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Optimal pricing and ordering policy for deteriorating items with price and stock dependent demand and partial backlogging

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A B S T R A C T

This study deals with an economic order quantity model to find out the optimal selling price and optimal ordering quantity for the products which deteriorates over time. The demand for the products depends on available stock level and selling price of the products. The shortages are allowed, and it is assumed that the occurring shortages are partially backlogged. Depending on the rate of backlogging two models are presented in this study. The first model assumes a constant rate of backlogging, while in second model the backlogging rate is assumed to be dependent on waiting time. Numerical example and sensitivity analysis are presented to illustrate the results of the proposed model.

1. Introduction

Traditionally, in inventory strategies, it is almost concentrated on solving the optimal order quantity and reorder point but ignoring the optimal selling price to optimize the total average cost. The stock level and selling price are two major factors that affect the demand of any product as well as the optimal results. An optimal lot size for a perishable good, under conditions of finite production and partial backordering and lost sale model was presented by Abad (2000). Abad (2001) also developed an optimal price and order size for a reseller under partial backordering model. Abad (2003) presented the pricing and lot sizing problem for a perishable good under finite production, exponential decay, partial backordering and lost sale. The backlogging phenomenon in the literature was often modeled using backordering and lost sale costs. Zhou et al. (2003) considered a variable production scheduling strategy for deteriorating items with variable demand and partial lost sale. In this policy, each cycle of a schedule starts with a period of shortages and then followed by continuous replenishment. Chu and Chung (2004) discussed the sensitivity of the inventory model with partial backorders. A general time-varying demand inventory lot-sizing model with waiting-time-dependent backlogging and a lot-size-dependent replenishment cost was developed by Zhou et al. (2004).

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Singh and Singh (2009) presented a production inventory model with variable demand rate for deteriorating items under permissible delay in payments. Singh et al. (2010a) developed an inventory model for deteriorating items with shortages and stock-dependent demand under inflation for two-shops under one management. Singh et al. (2010b) presented a policy for replenishment in relation to non-instantaneous deteriorating items. Author then also defined partial back logging and demand dependent stock and with the facility of two storage under inflationary environment. Shukla et al. (2013) developed an economic order quantity (EOQ) model for deteriorating items with exponential demand rate and shortages. Tayal et al. (2014a) introduced a production inventory problem with space restriction for deteriorating products. In this paper, due to limited stock capability, the spare ordered quantity was returned to supplier.

Singh and Sharma (2014) introduced an optimal trade-credit policy for perishable items deeming imperfect production and stock dependent demand. Tayal et al. (2014b) also presented another model of inventory, as an alternate of alternative market and in relation to seasonal goods and in terms of deteriorating nature. In this paper, at the end of the season, the retailer transfers all remaining stock in secondary market with the change in selling price. Tyagi et al. (2014c) presented an optimal replenishment policy for non-instantaneous deteriorating items with stock dependent demand and variable holding cost. Singh et al. (2014) presented three level supply chain of inventory with deterioration for multi items. Tayal (2015) presented an optimal decisions for deteriorating items with expiration date and uncertain lead time. Shastri et al. (2015) developed a supply chain management under the effect of trade credit for deteriorating items with ramp-type demand and partial backordering under inflationary environment. Next, the deteriorating products are the other major issues in the developments of different inventory models. Initially in the development of inventory models, it was assumed that the products have indefinite life time, which is not true. Almost all the products have deteriorating nature; difference is only in the rate of deterioration. Some products have a high rate of deterioration and some products deteriorate with a slow rate. Tayal et al. (2014d) considered a multi items inventory model for deteriorating items, keeping in view the expiration date & permissive shortages. A consolidated production inventory model for perishable products was devised by Tayal et al. (2015a). They also defined the preservation technology for investment. Tayal et al. (2015b) also developed an EPQ model for non-instantaneous deteriorating item with time dependent holding cost and exponential demand rate.

In general, during stock out two possible conditions occur: one is backlogging of customers and the other is lost sales. Backlogging means when the customers come back to fulfill their demands and lost sale means when the customers are impatient and made their purchases from any other supplier. Generally, it is assumed that the occurring demand, during shortages are completely backlogged or completely lost, but this is not always true. In practice, during stock out, some customers come back and other remaining other customers go to other vendors to complete their purchases. Tayal et al. (2015c) presented a model for deteriorating items with expiration date and uncertain lead time. In addition, Tayal et al. (2015d) investigated an inventory model for deteriorating items with seasonal products and an option of an alternative market. Singh and Saxena (2013) presented a closed loop supply chain system with flexible manufacturing and reverse logistics operation under shortages for deteriorating items. Singh et al. (2016) developed an economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology.

In this paper, an optimal pricing and ordering quantity model, in which demand is a function of available stock level and selling price is developed. The shortages are allowed and two different and possible cases of partial backlogging are discussed. For the first case, the rate of backlogging is constant and in the second case, the backlogging rate depends on the waiting time up to the arrival of next lot. The model has been exemplified numerically. The graphical interpretation of results has also been shown.

2. Assumptions and Notations

Assumptions

The following assumptions are used in the development of this model.

1. The products are deteriorating in nature.
2. The demand for the products is price and stock dependent.
3. Shortages are allowed.
4. The occurring shortages are partially backlogged.
5. The warehouse has unlimited capacity.
6. The deteriorated items are completely discarded.

Notations

The following notations are used in the development of this model.

a	initial demand rate for the products
p	selling price per unit
c	purchasing cost per unit
Q	initial stock level at t=0
Q_2	backordered quantity
T	cycle time
t_1	the time at which inventory level becomes zero
h	holding cost per unit
o	ordering cost per order
s	cost of shortage each unit
l	cost of lost sale each unit
d	deterioration cost per unit
θ	rate of backlogging
η	waiting time up to the arrival of next lot
K	a positive constant representing the deterioration rate, K<<1
α	positive constant, α<1
β	positive constant, β>1

3. Mathematical Modelling

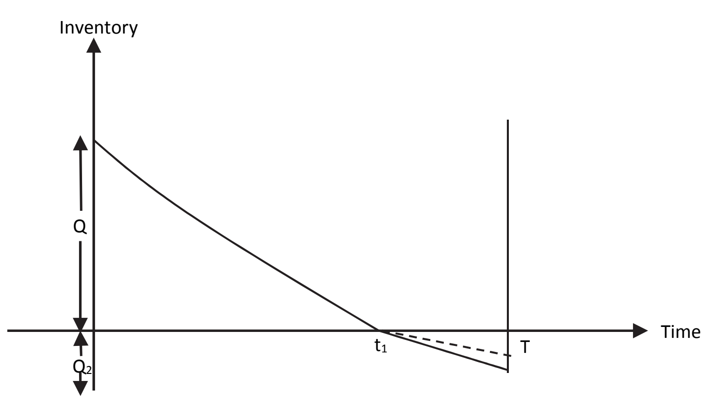

Fig. 1. Inventory time graph for the retailer

Fig. 1 shows the behavior of inventory level over time. Replenishment cycle is set at *T, Q* and Q_2 are the maximum inventory level and backordered quantity, respectively. At time t=0, the inventory level is maximum and as time t increases the inventory level depletes due to combined effect of demand and deterioration during [0, t_1]. At time t=t_1, the inventory level becomes zero and after that shortages occur. The occurring demand during shortages is partially backlogged and these unsatisfied demands are completed at the arrival of next lot. There are the following differential equations, which governing the transitions system:

$$\frac{dI_1(t)}{dt} = -KI_1(t) - \frac{(a + \alpha I_1(t))}{p^\beta} \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -\frac{a}{p^\beta} \qquad t_1 \le t \le T \tag{2}$$

with boundary conditions:

$$I_1(t_1) = I_2(t_1) = 0, \; I_1(0) = Q \tag{3}$$

$$Q = \frac{a}{\alpha + Kp^\beta}(e^{(\frac{\alpha}{p^\beta}+K)t_1} - 1) \tag{4}$$

There are solutions of the problems:

$$I_1(t) = \frac{a}{(\alpha + p^\beta K)}(e^{(\frac{\alpha}{p^\beta}+K)(t_1-t)} - 1) \qquad 0 \le t \le t_1 \tag{5}$$

$$I_2(t) = \frac{a}{p^\beta}(t_1 - t) \qquad t_1 \le t \le T \tag{6}$$

Now we will discuss this model for two different cases of backlogging:

(i) When the occurring shortages during stock out are partially backlogged at a constant rate.
(ii) When the rate of backlogging depends on the waiting time.

4. Cost Analysis

a) Purchasing Cost:

$$P.C\,(p,t_1) = (Q + Q_2)c \tag{7}$$

where

$$Q = \frac{a}{\alpha + Kp^\beta}(e^{(\frac{\alpha}{p^\beta}+K)t_1} - 1)$$

i) For constant rate of backlogging

$$Q_2 = \frac{\theta a}{p^\beta}(T - t_1) \tag{8}$$

ii) For waiting time dependent rate of backlogging

$$\theta(n) = 1 - \frac{T-t}{T} \quad \text{where } n = T-t$$

$$Q_2 = \frac{a}{2p^{\beta}T}(T^2 - t_1^{\,2}) \tag{9}$$

b) Holding Cost

$$\text{H.C }(p, t_1) = h\int_0^{t_1} I_1(t)dt$$

$$\text{H.C }(p, t_1) = \frac{hap^{\beta}}{(\alpha + Kp^{\beta})}\left(\frac{e^{(\frac{\alpha}{p^{\beta}}+K)t_1} - 1}{(\alpha + Kp^{\beta})} - t_1\right) \tag{10}$$

c) Ordering Cost

$$\text{O.C }(p, t_1) = \text{O} \tag{11}$$

d) Shortage Cost

Since during stock out the demand factor depending on stock level will be zero so the occurring shortage will be $\int_{t_1}^{T} \frac{a}{p^{\beta}}dt$

$$\text{S.C }(p, t_1) = s\int_{t_1}^{T} \frac{a}{p^{\beta}}dt \tag{12}$$

e) Lost Sale Cost

i) For constant rate of backlogging

$$\text{L.S.C }(p, t_1) = l\int_{t_1}^{T} \frac{a}{p^{\beta}}(1-\theta)dt$$

$$\text{L.S.C }(p, t_1) = (1-\theta)\frac{a}{p^{\beta}}(T - t_1) \tag{13}$$

(ii) For waiting time dependent rate of backlogging

$$\text{L.S.C }(p,t_1) = l\int_{t_1}^{T} \frac{a}{p^{\beta}}(1-\theta(\eta))dt = \frac{la}{2p^{\beta}T}(T - t_1)^2 \tag{14}$$

f) Deterioration Cost

Total deteriorated units = [Initial stock - total demand during positive inventory]

D.C (p, t1) = $d[Q-\int_0^{t_1}\frac{(a+\alpha I_1(t))}{p^\beta}dt\ ]$

$$\text{D.C (p, t}_1\text{)} = d[Q-\{\frac{\alpha}{p^\beta}t_1+\frac{\alpha a}{p^\beta(\alpha+p^\beta K)}(\frac{e^{(\frac{\alpha}{p^\beta}+K)t_1}-1}{(\frac{\alpha}{p^\beta}+K)}-t_1)\}] \tag{15}$$

Total average cost of the system is represented as under;

$$T.A.C\ (p,\ t_1) = \frac{1}{T}[H.C. + P.C. + O.C. + S.C. + L.S.C. + D.C.] \tag{16}$$

Next, we drive the objectives function for the above inventory system. The problem could be formulated as follows,

min $T.A.C.\ (p,\ t_1)$

subject to

$$p > c\,, 0 < t_1 \le T \tag{17}$$

According to Eq. (17), if p and t are the real numbers, then the total average cost is a function of the two variables p and t_1.

5. Solution Procedure

Since the T.A.C. is the function of two variables p and t_1 with the constraint $p > c$ and $0 < t_1 \le T$. Accordingly, we could deduce the optimal solution as to the following:

$$\frac{\partial T.A.C.(p,t_1)}{\partial p} = 0 \tag{18}$$

$$\frac{\partial T.A.C.(p,t_1)}{\partial t_1} = 0 \tag{19}$$

6. Numerical Example

In practice, the related parameters can be used for historical transaction data. The inputs values are T=50 days, $\alpha = 0.2$, $\beta = 2.5$, c=12 Rs./unit, h=0.2 Rs./unit, a=25 units, s=10 Rs./unit, l=4 Rs./unit, d=12 Rs./unit, K=0.001, θ=0.8

(i) For constant rate of backlogging

For these given parameters, the optimal value of p and t_1 are Rs. 42.9877 and 35.6827 days, respectively and corresponding to these the optimal value of T.A.C. and ordering quantity are Rs. 10.0353 and 137.2 units, respectively.

(ii) For waiting time dependent rate of backlogging

In this case, the optimal values of p and t_1 are Rs. 37.3236 and 31.1129 days, respectively and corresponding to these, the optimal value of T.A.C. and ordering quantity are Rs. 10.051 and 155.97 units, respectively.

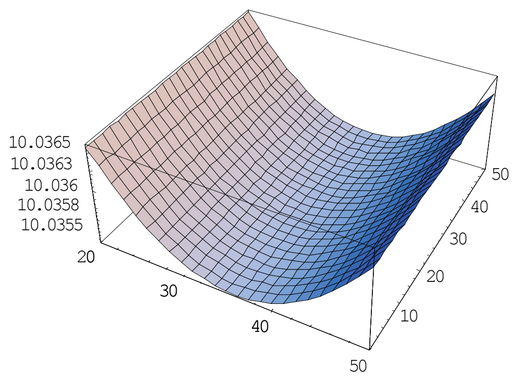

Fig. 2. Behavior of the T.A.C. function for case (i)

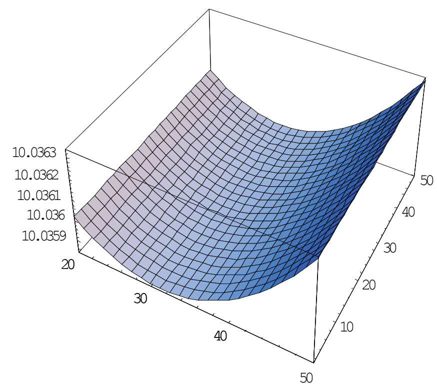

Fig. 3. Behavior of the T.A.C. function for case (ii)

7. Sensitivity Analysis

(i) For constant rate of backlogging

Table 1
Sensitivity Analysis with respect to demand parameter (a)

% variation in a	A	P	t_1	T.A.C
-20%	20	39.9567	33.2373	14.2154
-15%	21.25	39.9567	33.2373	14.4788
-10%	22.5	39.9567	33.2373	14.7423
-5%	23.75	39.9567	33.2373	15.0057
0%	25	39.9567	33.2373	15.2692
5%	26.25	39.9567	33.2373	15.5327
10%	27.5	39.9567	33.2373	15.7691
15%	28.75	39.9567	33.2373	16.0596
20%	30	39.9567	33.2373	16.323

Table 2
Sensitivity Analysis with respect to demand coefficient (α)

% variation in α	A	P	t_1	T.A.C
-20%	0.16	41.0393	34.1107	15.0816
-15%	0.17	40.77	33.8935	15.1272
-10%	0.18	40.4998	33.6755	15.1737
-5%	0.19	40.2287	33.4568	15.221
0%	0.2	39.9567	33.2373	15.2692
5%	0.21	39.6838	33.0171	15.3183
10%	0.22	39.4099	32.7962	15.3683
15%	0.23	39.135	32.5744	15.4193
20%	0.24	38.8592	32.3519	15.4713

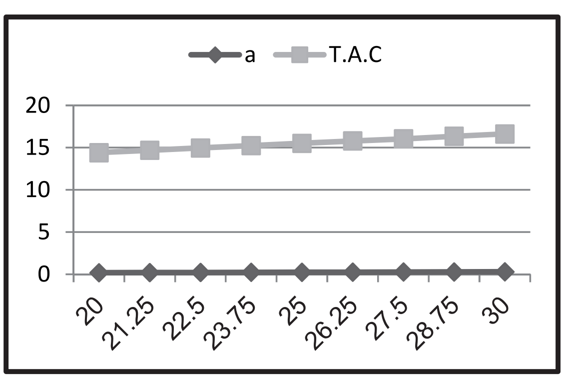

Fig. 4. Changes in T.A.C. versus a

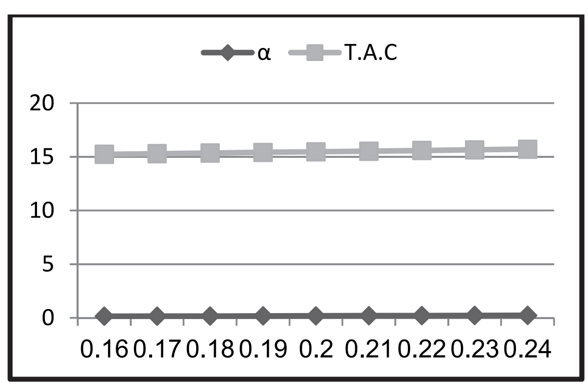

Fig. 5. Changes in T.A.C. versus α

Table 3

Sensitivity Analysis with respect to deterioration rate (K)

% variation in K	K	P	t_1	T.A.C
-20%	0.0008	40.8873	33.9881	15.1073
-15%	0.00085	40.6508	33.7973	15.1476
-10%	0.0009	40.4169	33.6086	15.1881
-5%	0.00095	40.1856	33.422	15.2286
0%	0.001	39.9567	33.2373	15.2692
5%	0.00105	39.7303	33.0547	15.3099
10%	0.0011	39.5063	32.874	15.3506
15%	0.00115	39.2847	32.6952	15.3914
20%	0.0012	39.0655	32.5183	15.4323

Table 4

Sensitivity Analysis with respect to backlogging rate (θ)

% variation in θ	θ	P	t1	T.A.C
-20%	0.64	32.618	27.3164	16.5568
-15%	0.68	34.4599	28.8025	16.1821
-10%	0.72	36.2966	30.2843	15.8465
-5%	0.76	38.1287	31.7624	15.5439
0%	0.8	39.9567	33.2373	15.2692
5%	0.84	41.7812	34.7093	15.0185
10%	0.88	43.6025	36.1788	14.5331
15%	0.92	45.421	37.646	14.5765
20%	0.96	47.2367	39.1111	14.3804

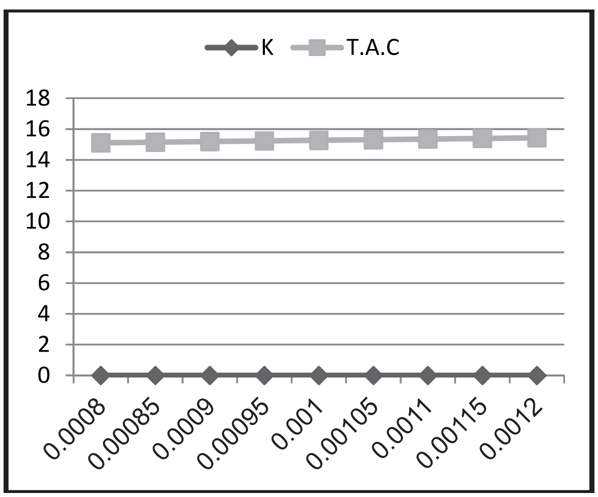

Fig. 6. Changes in T.A.C. versus K

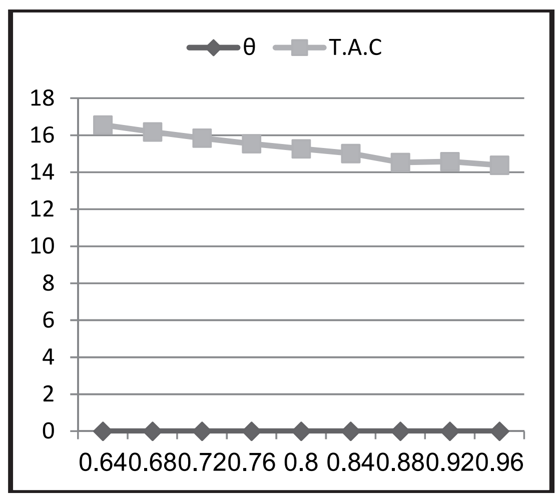

Fig. 7. Changes in T.A.C. versus θ

Table 5

Sensitivity Analysis with respect to holding cost (h)

% variation in h	h	P	t_1	T.A.C
-20%	0.16	49.2097	40.7027	13.9577
-15%	0.17	46.5356	38.5452	14.2773
-10%	0.18	44.1257	36.6009	14.6025
-5%	0.19	41.943	34.8399	14.9332
0%	0.2	39.9567	33.2373	15.2692
5%	0.21	38.1416	31.7729	15.6104
10%	0.22	36.4766	30.4295	15.9568
15%	0.23	34.9437	29.1928	16.3082
20%	0.24	33.528	28.0506	16.6647

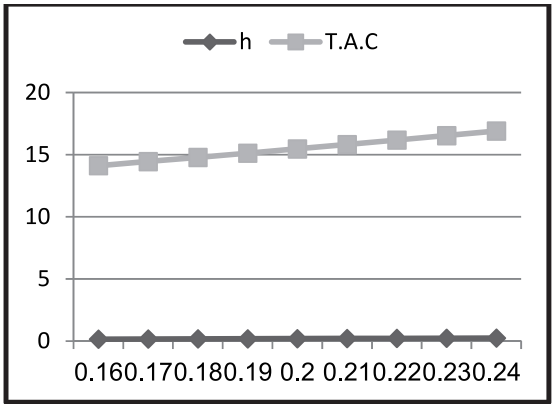

Fig. 8. Changes in T.A.C. versus h

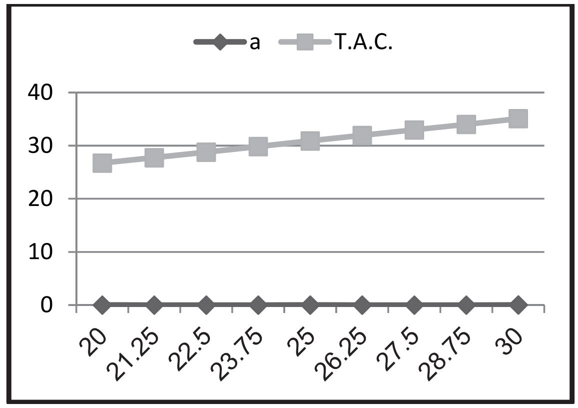

Fig. 9. Changes in T.A.C. versus a

(ii) For waiting time dependent rate of backlogging

Table 6
Sensitivity Analysis with respect to demand parameter (a)

% variation in a	a	P	t_1	T.A.C.
-20%	20	12.7812	11.3119	26.708
-15%	21.25	12.7812	11.3119	27.7522
-10%	22.5	12.7812	11.3119	28.7965
-5%	23.75	12.7812	11.3119	29.8407
0%	25	12.7812	11.3119	30.8849
5%	26.25	12.7812	11.3119	31.9292
10%	27.5	12.7812	11.3119	32.9734
15%	28.75	12.7812	11.3119	34.0177
20%	30	12.7812	11.3119	35.0619

Table 7
Sensitivity Analysis with respect to demand coefficient (α)

% variation in α	A	P	t_1	T.A.C
-20%	0.16	19.9921	17.1298	22.1293
-15%	0.17	18.5255	15.9465	23.3084
-10%	0.18	16.9209	14.6519	24.8585
-5%	0.19	15.0914	13.1759	27.0739
0%	0.2	12.7812	11.3119	30.8849

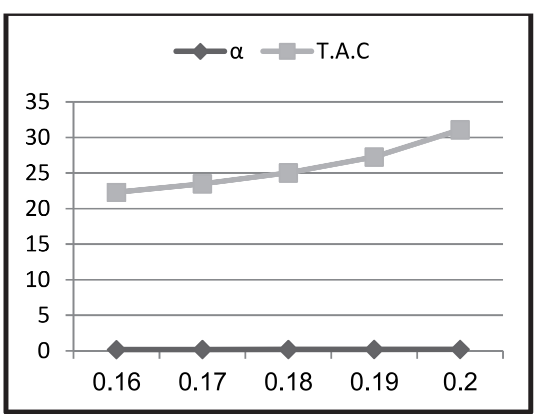

Fig. 10. Changes on T.A.C. versus α

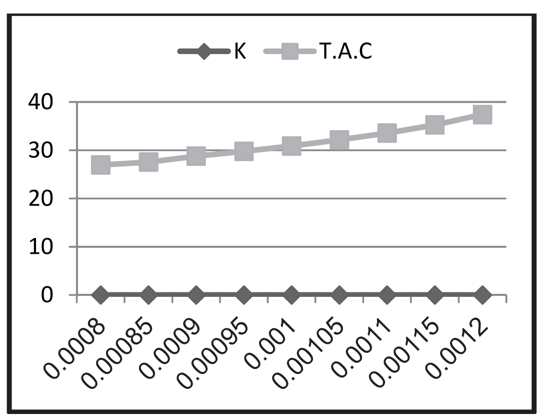

Fig. 11. Changes on T.A.C. versus K

Table 8
Sensitivity Analysis with respect to deterioration rate (K)

% variation in K	K	P	t_1	T.A.C
-20%	0.0008	15.1817	13.2503	26.9507
-15%	0.00085	14.7515	12.7564	27.5537
-10%	0.0009	13.9567	12.2708	28.7722
-5%	0.00095	13.3744	11.7906	29.7677
0%	0.001	12.7812	11.3119	30.8849
5%	0.00105	12.1834	10.8296	32.1326
10%	0.0011	11.5714	10.3359	33.5577
15%	0.00115	10.9281	9.81682	35.2462
20%	0.0012	10.2158	9.24217	37.3907

Table 9
Sensitivity Analysis with respect to holding cost (h)

% variation in h	h	P	t_1	T.A.C
-20%	0.18	56.4308	46.5288	13.3755
-15%	0.19	36.0135	30.0559	15.898
-10%	0.2	25.0367	21.1998	19.266
-5%	0.21	18.0281	15.5452	23.7654
0%	0.22	12.7812	11.3119	30.8849

Result summarized in mentioned tables and figures are as follows,

1) Table 1 and Table 6 show the changes in *p*, t_1 and T.A.C. with the variation in demand parameter (*a*). From these tables, it is observed that with the increment in (*a*), the values of *p* and t_1 remain unchanged and the value of T.A.C. increases.
2) Table 2 and Table 7 show the changes in critical time (t_1) in selling price (*p*) and in T.A.C. for variation in demand coefficient (α). It is observed that with the increment in *α*, the value of *p* and t_1 decrease and T.A.C. remains constant.
3) Table 3 and Table 8 show the deterioration rate (*K*) at different points under various cases and other variables do not change. It is depicted that as deterioration rate (*K*) enhances, the critical point (t_1) and selling price (*p*) diminish, but T.A.C. also enhances.
4) Table 4 indicates that the value of backlogging rate (θ) at different points and other variable are unaffected. It depicts that as backlogging rate (θ) enhances, the critical point (t_1) and selling price (*p*) enhances but T.A.C. gives the opposite effect.
5) Table 5 and Table 9 show the changes in holding cost (*h*) and the other variable which are not altered. It depicts that as (*h*) enhances the critical point (t_1) and the selling prices (*p*) diminishes but T.A.C. increases.

8. Conclusion

This research has addressed an optimal ordering and pricing policy. The system of price and stock sensitive demand and partial backlogging during shortages, we considered here, has incorporated more realistic feature of inventory control. The proposed model described clearly different conditions of backlogging and the effect of stock level and selling price dependent demand on the decisions of a model. Computational tests and analysis shown in the model have indicated that the model was practical and satisfactory. For future, the model can be extended for deterioration having time dependent rate and for different cases of permissible delay.

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