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A multivariate quality loss function approach for parametric optimization of non-traditional machining processes

Shankar Chakraborty[a*] and Partha Protim Das[b]

[a]*Department of Production Engineering, Jadavpur University, Kolkata, West Bengal, India*
[b]*Department of Mechanical Engineering, Sikkim Manipal Institute of Technology, Sikkim, India*

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A B S T R A C T

Due to various added advantages over the conventional material removal processes, non-traditional machining (NTM) processes have now been widely applied in different manufacturing industries. To achieve the desired response values, it is always recommended to operate these NTM processes at their optimal parametric settings. Various single response optimization techniques are already available to determine the optimal combinations of NTM process parameters for achieving maximum or minimum value of a single response. In this paper, a multivariate quality loss function approach is adopted for simultaneous optimization of responses for three NTM processes. It is observed that this approach outperforms the other multi-response optimization techniques, like desirability function, distance function and mean squared error methods with respect to the achieved response values. With modification of the corresponding objective function and constraints of the developed non-linear programming problem, it can be effectively applied to any non-traditional as well as conventional machining process as a multi-objective optimization tool.

1. Introduction

In other to meet the requirements of high dimensional accuracy and low surface roughness of the present day's manufacturing industries, the conventional machining methods are continuously being substituted by the non-traditional machining (NTM) processes due to their ability to achieve more consistent workpiece quality, higher production efficiency in processing of hard and tough materials, and capability of generating complex shapes. These NTM processes usually deploy mechanical, electrical, chemical or thermal energy or their combinations to remove material from the workpiece in the form of atoms/molecules enabling attainment of high degree of surface finish (Pandey & Shan, 1980). Like the conventional machining processes, they do not use sharp cutting tools and the material removal rate is also not constrained by the mechanical properties of the work materials. These NTM processes are quite suitable to generate complex shape geometries on different difficult-to-machine materials, like ceramic-based tool materials, fibre reinforced materials, tungsten carbides, stainless steels, high speed steels, titanium-based alloys etc. Complex shape features with internal and external profiles, small holes having micro- and

* Corresponding author.
E-mail address: s_chakraborty00@yahoo.co.in (S. Chakraborty)

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nano-dimensions, and components requiring high tolerance and surface finish can be precisely and economically machined using these processes. It is a well understood fact that the NTM processes can probably never replace the conventional machining processes currently employed in various manufacturing industries. However, these processes have ensured a steadily increasingly important role because of their improved machining capabilities and characteristics.

To exploit the fullest potential and benefits from these NTM processes for generating complex and intricate shape geometries having the required dimensional accuracy, tolerance and surface finish on various difficult-to-machine work materials, they are always required to operate at their optimal parametric settings. Identification of the most appropriate parametric mix for a specific NTM process to generate the desired shape feature on a given work material is often found to be a challenging task. For this purpose, the manufacturers' operating manuals are frequently consulted or the machining experts' opinions are regularly sorted. The existence of several conflicting machining responses, like maximization of material removal rate (MRR) and minimization of surface roughness (SR), maximization of machining efficiency and minimization of power requirement etc., also makes the identification of the optimal combination of various NTM process parameters a more difficult task. Thus, it is always recommended to deploy some mathematical tools and techniques that would provide optimal or near optimal settings of the NTM process parameters so as to meet the target response values.

2. Literature review

Antony (2001) applied Taguchi quality loss function approach for simultaneous optimization of multiple quality characteristics in manufacturing processes. While applying Taguchi approach and utility concept, Singh and Kumar (2006) predicted the optimal setting of turning process parameters for having the desired quality characteristics of EN24 steel turned components. Walia et al. (2006) integrated utility theory and Taguchi quality loss function for optimization of three responses, i.e. MRR, percentage improvement of surface finish and scatter of SR in a centrifugal force-assisted abrasive flow machining process. Datta and Mahapatra (2010) combined utility theory with principal component analysis and Taguchi robust design for simultaneous optimization of multiple correlated surface quality characteristics of mild steel in straight turning operation. Dave et al. (2012) proposed the application Taguchi loss function as a multi-response optimization tool for electro discharge machining (EDM) operation of Inconel 718 alloy. Gaitonde and Karnik (2012) employed Taguchi quality loss function approach to identify the best combination of cutting speed, feed, point angle and lip clearance angle for specified drill diameters to simultaneously minimize burr height and burr thickness during drilling of AISI 316L stainless steel work materials. Kumar and Kumari (2012) applied fuzzy logic and Taguchi quality loss function for modelling and simultaneous optimization of multiple performance characteristics, such as MRR and SR of polycrystalline diamond in an ultrasonic machining (USM) process. Periyanan et al. (2012) applied Taguchi loss function approach to study the influences of spindle speed, feed rate and depth of cut on MRR and SR in a micro-end milling process. Singh et al. (2012) adopted Taguchi quality loss function for identifying the optimal operating levels of wheel speed, current, pulse-on time and duty factor in an electro-discharge diamond grinding process. Three quality characteristics, i.e. MRR, wheel wear rate and average SR were subsequently optimized. Applying Taguchi quality loss function approach, Sahoo and Mohanty (2013) determined the optimal settings of cutting speed, feed and depth of cut so as to minimize cutting force and chip reduction coefficient in an orthogonal turning process. Bhuyan and Yadava (2014) considered applied voltage, electrolyte concentration, wire feed velocity and workpiece thickness as the input parameters, and SR and MRR as the responses in a travelling wire electro-chemical spark machining process. Taguchi quality loss function was then employed to determine the optimal levels of those process parameters. Das Mohapatra and Sahoo (2014) analyzed the effects of four input parameters of a wire electro discharge machining (WEDM) process, i.e. pulse-on time, pulse-off time, wire feed rate and wire tension on the geometry of machined spur gears (i.e. MRR and single pitch error). Those WEDM process parameters were subsequently optimized employing Taguchi loss function approach. Dhobe et al. (2014) performed parametric analysis and single objective optimization based on Taguchi methodology during electrochemical machining (ECM) of pure titanium material. Using quality loss function, multi-objective optimization of two responses, i.e. MRR and SR was also conducted. It was observed that electrolyte flow rate, applied voltage, electrolyte concentration and initial inter-electrode gap were the main process parameters affecting the considered responses. Hansda and Banerjee (2014) considered material thickness, cutting velocity and feed rate in drilling of glass fibre reinforced polyester composites, and employed Taguchi quality loss function as a multi-objective optimization tool. Koranne et al. (2014) investigated the performance of multi-layer coated tool in machining of hardened steel under high speed turning and optimized the process using quality loss function approach. Kumar (2014) studied the influences of various parameters of an USM process, i.e. type of tool material, abrasive type, slurry grit size and power rating on surface quality and micro-hardness of the machined components. Taguchi's robust design approach was effectively applied for optimizing the said process. While considering air pressure and stand-off distance as the control parameters, Padhy et al. (2014) simultaneously optimized overcut and MRR of glass materials in an abrasive jet machining process using quality loss function. Periyanan and Natarajan (2014) applied Taguchi quality loss function approach to investigate the effects of three micro-WEDM process parameters, i.e. feed rate, capacitance and voltage on MRR and SR. Rahul et al. (2017) determined the optimal settings for gap voltage, peak discharge current, pulse-on time, duty factor and flushing pressure while attaining the desired values of MRR, electrode wear rate, SR, surface crack density, white layer thickness and micro-hardness during EDM operation of Inconel 718 alloy. The derived results were also compared with those observed using principal component analysis and quality loss approach.

From the above-cited literature survey, it is clearly noticed that since few decades, parametric optimization of different NTM as well as conventional machining processes has been a topic of immense interest among the researchers. The concept of Taguchi loss function has been popularly employed for identifying the optimal parametric mixes for several machining processes to fulfil the end requirements. The Taguchi loss function is an effective tool for single-objective optimization of NTM processes. As almost all the NTM processes have multiple quality characteristics or responses, the application of Taguchi loss function miserably fails to simultaneously optimize all the considered responses. To overcome this drawback of Taguchi loss function, in this paper, a multivariate quality loss function approach is adopted which can concurrently optimize all the responses of the NTM processes under consideration.

3. Multivariate quality loss function

The concept of quality loss function was introduced by Genichi Taguchi in the early 1950's, which was later implemented by the major American manufacturers. In the Western Europe, it was also executed among the automotive and aircraft industries. The classical definition of quality control is based on lower and upper specification limits (LSL and USL) as boundaries between the acceptable and unacceptable performance.

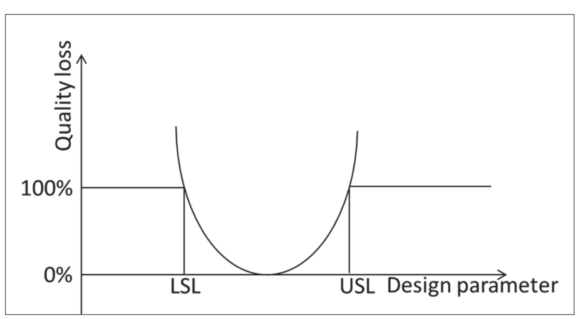

Fig. 1. Quadratic quality loss function

Performance of a product between these specification limits is deemed to be acceptable. Taguchi did not accept this traditional definition of quality. He defined 'quality' as the deviation from on-target performance. According to him, "quality of a manufactured product is total loss generated by that product to society from the time it is shipped, other than any losses caused by intrinsic functions". By the term 'loss' Taguchi referred to the loss caused by the variability of the function and loss caused by harmful side effects. It thus signifies that products with acceptable quality incur the lowest quality loss, particularly zero. When a product moves from its target value, it will cause loss even if the product lies or not within the set specification limits. On the other hand, the engineering experience shows that quality degrades, with some exceptions, continuously. Taguchi proposed that this performance degradation can be measured as a deviation from some target value as a quadratic quality loss function, as shown in Fig. 1 and asserted that the degradation can be related to a loss in value to the consumer.

Thus, the objective of quality loss function as a quality improvement tool is to minimize the total losses to society. Subsequently, the quadratic quality loss function approach has been applied in on-line and off-line quality control. The advantage of quadratic loss function mainly lies on its simplicity and comprehensiveness. In order to reduce the expected losses with respect to quadratic loss function, the process mean should be close to the target and process standard deviation should be small. Thus, if a quality characteristic concentrates on target value with minimum standard deviation, it can be said that the product has minimum quality loss. Quality loss function approach has already become a popular tool for evaluating the quality level of products, mainly in one-dimensional case (Târcolea & Paris, 2011). The concept of quality loss function thus forms the basis for a quality evaluation system and an appropriately chosen loss function should be competent in evaluating the quality of a given product. The idea of multivariate loss function was proposed by Artiles-León (1996-97), and Ma and Zhao (2004) then modified it to make it more suitable for real time applications. It has been observed that most of the quality characteristics are of 'nominal-the-best' type (N-type), where each product has an ideal value for each of its characteristics from the viewpoint of the customers/end users. This ideal value is often referred to as the target value and the corresponding loss is always proportional to the square of the deviation of the said quality characteristic from its target value. Taguchi proposed the following quadratic loss function to estimate the quality losses (Taguchi et al., 2004).

$$Loss(y) = k(y - t)^2, \tag{1}$$

where y is the quality characteristic of a product, t is the target value for x, the quality characteristic and k is a quality loss coefficient. It is required to determine the value of k so that the above-mentioned equation can be able to appropriately approximate the actual economical loss within the interest domain. In order to estimate the value of k so that the loss function remains insensitive to the system of units employed to measure the corresponding quality characteristic, Artiles-León (1996-97) assumed that the relationship between each quality characteristic and design variable could be predicted, the target value would belong to the centre of the specifications (symmetric loss function), and the loss function would become unity whenever the quality characteristic value would be at its upper or lower specification limit. Thus, the value of k can be expressed as follows:

$$k = \left(\frac{2}{USL - LSL} \right)^2 . \tag{2}$$

Substituting the value of k in Eq. (1), the loss function becomes:

$$L(y) = 4\left(\frac{y(x) - t}{USL - LSL} \right)^2 , \tag{3}$$

where USL and LSL are the upper and lower specification limits of the considered quality characteristic respectively. As this 'standardized' loss function is dimensionless, loss functions corresponding to several quality characteristics for a product can be added when the quality characteristics are uncorrelated, and Artiles-León (1996-97) defined the total standardized loss function ($TSLoss$) corresponding to quality characteristics, $Y_1, Y_2, \ldots, Y_n$, as below:

$$TSLoss(Y_1, Y_2, \ldots, Y_n, X, T) = 4\sum_{i=1}^{n} \left(\frac{Y_i(X) - t_i}{USL_i - LSL_i} \right)^2 , \tag{4}$$

where t_i, LSL_i and USL_i (i = 1,2,…,n) are the target value, lower specification limit and upper specification limit respectively, for i^{th} quality characteristic Y_i, in which the quality characteristic Y_i is a function of design variables X's. The loss function in Eq. (3) can be treated as a practical tool to multi-response problems and can be easily implemented. However, it has sudden drawbacks which need to be overcome. Firstly, it is not suitable for 'smaller-the-better' type (*S*-type) and 'larger-the- better' type (*L*-type) of quality characteristics. On the other hand, it does not consider the correlation structure between different responses. Thus, it becomes necessary to modify it to make it more suitable for real time applications. Ma and Zhao (2004) developed an improved multivariate loss function approach for different quality characteristics, i.e. 'larger-the-better' (*L*-type), 'smaller-the-better' (*S*-type) and 'nominal-the- best' (*N*-type) (Târcolea & Paris, 2011). For an *L*-type quality characteristic (Y_i), let Y_{L_i} be the minimum acceptable value and Y_{U_i} be the maximum value beyond which there is no further improvement in the product performance. Now, the dimensionless loss function for *L*-type quality characteristic can be expressed as follows:

$$L(Y_i(X), X, Y_{U_i}) = \left(\frac{Y_i(X) - Y_{U_i}}{Y_{U_i} - Y_{L_i}} \right)^2 . \tag{5}$$

Similarly, for an *S*-type quality characteristic, the loss function can be modelled as below:

$$L(Y_i(X), X, Y_{L_i}) = \left(\frac{Y_i(X) - Y_{L_i}}{Y_{U_i} - Y_{L_i}} \right)^2 . \tag{6}$$

Thus, as an extension of Artiles-León's loss function, all the three types of loss function now become dimensionless and can be added to form a more generalized multivariate loss function, as given below:

$$L(Y(X), X) = \sum_{i \in N} 4\left(\frac{Y_i(X) - t_i}{USL_i - LSL_i} \right)^2 + \sum_{j \in L} \left(\frac{Y_j(X) - Y_{U_j}}{Y_{U_j} - Y_{L_j}} \right)^2 + \sum_{l \in S} \left(\frac{Y_l(X) - Y_{L_l}}{Y_{U_l} - Y_{L_l}} \right)^2 , \tag{7}$$

where *N*, *L* and *S* are the numbers of *N*-type, *L*-type and *S*-type quality characteristics.

4. Illustrative examples

In this paper, the multivariate quality loss function approach is applied for multi-response optimization of NTM processes and its superiority over the other state-of-the-art optimization methods is well validated with the help of three illustrative examples.

4.1 Example 1

Mehrvar et al. (2017) considered four ECM process parameters, i.e. applied voltage, tool feed rate, electrolyte flow rate and electrolyte concentration to investigate their effects on MRR and SR. Each of those process parameters was set at five different levels, as shown in Table 1, and based on the central composite second order rotatable design plan, 31 experiments were conducted while taking stainless steel as the work material. The results of those 31 experiments are detailed out in Table 2. The machining time was fixed for 2 min and the inter-electrode gap was set at 0.6 mm.

Table 1
ECM process parameters along with their levels (Mehrvar et al., 2017)

Process parameter	Symbol	Unit	Level				
			-2	-1	0	1	2
Applied voltage	x_1	V	10	15	20	25	30
Tool feed rate	x_2	mm/min	0.2	0.3	0.4	0.5	0.6
Electrolyte flow rate	x_3	l/min	5	6	7	8	9
Electrolyte concentration	x_4	g/l	50	100	150	200	250

Table 2
Experimental results for ECM process (Mehrvar et al., 2017)

Exp. No.	Process parameter				Response	
	x_1	x_2	x_3	x_4	MRR (g/min)	SR (µm)
1	-1	-1	-1	-1	0.1253	0.76
2	1	-1	-1	-1	0.2134	1.08
3	-1	1	-1	-1	0.1547	0.89
4	1	1	-1	-1	0.2361	1.13
5	-1	-1	1	-1	0.1246	0.84
6	1	-1	1	-1	0.2107	1.16
7	-1	1	1	-1	0.1569	0.96
8	1	1	1	-1	0.2525	1.31
9	-1	-1	-1	1	0.1673	1.29
10	1	-1	-1	1	0.2921	1.94
11	-1	1	-1	1	0.1975	1.63
12	1	1	-1	1	0.3218	2.21
13	-1	-1	1	1	0.1779	1.47
14	1	-1	1	1	0.2979	2.15
15	-1	1	1	1	0.2019	1.78
16	1	1	1	1	0.3235	2.49
17	-2	0	0	0	0.1154	1.22
18	2	0	0	0	0.3379	2.17
19	0	-2	0	0	0.1989	1.12
20	0	2	0	0	0.2755	1.51
21	0	0	-2	0	0.1927	1.12
22	0	0	2	0	0.2194	1.35
23	0	0	0	-2	0.1365	0.72
24	0	0	0	2	0.2696	2.45
25	0	0	0	0	0.2351	1.00
26	0	0	0	0	0.2291	0.98
27	0	0	0	0	0.2250	1.02
28	0	0	0	0	0.2238	0.96
29	0	0	0	0	0.2220	1.04
30	0	0	0	0	0.2275	0.95
31	0	0	0	0	0.2232	1.02

Now, based on the experimental data, Mehrvar et al. (2017) developed the following two response surface methodology (RSM)-based equations to show the main, second order and interaction effects of the four ECM process parameters on MRR and SR.

$$
\begin{aligned}
Y(MRR) = & -0.3374 + 0.0055x_1 - 0.0496x_2 + 0.0755x_3 + 0.00072x_4 - 0.00001449x_1^2 + 0.2275x_2^2 \\
& - 0.00551x_3^2 - 0.00000250x_4^2 + 0.00049x_1x_2 + 0.00005875x_1x_3 + 0.000034875x_1x_3 \\
& + 0.0072x_2x_3 - 0.00020875x_2x_4 + 0.000009125x_3x_4
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
Y(SR) = & \, 8.75714 - 0.29921x_1 - 7.01905x_2 - 0.01042x_3 - 0.0234x_4 + 0.0069x_1^2 + 7.74256x_2^2 \\
& + 0.05743x_3^2 + 0.00005794x_4^2 - 0.01125x_1x_2 + 0.00338x_1x_3 + 0.00035x_1x_3 + 0.08125x_2x_3 \\
& + 0.01013x_2x_4 + 0.00051x_3x_4
\end{aligned}
\tag{9}
$$

Later, differential evolutionary algorithm was employed for solving the related single and multi-objective optimization problems while developing the optimal Pareto front. It was claimed that the adopted algorithm would be able to determine a set of ECM process parameters for having global optimal values of the responses with reasonable computational cost and time. While applying differential evolutionary algorithm for simultaneous optimization of the two responses, Mehrvar et al. (2017) derived the optimal combination of the process parameters as applied voltage = 24.78 V, tool feed rate = 0.49 mm/min, electrolyte flow rate = 6.84 l/min and electrolyte concentration = 134.15 g/l with a MRR value of 0.28 g/min and SR of 1.36 µm. Now, when the multivariate quality loss function approach is adopted for parametric optimization of the considered ECM process, the acceptable range for MRR is decided to lie between 0.25 g/min and 0.50 g/min. On the other hand, the acceptable range for SR is from 0.5 μm to 1.5 μm. These ranges are decided based on the observed MRR and SR values in Table 2. As MRR is an *L*-type quality characteristic, its higher value is always desired. Similarly, SR being an *S*-type quality characteristic, it is always preferable to have its lower value. Based on these considerations, the USL and LSL values for the two responses are set in the objective function of the developed multi-response optimization problem. Of course, the optimal settings should be constrained to reside on or within the sphere defined by the experimental design plan. Thus, according to the loss function provided in Eq. (7), the multi-response optimization problem can be expressed as follows:

$$
\begin{aligned}
& Min\,L(Y,X)=\left(\frac{y(MRR)-0.5}{0.25}\right)^2+\left(\frac{y(SR)-0.5}{1}\right)^2 \\
& \text{subject to} \\
& 0.25 \le y(MRR) \le 0.50 \\
& 0.50 \le y(SR) \le 1.50 \\
& x_1^2+x_2^2+x_3^2+x_4^2 \le 2^2
\end{aligned}
\tag{10}
$$

The above-mentioned non-linear optimization problem is now solved using LINGO software and the derived results are shown in Table 3. It is observed that a maximum value of MRR and a minimum value of SR are simultaneously achieved at the coded values of x_1, x_2, x_3 and x_4 as 0.2480, -1.6213, 0.9892 and 0.1942 respectively. After de-coding, the optimal parametric mix for the considered ECM process is thus attained at applied voltage = 21.24 V, tool feed rate = 0.2379 mm/min, electrolyte flow rate = 7.9892 l/min and electrolyte concentration = 159.71 g/l with a MRR of 0.40 g/min and SR of 1 μm. It is interestingly noticed that with this optimal parametric mix, the MRR value is increased by 30% and SR value is trimmed down by 26.47% as compared to those obtained by Mehrvar et al. (2017). From the single-objective optimization results of Mehrvar et al. (2017), it is observed that maximum MRR would occur at higher applied voltage, tool feed rate and electrolyte concentration, while moderate to low level of applied voltage and electrolyte concentration would be responsible for minimum SR. In order to have a trade-off between these two conflicting quality characteristics (responses), it is thus recommended to operate the considered ECM process at higher values of applied voltage, electrolyte flow rate and electrolyte concentration, and lower value of tool feed rate. Table 3 also exhibits a comparison of the derived optimal solutions with those achieved while employing the other popular multi-objective optimization approaches, i.e. desirability function method (Derringer and Suich, 1980), distance function method (Khuri. and Conlon, 1981; Jing and Yongfan, 2017) and mean squared error method (Vining, 1998). In all the cases, the achieved MRR and SR values are observed to be worst as compared to those derived using the multivariate quality loss function approach.

Table 3
Comparison of the optimal parametric settings for ECM process

Optimization approach	Variable				Response	
	x_1	x_2	x_3	x_4	MRR	SR
Multivariate loss function approach	0.2480	-1.6213	0.9892	0.1942	0.4	1
Differential evolutionary algorithm	0.9560	0.9	-0.16	-0.317	0.28	1.36
Desirability function method	2	2	2	-2	0.2589	1.2
Distance function method	0.7923	-1.5851	0.9091	-1.1820	0.3700	1.2
Mean squared error method	0.4002	-1.5522	1.0255	0.3548	0.3670	1.151

4.2 Example 2

While performing EDM operation on AISI 316LN stainless steel using copper as an electrode, Majumder et al. (2014) predicted the optimal settings of three process parameters, i.e. supply current, pulse-on time and pulse-off time for achieving maximum MRR and minimum volumetric electrode wear ratio (EWR). Each of those process parameters was set at three different levels, as provided in Table 4, and based on Box-Behnken response surface design plan, 15 experiments were carried out at different combinations of the considered process parameters. The detailed experimental results are exhibited in Table 5. Majumder et al. (2014) then employed desirability-based multi-objective particle swarm optimization (PSO) (original), desirability-based multi-objective PSO (inertia weight) and desirability-based multi-objective PSO (constriction factor) methods for multi-objective optimization of the responses. It was concluded that the desirability-based multi-objective PSO (constriction factor) method outperformed the other two approaches with respect to the achieved values of both the responses.

Table 4
EDM process parameters and their levels (Majumder et al., 2014)

Process parameter	Symbol	Unit	Level		
			−1	0	1
Supply current (A)	x_1	A	3	7	11
Pulse-on duration (ON)	x_2	μs	8000	10000	12000
Pulse-off duration (OFF)	x_3	μs	8000	10000	12000

Table 5
Experimental plan and results for EDM process (Majumder et al., 2014)

Exp. No.	Process parameter			Response	
	A	ON	OFF	MRR (mm^3/sec)	Volumetric EWR (%)
1	0	0	0	8.13	5.994
2	0	1	-1	2.3173	4.492
3	0	0	0	8.438	6.01
4	-1	-1	0	3.429	4.7926
5	1	0	-1	4.008	14.487
6	-1	0	1	3.625	5.496
7	0	-1	-1	5.003	4.13
8	1	1	0	4.919	6.331
9	0	1	1	5.82	4.5283
10	-1	1	-1	3.03	4.8789
11	0	0	0	8.42	5.98
12	-1	1	0	3.27	4.055
13	1	-1	0	4.743	16.32
14	1	0	1	10.011	4.1759
15	0	-1	1	4.2307	6.727

Based on the experimental data, the following two RSM-based equations were developed interrelating the responses with the three EDM process parameters which are later employed for the multivariate quality loss function-based approach.

$$Y(MRR) = 8.43060 + 1.54088x_1 + 0.11508x_2 + 1.16605x_3 - 2.75730x_1^2 - 3.58305x_2^2 - 2.00480x_3^2 + 0.08375x_1x_2 + 1.852x_1x_3 + 0.56875x_2x_3 \tag{11}$$

$$Y(EWR) = 5.9940 + 2.7614x_1 - 1.5704x_2 - 0.8826x_3 + 2.0854x_1^2 - 0.2047x_2^2 - 0.8199x_3^2 - 2.31228x_1x_2 - 2.7321x_1x_3 - 0.6402x_2x_3 \tag{12}$$

Now, based on the above-mentioned RSM-based equations and experimental data of Table 5, the following multi-response optimization problem in the form of multivariate loss function is formulated. It is worthwhile to mention here that MRR is an *L*-type and EWR is an *S*-type quality characteristics.

$$\min L(Y,X) = \left(\frac{y(MRR)-10}{2}\right)^2 + \left(\frac{y(EWR)-3}{1}\right)^2$$

subject to

$$8 \le y(MRR) \le 10 \tag{13}$$

$$3 \le y(EWR) \le 4$$

$$x_1^2 + x_2^2 + x_3^2 \le 1^2$$

While solving this multi-response optimization problem using LINGO software, it is observed that a maximum MRR value of 9.0776 mm^3/sec and a minimum EWR value of 3.5% are attained at the coded parametric settings of the considered EDM process as x_1 = 0.4509, x_2 = 0.0616 and x_3 = 0.5078. On the other hand, while applying the desirability-based multi-objective PSO (constriction factor) approach, Majumder et al. (2014) achieved a MRR value of 8.4306 mm^3/sec and an EWR value of 3.95% at the optimal settings of x_1 = 0, x_2 = 0 and x_3 = 1. It is noticed that in the multivariate loss function approach, there is an increase in MRR by 7.67% and EWR is decreased by 11.39%. In this approach, the actual parametric mix for the considered EDM process is observed as supply current = 8.8036 A, pulse-on duration = 10123.2 μs and pulse-off duration = 11015.6 μs. Thus, in this EDM operation, for having the maximum MRR and minimum EWR values, it is always recommended to approximately set higher values for all the considered process parameters. Table 6 compares the optimization performance of multivariate loss function approach with that of the other methods, like desirability function, distance function and mean squared method, and it is revealed that this adopted method supersedes others with respect to the achieved values of both MRR and EWR.

Table 6
Comparison of the optimal parametric settings for EDM process

Optimization approach	Variable			Response	
	x_1	x_2	x_3	MRR	EWR
Multivariate loss function approach	0.4509	0.0616	0.5078	9.0776	3.5
Desirability-based multi-objective PSO	0	0	1	8.4306	3.95
Desirability function method	0	1	1	6.5281	5.344
Distance function method	0.0382	0.0029	0.0290	8.52	3.60
Mean squared error method	0.3426	0.0339	0.3151	9.01	3.61

4.3 Example 3

In this example, the experimental data of Sivaprakasam et al. (2013) are considered for multi-objective optimization of the responses using the adopted approach. Based on a central composite design plan, Sivaprakasam et al. (2013) conducted 20 experiments in a micro-WEDM set-up while taking into account three process parameters, i.e. voltage, capacitance and feed rate, and three responses, i.e. MRR (in mm^3/min), kerf width (KW) (in μm) and SR (in μm). Each of those process parameters had three different levels, as shown in Table 7. The derived experimental data are exhibited in Table 8. Using desirability function approach, Sivaprakasam et al. (2013) determined the optimal values of MMR, KW and SR responses as 0.0260 mm^3/min, 87 μm and 0.97 μm respectively at a parametric combination of voltage = 80 V, capacitance = 0.01 μF and feed rate =15 μm/sec. In this example, MRR is an *L*-type response, whereas, KW and SR are *S*-type responses.

Table 7
Micro-WEDM process parameters and their levels (Sivaprakasam et al., 2013)

Process parameter	Symbol	Unit	Level		
			−1	0	1
Voltage	x_1	V	80	90	100
Capacitance	x_2	μF	0.01	0.1	0.4
Feed rate	x_3	μm/sec	5	10	15

Based on the RSM technique, three regression models were developed highlighting the influences of the micro-WEDM process parameters on the considered responses. Using these three regression equations and employing the multivariate loss function approach, the corresponding optimization problem is now formulated which is subsequently solved using LINGO software. It is interestingly noticed that in this approach, the value of MRR is increased to 0.0893 mm^3/min, and the values of KW and SR are decreased to 85.73 μm and 0.95 μm respectively. From the experimental investigations of Sivaprakasam et al. (2013), it is observed that an increase in voltage causes higher MRR value. Similarly, MRR also tends to increase with increase in capacitance. Increase in MRR is usually responsible for higher SR. Higher feed rate also causes attainment of higher MRR. On the other hand, minimum KW can be achieved at lower value of voltage.

Table 8
Experimental data for micro-WEDM process (Sivaprakasam et al., 2013)

Exp. No.	Variables			Response		
	x_1	x_2	x_3	MRR (mm^3/min)	KW (μm)	SR (μm)
1	0	0	0	0.025084	97	2.19
2	0	0	0	0.025694	97	1.77
3	1.68	0	0	0.027545	96	2.07
4	0	1.68	0	0.025992	94	3.2
5	1	1	-1	0.020042	96	3.06
6	-1	-1	-1	0.015232	87	1.1
7	0	-1.68	0	0.024028	88	0.92
8	0	0	0	0.026618	97	2.07
9	0	0	0	0.027551	97	2.17
10	0	0	0	0.026297	96	1.72
11	-1	-1	1	0.025925	87	0.97
12	1	1	1	0.033847	94	3.3
13	0	0	1.68	0.035446	96	1.97
14	-1	1	1	0.035714	95	2.27
15	1	-1	-1	0.021056	97	1.15
16	1	-1	1	0.03675	97	1.12
17	-1.68	0	0	0.022	90	1.16
18	0	0	0	0.025463	96	2.15
19	0	0	-1.68	0.017504	97	1.97
20	-1	1	-1	0.018968	98	2.55

With the increment in voltage, the SR tends to increase noticeably. The SR response also increases when capacitance is increased. Keeping all these findings in mind, it is thus advised to operate the considered micro-WEDM process at lower values of voltage and capacitance, and higher value of feed rate so as to have maximum MRR, and minimum KW and SR values. A comparison between the derived optimal parametric settings (coded) for the considered micro-WEDM process and those attained by the other approaches are provided in Table 9. These observations also assure the superiority of multivariate quality loss function approach over the other considered techniques as an effective multi-objective optimization tool.

$$Y(MRR) = 0.26194 + 0.017538\,x_1 + 0.12824\,x_2 + 0.060791\,x_3 - 0.021428\,x_1x_2 \tag{14}$$

$$Y(KW) = 96.69807 + 1.98367x_1 + 1.83723x_2 - 0.48926x_3 - 2.87500x_1x_2 - 0.62500x_2x_3 - 1.18272x_1^2 - 1.88983x_2^2 \tag{15}$$

$$Y(SR) = 2.03491 + 0.24533x_1 + 0.78748x_2 + 0.17750x_1x_2 - 0.12728x_1^2 \tag{16}$$

$$\min L(Y,X) = \left(\frac{y(MRR)-0.035}{0.01}\right)^2 + \left(\frac{y(KW)-85}{5}\right)^2 + \left(\frac{y(SR)-0.9}{0.1}\right)^2 \tag{17}$$

subject to

$$0.025 \le y(MRR) \le 0.035 \quad 85 \le y(KW) \le 90 \quad 0.9 \le y(SR) \le 1$$

$$x_1^2 + x_2^2 + x_3^2 \le 1.68^2$$

Table 9
Comparison of the optimal parametric settings for micro-WEDM process

Optimization approach	Variable			Response		
	x_1	x_2	x_3	MRR	KW	SR
Multivariate loss function approach	-0.7933	-0.8289	-1.2512	0.0893	85.73	0.95
Desirability function method	-1	-1	1	0.0260	87	0.97
Distance function method	-0.810	-0.856	-1.25	0.0876	86.31	0.9626
Mean squared error method	-0.0014	-0.012	0	0.035	86	0.96

5. Conclusions

In non-traditional as well as conventional machining processes, it is always desired to explore their fullest machining potential while setting the controllable process parameters at their optimal operating levels. Several mathematical approaches already exist which can determine the optimal settings of the NTM process parameters separately for each of the responses. As there is always more than one response in NTM processes, it is almost impossible/impractical to set different parametric settings for different responses all at a time in a particular machining set-up. Thus, in this paper, a multivariate quality loss function approach is adopted to identify a single parametric setting which would almost simultaneously optimize all the responses of a considered NTM process. The adopted approach would guide the concerned process engineers in identifying the desired combinations of various parameters for different NTM processes. The derived results are also compared with those obtained by the other popular methods which prove the superiority of the adopted approach over them as an efficient multi-objective optimization tool. This approach can also be employed to other NTM and conventional material removal processes to determine the optimal parametric mixes so as to achieve the desired responses under constrained machining environments.

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