}{\sqrt{2\pi}}\left[0.56(p_i + 3)\left[1 - e^{-\frac{1}{2}((1.1320+0.0026p_i)^2}\right] + 0.24(p_i + 3)\left[1 - e^{-\frac{1}{2}(0.8060+0.0035p_i)^2}\right]\right] - 3\Bigg]\end{aligned}$$

To obtain the optimum price and service level, we must first solve the below system based on game theory to help get the optimum amount of the price:

$$\frac{\partial \pi_{1r}}{\partial p_1} = 0,$$
$$\frac{\partial \pi_{2r}}{\partial p_2} = 0.$$

To obtain the equilibrium price and service level, we must first solve the below system based on game theory to help get the equilibrium amount of the price:

P_1=68.73 and P_2=68.7

According to $s_i^* = \frac{\gamma Q(p_i)}{\vartheta 1}$, the service level retailers are as follows:
S_1=0.9165 and S_2=0.9165

As shown in the sensitivity analysis in the previous chapter, when all variables of demand function are equal to all variables of demand function for other retailer, the retailer's price and service level both will be equal. We analyze the above case by changing the variables and show the changes in price and service levels of a retailer. In the first phase, we examine the impact of sensitivity of a retailer's demand to its own retail price on price and service levels:

Table 2
Computational result the impact of sensitivity of retailer's demand to its own retail price on price and service levels

$a_1=a_2=30, \gamma_1=\gamma_2=0.4, \vartheta_1=\vartheta_2=4, \theta_1=\theta_2=0.4, \rho_1=\rho_2=0.6$									
$m_1=m_2$	p_1	p_2	s_1	s_2	$m_1=m_2$	p_1	p_2	s_1	s_2
0.2	Int	int	int	int	0.8	56.69	56.69	0.674248	0.674248
0.3	Int	int	int	int	0.9	48.49	48.49	0.517856	0.517856
0.4	239.5	239.5	5.69244	5.69244	1	42.55	42.55	0.408982	0.408982
0.5	126.6	126.6	2.27868	2.27868	2	20.3	20.3	0.034639	0.034639
0.6	88.35	88.35	1.34265	1.34265	4	10.88	10.88	-0.10775	-0.10775
0.7	68.73	68.73	0.916508	0.916508	20	3.474	3.474	-0.21294	-0.21294

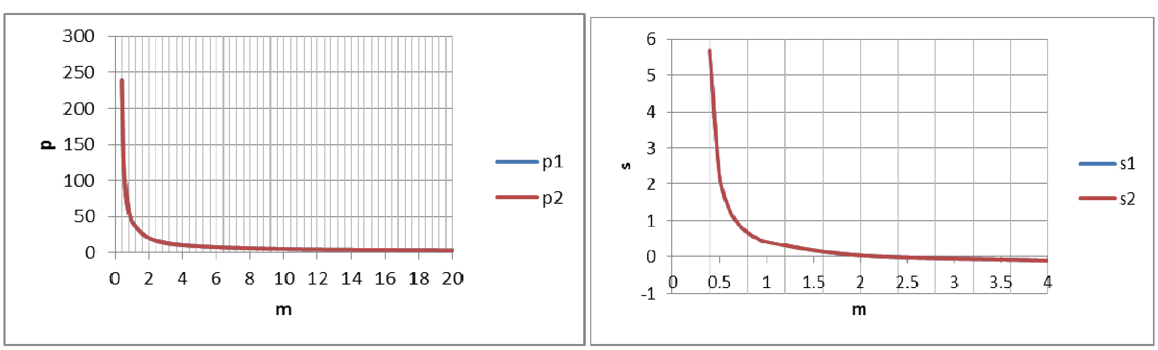

Fig. 1. Impact of sensitivity of their demand retailer to its own retail price on price and service levels

Table 2 shows an increase of the sensitivity of their retailer's demand with respect to its own retail price on price and service levels. If we look carefully into figures of Table 1 and Fig. 1, it is clear that when retail price is less than w_1-k, the service level is negative. In the second phase, we examine the impact of base market on price and service levels:

Table 3

Computational result the impact of base market on price and service levels

$m_1=m_2=0.7, \gamma_1=\gamma_2=0.4, \vartheta_1=\vartheta_2=4, \theta_1=\theta_2=0.4, \rho_1=\rho_2=0.6$									
$a_1=a_2$	p_1	p_2	s_1	s_2	$a_1=a_2$	p_1	p_2	s_1	s_2
1	15.47	15.47	-0.03957	-0.03957	30	68.73	68.73	0.9165	0.9165
2	18.64	18.64	0.0088	0.0088	40	83.02	83.02	1.2233	1.2233
5	26.29	26.29	0.13	0.13	70	123.47	123.47	2.1971	2.1971
10	36.5	36.5	0.3019	0.3019	120	188.07	188.07	4.035	4.035
20	53.55	53.55	0.6135	0.6135	-	-	-	-	-

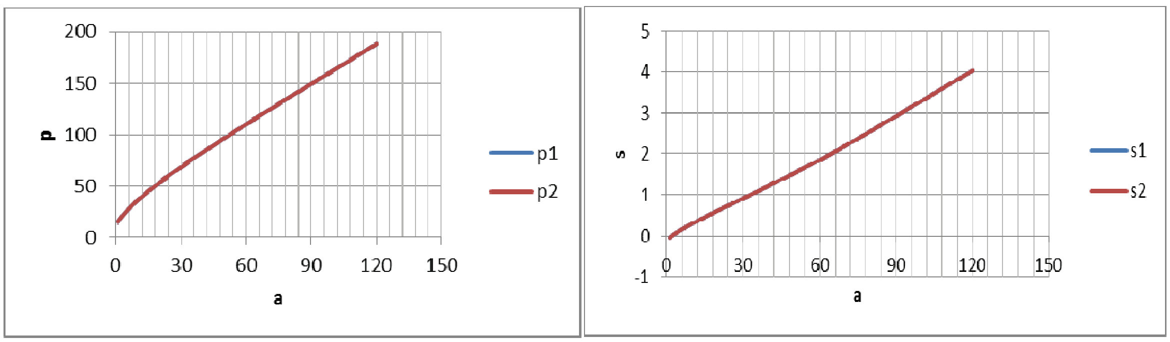

Fig. 2. Impact of base market on price and service levels

Table 3 shows an increase on the base market, price and service levels reduced. It is clear that when retail price is less than w_1-k, the service level is negative. In the third phase, we examine the impact of sensitivity of retailer 1 demand to its own retail price on price and service levels:

Table 4

Computational result the impact of sensitivity of retailer 1 demand to its own retail price

$m_2=0.7, a_1=a_2=30, \gamma_1=\gamma_2=0.4, \vartheta_1=\vartheta_2=4, \theta_1=\theta_2=0.4, \rho_1=\rho_2=0.6$									
m_1	p_1	p_2	s_1	s_2	m_1	p_1	p_2	s_1	s_2
0.2	int	int	int	int	0.8	60	64.65	0.74	0.832
0.3	int	int	int	int	0.9	53.47	61.6	0.612	0.77
0.4	131.61	97.2	2.411	1.547	1	48.35	59.15	0.515	0.722
0.5	99.65	82.9	1.6	1.22	2	26.12	48.38	0.127	0.515
0.6	80.96	74.37	1.17	1.03	4	14.815	42.71	-0.05	0.412
0.7	68.73	68.73	0.916508	0.916508	20	4.6	37.45	-0.19	0.318

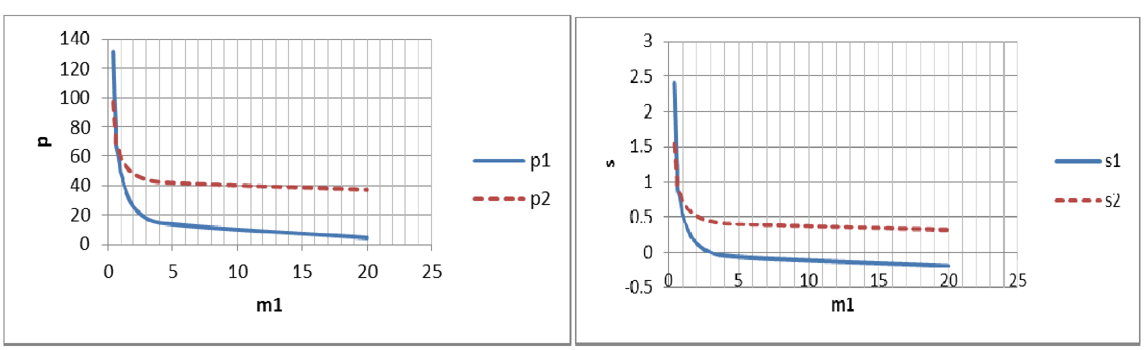

Fig. 3. Impact of the sensitivity of retailer 1 demand to its own retail price on price and service levels

Table 4 shows when only a retailer's demand to its own retail price changes, price and service levels for both the retailer decreases. However, as the retail price and service levels for that retailer increase, its coefficient are is reduced more than the arrival retailer. In the final phase, we examine the effect of changes in the probability of the disruption.

Table 5
Computational result the impact of changes in the probability of the disruption on price and service levels

$m_1 = m_2 = 0.7$, $a_1 = a_2 = 30$, $\gamma_1 = \gamma_2 = 0.4$, $\vartheta_1 = \vartheta_2 = 4$, $\theta_1 = \theta_2 = 0.4$, $\rho_1 = \rho_2 = 0.6$, $\beta = 0.7$					$m_1 = m_2 = 0.7$, $a_1 = a_2 = 30$, $\gamma_1 = \gamma_2 = 0.4$, $\vartheta_1 = \vartheta_2 = 4$, $\theta_1 = \theta_2 = 0.4$, $\rho_1 = \rho_2 = 0.6$, $\beta = 0.8$				
α	p_1	p_2	s_1	s_2	β	p_1	p_2	s_1	s_2
0.1	142.51	142.51	2.7085	2.7085	0.1	69.21	69.21	0.712	0.712
0.3	87.22	87.22	1.317	1.317	0.3	69.12	69.12	0.782	0.782
0.5	75.4	75.4	1.057	1.057	0.5	69	69	0.851	0.851
0.7	70.32	70.32	0.9496	0.9496	0.7	68.73	68.73	0.916	0.916
0.8	68.73	68.73	0.9165	0.9165	0.8	68.52	68.52	0.9476	0.9476
0.95	66.98	66.98	0.88	0.88	20	4.6	37.45	-0.19	0.318

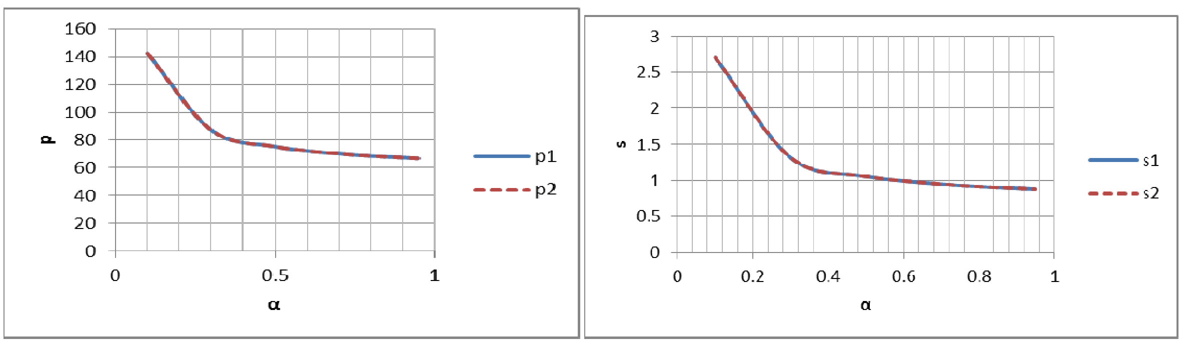

Fig. 4. Impact of changes in the probability of the disruption on price and service levels

5. Conclusions and further research

In this paper, we have considered two suppliers and two competing retailers in the decentralized of a supply chain. In this supply chain, there have been two retailers under an environment of supply disruption. We have investigated the sourcing and pricing strategies for them. These retailers have faced their individual stochastic demand markets. Retailers have competed with each other through a two-stage operation of the price and the service. Because the chain has been decentralized, members of the chain have followed for the maximum profit to themselves; therefore, in this study based on the game theory, a model has presented to achieve balance. According to various conditions, the retailer might prefer to supply his/her own order quantity from one supplier or two suppliers or to pay stock-out penalty cost but not to order from suppliers. Because risk management was considered in recent decades, and according to not been completely investigated risk in a supply chain for all elements of the chain, for future studies, researchers can evaluate disruption in production, sales, network and locating risks in chain for elements of the supply chain.

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