Uncertain Supply Chain Management 6 (2018) 321–334

Contents lists available at GrowingScience

Uncertain Supply Chain Management

homepage: www.GrowingScience.com/uscm

Vendor managed inventory model for non-instantaneous deteriorating product with quadratic demand allowing partial backlogging

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C H R O N I C L E

Article history:
Received March 2, 2017
Received in revised format October 10, 2017
Accepted October 18 2017
Available online October 18 2017

Keywords:
Vendor managed inventory (VMI)
Non-instantaneous deteriorating item
Single vendor
Single buyer
Quadratic demand
Generalized deterioration rate

A B S T R A C T

Vendor managed inventory system is vendor centric inventory system in which all inventory decisions are taken by vendor rather than buyer. This study considers non-instantaneous deteriorating items with generalized type time dependent deterioration rate when demand rate is quadratic function of time and shortages are allowed with partial backlogging. The paper aims to compare traditional inventory system and vendor managed inventory system by analyzing total cost function. To validate the model, numerical example and sensitivity analysis are performed. The analysis shows that vendor managed inventory is more beneficial compare with the traditional inventory system.

1. Introduction

Most of the inventories lose their characteristics over time, which is known as deterioration. For example photography film, food, cloths, drugs, etc. (Khanlarzade et al., 2014). Whitin (1957) is believed to be the first who introduced this concept. Behavior of deteriorating inventory was first modeled by Ghare and Schrader (1963). After Ghare and Schrader (1963)'s work, many studies have been considered deteriorating items as a central agenda. Tripathy and Mishra (2011) formulated economic order quantity (EOQ) model with two variable distribution deterioration rate and ramp type demand rate. Tripathy and Pradhan (2012) developed EOQ model adopting three variable Weibull distribution deterioration function by optimizing cost function including salvage value. Shukla et al. (2013) studied EOQ model adopting exponential demand rate with constant deterioration rate allowing partial backlogging. Singhal and Singh (2015) optimized total cost by considering multi variate demand

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doi: 10.5267/j.uscm.2017.10.001

and permitting shortages partially. More detailed reviews are given by Goyal and Giri, (2001), Bakker et al. (2012), Khanlarzade et al. (2014) and Janssen et al. (2016).

The present study is concentrated on deteriorating items, which are fresh for some time span. These products are known as 'non-instantaneous' deteriorating items. Wu et al. (2006) first developed the model for non-instantaneous deteriorating items, allowing stock dependent demand and partial backlogging. There have been many extensions after Wu et al. (2006) model by applying different assumptions. Shah et al. (2013) considered marketing policy for non-instantaneous with generalized type deterioration, permitting no shortages and demand rate is power function of price and frequency of advertisement. Palanivel and Uthayakumar (2014) established model for non-instantaneous deteriorating products with time dependent two variable Weibull deterioration rate, where demand rate is power function of time and permitting partial backlogging. While, Zhang et al. (2015) developed pricing model for non-instantaneous deteriorating item by considering constant deterioration rate and stock sensitive demand. Further, Farughi et al. (2014) modeled pricing and inventory control policy for non-instantaneous deteriorating items with price and time dependent demand permitting shortages with partial backlogging. Mashud et al. (2018) worked on non-instantaneous deteriorating item having different demand rates allowing partial backlogging.

When cross-examining previous literature, there are very few studies, which have considered generalized type deterioration rate and demand as quadratic function of time. Sett et al. (2012) formulated two-warehouse inventory model with time dependent deterioration and quadratic demand. Shah et al. (2012) computed pricing model of single manufacturer-single buyer by optimizing cost function for deteriorating items having quadratic demand and deterioration rate follows two variable Weibull distributions. According to Shah et al. (2012), quadratic demand is suitable when product is new to the market.

The above studies have comprehensively focused on buyers' perspective and overlook the role of vendors in coordinating the functions of entire supply chain that makes it more uneconomical. Henceforth, it is equally important to have more number of vendor centric studies. Vendor managed inventory (VMI) system is the kind of inventory system in which system buyer should inform the vendor regarding status of system, thus supplier is responsible for deciding replenishment policy and timing of replenishment according to the flow of information (Tat et al. 2015). Jindal and Solanki (2016) presented integrated single vendor single buyer inventory model to minimize cost function for non-deteriorating item. VMI models are developed for non-deteriorating items with single vendor and multiple buyers by Zhang et al. (2007) and Darwish and Odah (2010) under different conditions. Bani-Asadi and Zanjani (2017) developed VMI model for three level supply chain system by optimizing cost function using genetic and PSO algorithm. Rabbani et al. (2018) developed VMI model for deteriorating inventory adopting different algorithms. Very few studies have been established for deteriorating products with VMI system.

In this context, Yu et al. (2012) developed VMI model by optimizing cost function with deteriorating products and raw materials assuming constant deterioration rate and constant demand by permitting no shortages. Taleizadeh et al. (2015) established pricing and replenishment policy in VMI system with constant deterioration. Furthermore, Tat et al. (2015) investigated performance of VMI system and traditional system for non-instantaneous deteriorating item by assuming constant demand and constant deterioration rate under two conditions: No shortages and permitted shortages with complete backlogging. But in real conditions, it is often seen that inventory problems studying constant demand, constant deterioration rate and complete backlogging is yet to be explored fully and requires attention. This study aims to fill this gap.

By considering aforementioned studies, the present paper investigates inventory model for a non-instantaneously deteriorating products. In this work, the deterioration rate is taken to be time-dependent and demand is time varying and increasing at quadratic rate and by considering partial backlogging. Following this, mathematical model is formulated for traditional system and VMI system considers total cost as an objective function. Some useful theoretical results are developed to identify optimal replenishment policy under both systems followed by presenting two numerical examples. Sensitivity analysis is exhibited to search key parameters. The current paper has the following sections: introduction, notations and assumptions, mathematical model, numerical examples, sensitivity analysis and conclusion with future scope of the study.

2. Notation and assumptions

Model is developed with following notations and assumptions.

2.1 Notations

t_1	the length of time when there is positive inventory in traditional system
$t_{1_{VMI}}$	the length of time when there is positive inventory in VMI system
t_2	the length of time when there is negative inventory in traditional system
$t_{2_{VMI}}$	the length of time when there is negative inventory in VMI system
v	the length of time when there is no deterioration
c_h	holding cost of buyer ($/unit/unit time)
c_d	deterioration cost ($/unit)
A_s	supplier's ordering cost ($/order)
A_b	buyer's ordering cost ($/order)
s	backorder cost ($/unit/unit time)
o	cost of lost sales ($/unit)
TC	total inventory cost before VMI
TC_{VMI}	total inventory cost of VMI system
D_1	constant demand rate at time $t,(0 \le t \le v)$
$ct^2 + bt + a$	quadratic demand at time $t,(v \le t \le t_1)$, where a is the initial rate of demand, b is the rate with which the demand rate increases. The rate of change in the demand itself increases at a rate c $(a > 0, b > 0, c > 0)$
D_2	constant demand rate at time $t,(0 \le t \le t_2)$
KB	buyer's inventory cost before VMI
KB_{VMI}	buyer's inventory cost after VMI
KS	supplier's inventory cost before VMI
KS_{VMI}	supplier's inventory cost after VMI
W	maximum inventory level at each cycle
S	maximum shortage
Q	order quantity per cycle, where $Q = W + S$
$I_1(t)$	the inventory level at time $t,(0 \le t \le v)$ in which the product has no deterioration
$I_2(t)$	the inventory level at time $t,(v \le t \le t_1)$ in which the product has deterioration
$I_3(t)$	the level of negative inventory at time $t,(0 \le t \le t_2)$

2.2 Assumptions

1) Replenishment rate is infinite.
2) Single vendor and single buyer for a single non-instantaneous deteriorating item deals with the system.
3) After time $v(0 \le v \le t_1)$ which is assumed to be a constant, inventory deteriorate at the variable rate $\theta(t)$, where $0 < \theta(t) < 1$.
4) Deteriorated product cannot be repaired or replaced.
5) Shortages are not allowed at vendor side but allowed at buyer side with partial backlogging and backlogged at a rate $\beta(x) = e^{-\delta x}$ with $0 \le \delta \le 1$ up to next replenishment.
6) Demand rate is different and followed by $D = \begin{cases} D_1 & ,0 \le t \le v \\ ct^2 + bt + a, & v \le t \le t_1 \\ D_2 & ,0 \le t \le t_2 \end{cases}$

3. Mathematical Model

In this section, we develop traditional model and VMI model for non-instantaneous deteriorating item. In this system Q units arrives at the beginning of each cycle. The inventory level decreases since demand is located only in the interval $[0, v]$ then deterioration starts and inventory level decreases due to deterioration and demand is located in the time interval $[v, t_1]$ and at the time t_1 inventory level reaches to zero and then shortage starts during the time interval $[0, t_2]$. Behavior of system is shown in the Fig. 1.

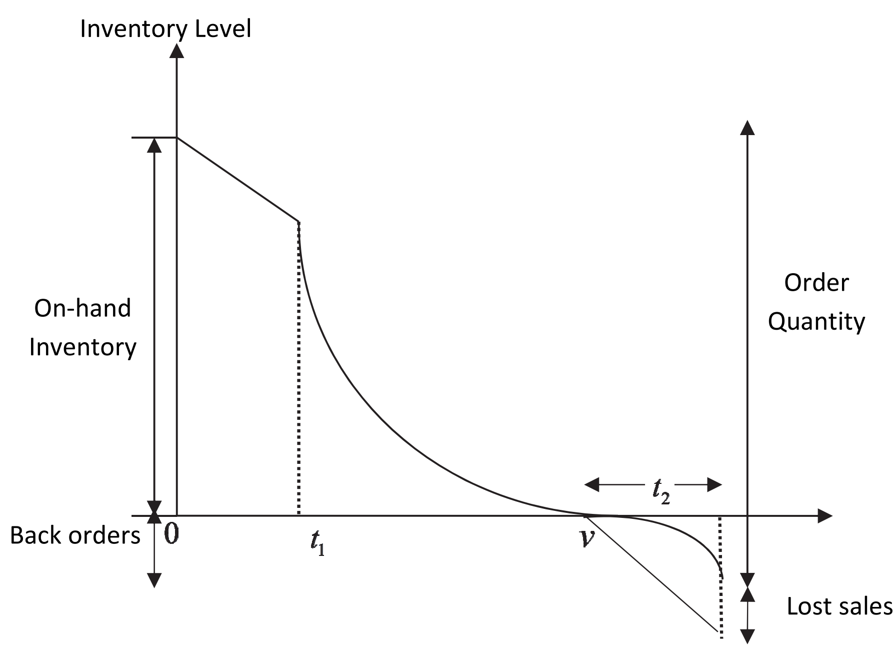

Fig. 1. The graphical representation for the inventory system

The instantaneous state of the system is given by

$$\frac{dI_1(t)}{dt} = -D_1, 0 \le t \le v \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta(t) I_2(t) = -\left(a + bt + ct^2\right), v \le t \le t_1 \tag{2}$$

$$\frac{dI_3(t)}{dt} = -D_2 e^{-\delta(t_2 - t)}, 0 \le t \le t_2 \tag{3}$$

With the boundary conditions $I_1(0) = W, I_2(t_1) = 0 = I_3(t_1)$.

Solution of equations (1) to (2) is given by

$$I_1(t) = W - D_1 t, \quad 0 \le t \le v \tag{4}$$

$$I_2(t) = e^{-g(t)} \int_t^{t_1} \left(cu^2 + bu + a\right) e^{g(u)} du, \quad v \le t \le t_1 \tag{5}$$

$$I_3(t) = -D_2 \int_0^t e^{-\delta(t_2 - u)} du \tag{6}$$

where $g(x) = \int_v^x \theta(u) du$

As $I_1(v) = I_2(v), W = D_1 v + \int_v^{t_1} \left(cu^2 + bu + a\right) e^{g(u)} du$

Therefore

$$I_1(t) = D_1(v - t) + \int_v^{t_1} \left(cu^2 + bu + a\right) e^{g(u)} du \tag{7}$$

The cost elements of the system are given as below:

- Buyer's ordering cost: A_B
- Supplier's ordering cost: A_S
- Product holding cost: $HC = h\left[\int_0^v I_1(t) dt + \int_v^{t_1} I_2(t) dt\right]$
- Product deteriorating cost:

$DC = c_d \times$ the amount of deteriorated units during $[v, t_1]$ $= c_d \left(I_2(v) - \int_v^{t_1} \left(ct^2 + bt + a\right) dt \right)$

- Shortage cost due to backlog: $SC = s \int_0^{t_2} \left(-I_3(t)\right) dt$
- The opportunity cost due to lost sales:

$LC = oD_2 \int_0^{t_2} \left(1 - e^{-\delta(t_2 - t)}\right) \mathrm{dt}$

3.1 Traditional Model

Total cost of the supplier and buyer in traditional inventory system are respectively given by

$$KB(t_1,t_2)=\frac{Z(t_1,t_2)}{t_1+t_2} \tag{8}$$

where

$$Z(t_1,t_2)=HC+DC+SC+LC+A_B$$

$$=c_h\left[\begin{array}{l}\int_0^v\left(D_1(v-t)+\left(\int_v^{t_1}(cu^2+bu+a)e^{g(u)}\,du\right)\right)dt+\\ \int_v^{t_1}e^{-g(t)}\left(\int_t^{t_1}(cu^2+bu+a)e^{g(u)}\,du\right)dt\end{array}\right] \tag{9}$$

$$+c_d\left[\left(\int_v^{t_1}(cu^2+bu+a)e^{g(u)}\,du\right)-\frac{c\left(t_1^{\,3}-v^3\right)}{3}-\frac{b\left(t_1^{\,2}-v^2\right)}{2}-a(t_1-v)\right]$$

$$-\frac{sD_2}{\delta^2}\left(e^{-\delta t_2}\delta t_2+e^{-\delta t_2}-1\right)+\frac{oD_2}{\delta}\left(\delta t_2+e^{-\delta t_2}-1\right)+A_B$$

and

$$KS(t_1,t_2)=\frac{A_S}{t_1+t_2}$$

Thus, total average cost is $TC(t_1,t_2)=KB+KS$

For the buyer, the optimization problem in tradition supply chain system is

$$\underset{t_1,t_2}{Minimize}\quad KB(t_1,t_2) \tag{10}$$

subject to $\quad 0\le v\le t_1$

The necessary conditions for the total average cost of the buyer $KB(t_1,t_2)$ to be the minimum are $\frac{\partial KB(t_1,t_2)}{\partial t_1}=0$ and $\frac{\partial KB(t_1,t_2)}{\partial t_2}=0$.

From Eq. (8), we have

$$\frac{\partial KB(t_1,t_2)}{\partial t_1}=-\frac{Z(t_1,t_2)}{(t_1+t_2)^2}+\frac{1}{t_1+t_2}\frac{\partial Z(t_1,t_2)}{\partial t_1}, \tag{11}$$

$$\frac{\partial^2 KB(t_1,t_2)}{\partial t_1^2}=\frac{2Z(t_1,t_2)}{(t_1+t_2)^3}-\frac{2}{(t_1+t_2)^2}\frac{\partial Z(t_1,t_2)}{\partial t_1}+\frac{1}{t_1+t_2}\frac{\partial^2 Z(t_1,t_2)}{\partial t_1^2}, \tag{12}$$

$$\frac{\partial KB(t_1,t_2)}{\partial t_2}=-\frac{Z(t_1,t_2)}{(t_1+t_2)^2}+\frac{1}{t_1+t_2}\frac{\partial Z(t_1,t_2)}{\partial t_2}, \tag{13}$$

$$\frac{\partial^2 KB(t_1,t_2)}{\partial t_2^2}=\frac{2Z(t_1,t_2)}{(t_1+t_2)^3}-\frac{2}{(t_1+t_2)^2}\frac{\partial Z(t_1,t_2)}{\partial t_2}+\frac{1}{t_1+t_2}\frac{\partial^2 Z(t_1,t_2)}{\partial t_2^2}. \tag{14}$$

From the necessary conditions $\frac{\partial KB(t_1,t_2)}{\partial t_1}=0=\frac{\partial KB(t_1,t_2)}{\partial t_2}$ and using Eqs. (11-14), one has

$$Z(t_1,t_2)=(t_1+t_2)\frac{\partial Z(t_1,t_2)}{\partial t_1}, \tag{15}$$

$$Z(t_1,t_2)=(t_1+t_2)\frac{\partial Z(t_1,t_2)}{\partial t_2}, \tag{16}$$

$$\frac{\partial^2 KB(t_1,t_2)}{\partial t_1^2}=\frac{1}{t_1+t_2}\frac{\partial^2 Z(t_1,t_2)}{\partial t_1^2}, \tag{17}$$

and

$$\frac{\partial^2 KB(t_1,t_2)}{\partial t_2^2}=\frac{1}{t_1+t_2}\frac{\partial^2 Z(t_1,t_2)}{\partial t_2^2}. \tag{18}$$

From Eq. (15) and Eq. (16), we can derive

$$\frac{\partial Z(t_1,t_2)}{\partial t_1}=\frac{\partial Z(t_1,t_2)}{\partial t_2} \tag{19}$$

Taking the first and the second order derivative of Eq. (9) partially with respect to t_1 and t_2 respectively, yields

$$\frac{\partial Z(t_1,t_2)}{\partial t_1}=c_h e^{g(t_1)}\left(ct_1^2+bt_1+a\right)\left(v+\int_v^{t_1}e^{-g(t)}dt\right)+c_d\left(e^{g(t_1)}-1\right) \tag{20}$$

$$\frac{\partial Z(t_1,t_2)}{\partial t_2}=sD_2e^{-\delta t_2}t_2+oD_2\left(1-e^{-\delta t_2}\right) \tag{21}$$

$$\begin{aligned}\frac{\partial Z^2(t_1,t_2)}{\partial t_1^2}&=e^{g(t_1)}\left(2ct_1+b+\left(ct_1^2+bt_1+a\right)\theta(t_1)\right)\left[c_hv+c_h\int_v^{t_1}e^{-g(t)}dt\right]+\\&+c_d\left(2ct_1+b\right)\left(e^{g(t_1)}-1\right)+\left(ct_1^2+bt_1+a\right)\left[c_d\theta(t_1)e^{g(t_1)}+c_h\right]\end{aligned} \tag{22}$$

$$\frac{\partial Z^2(t_1,t_2)}{\partial t_2^2}=\left((1-\delta t_2)sD_2+sD_2+oD_2\delta\right)e^{-\delta t_2} \tag{23}$$

Also

$$\frac{\partial^2 Z(t_1,t_2)}{\partial t_1\partial t_2}=0=\frac{\partial^2 Z(t_1,t_2)}{\partial t_2\partial t_1} \tag{24}$$

From Eq. (19), one has $M(t_1)=N(t_2)$ where

$$M(t_1) = c_h e^{g(t_1)}\left(ct_1^2 + bt_1 + a\right)\left(v + \int_v^{t_1} e^{-g(t)}dt\right) + c_d\left(e^{g(t_1)} - 1\right) \text{ and} \tag{25}$$

$$N(t_2) = sD_2 e^{-\delta t_2} t_2 + oD_2\left(1 - e^{-\delta t_2}\right) \tag{26}$$

Here $\dfrac{dN(t_2)}{dt_2} = \left[(1-\delta t_2)s + (s + o\delta)\right]e^{-\delta t_2} D_2$

Therefore

$\dfrac{dN(t_2)}{dt_2} \geq 0, \text{if } t_2 \leq \dfrac{2s+o\delta}{s\delta} = \tilde{t_2}$ and

$\dfrac{dN(t_2)}{dt_2} \leq 0, \text{if } t_2 \geq \dfrac{2s+o\delta}{s\delta} = \tilde{t_2}$

Hence $N(t_2)$ is monotonic increasing function of $t_2 \in \left(0, \tilde{t_2}\right)$ and is monotonic decreasing function of $t_2 \in \left(\tilde{t_2}, \infty\right)$. Hence maximum value of N, $N_{\max} = sD_2 e^{-\left(2+\frac{o\delta}{s}\right)}\left(\dfrac{2s+o\delta}{s\delta}\right) + oD_2\left(1 - e^{-\left(2+\frac{o\delta}{s}\right)}\right)$. On other side, $M(t_1)$ is strictly increasing function of t_1 and it tends to infinite as $t_1 \to \infty$. Hence there exists a unique $\tilde{t_1}$ such that $M\left(\tilde{t_1}\right) = N_{\max}$. Therefore for any given $t_2^* \in \left(0, \tilde{t_2}\right)$, there exists a unique $t_1^* \in \left(0, \tilde{t_1}\right)$ such that $M\left(t_1^*\right) = N\left(t_2^*\right)$. Consequently t_1^* can be uniquely determined as a function of t_2^*.

Let $\left(t_1^*, t_2^*\right)$ be the optimal value of (t_1, t_2) then we can obtain following result.

Theorem 1

Buyer's total average cost function $KB(t_1, t_2)$ is convex and reaches its global minimum at $(t_1, t_2) = \left(t_1^*, t_2^*\right)$.

Proof: We use the second order sufficient condition for global optimal solution.

From Eq. (22) and Eq. (23), it is clear that $\dfrac{\partial Z^2(t_1,t_2)}{\partial t_1^2} > 0$ and $\dfrac{\partial Z^2(t_1,t_2)}{\partial t_2^2} > 0$ respectively.

Using the above fact, from Eq. (17), Eq. (18) and Eq. (24), one has

$$\left(\left.\frac{\partial^2 KB(t_1,t_2)}{\partial t_1^2}\right|_{\left(t_1^*,t_2^*\right)}\right) \times \left(\left.\frac{\partial^2 KB(t_1,t_2)}{\partial t_2^2}\right|_{\left(t_1^*,t_2^*\right)}\right) - \left[\left.\frac{\partial^2 KB(t_1,t_2)}{\partial t_1 \partial t_2}\right|_{\left(t_1^*,t_2^*\right)}\right]^2 > 0 \tag{27}$$

Hence Hessian matrix is negative definite at point $(t_1, t_2) = \left(t_1^*, t_2^*\right)$. Consequently, the stationary point $\left(t_1^*, t_2^*\right)$ is a global minimum for the problem (8).

This completes the proof.

3.2 VMI Model

In vendor managed inventory system, buyer's cost is paid by supplier. So the buyer's cost and supplier's cost in VMI system are respectively as follows,

$$KB_{VMI} = 0 \text{ and } KB_{VMI} = \frac{\left(HC + DC + SC + LC + A_B + A_S\right)}{t_{1_{VMI}} + t_{2_{VMI}}}$$

And total average cost function is $TC_{VMI} = KB_{VMI} + KS_{VMI}$

where

$$\therefore TC_{VMI}\left(t_{1_{VMI}}, t_{2_{VMI}}\right) = \frac{Z_{VMI}\left(t_{1_{VMI}}, t_{2_{VMI}}\right)}{t_{1_{VMI}} + t_{2_{VMI}}} \text{ where}$$

$$Z_{VMI}\left(t_{1_{VMI}}, t_{2_{VMI}}\right) = c_h \left[\begin{array}{l} \int_0^v \left(D_1\left(v-t\right) + \left(\int_v^{t_{1_{VMI}}} \left(cu^2 + bu + a\right) e^{g(u)}\, du \right) \right) dt + \\ \int_v^{t_{1_{VMI}}} e^{-g(t)} \left(\int_t^{t_{1_{VMI}}} \left(cu^2 + bu + a\right) e^{g(u)}\, du \right) dt \end{array} \right] \tag{28}$$

$$+ c_d \left[\left(\int_v^{t_{1_{VMI}}} \left(cu^2 + bu + a\right) e^{g(u)}\, du \right) - \frac{c\left({t_{1_{VMI}}}^3 - v^3\right)}{3} - \frac{b\left({t_{1_{VMI}}}^2 - v^2\right)}{2} - a\left(t_{1_{VMI}} - v\right) \right]$$

$$- \frac{sD_2}{\delta^2}\left(e^{-\delta t_{2_{VMI}}}\delta t_{2_{VMI}} + e^{-\delta t_{2_{VMI}}} - 1\right) + \frac{oD_2}{\delta}\left(\delta t_{2_{VMI}} + e^{-\delta t_{2_{VMI}}} - 1\right) + A_B + A_S$$

The optimization problem in VMI system is

$$\begin{array}{ll} \underset{t_{1_{VMI}},\, t_{2_{VMI}}}{Minimize} & TC\left(t_{1_{VMI}}, t_{2_{VMI}}\right) \\ \text{subject to} & 0 \le v \le t_{1_{VMI}} \end{array} \tag{29}$$

Again, as discussed in traditional model, the existence of optimal value $\left({t_{1_{VMI}}}^*, {t_{2_{VMI}}}^*\right)$ in VMI system can be established.

4. Numerical examples

Existing model consists of generalized type and time dependent deterioration rate. To illustrate the above theoretical result, we solve the following numerical example by assuming different kinds of particular form of deterioration rate.

4.1 Example 1: Constant deterioration rate

Let $\theta(t) = \alpha$ (constant). Parameters values are given as below:

$A_S = \$200$ /order, $A_B = \$50$ /order, $\alpha = 0.07$, $a = 4000, b = 0.05, c = 0.1$, $c_h = \$4$ /unit/year, $v = \frac{20}{365}$,

$\delta = 0.9$, $c_d = \$50$ /unit, $s = \$10$ /unit/year, o = \$15 /unit, $D_1 = 1500$ Unit/year, $D_2 = 1000$ unit/year.

The result of solution procedure is as under:

Table 1
Results of computation for constant deterioration rate

Q	Q_{VMI}	t_1	$t_{1_{VMI}}$	t_2	$t_{2_{VMI}}$	TC	TC_{VMI}
456	589	0.1254	0.1522	0.0946	0.1221	3910	3810

4.2 Example 2: Three variable Weibull distribution deterioration rate

Let $\theta(t) = \alpha\beta(t-v)^{\beta-1}$ be a three variable Weibull distribution function where α is scale parameter and β represents the shape parameter where $\alpha = 0.07, \beta = 2$ and all other parameters values are the same as example 1. The result of solution procedure is as under:

Table 2
Results of computation for three variable Weibull distribution deterioration rate

Q	Q_{VMI}	t_1	$t_{1_{VMI}}$	t_2	$t_{2_{VMI}}$	TC	TC_{VMI}
601	768	0.1635	0.1998	0.0874	0.1113	3580.43	3495

From Table 1 and Table 2, we can say that vendor managed inventory system is more beneficial compared with individual system as cost with VMI system is less than to cost with traditional system in both examples: constant deterioration rate and three variable Weibull distribution deterioration rate.

Though the convexity is established for the objective function of both traditional system and VMI system, behavior of the function with respect to decision variables followed by numerical example 2 is shown in Figs. (2-3), respectively.

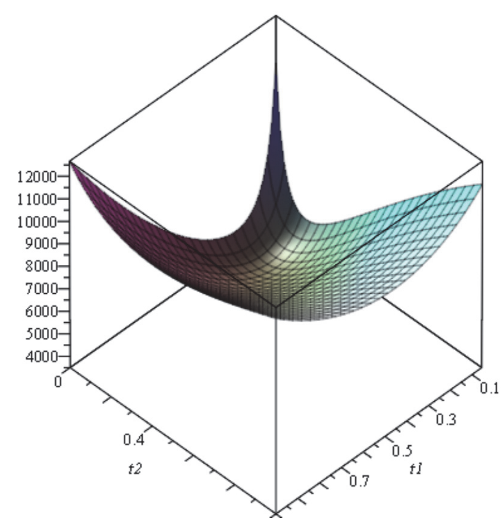

Fig. 2. Convexity of traditional system

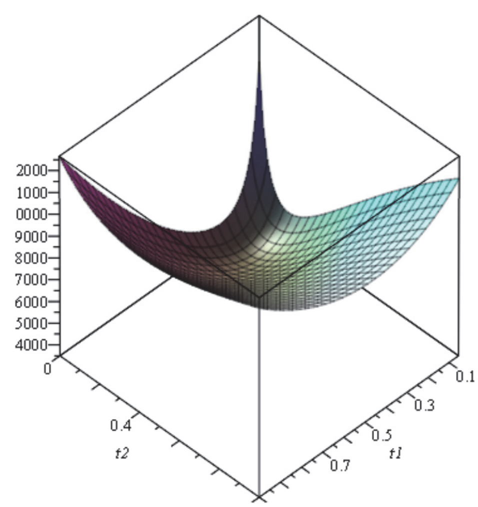

Fig. 3. Convexity of VMI system

5. Sensitivity analysis

In this section, we study the effect of the changes in decision variable Q, t_1, t_2 and TC in traditional system and VMI system with respect to parameters by changing -40%, 20%, 20% and 40% in original values as per example 2, considering deterioration rate as three variable Weibull distribution function.

Table 3
Sensitivity analysis

Parameter	% changes	t_1	$t_{1_{VMI}}$	t_2	$t_{2_{VMI}}$	Q	Q_{VMI}	TC	TC_{VMI}
A_B	-40	0.1593	0.1966	0.0847	0.1091	582	753	3522.21	3430.00
	-20	0.1614	0.1982	0.0861	0.1102	592	761	3551.36	3462.57
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1656	0.2014	0.0887	0.1124	611	776	3609.42	3526.87
	+40	0.1676	0.2030	0.0900	0.1134	620	783	3638.32	3558.60
D_1	-40	0.1323	0.1771	0.0684	0.0961	426	631	3197.95	3044.74
	-20	0.1489	0.1889	0.0784	0.1039	518	702	3388.27	3277.09
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1766	0.2100	0.0958	0.1183	678	832	3769.14	3700.55
	+40	0.1884	0.2195	0.1036	0.1250	749	892	3952.64	3896.03
a	-40	0.2225	0.2696	0.0740	0.0940	557	688	3050.30	2981.50
	-20	0.1879	0.2287	0.0816	0.1037	587	738	3348.60	3270.78
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1450	0.1779	0.0920	0.1174	604	785	3766.56	3674.07
	+40	0.1305	0.1607	0.0958	0.1224	598	791	3919.28	3820.47
b	-40	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	-20	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1635	0.1998	0.0874	0.1113	601	768	3580.44	3494.86
	+40	0.1635	0.1998	0.0874	0.1113	601	768	3580.44	3494.86
c	-40	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	-20	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1635	0.1998	0.0874	0.1113	601	768	3580.44	3494.86
	+40	0.1635	0.1998	0.0874	0.1113	601	768	3580.44	3494.86
D_2	-40	0.1485	0.1814	0.1345	0.1719	533	684	3207.31	3129.22
	-20	0.1574	0.1923	0.1058	0.1349	573	734	3427.07	3344.56
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1680	0.2053	0.0745	0.0948	622	794	3693.78	3605.96
	+40	0.1714	0.2095	0.0650	0.0826	637	813	3781.07	3691.52
c_h	-40	0.1764	0.2294	0.0587	0.0824	626	860	2752.23	2631.87
	-20	0.1696	0.2129	0.0734	0.0973	613	808	3179.85	3079.35
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1580	0.1890	0.1008	0.1246	592	737	3957.58	3883.55
	+40	0.1530	0.1798	0.1136	0.1373	583	712	4314.19	4249.29
c_d	-40	0.1667	0.2051	0.0871	0.1106	614	789	3561.67	3475.70
	-20	0.1650	0.2024	0.0872	0.1110	607	778	3571.25	3485.50
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1621	0.1974	0.0876	0.1116	596	759	3589.25	3503.81
	+40	0.1607	0.1952	0.0877	0.1119	590	750	3597.72	3512.39
s	-40	0.1560	0.1906	0.1100	0.1402	592	757	3392.66	3310.90
	-20	0.1601	0.1957	0.0974	0.1241	597	763	3495.65	3411.78
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1663	0.2033	0.0793	0.1009	605	773	3651.47	3564.47
	+40	0.1687	0.2062	0.0725	0.0923	608	777	3711.85	3623.64
o	-40	0.1587	0.1939	0.1019	0.1300	595	761	3459.42	3376.20
	-20	0.1613	0.1970	0.0941	0.1199	598	765	3523.93	3439.45
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1655	0.2022	0.0816	0.1039	604	771	3630.38	3543.82
	+40	0.1672	0.2044	0.0766	0.0974	606	774	3674.86	3587.43
δ	-40	0.1592	0.1946	0.0997	0.1263	597	764	3473.31	3390.28
	-20	0.1615	0.1974	0.0931	0.1183	599	766	3530.00	3445.61
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1653	0.2020	0.0824	0.1050	603	771	3625.61	3538.98
	+40	0.1670	0.2040	0.0779	0.0995	605	773	3666.32	3578.76
v	-40	0.1340	0.1767	0.0712	0.0986	523	719	3263.47	3120.78
	-20	0.1497	0.1887	0.0798	0.1052	566	746	3424.50	3316.82
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1760	0.2101	0.0942	0.1168	630	787	3728.24	3657.69
	+40	0.1874	0.2198	0.1004	0.1220	654	803	3867.19	3807.42
α	-40	0.1667	0.2052	0.0871	0.1106	614	789	3561.48	3475.49
	-20	0.1651	0.2024	0.0872	0.1109	607	778	3571.16	3485.40
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1621	0.1974	0.0876	0.1116	596	759	3589.33	3503.90
	+40	0.1607	0.1951	0.0877	0.1119	590	750	3597.87	3512.56
β	-40	0.1378	0.1669	0.0918	0.1181	503	644	3787.86	3694.17
	-20	0.1544	0.1875	0.0887	0.1134	566	721	3645.73	3558.18
	0	0.1635	0.1998	0.0874	0.1113	601	768	3580.43	3494.86
	+20	0.1682	0.2070	0.0869	0.1103	619	796	3551.35	3465.93
	+40	0.1704	0.2109	0.0867	0.1098	628	811	3538.73	3452.90

- As ordering cost increases, cycle length, order quantity and hence total cost increase in both the system and there is a big difference between order quantities of both systems. However, total cost in vendor managed inventory system is low compared with the individual system.

- As D_1 increases, when there is no deterioration, cycle length, order quantity and total cost increases in both the system. When deterioration starts, increases in constant demand rate a, cycle length decreases at the time of positive inventory, cycle length increases when there is a negative inventory, order quantity and total cost also increase in both traditional and VMI systems. On changing in linear demand rate b and exponential demand rate c, there is no changes in any decision variables in both systems. Hence b and c are very less sensitive in both systems. At the time of shortages, on rising constant demand D_2, t_1 and $t_{1_{VMI}}$ increase, t_2 and $t_{2_{VMI}}$ decrease, Q and Q_{VMI} increase and so total cost TC and TC_{VMI} increase. However, TC_{VMI} is less than TC on changing in each demand parameters.
- When buyer's holding cost increases, cycle length at the time of positive inventory and ordering cost decrease and cycle length at the time of negative inventory and total cost increase in both systems. Here, it is obvious that when the holding cost increases, total cost also increases. Although, VMI system is beneficial in this case compared with the traditional system.
- When deterioration cost increases, there is a decrease in t_1 and $t_{1_{VMI}}$, negligible increases in t_2 and $t_{2_{VMI}}$, negligible decrease in Q and Q_{VMI} and increase in TC and TC_{VMI}. Here it is noted that total cost is low in VMI system compare with the traditional system.
- When there is an increase in s and o, there is an increase in t_1, $t_{1_{VMI}}$, Q, Q_{VMI}, TC and TC_{VMI} while there is a decrease in t_2 and $t_{2_{VMI}}$. It is obvious that when the cost parameters increase, total cost increases too. However total cost is higher in traditional policy compared with the VMI policy.
- When δ decreases, there is a decrease in $t_1, t_{1_{VMI}}, Q$, Q_{VMI}, TC and TC_{VMI} while there is an increase in t_2 and $t_{2_{VMI}}$. Here total cost in supply chain with traditional policy is lower than the total cost in supply chain with vendor managed policy.
- When there is an increase in v, there is a significant increase in cycle length with increase in ordering cost and total cost in both systems. Although cost in VMI system is lower than traditional system.
- When there is an increase in α, it is found that cycle length in positive inventory and order quantity decrease, cycle length at the time of shortages increases negligibly and total cost increases. As β increases, there is a decrease in t_2, $t_{2_{VMI}}$, TC and TC_{VMI}. Additionally, t_1, $t_{1_{VMI}}$, Q and Q_{VMI} increase with increase in β. It is noticed that an increase in beta value decreases the cost and total cost in VMI system which is lower than individual supply chain system.

Sensitivity analysis shows that gap between total cost in vender managed inventory supply chain system and traditional supply chain system is very high. Hence VMI system is beneficial in comparison with individual supply chain system.

6. Conclusion and future scope

VMI system and individual system have been developed for single vendor and single buyer for non-instantaneous deteriorating item having generalized time dependent deterioration rate with quadratic demand with respect to time and permitting shortages with partial backlogging. The study has considered total cost function as an objective function. Results have been shown analytically and numerically with different deterioration rates. Sensitivity analysis has been exhibited to show the liability of the model. The major contribution of this study is the consideration of quadratic demand and time dependent generalized deterioration rate in traditional supply chain system and VMI supply chain system. The primary objective of the study was to compare decision variable in traditional model and VMI models. It was noticed that total average cost decreases significantly in VMI system compared with the system developed by individual effort of vendor and buyer with time varying generalized deterioration rate. Hence VMI policy is more beneficial than traditional policy. One can extend this model with stock dependent demand, single vendor-multiple buyer etc.

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