Management Science Letters 2 (2012) 1083–1096

Contents lists available at GrowingScience

Management Science Letters

homepage: www.GrowingScience.com/msl

Supercritical boiler material selection using fuzzy analytic network process

Saikat Ranjan Maity[a] and **Shankar Chakraborty**[b*]

[a]*Department of Production Engineering, Haldia Institute of Technology, Haldia - 721657, East Midnapur, India*
[b]*Department of Production Engineering, Jadavpur University, Kolkata - 700032, India*

ARTICLE INFO

Article history:
Received October 15, 2011
Accepted 10 March 2012
Available online
12 March 2012

Keywords:
Supercritical boiler material
Fuzzy theory
Analytic network process
Supermatrix
Priority vector

ABSTRACT

The recent development of world is being adversely affected by the scarcity of power and energy. To survive in the next generation, it is thus necessary to explore the non-conventional energy sources and efficiently consume the available sources. For efficient exploitation of the existing energy sources, a great scope lies in the use of Rankin cycle-based thermal power plants. Today, the gross efficiency of Rankin cycle-based thermal power plants is less than 28% which has been increased up to 40% with reheating and regenerative cycles. But, it can be further improved up to 47% by using supercritical power plant technology. Supercritical power plants use supercritical boilers which are able to withstand a very high temperature (650-720˚C) and pressure (22.1 MPa) while producing superheated steam. The thermal efficiency of a supercritical boiler greatly depends on the material of its different components. The supercritical boiler material should possess high creep rupture strength, high thermal conductivity, low thermal expansion, high specific heat and very high temperature withstandability. This paper considers a list of seven supercritical boiler materials whose performance is evaluated based on seven pivotal criteria. Given the intricacy and difficulty of this supercritical boiler material selection problem having interactions and interdependencies between different criteria, this paper applies fuzzy analytic network process to select the most appropriate material for a supercritical boiler. Rene 41 is the best supercritical boiler material, whereas, Haynes 230 is the worst preferred choice.

1. Introduction

A boiler is a closed vessel in which water or other fluid is heated. The heated or vaporized fluid exits from the boiler for use in various processes or heating applications. When the operating pressure is around 19 MPa and temperature is below 374˚C in the evaporator part of the boiler, it is called subcritical. It means that there is a non-homogeneous mixture of water and steam in the evaporator part of the boiler. In this case, a drum-type boiler is used because the steam needs to be separated from water in the drum of the boiler before it is superheated and led into the turbine (Steingress,

---

* Corresponding author. Tel. + 091-033-2414-6153
E-mail addresses: s_chakraborty00@yahoo.co.in (S. Chakraborty)

doi: 10.5267/j.msl.2012.03.012

2001; Steingress et al., 2003). Supercritical boiler is able to produce the state of water where there is no clear distinction between the liquid water and water vapour (they become a homogenous fluid). Water reaches this state at a pressure above 22.1 MPa and temperature above 374˚C. The thermal efficiency of a supercritical power plant greatly depends on how much of energy fed into the supercritical boiler is converted with the help of steam turbine which is coupled with the generator into electrical energy. If the energy input to a supercritical boiler is kept constant, selecting elevated pressures and temperatures for the water-steam cycle can increase the output. The efficiency of Rankin cycle-based thermal power plants is less than 28% which has been increased up to 40% with reheating and regenerative cycles. Now, these power plants are using supercritical boilers with improved process and component quality which are able to generate elevated steam conditions while increasing efficiencies up to 47% with much lower emission of green house gases. An increase in cycle efficiency from 28% to 47% in supercritical boilers decreases CO_X and NO_X emissions by more than 30% (Perrine and Fishburn, 2008). This huge effect on environment makes a compelling case for both the developed and developing countries to switch to supercritical power plants.

A supercritical boiler consists of several components, like evaporator, superheater and reheater as primary heating elements, whereas, secondary heating elements include preheater and economiser. The entire supercritical boiler can work in a severe environment because all of its components are able to withstand a very high temperature and pressure. Older supercritical boilers are generally made of ferritic and austenitic stainless steels. But these materials have low thermal conductivity and high thermal expansion resulting in high thermal stress and fatigue cracking (Shingledecker et al., 2005). Currently, different types of heat-resistant materials, like super alloys and refractory alloys are extensively used for making supercritical boilers. Super alloys have different types which are referred to as iron-base, cobalt-base and nickel-base super alloys. Recent advancements in technology have resulted in modern supercritical boiler designs being increasingly more competent, cost effective and consistent. The basic design aspects for a supercritical boiler are selection of its material and the corresponding improved thermal efficiency. The supercritical boiler material should have high stiffness at elevated temperature, high creep strength, high specific heat, high thermal conductivity and high melting temperature.

To meet such varied requirements, super alloys of different chemical compositions, often produced by special metallurgical processes, have been developed. These super alloys are generally available with proprietary nomenclature and designations. So, it is rather difficult for the boiler designer to select the most suitable super alloy for supercritical boiler design. The challenge for the designer is thus to go away from the simple ferritic or austenitic stainless steels and select the most suitable supercritical boiler material for maximum thermal efficiency and cost effective production.

Thus to fulfill the necessities of the thermal power plants, researchers have paid continual attention to develop suitable heat-resistant materials for supercritical and ultrasuper critical boilers. Viswanathan and Bakker (2001) observed that for higher temperatures, austenitic steels and Ni-base super alloys were the most suitable materials. Advanced austenitic stainless steels have been found available for super and reheater tubing to operate at temperatures up to 650˚C and possibly 700˚C. None of those steels have been approved by the ASME Boiler Code Group. Higher strength materials have been needed for upper water walls of boilers with steam pressure above 24 MPa. A high-strength 2% Cr steel, recently approved by ASME as T-23, has been the preferred material for supercritical boiler. Blum and Vanstone (2003) listed a wide range of alloys available for high temperature applications and concluded that the selection of a specific alloy would depend on its end use requirements. Ferritic steels have been used up to approximately 566˚C and Ni-base Nimonic alloys have been typically used for higher temperatures. Nimonic 80A and a few proprietary alloys (such as, Refractaloy 26) have been appeared to be good candidates for temperatures up to 593˚C. Waspaloy has been apparently preferred up to 700/720˚C. Two other relatively new alloys, e.g. Inconel 740 and Allvac 718-Plus could be considered for ultrasupercritical temperature applications, based on their good combination of creep strength and ductility. Allvac 718-Plus has been developed as an alloy intermediate in composition between standard alloy 718 and Waspaloy, with more temperature withstandability and creep strength at 700˚C. Viswanathan et al. (2006) aimed at identifying, evaluating and selecting materials needed for construction of the critical components of coal-fired boilers capable of operating at 760˚C temperature and 35 MPa steam pressure. The economic viability of such a plant was explored and the candidate alloys applicable to various ranges of temperature were identified.

It is clearly revealed that the precedent researchers have mainly paid their attention on the development of high-temperature materials suitable for supercritical boilers using mechanical and chemical properties characterization approaches. So, there is a scarcity of mathematical models in the domain of supercritical boiler material selection and correspondingly, an ardent need is felt to have sound mathematical techniques to search out the most appropriate material for a supercritical boiler from a list of several feasible alternatives. This paper proposes the application of fuzzy analytic network process for selection of the most suitable material for a supercritical boiler while considering interactions and interdependencies between the considered selection criteria. The obtained results are fairly acceptable, having potential for supercritical boiler design and manufacture.

2. Fuzzy analytic network process

Many decision-making problems cannot be structured hierarchically because they involve the interaction and dependence of higher level elements on a lower level element (Saaty, 1996). Structuring a problem involving functional dependence allows for feedback among the clusters. This is a network system. Saaty (1996) suggested the use of analytic hierarchy process (AHP) to solve the problem of independence on alternatives or criteria, and the use of analytic network process (ANP) to deal with the problem of dependence among alternatives or criteria. The ANP is a generalization of AHP. The AHP represents a framework with a uni-directional hierarchical relationship, whereas, the ANP allows for complex interrelationships among the decision levels and attributes. The ANP feedback approach replaces hierarchies with networks in which the relationships between levels are not easily represented as higher or lower, dominated or being dominated, and directly or indirectly (Meade and Sarkis, 1999). For instance, not only does the importance of the criteria determine the importance of the alternatives as in a hierarchy, but also the importance of the alternatives may have impact on the importance of the criteria (Saaty, 1996). Therefore, a hierarchical structure with a linear top-to-bottom approach is not applicable for a complex system.

2.1 Fuzzy numbers

The fuzzy sets are defined in terms of membership functions. Membership functions relative to X represent fuzzy subsets of X. The membership function representing a fuzzy set is usually denoted as μ_A. For an element x of X, the value $\mu_A(x)$ is called the membership degree of x in the fuzzy set. This function assigns to each element x of the universal set X a number $\mu_A(x)$ in the unit interval [0,1]. The membership degree $\mu_A(x)$ quantifies the grade of membership of the element x to the fuzzy set. An element x really belongs to A if $\mu_A(x) = 1$ and clearly does not if $\mu_A(x) = 0$. Fuzzy numbers may be of almost any shape (though conventionally they are required to be convex and to have finite area), but frequently they are triangular (piecewise linear), s-shaped (piecewise quadratic) or normal (bell shaped). Fuzzy numbers may also be trapezoidal, with an interval within which the membership is 1; such numbers are called fuzzy intervals (Siler and Buckley, 2005). A triangular fuzzy number $\bar{A}$ can be denoted by three real numbers (l, m, u). The parameters l, m and u respectively denote the smallest possible value, the most promising value and the largest possible value. Then, the membership function $\mu(x)$ of a triangular fuzzy number is expressed by the following equation:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \ x < l, x > u \\ \dfrac{x-l}{m-l}, l \le x < m \\ \dfrac{x-u}{m-u}, m \le x \le u \end{cases} \tag{1}$$

Each fuzzy number is defuzzified using the centroid method (Vahdani et al., 2011). For triangular fuzzy numbers, $\tilde{A}_1 = (l_1, m_1, u_1)$, the defuzzified centroid value is obtained applying the following expression:

$$\overline{A_1} = \frac{\int A_1 \mu(A_1) dA_1}{\int \mu(A_1) dA_1} = \frac{1}{3}(l_1 + m_1 + u_1) \tag{2}$$

2.2 Analytic network process

The ANP provides a general framework to deal with the decisions without making assumptions about the independence of higher level elements from lower level elements and about the independence of the elements within a level. In fact, ANP uses a network without the need to specify levels as in a hierarchy. A system with feedback can be represented by a network where nodes correspond to the levels or components. There is wide structural difference between a hierarchy and a network (Saaty, 1980). The elements in a node (level) may influence some or all the elements of any other node. In a network, there can be source nodes, intermediate nodes and sink nodes. Relationships in a network are represented by arcs and the directions of arcs signify dependence. Interdependency between two nodes, termed as outer dependence, is represented by a two-way arrow, and the inner dependencies among elements in a node are represented by a looped arc (Sarkis, 2002).

The application of ANP comprises of the following four major steps (Chung et al, 2006):
Model construction and problem structuring: The problem should be clearly stated and decomposed into a rational system, like a network. The structure can be obtained by the opinion of the decision makers through brainstorming or other appropriate methods.

Pair-wise comparison matrices and priority vectors: In ANP, like AHP, decision elements at each component are compared pair-wise with respect to their importance towards their control criterion, and the components themselves are also compared pair-wise with respect to their contribution to the goal (objective). The decision makers are asked to respond to a series of pair-wise comparisons where two elements or two components at a time are compared in terms of how they contribute to their particular upper level criterion (Meade and Sarkis, 1999). In addition, if there are interdependencies among the elements of a component, pair-wise comparisons also need to be performed, and an eigenvector (priority vector) is obtained for each element to show the influence of other elements on it. The relative importance values are determined using Saaty's 1-9 scale, where a score of 1 represents equal importance between the two elements and a score of 9 indicates the extreme importance of one element (row component in the matrix) compared to the other one (column component in the matrix). A reciprocal value is assigned to the inverse comparison, i.e., $a_{ij} = 1/a_{ji}$, where a_{ij} (a_{ji}) denotes the importance of i^{th} (j^{th}) element. Like AHP, the pair-wise comparison in ANP is made in the framework of a matrix, and a local priority vector can be derived as an estimate of relative importance associated with the elements (components) being compared by solving the following equation:

$$A \times w = \lambda_{max} \times w \tag{3}$$

where A is the matrix of pair-wise comparison, w is the eigenvector and λ_{max} is the largest eigenvalue of A. Saaty (1980) proposed several algorithms for approximating the values of w. In this paper, the following three step procedure is adopted to synthesize these priority values (Chung et al., 2006).

a) Sum the values in each column of the pair-wise comparison matrix.
b) Divide each element in a column by the sum of its respective column. The resultant matrix is referred to as the normalized pair-wise comparison matrix.

c) Sum the elements in each row of the normalized pair-wise comparison matrix and divide the sum by the n elements in the row. This provides an estimate of the relative priorities for the elements being compared with respect to its upper level criterion. Priority vectors must be derived for all the comparison matrices.

Supermatrix formation: The supermatrix concept is similar to the Markov chain process (Saaty, 1996). To obtain the global priorities in a system with interdependent influences, the local priority vectors are entered in the appropriate columns of the matrix. As a result, a supermatrix is actually a partitioned matrix, where each matrix segment represents a relationship between two nodes (components or clusters) in a system (Meade and Sarkis, 1999).

As an example, the supermatrix for a hierarchy with three levels, as shown in Figure 1(a), can be represented as below:

$$W_h = \begin{bmatrix} 0 & 0 & 0 \\ w_{21} & 0 & 0 \\ 0 & w_{32} & 1 \end{bmatrix} \tag{4}$$

where w_{21} is a vector that represents the impact of the goal on the criteria, W_{32} is a matrix that represents the impact of criteria on each of the alternatives, I is the identity matrix and entries of zero correspond to those elements that have no influence.

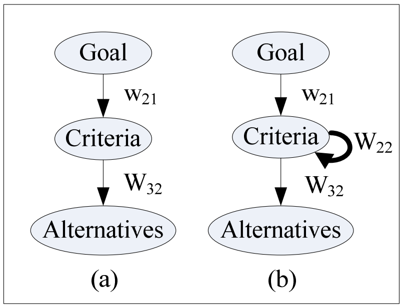

Fig. 1. A hierarchy and a network

Now, if the criteria are interrelated among themselves, the hierarchy is replaced by a network, as shown in Fig. 1(b) (Momoh & Zhu, 2003). This interdependency is exhibited by the presence of the matrix element W_{22} of the supermatrix yielding the following matrix:

$$W_n = \begin{bmatrix} 0 & 0 & 0 \\ w_{21} & W_{22} & 0 \\ 0 & W_{32} & I \end{bmatrix} \tag{5}$$

Any zero value in the supermatrix can be replaced by a matrix if there is an interrelationship of the elements in a component or between two components. Since there is interdependence among the clusters in a network, the columns of a supermatrix usually sum to more than one. The supermatrix must be first transformed to make it stochastic which means that each column of the matrix sums to unity. For this, the recommended approach (Saaty, 1996) is to determine the relative importance of the clusters in the supermatrix with the column cluster (block) as the controlling component. That is, the row components with non-zero entries for their blocks in that column block are compared according to their impact on the component of that column block. With pair-wise comparison matrix of the row components with respect to the column component, an eigenvector can be obtained. This process gives rise to an eigenvector for each column block. For each column block, the first entry of the respective eigenvector is multiplied by all the elements in the first block of that column, the second by all the elements in the second block of that column and so on. In this way, the blocks in each column of the supermatrix are weighted and the result is known as the weighted supermatrix which is stochastic.

Selection of the best alternative: If the developed supermatrix covers the entire network, the priority values of the alternatives can be established in the column of alternatives in the convergence supermatrix. The selection of the best alternative is based on the value of desirability index. The desirability index, D_i for i^{th} alternative is defined as follows:

$$D_i = \sum_{j=1}^{n} P_j \text{x} A_{ij} \text{x} M_{ij}, \tag{6}$$

where P_j is the relative importance (weight) of j^{th} criterion, A_{ij} is the stabilized relative importance of i^{th} alternative on j^{th} criterion and M_{ij} denotes the impact of i^{th} alternative on j^{th} criterion. When the desirability indices are arranged in descending order, the best alternative is that having the maximum desirability index value.

3. Supercritical boiler material selection

It has been already become a well established fact that the robustness of a supercritical boiler along with its performance and thermal efficiency greatly depends on its material properties. Hence, it is extremely indispensable to select the most suitable supercritical boiler material that can operate at high temperature and pressure with maximum thermal efficiency and minimum green house gas emission. Now-a-days, various heat-resistant materials, super alloys and refractory alloys are available in the market. The past researchers have intensely tried to develop newer materials while varying the alloying composition, and applying the mechanical and chemical properties characterization approaches to solve the supercritical boiler material selection problem. Even though immense experimentation techniques on different heat-resistant alloys have been adopted to solve this supercritical boiler material selection problem, mathematical tools, like fuzzy ANP can be considered to be the most effective approach to solve this problem as it can deal with the interactions and interdependencies that exist between the considered selection criteria. Hence, in order to apply the fuzzy ANP method for this boiler material selection problem and prove its potentiality, the corresponding decision matrix of Table 1 is developed.

Table 1

Decision matrix for supercritical boiler material selection problem

Sl. No.	C_1	C_2	C_3	C_4	C_5	C_6	C_7
M_1	1370	395	200	8.9	14.8	397	10
M_2	1446	205	290	21.4	18.9	500	65
M_3	1336	875	204	22.2	15.5	435	100
M_4	1355	423	282	22.9	14	607	27.9
M_5	1427	207	511	10.5	14.6	515	45
M_6	1360	910	550	24.5	17.8	553	53.3
M_7	1371	1062	689	25.2	16.7	452	10

This decision matrix consists of seven supercritical boiler materials and seven pivotal selection criteria. All the criteria values of Table 1 are accumulated from different handbooks (Davis, 1997; Donachie & Donachie, 2002). The details of these seven selection criteria are given in Table 2.

Table 2
Criteria for supercritical boiler material selection

Properties of supercritical boiler materials	Symbol
Melting point (˚C)	C_1
Yield strength (MPa)	C_2
High temperature stress rupture strength (MPa)	C_3
Thermal conductivity (W/m-K)	C_4
Mean coefficient of thermal expansion (μm/m-˚C)	C_5
Specific heat (J/kg-K)	C_6
Cost (USD/kg)	C_7

3.1 Criteria for supercritical boiler material selection

The melting point of a supercritical boiler material is the temperature at which it changes its state from solid to liquid. At the melting point, the solid and liquid phases exist in equilibrium. The melting point of a supercritical boiler material depends (usually slightly) on pressure and is usually specified at standard pressure. The melting point of the supercritical boiler material should be as high as possible. The yield strength of a supercritical boiler material is defined as the stress at which it begins to deform plastically. Before the yield point, it will deform elastically, and will come back to its original shape and size when the applied stress is removed. Once the yield point is passed, some fraction of the deformation will be everlasting and non-reversible. For supercritical boiler material, the yield strength value should be as high as possible. High temperature stress rupture strength or creep strength of a supercritical boiler material can be defined as its ability to withstand a constant weight or force at supercritical conditions. While selecting the material for a supercritical boiler, the high temperature stress rupture strength is considered as a beneficial criterion. Thermal conductivity is the intrinsic property of a supercritical boiler material which relates its ability to conduct heat. Heat transfer by conduction involves transfer of energy within a supercritical boiler material without its any motion as a whole. Thermal conductivity of a supercritical boiler material is defined as the quantity of heat transmitted through a unit thickness in a direction normal to a surface of unit area due to a unit temperature gradient under steady state conditions and when the heat transfer is dependent only on the temperature gradient. Correspondingly, materials with high thermal conductivity are preferred for supercritical boiler applications. A supercritical boiler material expands because an increase in temperature leads to greater thermal vibration of its atoms, and hence, to an increase in the average separation distance between the adjacent atoms. The mean coefficient of thermal expansion for a supercritical boiler material describes how much it will expand for a unit degree of temperature rise. Its value should be as low as possible while selecting materials for supercritical boiler applications. Specific heat of a supercritical boiler material is defined as the amount of heat per unit of its mass required to raise its temperature by one degree. Specific heat of a supercritical boiler material should be as high as possible. The cost of a supercritical boiler material indicates its current market price which greatly influences the final boiler cost. It is expressed in terms of the price value per unit weight of the supercritical boiler material. So, its cost should be as low as possible and is taken as a non-beneficial criterion.

3.2 Supercritical boiler materials

Table 3 shows an exhaustive list of the seven alternative supercritical boiler materials. Haynes 230 is a high-performance, industrial heat-resistant alloy for applications demanding high strength as well as resistance to oxidation, corrosion and erosion. It has substantial improvement in performance from the common iron-nickel-chromium and nickel-chromium alloys, and displays the best combination of strength, stability, environment-resistance and fabricability of any commercial nickel-base alloy. It can be utilized at temperatures as high as 1150˚C for continuous operation. Its resistance to oxidation, combustion environment and nitriding property highly recommends it for applications, such as nitric acid catalyst grids, high-temperature bellows, industrial furnace fixtures, strand annealing tubes, thermocouple protection tubes and many more. It contains Ni 57%, Co 5%, Cr 22%, Mo 2%, W 14%, Fe 3%, Si 0.4%, Mn 0.5%, C 0.10%, Al 0.3%, B 0.015% and La 0.02%. RA 602 CA is a nickel-base alloy offering significantly high strength, excellent resistance to grain growth and low density. It contains Cr 26%, Ni 59.15%, Cu 0.1%, P 0.02%, S 0.01%, Fe 11%, C 0.25%, Al 2.4%, Ti 0.2%, Yb 0.12%, Zr 0.1%, Si 0.5% and Mn 0.15%. Incoloy 718 is a high-strength, corrosion-resistant nickel-chromium alloy. Its chemical composition is Ni 55%, Cr 21%, Fe 11.134%, (Nb + Ta) 5.50%, Mo 3.30%, Ti 1.15%, Al 0.80%, Co 1%, C 0.08%, Mn 0.35%, Si 0.35%, P 0.015% S 0.015%, B 0.006% and Cu 0.30%.

This age-hardnable alloy can be readily fabricated, even into complex parts. Its welding characteristics, especially its resistance to post-weld cracking, are outstanding. Hastelloy X is a nickel-chromium-iron-molybdenum alloy which possesses an exceptional combination of oxidation resistance, fabricability and high-temperature strength. It has also been found to be exceptionally resistant to stress-corrosion cracking, and has excellent forming and welding characteristics. It can be forged, and because of its good ductility, can be cold worked. Hastelloy X exhibits good ductility after prolonged exposure at temperatures of 650, 760 and 870˚C for 16,000 hours. The nominal chemical composition of this alloy is Ni 46.792%, Cr 22%, Fe 18%, Mo 9%, Co 1.5% W 0.6%, C 0.10%, Mn 1%, Si 1% and B 0.008%. Inver 36, also known as FeNi36, is a nickel-steel alloy, notable for its uniquely low coefficient of thermal expansion. Invar 36 can be heat treated using special types of annealing methods. The heating and cooling rates need to be controlled to prevent damage of the components from cracking, warpage etc. Conventional welding methods can be used to fabricate Invar 36. It contains C 0.15%, Ni 36%, P 0.06%, Fe 62.036%, Si 0.40%, Mn 0.60%, S 0.004%, Cr 0.25% and Co 0.50%. Waspaloy is an age-hardnable, nickel-base super alloy with good strength temperature at up to 980˚C. It is widely used as a wrought material for forged and fabricated gas turbine and aerospace components. It can be cold formed in annealed condition and may also be hot formed at a temperature of 1040˚C or above. Its weldability is somewhat constrained due to its susceptibility to strain age cracking under condition of heavy restraint. It contains Ni 56.214%, Co13.5%, Fe 2%, Cr 19%, Mo 4.3%, Al 1.5%, Ti 3%, C 0.08%, Mn 0.1%, Si 0.15%, B 0.006%, Cu 0.1% and Zr 0.05%. Rene 41 is an age-hardening nickel-base super alloy with exceptional strength at high temperatures. This alloy is sensitive to strain age cracking during welding. However, sound welds can be made by the resistance and electron beam welding methods. Rene 41 should be in a fully solution treated condition prior to welding. After welding, the assembly should be solution treated at rapid heating and cooling rates through the 649-870˚C temperature range, followed by aging. Its mechanical properties vary with the solution and aging treatments. Higher solution temperatures result in its better room temperature ductility and elevated temperature creep rupture strength. Lower solution temperatures give higher tensile strengths. The typical chemical composition of this alloy is Cr 20%, Ni 46.155%, Mo 10.5%, Co 12%, Al 1.8%, Ti 3.3%, B 0.01%, C 0.12%, Fe 5%, Mn 0.1%, Si 0.5%, S 0.015% and Cu 0.5%.

Table 3

Supercritical boiler materials

Material	Haynes 230	RA 602 CA	Incoloy 718	Hastelloy X	Inver 36	Waspaloy	Rene 41
Symbol	M_1	M_2	M_3	M_4	M_5	M_6	M_7

3.3 Selection of the supercritical boiler material

For solving this supercritical boiler material selection problem using fuzzy ANP method, the decision matrix of Table 1 is first converted into a fuzzy decision matrix of Table 4 using triangular fuzzy membership function. Now, a pair-wise comparison matrix for the considered selection criteria is constructed and the corresponding priority vector is obtained, as shown in Table 5. It is observed that the melting point of the candidate material attains the highest priority, followed by the yield strength. These priority weights are used as the P_j values to calculate the desirability indices for the alternative supercritical boiler materials. In the subsequent step, the interdependency relationship among the seven selection criteria is considered and seven pair-wise comparison matrices of criteria are constructed, as shown in Tables 6-12. The priority vectors of these pair-wise comparison matrices for different criteria showing the importance on other criteria are essential for the development of the supermatrix. Now, the pair-wise comparison matrices of the considered alternative materials exhibiting the importance of different criteria are developed to establish the interdependency relationship among the alternatives. In this step, seven pair-wise comparison matrices are constructed for the seven alternatives. Table 13 shows such a pair-wise comparison matrix when the performance of all the seven alternatives is studied with respect to the importance of 'melting point' criterion. The priority vectors and the equivalent defuzzified priority vectors for the seven alternative materials are shown in Tables 14-20. The values of defuzzified priority vectors are obtained using centroid method (Vahdani et al., 2011). These defuzzified priority vectors are used as the M_{ij} values to compute the desirability indices for the alternative materials. The supermatrices before and after convergence are respectively shown in Tables 21 and 22. In the convergence supermatrix, the values of any column can be treated as A_{ij} values. Finally, the desirability indices are calculated and the ranking of the alternatives based on the desirability indices is obtained, as shown in Tables 23 and 24. It is revealed from the results that Rene 41 is the most appropriate choice as the supercritical boiler material. Waspaloy and Incoloy 718 may also be used as supercritical boiler materials because they respectively obtain the second and third ranks. Haynes 230 is the worst chosen material for supercritical boiler design.

Table 4

Fuzzy decision matrix for supercritical boiler material selection problem

Sl. No.	C_1	C_2	C_3	C_4	C_5	C_6	C_7
M_1	(1220,1370,1520)	(295,395,495)	(150,200,250)	(5.9,8.9,11.9)	(10.3,14.8,19.3)	(322,397,472)	(7,10,13)
M_2	(1296,1446,1596)	(155,205,255)	(220,290,360)	(14.4,21.4,28.4)	(14.4,18.9,23.4)	(400,500,600)	(50,65,80)
M_3	(1186,1336,1486)	(725,875,1025)	(154,204,254)	(15.2,22.2,29.2)	(11,15.5,20)	(360,435,510)	(80,100,120)
M_4	(1205,1355,1505)	(298,423,548)	(232,282,332)	(15.4,22.9,30.4)	(9.5,14,18.5)	(507,607,707)	(21.4,27.9,34.4)
M_5	(1277,1427,1577)	(157,207,257)	(411,511,611)	(7.5,10.5,13.5)	(10.1,14.6,19.1)	(440,515,590)	(36,45,54)
M_6	(1210,1360,1510)	(760,910,1060)	(450,550,650)	(17.3,24.5, 1.7)	(13.3,17.8,22.3)	(478,553,628)	(43.8,53.3,62.8)
M_7	(1221,1371,1521)	(887,1062,1237)	(564,689,814)	(18.2,25.2,32.2)	(12.2,16.7,21.2)	(377,452,527)	(7,10,13)

Table 5

Pair-wise comparison matrix for different criteria

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Priority vector
C_1	1	2	3	3	3	2	3	0.2855
C_2	1/2	1	3	3	1/2	3	5	0.2067
C_3	1/3	1/3	1	1	1/2	3	3	0.1132
C_4	1/3	1/3	1	1	1	1/2	3	0.0968
C_5	1/3	2	2	1	1	2	2	0.1588
C_6	1/2	1/3	1/3	2	1/2	1	1	0.0827
C_7	1/3	1/5	1/3	1/3	1/2	1	1	0.0562

Table 6

Pair-wise comparison matrix for different criteria on 'cost'

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	Priority vector
C_1	1	5	7	7	2	9	0.4358
C_2	1/5	1	5	5	1/2	7	0.1734
C_3	1/7	1/5	1	1	1/5	3	0.0546
C_4	1/7	1/5	1	1	1/5	3	0.0546
C_5	1/2	2	5	5	1	7	0.2545
C_6	1/9	1/7	1/3	1/3	1/7	1	0.0270

Table 7

Pair-wise comparison matrix for different criteria on 'specific heat'

Criteria	C_1	C_2	C_3	C_4	C_5	C_7	Priority vector
C_1	1	5	7	7	1/2	9	0.3636
C_2	1/5	1	5	3	1/2	7	0.1674
C_3	1/7	1/5	1	2	1/5	3	0.0645
C_4	1/7	1/3	1/2	1	1/5	3	0.0557
C_5	2	2	5	5	1	5	0.3187
C_7	1/9	1/7	1/3	1/3	1/5	1	0.0301

Table 8

Pair-wise comparison matrix for different criteria on ‘mean coefficient of thermal expansion’

Criteria	C_1	C_2	C_3	C_4	C_6	C_7	Priority vector
C_1	1	3	5	5	1/3	7	0.2736
C_2	1/3	1	5	5	1/2	7	0.2030
C_3	1/5	1/5	1	1	1/5	3	0.0621
C_4	1/5	1/5	1	1	1/5	3	0.0621
C_6	3	2	5	5	1	7	0.3688
C_7	1/7	1/7	1/3	1/3	1/7	1	0.0303

Table 9

Pair-wise comparison matrix for different criteria on ‘thermal conductivity’

Criteria	C_1	C_2	C_3	C_5	C_6	C_7	Priority vector
C_1	1	1/2	7	7	2	5	0.3114
C_2	2	1	3	3	2	5	0.2958
C_3	1/7	1/3	1	1	1/5	1/2	0.0511
C_5	1/7	1/3	1	1	1/5	3	0.0688
C_6	1/2	1/2	5	5	1	5	0.2209
C_7	1/5	1/5	2	1/3	1/5	1	0.0520

Table 10

Pair-wise comparison matrix for different criteria on ‘high temperature stress rupture strength’

Criteria	C_1	C_2	C_3	C_5	C_6	C_7	Priority vector
C_1	1	1/2	7	7	2	5	0.3839
C_2	2	1	3	3	2	5	0.2103
C_3	1/7	1/3	1	1	1/5	1/2	0.0612
C_5	1/7	1/3	1	1	1/5	3	0.0486
C_6	1/2	1/2	5	5	1	5	0.2649
C_7	1/5	1/5	2	1/3	1/5	1	0.0311

Table 11

Pair-wise comparison matrix for different criteria on ‘yield strength’

Criteria	C_1	C_3	C_4	C_5	C_6	C_7	Priority vector
C_1	1	5	3	3	2	9	0.3721
C_3	1/5	1	5	5	1/2	7	0.1964
C_4	1/3	1/5	1	1	1/5	3	0.0713
C_5	1/3	1/5	1	1	1/3	3	0.0776
C_6	1/2	2	5	3	1	5	0.2503
C_7	1/9	1/7	1/3	1/3	1/5	1	0.0324

Table 12

Pair-wise comparison matrix for different criteria on ‘melting point’

Criteria	C_2	C_3	C_4	C_5	C_6	C_7	Priority vector
C_2	1	5	7	7	2	7	0.4247
C_3	1/5	1	5	5	1/2	7	0.1762
C_4	1/7	1/5	1	1/2	1/5	3	0.0495
C_5	1/7	1/5	2	1	1/5	3	0.0623
C_6	1/2	2	5	5	1	7	0.2586
C_7	1/7	1/7	1/3	1/3	1/7	1	0.0287

Table 13

Pair-wise comparison of importance of ‘melting point’ on alternatives

Alt.	M_1	M_2	M_3	M_4	M_5	M_6	M_7
M_1	(1,1,1)	(0.94,0.95,0.95)	(1.03,1.03,1.02)	(1.01,1.01,1.01)	(0.96,0.96,0.96)	(1.01,1.01,1.01)	(1,1,1)
M_2	(1.06,1.06,1.05)	(1,1,1)	(1.09,1.08,1.07)	(1.08,1.07,1.06)	(1.01,1.01,1.01)	(1.07,1.06,1.06)	(1.06,1.05,1.05)
M_3	(0.97,0.98,0.98)	(0.92,0.92,0.93)	(1,1,1)	(0.98,0.99,0.99)	(0.93,0.94,0.94)	(0.98,0.98,0.98)	(0.97,0.97,0.98)
M_4	(0.99,0.99,0.99)	(0.93,0.94,0.94)	(1.02,1.01,1.01)	(1,1,1)	(0.94,0.95,0.95)	(1,1,1)	(0.99,0.99,0.99)
M_5	(1.05,1.04,1.04)	(0.99,0.99,0.99)	(1.08,1.07,1.06)	(1.06,1.05,1.05)	(1,1,1)	(1.06,1.05,1.04)	(1.05,1.04,1.04)
M_6	(0.99,0.99,0.99)	(0.93,0.94,0.95)	(1.02,1.02,1.02)	(1,1,1)	(0.95,0.95,0.96)	(1,1,1)	(0.99,0.99,0.99)
M_7	(1,1,1)	(0.94,0.95,0.95)	(1.03,1.03,1.02)	(1.01,1.01,1.01)	(0.96,0.96,0.96)	(1.01,1.01,1.01)	(1,1,1)

Table 14
Priority and defuzzified priority vectors of importance of 'melting point' on the alternatives

Alternative	Priority vector	Defuzzified priority vector
M_1	(0.1416,0.1417,0.1419)	0.1417
M_2	(0.1504,0.1496,0.1490)	0.1497
M_3	(0.1377,0.1382,0.1387)	0.1382
M_4	(0.1399,0.1402,0.1405)	0.1402
M_5	(0.1482,0.1476,0.1472)	0.1477
M_6	(0.1405,0.1407,0.1409)	0.1407
M_7	(0.1417,0.1419,0.1420)	0.1418

Table 15
Priority and defuzzified priority vectors of importance of 'yield strength' on the alternatives

Alternative	Priority vector	Defuzzified priority vector
M_1	(0.0900,0.0969,0.1015)	0.0961
M_2	(0.0473,0.0503,0.0523)	0.0500
M_3	(0.2212,0.2146,0.2102)	0.2153
M_4	(0.0909,0.1038,0.1124)	0.1024
M_5	(0.0479,0.0508,0.0527)	0.0505
M_6	(0.2319,0.2232,0.2173)	0.2242
M_7	(0.2707,0.2605,0.2536)	0.2616

Table 16
Priority and defuzzified priority vectors of importance of 'high temperature stress rupture strength' on the alternatives

Alternative	Priority vector	Defuzzified priority vector
M_1	(0.0694,0.0734,0.0760)	0.0729
M_2	(0.1016,0.1064,0.1095)	0.1059
M_3	(0.0712,0.0748,0.0772)	0.0744
M_4	(0.1060,0.1034,0.1018)	0.1037
M_5	(0.1883,0.1875,0.1869)	0.1876
M_6	(0.2057,0.2018,0.1992)	0.2022
M_7	(0.2578,0.2528,0.2494)	0.2533

Table 17
Priority and defuzzified priority vectors of importance of 'thermal conductivity' on the alternatives

Alternative	Priority vector	Defuzzified priority vector
M_1	(0.0628,0.0656,0.0671)	0.0652
M_2	(0.1534,0.1578,0.1602)	0.1571
M_3	(0.1619,0.1637,0.1647)	0.1634
M_4	(0.1640,0.1689,0.1715)	0.1681
M_5	(0.0799,0.0774,0.0761)	0.0778
M_6	(0.1842,0.1807,0.1788)	0.1812
M_7	(0.1938,0.1858,0.1816)	0.1871

Table 18
Priority and defuzzified priority vectors of importance of 'mean coefficient of thermal expansion' on the alternatives

Alternative	Priority vector	Defuzzified priority vector
M_1	(0.1275,0.1318,0.1342)	0.1312
M_2	(0.1782,0.1683,0.1627)	0.1697
M_3	(0.1361,0.1380,0.1391)	0.1377
M_4	(0.1176,0.1247,0.1287)	0.1236
M_5	(0.1250,0.1300,0.1328)	0.1293
M_6	(0.1646,0.1585,0.1551)	0.1594
M_7	(0.1510,0.1487,0.1474)	0.1490

Table 19

Priority and defuzzified priority vectors of importance of ‘specific heat’ on the alternatives

Alternative	Priority vector	Defuzzified priority vector
M_1	(0.1117,0.1148,0.1170)	0.1145
M_2	(0.1387,0.1446,0.1487)	0.1440
M_3	(0.1248,0.1258,0.1264)	0.1257
M_4	(0.1758,0.1755,0.1753)	0.1755
M_5	(0.1526,0.1489,0.1463)	0.1492
M_6	(0.1657,0.1599,0.1557)	0.1604
M_7	(0.1307,0.1307,0.1306)	0.1307

Table 20

Priority and defuzzified priority vectors of importance of ‘cost’ on the alternatives

Alternative	Priority vector	Defuzzified priority vector
M_1	(0.0285,0.0321,0.0345)	0.0317
M_2	(0.2039,0.2089,0.2121)	0.2083
M_3	(0.3263,0.3213,0.3181)	0.3219
M_4	(0.0873,0.0897,0.0912)	0.0894
M_5	(0.1468,0.1446,0.1432)	0.1449
M_6	(0.1786,0.1713,0.1665)	0.1721
M_7	(0.0285,0.0321,0.0345)	0.0317

Table 21

Supermatrix before convergence

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	0	0.3721	0.3839	0.3114	0.2736	0.3636	0.4358
C_2	0.4247	0	0.2103	0.2958	0.2030	0.1674	0.1734
C_3	0.1762	0.1964	0	0.0511	0.0621	0.0645	0.0546
C_4	0.0495	0.0713	0.0612	0	0.0621	0.0557	0.0546
C_5	0.0623	0.0776	0.0486	0.0688	0	0.3187	0.2545
C_6	0.2586	0.2503	0.2649	0.2209	0.3688	0	0.0270
C_7	0.0287	0.0324	0.0311	0.0520	0.0303	0.0301	0

Table 22

Supermatrix after convergence

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	0.2615	0.2615	0.2615	0.2615	0.2615	0.2615	0.2615
C_2	0.2150	0.2150	0.2150	0.2150	0.2150	0.2150	0.2150
C_3	0.1134	0.1134	0.1134	0.1134	0.1134	0.1134	0.1134
C_4	0.0557	0.0557	0.0557	0.0557	0.0557	0.0557	0.0557
C_5	0.1163	0.1163	0.1163	0.1163	0.1163	0.1163	0.1163
C_6	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075	0.2075
C_7	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306

Table 23

Calculation of desirability indices

Criteria	P_j	A_{ij}	M_{1j}	M_{2j}	M_{3j}	M_{4j}	M_{5j}	M_{6j}	M_{7j}
C_1	0.2855	0.2615	0.1417	0.1497	0.1382	0.1402	0.1477	0.1407	0.1418
C_2	0.2067	0.2150	0.0961	0.0500	0.2153	0.1024	0.0505	0.2242	0.2616
C_3	0.1132	0.1134	0.0729	0.1059	0.0744	0.1037	0.1876	0.2022	0.2533
C_4	0.0968	0.0557	0.0652	0.1571	0.1634	0.1681	0.0778	0.1812	0.1871
C_5	0.1588	0.1163	0.1312	0.1697	0.1377	0.1236	0.1293	0.1594	0.1490
C_6	0.0827	0.2075	0.1145	0.1440	0.1257	0.1755	0.1492	0.1604	0.1307
C_7	0.0562	0.0306	0.0317	0.2083	0.3219	0.0894	0.1449	0.1721	0.0317

4. Discussions

The primary function of Hastelloy X is to survive under high-temperature, high-stress operation in a moderately corrosive and erosive environment, such as pressure vessels of some nuclear reactors, chemical reactors, supercritical boiler components, distillation equipment, and pipes and valves in chemical industry where more common and less expensive iron-based alloys are used to fail. Hastelloy X experiences degradation during fabrication and handling.

Table 24
Desirability indices of the alternatives

Criteria	M_1	M_2	M_3	M_4	M_5	M_6	M_7
C_1	0.0106	0.0112	0.0103	0.0105	0.0110	0.0105	0.0106
C_2	0.0043	0.0022	0.0096	0.0045	0.0022	0.0100	0.0116
C_3	0.0009	0.0014	0.0010	0.0013	0.0024	0.0026	0.0033
C_4	0.0004	0.0008	0.0009	0.0009	0.0004	0.0010	0.0010
C_5	0.0024	0.0031	0.0025	0.0023	0.0024	0.0029	0.0028
C_6	0.0020	0.0025	0.0022	0.0030	0.0026	0.0028	0.0022
C_7	0.0001	0.0004	0.0006	0.0002	0.0002	0.0003	0.0001
Sum of the desirability indices	0.0206	0.0216	0.0270	0.0227	0.0213	0.0300	0.0315
Ranking of the alternatives	7	5	3	4	6	2	1

Electro-polishing or passivation of Hastelloy X can improve its corrosion resistance property. Incoloy 718 is a very good oxidation and corrosion-resistant supercritical boiler material, well suited for service in extreme environments subjected to pressure and heat. When heated, Incoloy 718 forms a thick, stable, passivating oxide layer protecting the surface from further damage. It retains its strength over a wide temperature range, attractive for high temperature applications where aluminum and steel usually succumb to high-temperature stress rupture strength as a result of thermally-induced crystal vacancies. Its high temperature strength is developed by solid solution strengthening or precipitation strengthening, depending on the amount of alloying element. In age-hardening or precipitation strengthening, small amounts of niobium combine with nickel to form an inter-metallic compound (Ni_3Nb) which forms small cubic crystals that inhibit slip and stress rupture strength effectively at elevated temperatures. Waspaloy is an age-hardnable, nickel-base super alloy with excellent strength properties through temperatures of approximately 980˚C. Its other characteristics include good corrosion resistance as well as being relatively impervious to oxidation, making it well suited for service in extreme environments. Waspaloy has the useful strength and good oxidation resistance in supercritical boiler atmospheres up to 870˚C. The high temperature stress rupture strength of Waspaloy is superior to that of Incoloy 718 at temperatures above 620-650˚C. Rene 41 is a nickel-base high temperature alloy retaining its high strength in the 649-982˚C temperature range which makes it the best choice as the supercritical boiler material. It has very good high temperature stress rupture strength, high specific heat and low cost. This super alloy is widely used in jet engine and missile components, supercritical boiler components and other applications requiring high strength at extreme temperatures (Sims et. al., 1987). Inver 36 is a nickel-iron, low thermal expansion alloy containing 36% nickel. It has a low coefficient of expansion from cryogenic temperatures to about 260˚C. It also retains good strength and toughness at cryogenic temperatures as well as high temperature. Its common applications include tooling for aerospace composites, different components of supercritical boiler, standards of length measuring devices, thermostat rods, laser components, and tanks and piping for storage and transportation of liquefied gases. Haynes 230 is designed to have superior physical properties than the traditional stainless steels, nickel-chromium alloys and iron-nickel-chromium alloys. It is both readily fabricable and repairable. Its retention of ductility after being in service for several years helps it for better reforming and weld repair operations without any need for pre- or post-repair treatment. But it has low yield strength, very low high temperature stress rupture strength, low thermal conductivity and low specific heat which makes it as the worst choice for supercritical boiler. RA 602 CA is one of the most oxidation resistant nickel-base super alloys. Its high chromium content, along with aluminum and ytterbium additions, permits it to develop a tightly adherent oxide scale. Its relatively high carbon content combined with titanium and zirconium additions results in high creep rupture strength. It may be considered for a wide variety of high temperature applications, and particularly for applications where it is important to minimize product contamination while maintaining high mechanical integrity at extreme temperatures (Kelly and Wilson, 1995). Based on this comparative study of the physical characteristics of the considered alternatives, it can be claimed that Rene 41 is the best choice as the supercritical boiler material.

5. Conclusions

The earlier researches have already attempted to develop new heat-resistant materials by changing the alloying composition and taken in consideration different material characterization approaches for identifying the most suitable supercritical boiler material. However, there is scarcity of sound mathematical techniques in this domain. To fill up this gap, this paper attempts to solve the supercritical boiler material selection problem using fuzzy analytic network process. It is revealed that this method has enough potential to deal with such types of complex decision-making problems having interactions and interdependencies between the considered selection criteria. The observed results are precisely in compliance with the predictable choices. This method can be applied to those decision-making situations where the criteria values in the decision matrix cannot be determined as crisp numbers. Thus, it can assist and direct the designers to select the best materials for varying engineering applications.

References

Blum, R., & Vanstone, R.W. (2003). Materials development for boilers and steam turbines operating at 700°C. Proc. of 6th International Charles Parsons Turbine Conference. Institute of Materials, Minerals, and Mining, London, 489-510.

Chung, S.H., Lee, A.H.L., & Pearn, W.L. (2006). Analytic network process (ANP) approach for product mix planning in semiconductor fabricator. *International Journal of Production Economics*, 96, 15-36.

Davis, J.R. (1997). Heat-resistant Materials. Ohio: ASM International.

Donachie, M.J. & Donachie, S.J. (2002). *Superalloys*: *A Technical Guide*. Ohio: ASM International.

Kelly, J.C., & Wilson, J.D. (1995). *Oxidation Rates of Some Heat Resistant Alloys*. ASM Proc. of 2nd International Conference on Heat-Resistant Materials, USA.

Meade, L.M., & Sarkis, J. (1999). Analyzing organizational project alternatives for agile manufacturing processes: An analytical network approach. *International Journal of Production Research*, 37, 241-261.

Momoh, J.A., & Zhu, J. (2003). Optimal generation - Scheduling based on AHP/ANP. *IEEE Transactions on Systems, Man, and Cybernetics - Part B*: *Cybernetics*, 33, 531-535.

Perrine, I.J., & Fishburn, J.D. (2008). A perspective on the design of high-temperature boiler components. *International Journal of Pressure Vessels and Piping*, 85, 14-21.

Saaty, T. L. (1980). *The Analytic Hierarchy Process*. New York: McGraw-Hill.

Saaty, T.L. (1996). *Decision Making with Dependence and Feedback*: *The Analytic Network Process*. Pittsburgh: RWS Publications.

Sarkis, J. (2002). Quantitative models for performance measurement systems - Alternate considerations. *International Journal of Production Economics*, 86, 81-90.

Sims, C., Stoloff, N., & Hagel, W. (1987). *Superalloys II: High Temperature Materials for Aerospace and Industrial Power*. New York: John Wiley & Sons.

Shingledecker, J.P., Swindeman, R.W., Wu, Q., & Vasudevan, V.K. (2005). Creep strength of high temperature alloys for ultra-supercritical steam boilers. Proc. to 4th International Conference on Advances in Materials Technology for Fossil Power Plants, Ohio, 1198-1212.

Siler, W. and Buckley, J.J. (2005). *Fuzzy Expert Systems and Fuzzy Reasoning*. New Jersey: John Wiley & Sons.

Steingress, F.M. (2001). *Low Pressure Boilers*. New York: American Technical Publishers.

Steingress, F.M., Frost, H. J. & Walker, D.R. (2003). *High Pressure Boilers*. Georgia: American Technical Publishers.

Vahdani, B., Mousavi, S. M., & Tavakkoli-Moghaddam, R. (2011). Group decision making based on novel fuzzy modified TOPSIS method. *Applied Mathematical Modelling*, 37, 72-93.

Viswanathan, R. & Bakker, W. (2001). Materials for ultrasupercritical coal power plants - Boiler materials: Part 1. *Journal of Materials Engineering and Performance*, 10, 81-95.

Viswanathan, R., Coleman, K., & Rao, U. (2006). Materials for ultra-supercritical coal-fired power plant boilers. *International Journal of Pressure Vessels and Piping*, 83, 778-783.