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Integrated production inventory model: multi-item, multiple suppliers and retailers, exponential demand rate

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In this paper, an integrated production-inventory model with multi-item is developed from the perspectives of single producer, multiple suppliers and retailers. In this three-layer supply chain, the retailers are non-competing. Every supplier delivers only single type of raw material to the producer. The producer manufactures finished goods from the combination of fixed percentage of different types of raw materials. The producer manufactures various types of objects and supplies them to retailers according to the demand of multiple retailers. This paper studies the impact of different types of business policies such as exponential demand rate, demand dependent production rate, optimum order size of raw materials, and unit production cost at each stage of integrated marketing system. Mathematica is used to develop the model and to optimize the integrated profit function. A numerical example and sensitivity analysis is illustrated to justify the feasibility of the proposed model.

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1. Introduction

In recent years, there has been significant amount of competition among companies in marketing system. Supply chain plays an important role to deliver the items at right place, in right quantities, at right time and in minimum cost according to customer's satisfaction. These days, multi-retailers shops are increased because people become professional to achieve their needs in minimum time. Now, multiple suppliers' supplies different types of raw materials to producer and producer delivers different types of products according to the demand of multiple retailers. Muckstadt (1973) considered a mathematical inventory model with a multi-item, multi-level and multi-convention organism for reworkable objects. Graves (1979) developed lot arrangement problem in particular machine for deterministic demand of multi-products in which the solitary manufacturing ability produced n products independently. Benton (1991) developed an amount reduction inventory model where various objects, reserve limits and various suppliers were considered. Bhattacharya (2005) developed a multi-

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item inventory model for worsening objects with a linear stock dependent demand rate. Brandimarte (2006) developed a stochastic edition of the traditional multi-item capacitated lot-sizing problem where a multi-stage mixed-integer stochastic programming model was discussed for uncertain demand. Haksever and Moussourakis (2008) presented an inventory system concerning various products with identified and stable autonomous demand, immediate replacement and stable lead era where no shortages were permitted. Caggiano et al. (2009) developed a multi-item, multi-level allotment organism with time based overhaul echelon supplies where seal charges of the channel were computed correctly and proficiently. Tsao (2010) discussed a multi-level multi-objects channel with suppliers' praise era and retailer's promotional attempt. Taleizadeh et al. (2011) considered a multi-buyer multi-vendor supply chain problem with numerous goods, ability limitation of buyer and store constraint of retailer. Sana (2011) developed an integrated multi-level production-inventory model of good and defective quality goods where supplier, producer and retailer were the members of the chain. Sana et al. (2012a) introduced a multi-echelon inventory model considering good and defective quality objects, product consistency and reworking of imperfect objects in the position of supply chain running. Hill and Pakkala (2007) extended a multi-item inventory model where each customer orders a particular list of items. They select at least one item is being order independently from the list of items. Li et al. (2011) developed an inventory model for multi-item with dynamic lot-size taking both single-level and multi-level cases. They considered limited production resources and each item faces a particular demands. Kamali et al. (2011) developed a multi-item mixed integer nonlinear programming model. They considered single customer and multiple retailers under discount policy for the retailers. Freshly, Sana et al. (2012b) developed a three layer multi-item production-inventory model for various suppliers and retailers.

In this paper, we have developed an integrated multi-item production inventory model where single producer, multiple suppliers and retailers are considered. Producer manufactured different types of finished items by a combination of fixed percentage of different types of raw materials. Each supplier delivers one type of raw material to the producer. Multiple retailers orders various types of products to producer and sell these products to customers in the market according to their demands. The integrated profit function of the integrated supply chain is maximized with respect to the ordering lot size of raw materials. We also find the optimal solution of the model including a numerical example and sensitivity analysis.

2. Essential Assumptions:

We used the following assumptions in the model:

1. The demand rates are exponential increasing function of time for each member of three layer supply chain.
2. Production rate is demand dependent i.e.

   $P(t) = \lambda D(t)$, where $D(t) = be^{at}$ *and* $\lambda > 1$

3. Holding cost per unit item varies with manufactured items, raw materials and each member of the supply chain.
4. Multiple (m) items are considered for joint effect of suppliers, producer and retailers in a three layer supply chain.
5. Single producer, multiple suppliers (n) and multiple retailers (l) are considered.
6. Producer produces various types of products on the basis of fixed percentage of different types of raw materials.
7. No shortages at each stage of three-layer supply chain.

8. Insignificant lead time.
9. Retailers purchases different types of manufactured items from the producer according to their demands.

3. Essential Notations

We used the following notations in the model:

We considered $i = 1, 2, 3, ..., n$ for suppliers, $j = 1, 2, 3, ..., m$ for products and $k = 1, 2, 3, ..., l$ for retailers.

R	Supplier's replenishment lot size,
λbe_s^{at}	Producer's demand rate for multi-item,
α	Supplier's percentage of raw material to produce products,
T_s	Supplier's cycle length,
h_s	Supplier's cost of holding per unit per unit time,
C_s	Supplier's purchasing cost per unit item,
w_s	Supplier's selling price per unit item,
λbe_p^{at}	Production rate of multi products,
T_{p_j}	Producer's cycle length for multi products,
h_p	Producer's cost of holding per unit per unit time for multi products,
w_p	Producer's selling price per unit item for multi products,
β	Product's demand percentage for multi-item to fulfill the demand of the retailers,
T_p	Manufacturing run-time of multi products,
L	Labor, energy and technology fixed cost,
$C(P)$	Per unit item production cost for multi-item for the producer,
γ	The variation constant used for tool and die costs,
be_r^{at}	Retailer's demand rate for multi products of producer,
be_c^{at}	Customer's demand rate for multi products,
T_k	Time for collecting multi products from producer for the retailers,
h_r	Retailer's cost of holding per unit per unit time for multi products,
w_r	Retailer's selling price per unit item for multi products,
T_r	Retailer's cycle length for multi products.

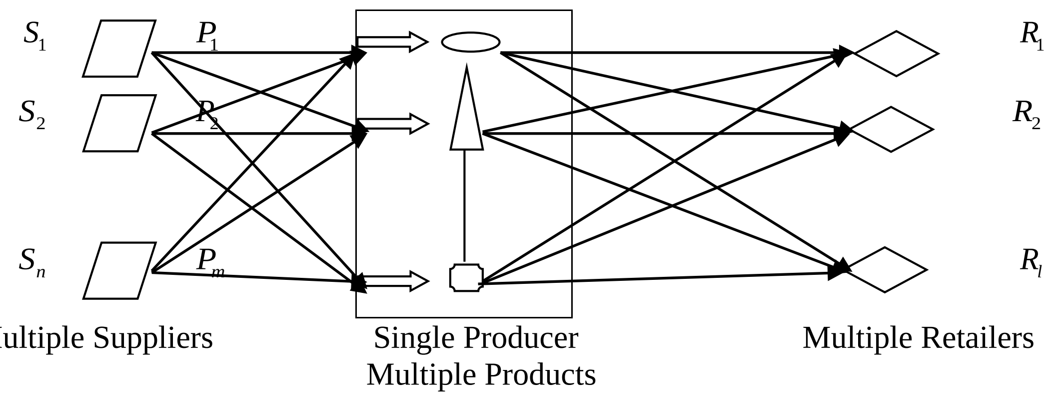

Fig.1. Integrated Logistic System

4. Mathematical Model

We develop a three layer supply chain production inventory model for multi-item with single producer, multiple suppliers and retailers. The producer orders different types of raw materials to different suppliers but one supplier delivers one type of raw material only. The producer produces multi-items where every object is produced by the amalgamation of a definite proportion of the raw materials. The retailer purchase different types of products from the producer and sells to the customers according to their demands which depends on locations and environment of society. Shortages are not allowed at any stage of the model (Fig.1).

4.1. *Inventory model of supplier*

We develop this segment with (n) suppliers where every supplier supplies only one type of raw material. We assume that multiple supplier supplies multiple raw materials. The producer orders R quantity of multiple raw materials at the opening of the manufacturing procedure and collects at a rate $\lambda b{e_s}^{at}$ from the multiple suppliers (Fig.2).

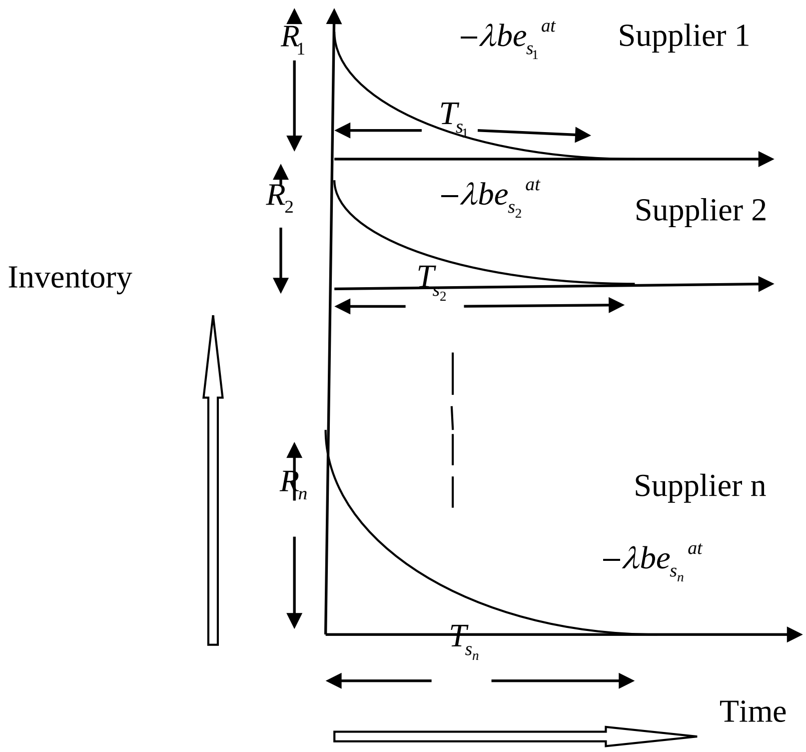

Fig. 2. Supply Chain for Suppliers

The leading differential equation for multi-item is

$$\frac{dQ_s(t)}{dt} = -\lambda b{e_s}^{at}, \text{with}\, Q_S(0) = R \text{ and } Q_S(T_s) = 0\,, 0 \le t \le T_s \tag{1}$$

From Eq. (1) we have,

$$Q_s(t) = \frac{\lambda b}{a}\left(1 - {e_s}^{at}\right) + R, 0 \le t \le T_s \tag{2}$$

Now, we have

$$Q_S\left(T_s\right)=0,\Rightarrow T_s=\frac{1}{a}\log\left(1+\frac{aR}{\lambda b}\right) \tag{3}$$

Supplier's inventory cost of multi-item is

$$C_I=h_s\int_0^{T_s}Q_s\left(t\right)dt=h_s\int_0^{T_s}\left(\frac{\lambda b}{a}\left(1-e_s^{\,at}\right)+R\right)dt=\frac{h_s}{a}\left[\left(\frac{\lambda b}{a}+R\right)\log\left(1+\frac{aR}{\lambda b}\right)-R\right] \tag{4}$$

using Eq. (3).

The supplier's total profit of multi-item = Total revenue from sales-Total inventory cost

$$STP=\left[\left(w_s-C_s\right)R-\frac{h_s}{a}\left(\frac{\lambda b}{a}+R\right)\log\left(1+\frac{aR}{\lambda b}\right)+\frac{h_sR}{a}\right] \tag{5}$$

4.2. Inventory model of producer

The producer manufactured multi-item where the production rate of multi-item is $\lambda be_p^{\,at}$. One unit of multi-item is manufactured by the amalgamation of α percentage of multi raw materials for $i=1,2,3,...,n$. Therefore, the manufacturing of multi-item is $\lambda be_p^{\,at}=\sum_{i=1}^{n}\alpha_{ji}\lambda be_s^{\,at}, 0\le\alpha\le1$ and $\sum_{j=1}^{m}\alpha_{ji}=1$ and the manufacturing run-time of multi-item by putting the value of T_s from Eq. (3) is (Fig.3).

$$T_{p_j}=\frac{\sum_{i=1}^{n}\alpha_{ji}\lambda be_s^{\,at}T_s}{\lambda be_p^{\,at}}=\frac{\sum_{i=1}^{n}\alpha_{ji}e_s^{\,at}\log\left(1+\frac{aR}{\lambda b}\right)}{ae_p^{\,at}}=\frac{1}{a}\log\left(1+\frac{aR}{\lambda b}\right) \tag{6}$$

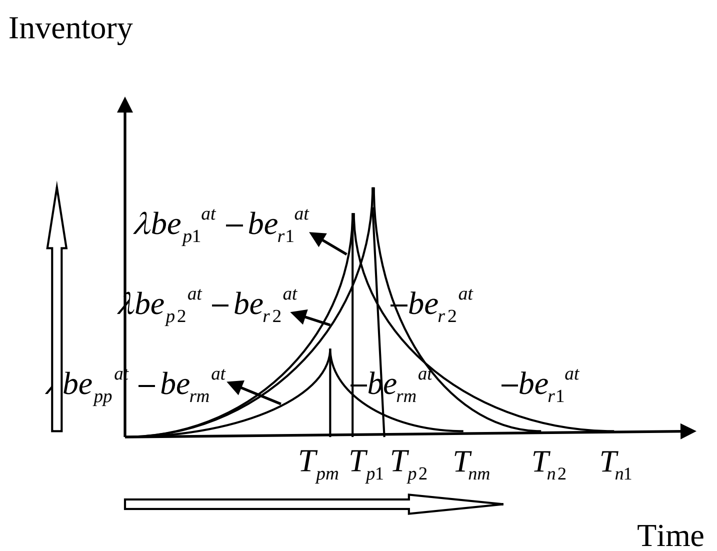

Fig. 3. Supply Chain for Producer

The producer supplies the finished goods to the retailers. Producer delivers multi-item the retailers according to their demands at a rate $be_r^{\,at}$. Then, the leading differential equation with multi-item is

$$\frac{dQ_p(t)}{dt} = \lambda b e_p^{\,at} - b e_r^{\,at}, \text{ with } Q_p(0) = 0,\ 0 \le t \le T_{p_j} \tag{7}$$

and $\frac{dQ_p(t)}{dt} = -b e_r^{\,at}$, with

$$Q_p(T_p) = 0 \text{ and } Q_p\left(T_{p_j}\right) = \left(\lambda b e_p^{\,at} - b e_r^{\,at}\right)T_{p_j},\ T_{p_j} \le t \le T_p \tag{8}$$

From Eq. (7) and Eq. (8), we get

$$Q_p(t) = \left\{\frac{b}{a}\left(1 - e_r^{\,at}\right) - \frac{\lambda b}{a}\left(1 - e_p^{\,at}\right)\right\},\ 0 \le t \le T_{p_j} \tag{9}$$

and

$$Q_p(t) = \frac{b}{a}\left(1 - e_r^{\,at}\right) - \frac{\lambda b}{a}\left(1 - e_p^{\,aT_{p_j}}\right),\ T_{p_j} \le t \le T_p \tag{10}$$

Now, we have

$$Q_p\left(T_{p_j}\right) = \left(\lambda b e_p^{\,at} - b e_r^{\,at}\right)T_{p_j} \tag{11}$$

$$T_p = \frac{1}{a}\log\left(1 + \frac{a}{b}R\right) = \frac{1}{a}\log\left(1 + \frac{aR}{\lambda b}\right)$$

using Eq. (6) for multi-item.

Producer’s inventory cost for multi-item is

$$C_p = h_p\left[\int_0^{T_{p_j}} Q_p(t)\,dt + \int_{T_{p_j}}^{T_p} Q_p(t)\,dt\right] = h_p\frac{b}{a^2}\left[\left(\frac{a}{b} - \frac{a}{\lambda b}\right)R + (1-\lambda)\log\left(1 + \frac{aR}{\lambda b}\right)\right] \tag{12}$$

The unit manufacturing cost for multi-item product is

$$C(P) = \frac{L}{\lambda b e_p^{\,at}} + \gamma\lambda b e_p^{\,at} \text{ where } \lambda b e_p^{\,at} = \sum_{i=1}^{n}\alpha\lambda b e_s^{\,at} \tag{13}$$

$\frac{L}{\lambda b e_p^{\,at}}$ is equally distributed labor/energy cost over manufacturing lot size $\left(\lambda b e_p^{\,at}\right)$. Therefore, unit manufacturing cost reduces with increases in manufacturing lot-size.

$\gamma\lambda b e_p^{\,at}$ is per unit instrument cost of finished goods which is proportional to the manufacturing lot-size and the cost for raw-material per unit item is predetermined which is taken independently in the particular profit function.

Producer’s total profit for multi-item =Total revenue from sales – Total inventory cost

$$PTP = \sum_{j=1}^{m}\left(w_p \lambda b e_p^{at} T_p - C(P)\lambda b e_p^{at} T_p\right) - \sum_{i=1}^{n} R w_s - \sum_{j=1}^{m} C_p$$
$$= \left(\frac{mh_p}{a\lambda} - \frac{mh_p}{a} - nw_s\right)R + \left(\frac{mw_p \lambda b e_p^{at}}{a} - \frac{mh_p b(1-\lambda)}{a^2} - \frac{mL + m\gamma\lambda^2 b^2 e_p^{2at}}{a}\right)\log\left(1+\frac{aR}{\lambda b}\right) \quad (14)$$

4.3. Inventory model of retailer

According to the location of the retailers, the demand rates of the finished goods for the retailers are dissimilar. The multiple retailers collect the multi-item from the producer at a rate $\beta b e_r^{at}$ where β is the quantity of multi-item product for multiple retailers, $0 \le \beta \le 1$ and $\sum_{k=1}^{l}\beta = 1$ for $k = 1,2,3,\ldots,l$. Multiple retailers facing the customer's demand $b e_c^{at}$ for the multi-item. The producer supplies the multi-item to the multiple retailers up to time T_p (Fig.4).

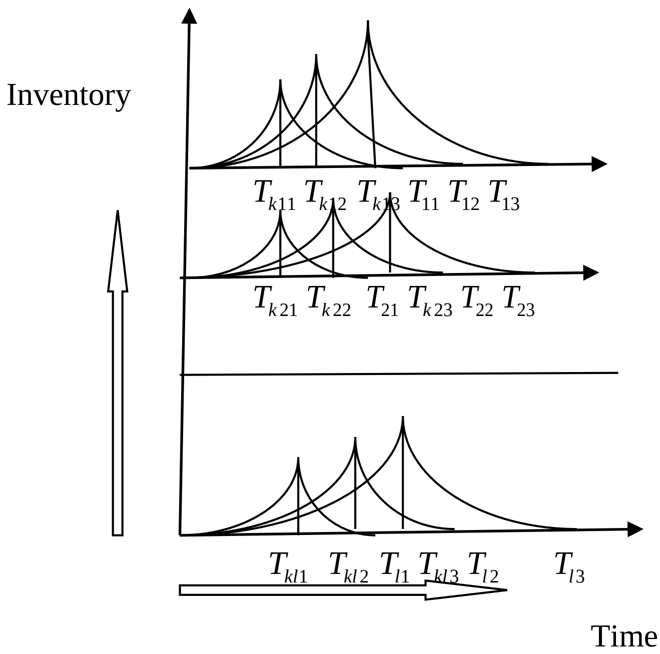

Fig. 4. Supply Chain for Retailers

Then, the leading differential equations for multi-item is

$$\frac{dQ_r(t)}{dt} = b\left(\beta e_r^{at} - e_c^{at}\right), \text{with } Q_r(0) = 0, \qquad 0 \le t \le T_k \quad (15)$$

$$\frac{dQ_r(t)}{dt} = -b e_c^{at}, \text{with } Q_r(T_k) = b\left(\beta e_r^{at} - e_c^{at}\right)T_k \text{ and } Q_r(T_r) = 0, T_k \le t \le T_r \quad (16)$$

From Eq. (15) and Eq. (16), we have

$$Q_r(t) = \frac{b}{a}\left(1 - e_c^{at}\right) - \frac{b}{a}\beta\left(1 - e_r^{at}\right), \; 0 \le t \le T_k \quad (17)$$

$$\text{And}\, Q_r(t) = \frac{b}{a}\left(e_c^{aT_r} - e_c^{at}\right),\ T_k \le t \le T_r \tag{18}$$

Now, we have

$$Q_r(T_k) = b\left(\beta e_r^{at} - e_c^{at}\right)T_k$$

$$T_r = \frac{\beta b e_r^{at} T_k}{b e_c^{at}} = \frac{\beta\lambda b e_p^{at}\frac{1}{a}\log\left(1+\frac{aR}{\lambda b}\right)}{b e_c^{at}} = \frac{\beta}{a}\log\left(1+\frac{aR}{\lambda b}\right) \tag{19}$$

using Eq. (11), according to model assumptions.

Retailer's inventory cost for multi-item is

$$C_r = h_r\left[\int_0^{T_k} Q_r(t)\,dt + \int_{T_k}^{T_r} Q_r(t)\,dt\right] = \left[\left(\frac{\beta h_r}{a\lambda} - \frac{h_r}{a\lambda}\right)R + \left(\frac{\beta h_r}{a\lambda} - \frac{h_r}{a\lambda}\right)R\log\left(1+\frac{aR}{\lambda b}\right)\right] \tag{20}$$

using Eq. (11).

Retailer's total profit for multi-item is

$$RTP = \sum_{j=1}^{m}\left[\begin{array}{l} w_r b e_c^{at}\frac{1}{a}\log\left(1+\frac{aR}{\lambda b}\right) - \beta w_p b e_r^{at}\frac{1}{a}\log\left(1+\frac{aR}{\lambda b}\right) - \left(\frac{\beta h_r}{a\lambda} - \frac{h_r}{a\lambda}\right)R \\ -\left(\frac{\beta h_r}{a\lambda} - \frac{h_r}{a\lambda}\right)R\log\left(1+\frac{aR}{\lambda b}\right) \end{array}\right]$$

$$= \left(\frac{mh_r}{a\lambda} - \frac{m\beta h_r}{a\lambda}\right)R + \left(\frac{mw_r b e_c^{at}}{a} - \frac{m\beta w_p b e_r^{at}}{a} - \left(\frac{m\beta h_r}{a\lambda} - \frac{mh_r}{a\lambda}\right)R\right)\log\left(1+\frac{aR}{\lambda b}\right) \tag{21}$$

5. Integrated total profit

The integrated total profit for multi-suppliers, single producer and multi-retailers, with the help of Eq. (5), Eq. (6), Eq. (14), Eq. (19) and Eq. (21) is

$$ITP(R) = \sum_{i=1}^{n} STP + PTP + \sum_{k=1}^{l} RTP = \left(\frac{mnh_s h_p}{a^2\lambda} - \frac{mh_p}{a} + \frac{mlh_r}{a\lambda} - \frac{ml\beta h_r}{a\lambda} - nC_s\right)R$$

$$+\left(\begin{array}{l} \frac{mw_p\lambda b e_p^{at}}{a} - \frac{mh_p b(1-\lambda)}{a^2} - \frac{mL + m\gamma\lambda^2 b^2 e_p^{2at}}{a} + \frac{mlw_r b e_c^{at}}{a} - \frac{ml\beta w_p b e_r^{at}}{a} \\ -\frac{nh_s\lambda b}{a^2} - \left(\frac{nh_s}{a} + \frac{ml\beta h_r}{a\lambda} - \frac{mlh_r}{a\lambda}\right)R \end{array}\right)\log\left(1+\frac{aR}{\lambda b}\right) \tag{22}$$

$$ITP(R) = AR + (B - CR)\log(1 + DR)$$

where

$$A = \frac{mnh_s h_p}{a^2\lambda} - \frac{mh_p}{a} + \frac{mlh_r}{a\lambda} - \frac{ml\beta h_r}{a\lambda} - nC_s,\ C = \frac{nh_s}{a} + \frac{ml\beta h_r}{a\lambda} - \frac{mlh_r}{a\lambda},\ D = \frac{a}{\lambda b},$$

$$B = \frac{mw_p\lambda b e_p^{at}}{a} - \frac{mh_p b(1-\lambda)}{a^2} - \frac{mL + m\gamma\lambda^2 b^2 e_p^{2at}}{a} + \frac{mlw_r b e_c^{at}}{a} - \frac{ml\beta w_p b e_r^{at}}{a} - \frac{nh_s\lambda b}{a^2}$$

Solution Procedure:

Differentiating Eq. (22) partially with respect to R for $i = 1, 2, 3, ..., n$, we have

$$\frac{\partial(ITP)}{\partial R} = A + \frac{D(B-CR)}{1+DR} - C\log(1+DR) \tag{23}$$

Now, $\frac{\partial(ITP)}{\partial R} = 0,\ A + \frac{D(B-CR)}{1+DR} - C\log(1+DR) = 0$

We get,

$$R = \frac{AD - CD - 2CLogD + D\sqrt{A^2 - 2AC + C^2 + 4(LogD)^2 + 4BCLogD}}{2C(LogD)^2} \tag{24}$$

Again differentiate Eq. (23) partially with respect to R for $i = 1, 2, 3, ..., n$ we have

$$\frac{\partial^2(ITP)}{\partial R^2} = -\frac{2CD}{1+DR} - \frac{D^2(B-CR)}{(1+DR)^2} \tag{25}$$

Thus $\frac{\partial^2(ITP)}{\partial R^2} < 0$ holds at R satisfying necessary and sufficient condition, and then $ITP(R)$ is maximum.

6. Numerical Examples

We consider three suppliers, single producer and three retailors in this integrated production inventory model. Each supplier delivers one type of raw material to the producer. There are three types of raw materials. Producer manufactures three types of products by the combination of three raw materials. Producer supplies three types of products to three retailors according to their demands. The following parameters and numerical data are considered as:

$$m = 3, n = 3, l = 3, \lambda = 2, a = 0.7, b = 0.1, \lambda be_{p1}{}^{at} = 200, \lambda be_{p2}{}^{at} = 220, \lambda be_{p3}{}^{at} = 250$$

Table 1
Supplier's numerical data

Suppliers i	Raw Materials i	h_s	$\lambda be_s{}^{at}$	w_s	C_s
1	1	5	550	18	8
2	2	3	680	16	7
3	3	6	450	22	11

Table 2
Producer's numerical data

Products j	α	L	γ	h_p	$be_r{}^{at}$	w_p
1	0.3	4000	0.01	9	200	60
2	0	4800	0.02	10	250	70
3	0.3	4500	0.01	8	280	65

Table 3

Retailer's numerical data

Retailers k	1	2	3
β	0.4	0.3	0.3
be_c^{at}	250	300	350
w_r	85	85.5	85
h_r	1	1.05	1.1

Applying these numerical values in the Eq. (22) and we have the solutions $R_1 = 1305.5, R_2 = 2077.3$ and $R_3 = 1423.2$. The integrated total profits on the basis of these lot sizes are $ITP_1 = 1280848, ITP_2 = 1567821.5$ and $ITP_3 = 1761590.4$

7. Sensitivity Analysis

To know, how the optimal solution is affected by the parameters, we derive the sensitivity analysis for holding costs of all members of integrated supply chain, purchasing cost of suppliers and production cost of producer. The best values of the parameters of optimal solution are increases or decreases by $25\%, -25\%$ and $50\%, -50\%$. The results of integrated total profits are existing in Table 4 as follows.

Table 4

The results of integrated total profits

Parameters	Values	25% increase	25% Decrease	50% Increase	50% Decrease
h_{s_1}	5	6.25	3.75	7.5	2.5
h_{s_2}	3	3.75	2.25	4.5	1.5
h_{s_3}	6	7.5	4.5	9	3
h_{p_1}	9	11.25	6.75	13.50	4.50
h_{p_2}	10	12.50	7.50	15	5
h_{p_3}	8	10	6	12	4
h_{r_1}	1	1.25	0.75	1.5	0.5
h_{r_2}	1.05	1.31	0.79	1.56	0.53
h_{r_3}	1.1	1.38	0.83	1.65	0.55
C_{s_1}	8	10	6	12	4
C_{s_2}	7	8.75	5.25	10.5	3.5
C_{s_3}	11	13.75	8.25	16.5	5.5
L_1	4000	5000	3000	6000	2000
L_2	4800	6000	3600	7200	2400
L_3	4500	5625	3375	6750	2250
R_1	1305.5	1221.3	1447	1173.8	1728.1
R_2	2077.3	5271.5	2314.1	1608.4	2417.8
R_3	1423.2	1093.8	1310.9	1047.7	1546.7
ITP_1	1280848	1465448.3	1121543.6	1724226.7	1030120.6
ITP_2	1567821.5	2635599.8	2764156.6	1647467.7	1031431.4
ITP_3	1761590.4	1415153.7	1098591.8	1654866.8	991455.9

8. Conclusion

We have developed an integrated multi-item production inventory model where multiple suppliers, single producer and multiple retailors were considered. Producer manufactured different types of finished items by a combination of fixed percentage of different types of raw materials. Each supplier delivered one type of raw material to the producer. Multiple retailors ordered different types of products to producer and sell these products to customers in the market according to their demands. We assume that demand rates of each members of integrated supply chain were exponential increasing function of time. The holding costs of raw materials and finished goods were different according to members of integrated supply chain. The integrated profit function of the integrated supply chain has been maximized with respect to the ordering lot size of raw materials. We have also found the optimal solution of the model including a numerical example and sensitivity analysis. Future research can be done for deterioration under inflation, reverse logistics and shortages are allowed.

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