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A memo on stock model with partial backlogging under delay in payments

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A B S T R A C T

In this paper, we presents an inventory problem where initially, a retailer purchases $Q(=P+R)$ units and after fulfilling the backlogged quantities, there is a P unit of the on-hand inventory. It continuously declines to meet the customer's demand, which depends on the on-hand inventory level up to the time $t = t_1$. After that the inventory level declines by constant demand up to $t = t_2$. Thereafter, shortage occurs and it accumulates at the rate $\psi(T-t)$ till $t = T$ when the next batch arrives. This whole cycle repeats itself after the cycle length T. The proposed model of this paper is investigated under various conditions and the implementation of the proposed model is presented through some numerical examples.

1. Introduction

During the past few years, there have been tremendous efforts on inventory models having stock dependent demand. Whitin (1957) stated "For trade stores, the internal control downside for vogue merchandise is more difficult by the actual fact that inventory and sales don't seem to be freelance of one another". A rise in inventory might transport hyperbolic sale of things. Wolfe (1968) bestowed associate empirical proof of this bond, note that the sales of fashion merchandise such as women's dresses or sport garments square measure proportional to the number of stock displayed. Thus, hyperbolic inventory levels offer the client a wider choice and increase the likelihood of constructing a buyer deal. However, stocking an excessive amount of inventory might hold up retailer's capital. Within the internal control models, ancient EOQ (Economic inventory model) and EPQ (Economic Produce Quantity) solely thought-about one aspect best. Every model has some artificial assumptions, like no deterioration, no shelf space area and best-known demand.

Time-honored inventory models have shaped under the idea of constant demand, time-dependent demand etc. Variety of inventory models are shown that the demand relies on the inventory level. Levin et al. (1972) ascertained that "large piles" of client merchandise displayed in an exceedingly

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grocery store escort the purchasers to shop for additional. Silver and Peterson in (1985) additionally noted that sales at the retail level tend to be proportional to the quantity of inventory displayed. Baker and Urban (1988) thought about an influence sort of demand rate in inventory model, which might flip down at the side of stock level throughout the whole cycle. Mandal and Phaujdar (1989) enclosed the deteriorating things with the linearly stock-dependent demand. Datta and Pal (1990) custom the model of Baker and Urban (1988) by assure that the stock-dependent demand was downward to a provide level of stock, beyond which it is a stable. By their assumption, not all the shoppers square measure fascinated to get merchandise by the vast stock. Once the stock level decline to an exact level, customers arrive to get merchandise as a result of its goodwill, high quality or facilities. Since then, the analysis articles that handled stock-dependent demand square measure Urban (1992), Pal et al. (1993), Padmanabhan and Vrat (1995), Datta and Paul (2001), Chang (2004), Hou and Lin (2006), Goyal and Chang (2009) and others.

For the amount of the shortages the partially backlogging is taken into account. And, we've to differentiate between the backorders and lost sale cases. The price arises from losing some sales leading to lost profits and annoyed customers. With a nonexistent sale, it will be thought about because the loss of profit on the sales. Or, it additionally includes the price of losing the purchasers, loss of goodwill, and of building a poor record of service. Therefore, if we tend to omit the lost sales from the profit perform then the profit are going to be overrated. It's true that the cost of stockout is incredibly tough to see. However, this doesn't mean that the unit doesn't have some precise values. In observe, the stockout price will be simple to get from clerking knowledge. Tsu-pang et al. (2010) extended the model of Datta and Pal's (1990) permitting the shortages with partially backlogging. Recently Pal et al. (2005), Bhunia and Shaikh (2011), Bhunia et al. (2013) and Bhunia and Shaikh (2014) developed different type's inventory model considering partially backlogged shortage.

In this paper, an endeavor has been created to review a scenario, once the demand rate declines with stock-level and right down to a particular level of inventory, and after this demand rate becomes constant. Shortages area unit permissible with partial backlogging rate underneath the condition of permissible delay in payments. Considering totally different situations are investigated and also the corresponding optimization issues have developed and resolved by the standard software LINGO 10.0. to exemplify the model, a numerical example has been resolved. A sensitivity analysis is additionally checked on the various parameters of the model for the best policy.

2. Notations and Assumptions

2.1 Notations

A	Replenishment cost per order
c	Purchasing cost per unit
m_1	Mark up rate ($m_1>1$)
s	Selling price per unit, where $s = m_1 c$
I_c	Rate of interest payable to the supplier
I_e	Rate of interest earned by the retailer
Q	Order quantity per cycle
P	The maximum inventory level per cycle
R	Maximum shortage quantity per cycle
c_1	The holding cost per unit per unit time
c_2	Backorder cost per unit per unit time
c_3	Opportunity cost per unit
t_1	Time point at which the inventory level reaches Q_0, where Q_0 is Known
t_2	Time point at which the shortages are allowed
T	Length of the inventory cycle
M	Period of permissible delay in payments offered by the supplier
$I^1(t)$	Level of positive inventory at time t, where $0 \le t \le t_1$

$I^2(t)$ Level of positive inventory at time t, where $t_1 < t \le t_2$
$I^3(t)$ Level of negative inventory at time t, where $t_2 < t \le T$
$\pi^{(1)}(t_2,T)$ the total profit per unit time when $0<M<t_1$
$\pi^{(2)}(t_2,T)$ the total profit per unit time when $t_1<M<t_2$
$\pi^{(3)}(t_2,T)$ the total profit per unit time when $t_2<M<T$

2.2 Assumptions

- The Replenishment rate is infinite and lead time is to be constant.
- The time horizon of the inventory system is consider to being infinite and system involves only one item.
- The demand rate of the system is dependent on the on-hand inventory (stock) and down to a certain inventory level Q_0, where Q_0 is fixed and known, beyond that level it is assumed to be a constant, i.e, when the on-hand inventory (stock) level is $I(t)$, and the demand rate $S(I(t))$ of the item is to be considered in this form.

$$S(I(t)) = \begin{cases} \alpha[I(t)]^{\beta}, & I(t) \ge Q_0 \\ W, & 0 \le I(t) < Q_0 \end{cases}$$

where $\alpha > 0$ and $0 < \beta < 1$ are termed as scale and shape parameters respectively, W (>0) is

constant such that $W = \alpha Q_0^{\beta}$.

- Shortages if any, are allowed then it is partially backordered, that is., only a fraction of shortages backordered is a function of time t denoted by $\psi(t)$, where t is the waiting time up to the next replenishment with $0 \le \psi(t) < 1$. Let the fraction is given by $\psi(t) = \frac{1}{1+\delta(t)}$, $\delta > 0$. It is to be noted that the partial backlogging reduces to a completely backlogging when $\delta \to 0$ i.e., $\psi(t) \to 1$.

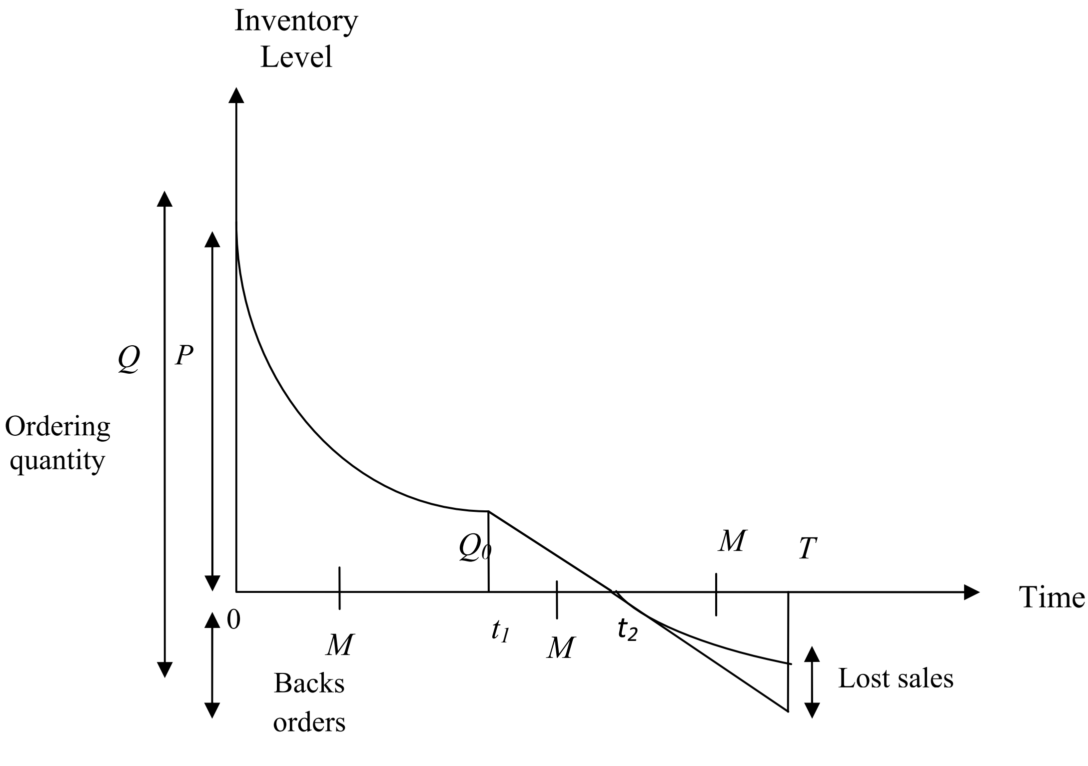

Fig. 1. Pictorial Representation of Inventory system

3. Mathematical formulation

Let us assume that initially, a retailer purchases *Q(=P+R)* units of the item. Then after fulfilling the backlogged quantities, there is a *P* unit of the on-hand inventory. It continuously declines to meet the customer's demand, which depends on the on-hand inventory level up to the time $t = t_1$. After that the inventory level declines by constant demand up to $t = t_2$. Thereafter, shortage occurs and it accumulates at the rate $\psi(T-t)$ till $t = T$ when the next batch arrives. This whole cycle repeats itself after the cycle length *T*. The pictorial representation of the inventory system is shown in Fig. 1. For the period of $(0,\ t_1)$, the stock is depleted due to the effect of demand, which depends on the on-hand inventory level and reaches the level Q_0 at time $t = t_1$. Hence, the inventory level is governed by the following differential equation:

$$\frac{dI^1(t)}{dt} = -\alpha\left(I^1(t)\right)^{\beta}, \qquad 0 \le t \le t_1 \tag{1}$$

with the help of initial and boundary conditions, we have $I^1(t)=P$ at $t=0$ *and* $I^1(t)=Q_0$ *at* $t = t_1$.

Then, the solution of Eq. (1) is in this form

$$I^1(t) = \left\{Q_0^{1-\beta} + \alpha(1-\beta)(t_1 - t)\right\}^{\frac{1}{1-\beta}}, \qquad 0 \le t \le t_1 \tag{2}$$

where the time is $t = t_1$, the demand rate becomes constant i.e. W and the inventory level becomes zero at time the $t = t_2$. In the interval (t_1, t_2), the inventory is depleted due to the effect of demand. Hence, the inventory level is governed by the following differential equation:

$$\frac{dI^2(t)}{dt} = -W, \qquad t_1 < t \le t_2 \tag{3}$$

with the help of this boundary condition $I^2(t) = 0$ at $t = t_2$. Hence, the solution of Eq. (3) is given by

$$I^2(t) = W(t_2 - t) \qquad t_1 < t \le t_2 \tag{4}$$

As there is continuity in between these two level ($I^1(t)$ and $I^2(t)$) at the point $t = t_1$ and it follows from the Eq. (2) and Eq. (4), i.e.,

$$W(t_2 - t_1) = Q_0 \tag{5}$$

which gives,

$$t_2 - t_1 = \frac{Q_0}{W} \Rightarrow \frac{Q_0}{W} + t_1 = t_2 \tag{6}$$

Furthermore, at time $t = t_2$, shortages occur and the inventory level starts dropping below *0*. During the period of shortage, the interval (t_2, T) in this the demand at the time *t* goes to partially backlogged at a fraction *(T-t),* Thus, the inventory level at time *t* is governed by the following equation

$$\frac{dI^3(t)}{dt} = -\frac{W}{1+\delta(T-t)} \qquad t_2 < t \le T \tag{7}$$

with this conditions $I^3(t) = 0$ *at* $t = t_2$, hence the solution of the Eq. (7) is

$$I^3(t) = -\frac{W}{\delta}\left\{\ln\left|1+\delta(T-t_2)\right| - \ln\left|1+\delta(T-t)\right|\right\}, \qquad t_2 < t \le T \tag{8}$$

Now, the maximum level of inventory per cycle is that

$$P = I^1(0) = \left[Q_0^{1-\beta} + \alpha(1-\beta)t_1\right]^{\frac{1}{1-\beta}} \tag{9}$$

The maximum amount of demand, which is backlogged in per cycle is given in this form

$$R = -I^3(T) = \frac{W}{\delta}\left\{\ln\left|1+\delta(T-t_2)\right|\right\} \tag{10}$$

Hence, the total order quantity per cycle is given as follows,

$$Q = P+R = \left[Q_0^{1-\beta}+\alpha(1-\beta)t_1\right]^{\frac{1}{1-\beta}}+\frac{W}{\delta}\left[\ln\left|1+\delta(T-t_2)\right|\right] \tag{11}$$

Now, the holding cost in the entire cycle is given as follows,

$$IHC = c_1\int_0^{t_1} I^1(t)dt + c_1\int_{t_1}^{t_2} I^2(t)dt = \frac{c_1}{\alpha(\beta-2)}\left[\left\{Q_0^{1-\beta}+\alpha(1-\beta)t_1\right\}^{\frac{2-\beta}{1-\beta}} - Q_0^{2-\beta}\right] + \frac{c_1 Q_0^{2}}{2W} \tag{12}$$

The backorder cost in the entire cycle is given as follows,

$$BC = c_2\int_{t_2}^{T}\left\{-I^3(t)\right\}dt = c_2\left[\left(R+\frac{W}{\delta}\right)(T-t_2)-\frac{W}{\delta^2}\left\{1+\delta(T-t_2)\right\}\log\left|1+\delta(T-t_2)\right|\right] \tag{13}$$

The opportunity cost due to lost sales in the entire cycle is given in this form

$$LS = c_3W\int_{t_2}^{T}\left\{1-\frac{1}{1+\delta(T-t)}\right\}dt = \frac{c_3W}{\delta}\left\{\delta(T-t_2)-\ln\left|1+\delta(T-t_2)\right|\right\} \tag{14}$$

The total purchase cost per in the entire cycle is given in this form

$$PC = cQ = c\left[\left\{Q_0^{1-\beta}+\alpha(1-\beta)t_1\right\}^{\frac{1}{1-\beta}}\right] + \frac{cW}{\delta}\ln\left|1+\delta(T-t_2)\right| \tag{15}$$

The total Sales revenue in the entire cycle is given in this form

$$SR = sQ = s\left[\left\{Q_0^{1-\beta}+\alpha(1-\beta)t_1\right\}^{\frac{1}{1-\beta}}\right] + \frac{sW}{\delta}\ln\left|1+\delta(T-t_2)\right| \tag{16}$$

As M be the period of permissible delay in payments offered by the supplier, there may arise different scenarios as follows:

Scenario 1: $0 < M \le t_1$

Scenario 2: $t_1 < M \le t_2$

Scenario 3: $t_2 < M \le T$

3.1 Scenario 1: When $0 < M \le t_1$

Since the length of cycle T is greater than the credit period (M), which is offered by the supplier therefore, the buyer use the sales revenue to earn the interest from the time *0 to M*. Beyond this credit period M, the unsold stock is to be assumed for financed with the rate of I_c. Then the interest earned in this case is as follows :

Interest earned for the period $0 < t \le t_2$

$$= sIe\int_0^{t_1}\int_0^{t}\alpha(I^1(t))^{\beta}dudt + sIe\int_{t_1}^{t_2}\int_{t_1}^{t}W dudt$$

$$= sI_e\left[\frac{1}{\alpha(2-\beta)}\left\{Q_0^{2-\beta}-\left(Q_0^{1-\beta}+\alpha(1-\beta)(t_1)\right)^{\frac{2-\beta}{1-\beta}}\right\}+\left\{Q_0^{1-\beta}+\alpha(1-\beta)t_1\right\}^{\frac{1}{1-\beta}}t_1\right]+sI_e\left[\frac{W}{2}\left(t_2-t_1\right)^2\right]$$

Interest earned on the revenue from the amount generated through shortage units which occurred in the previous cycle $= \frac{sI_e t_2 W}{\delta}\left[\ln\left|1+\delta(T-t_2)\right|\right]$. Hence, the total interest earned in the entire cycle is given in this form

$$IE_1 = sI_e\left[\frac{1}{\alpha(2-\beta)}\left\{Q_0^{\,2-\beta} - \left(Q_0^{\,1-\beta}+\alpha(1-\beta)(t_1)\right)^{\frac{2-\beta}{1-\beta}}\right\} + \left\{Q_0^{\,1-\beta}+\alpha(1-\beta)t_1\right\}^{\frac{1}{1-\beta}} t_1\right] + sI_e W\left[\frac{(t_2-t_1)^2}{2}\right] + \frac{sI_e t_2 W}{\delta}\left[\ln\left|1+\delta(T-t_2)\right|\right] \quad (17)$$

and for this case, the interest paid to the supplier is given by

$$IC_1 = cI_c\int_M^{t_1} I^1(t)dt + cI_c\int_{t_1}^{t_2} I^2(t)dt = \frac{cI_c\alpha}{(2-\beta)}\left[\left\{Q_0^{\,1-\beta}+\alpha(1-\beta)(t_1-M)\right\}^{\frac{2-\beta}{1-\beta}} - Q_0^{\,2-\beta}\right] + cI_c W\left[\frac{(t_2-t_1)^2}{2}\right] \quad (18)$$

Now, the total profit per unit time becomes

$$\pi^{(1)}(t_2,T) = \frac{1}{T}\left(SR-PC-OC-IHC-BC-LS+IE_1-IC_1\right).$$

Hence, the corresponding optimization problem is as follows:

Problem 1:

Maximize $\pi^{(1)}(t_2,T)$
subject to $0<M\le t_1$

3.2 Scenario 2: When $t_1<M\le t_2$

The interest earned in this case is as follows:

Interest earned for period $0<t\le t_2$

$$= sIe\int_0^{t_1}\int_0^{t}\alpha(I^1(t))^\beta du\,dt + sIe\int_{t_1}^{t_2}\int_{t_1}^{t} W\,du\,dt$$

$$= sI_e\left[\frac{1}{\alpha(2-\beta)}\left\{Q_0^{\,2-\beta} - \left(Q_0^{\,1-\beta}+\alpha(1-\beta)(t_1)\right)^{\frac{2-\beta}{1-\beta}}\right\} + \left\{Q_0^{\,1-\beta}+\alpha(1-\beta)t_1\right\}^{\frac{1}{1-\beta}} t_1\right] + sI_e\left[\frac{W}{2}(t_2-t_1)^2\right]$$

Interest earned on the revenue from the amount generated through shortage units which occurred in the previous cycle $= \frac{sI_e t_2 W}{\delta}\left[\ln\left|1+\delta(T-t_2)\right|\right]$. Hence, the total interest earned in the entire cycle is given in this form

$$IE_2 = sI_e\left[\frac{1}{\alpha(2-\beta)}\left\{Q_0^{\,2-\beta} - \left(Q_0^{\,1-\beta}+\alpha(1-\beta)(t_1)\right)^{\frac{2-\beta}{1-\beta}}\right\} + \left\{Q_0^{\,1-\beta}+\alpha(1-\beta)t_1\right\}^{\frac{1}{1-\beta}} t_1\right] + sI_e\left[\frac{W}{2}(t_2-t_1)^2\right] + \frac{sI_e t_2 W}{\delta}\left[\ln\left|1+\delta(T-t_2)\right|\right] \quad (18)$$

and, for this case, the interest paid to the supplier is given by

$$IC_2 = cI_c\int_M^{t_2} I^2(t)\,dt = cI_c W\left[\frac{(t_2-M)^2}{2}\right] \quad (19)$$

Now, the total profit per unit time becomes

$$\pi^{(2)}(t_2,T) = \frac{1}{T}\left(SR - PC - OC - IHC - BC - LS + IE_2 - IC_2\right)$$

Hence, the corresponding optimization problem is as follows:

Problem 2:

Maximize $\pi^{(2)}(t_2,T)$
subject to $t_1 < M \le t_2$

3.3 Scenario 3: When $t_2 < M \le T$

The interest earned in this case is follow as:

$$\text{Interest earned for period } = sIe\int_0^{t_1}\int_0^{t}\alpha(I^1(t))^{\beta}du\,dt + sIe\int_{t_1}^{t_2}\int_{t_1}^{t} W\,dudt$$

$$= sI_e\left[\frac{1}{\alpha(2-\beta)}\left\{Q_0^{\,2-\beta} - \left(Q_0^{\,1-\beta} + \alpha(1-\beta)(t_1)\right)^{\frac{2-\beta}{1-\beta}}\right\} + \left\{Q_0^{\,1-\beta} + \alpha(1-\beta)t_1\right\}^{\frac{1}{1-\beta}} t_1\right] + sI_eW\left[\frac{(t_2-t_1)^2}{2}\right]$$

Interest earned on the revenue from the amount generated through shortage units, which occurred in the previous cycle $= \dfrac{sI_eMW}{\delta}\left[\ln\left|1+\delta(T-t_2)\right|\right]$

Hence, the total interest earned in the entire cycle is given in this form

$$IE_3 = sI_e\left[\frac{1}{\alpha(2-\beta)}\left\{Q_0^{\,2-\beta} - \left(Q_0^{\,1-\beta} + \alpha(1-\beta)(t_1)\right)^{\frac{2-\beta}{1-\beta}}\right\} + \left\{Q_0^{\,1-\beta} + \alpha(1-\beta)t_1\right\}^{\frac{1}{1-\beta}} t_1\right] \quad (21)$$

$$+ sI_eW\left[\frac{(t_2-t_1)^2}{2}\right] + \frac{sI_eMW}{\delta}\left[\ln\left|1+\delta(T-t_2)\right|\right]$$

Now, the total profit per unit time becomes

$$\pi^{(3)}(t_2,T) = \frac{1}{T}\left(SR - PC - OC - IHC - BC - LS + IE_3\right)$$

Hence, the corresponding optimization problem is as follows:

Problem 3:

Maximize $\pi^{(3)}(t_2,T)$
subject to $t_2 < M \le T$

Now our objective is to obtain the optimal solution of the proposed inventory system. This can be accomplished by solving Problem 1-3 and then comparing the average profit of all the scenarios. Hence, the optimal average profit of the system is given by $\mathbf{Z^*}$ = **max** $\left\{\pi^{(1)}(t_2,T), \pi^{(2)}(t_2,T), \pi^{(2)}(t_2,T)\right\}$. The corresponding values of t_1, t_2, T, R and P be denoted by t_1^*, t_2^*, T^*, R^* and P^* which will be the optimal solution of the problem. In this work, we have solved all the problems by using the well known LINGO 10.0 software.

4. Numerical Example

To illustrate the model, we have considered an inventory system with the following,
$A = \$250,\ c = \$8,\ c_1 = \$2,\ c_2 = \$6,\ c_3 = \$5,\ \alpha = 50,\ \beta = 0.3,\ Q_0 = 15,\ \delta = 1.5,\ I_e = \$0.10/12,$
$I_c = \$0.12/12,\ M = 3.0,\ m = 1.40$

The values of the parameters thought have not been picked from any world case study but they represent a real-world case study. For the given constant values, the sub issues of various scenarios are solved for every scenario to search about the optimum result. The computational results have been shown in Table 1. From Table 1, it is experiential that the optimal solution happens for Scenario 3. Conjointly, we have solved the matter for various values markup rate (m_1) and therefore the results are shown in Table 2.

Table 1
Computational results for different cases

Scenario	*P*	*R*	t_1	t_2	*T*	*Average profit*
1	842.46	16.03	3.0000	3.1331	3.2917	123.03
2	792.68	14.68	2.8668	3.0000	3.1439	142.39
3	373.02	8.36	1.6134	1.7466	1.8251	243.90

Table 2
Computational results for different values of mark-up rate (m)

Different values of m_1	*P*	*R*	t_1	t_2	*T*	*Average profit*	*Results obtained from*
1.25	285.84	16.50	1.3068	1.4399	1.6036	3.57	*Scenario 3*
1.30	326.65	13.68	1.4048	1.5380	1.6712	80.91	*Scenario 3*
1.35	342.01	10.97	1.5071	1.6402	1.7451	161.03	*Scenario 3*
1.40	373.02	8.36	1.6134	1.7466	1.8251	243.90	*Scenario 3*
1.45	406.03	5.85	1.7237	1.8568	1.9108	329.47	*Scenario 3*
1.50	441.06	3.42	1.8378	1.9710	2.0021	417.72	*Scenario 3*
1.55	478.13	1.08	1.9557	2.0888	2.0985	508.58	*Scenario 3*

4. Sensitivity Analysis

For the given numerical example mentioned earlier (with m_1=1.25), sensitivity analyses have been performed to check the result of underneath or over estimation of system parameters on the best values of initial stock, most shortage level, cycle length beside the most average profit of the inventory system. The percentage changes within the best values of best values are taken as measures of sensitivity. These analyses are administered by dynamic (increasing and decreasing) the parameters by -20% to +20%. The results are obtained by dynamic one parameter at a time and keeping the opposite parameters at their original values. These results are shown in Table 3 that square measure self-instructive.

Table 3
Sensitivity analyses with respect to different parameters

Parameters	*% changes*	*% changes in* Z^*	*% changes in* t_1^*	t_2^*	T^*	P^*	R^*
c_1	-20	28.3313	20.9000	19.3072	16.3443	27.7385	-48.0861
	-10	13.2267	9.2847	8.5772	7.1832	12.0691	-22.7273
	10	-11.7589	-7.6175	-7.0369	-5.786	-9.592	20.6938
	20	-22.3206	-14.0077	-12.9402	-10.5364	-17.4093	39.7129
c_2	-20	0.0943	-0.0434	-0.0401	0.2849	-0.0536	7.0574
	-10	0.0410	-0.0186	-0.0172	0.137	-0.0268	3.4689
	10	-0.0410	0.0186	0.0172	0.2027	0.0268	-3.11
	20	-0.0820	0.0372	0.0401	-0.2466	0.0483	-6.2201
c_3	-20	0.1148	-0.0496	-0.0458	0.3671	-0.0697	9.0909
	-10	0.0533	-0.0248	-0.0229	0.1753	-0.0322	4.4258
	10	-0.0533	0.0248	0.0286	-0.1589	0.0322	-3.9474
	20	-0.0984	0.0496	0.0458	-0.3068	0.059	-7.6555
A	-20	11.5621	-5.4729	-5.0558	-5.7367	-6.9272	-19.8565
	-10	5.6950	-2.6590	-2.4563	-2.7944	-3.3805	-9.8086
	10	-5.5433	2.5226	2.3304	2.6683	3.2384	9.8086
	20	-10.9430	4.9213	4.5520	5.2326	6.3509	19.3780

Table 3
Sensitivity analyses with respect to different parameters (continued)

Parameters	% changes	% changes in Z^*	% changes in t_1^*	t_2^*	T^*	P^*	R^*
α	-20	-41.2464	5.3118	6.8079	8.7273	-19.5083	17.9426
	-10	-21.2669	2.4173	3.0862	3.9613	-9.8467	9.8086
	10	22.4641	-2.0516	-2.5880	-3.3587	10.0343	-11.3636
	20	45.3875	-3.5825	-4.5749	-8.6845	20.6584	-100
β	-20	-50.3567	-2.3863	-0.8589	2.7615	-31.3308	49.2823
	-10	-28.6347	-1.6611	-0.8875	1.0136	-18.1706	29.3062
	10	38.5363	2.7767	1.9697	-0.2027	25.9477	-42.4641
	20	91.7753	6.8551	5.1932	0.5205	64.4041	-104.1866
C	-20	-54.7519	-10.9954	-10.1575	-7.4188	-13.7473	49.2823
	-10	-27.7655	-5.6155	-5.1818	-3.8464	-7.0961	24.2823
	10	28.5445	5.8448	5.3994	4.1039	7.5546	-23.6842
	20	57.8680	11.9251	11.0163	8.4653	15.5809	-46.7703
Q_0	-20	-0.6109	2.1321	0.8646	0.3342	-6.1578	-16.6268
	-10	-0.3034	1.0475	0.4294	0.1808	0.2681	-7.8947
	10	0.2952	-1.0165	-0.4123	-0.1972	-0.26	7.4163
	20	0.5863	-2.0082	-0.8131	-0.4055	-0.5201	14.4737

5. Conclusions

We know very well that the extent of stock includes a psychological feature impact on the purchasers, however expertise shows that some customers forever arrive to get merchandise for different reasons particularly, goodwill, real price, high-quality of the item, etc. In this paper, we have contemplated a settled inventory model that have an influence kind of the stock dependent demand however once a while it followed as a continuing demand rate, with shortages which is partially backlogging beneath the condition of permissible delay. To indicate the validity of the model a numerical example has been presented. From the numerical examples, it has observed that, the lot of optimum average profit was found in Scenario 3 compared with Scenario 1 & 2. A sensitivity analysis is additionally conducted to indicate the result of modification on the key parameters such as- mark-up rate, ordering value, shortage value, holding value, etc. The projected model has been extended in additional than a couple of ways that, as an example, one might contemplate the impact of inflation and time dependent demand etc.

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